Long-range phase-coherent effects in the transport properties of mesoscopic superconductor–normal-metal structures

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We present a theory of long-range phase-coherent effects in the transport properties of a normal-metal film contacting two superconductors. It is shown that the phase-coherent correction to the conductance exists even though the spacing between the two superconductors largely exceeds the coherence length and the critical Josephson current is exponentially small. This effect can appear as a Shapiro step or large conductance oscillation caused by the magnetic field. Moreover, it is shown that in the case of negligible Josephson coupling between superconductors, the Josephson effect can arise in the system if an additional current flows through the normal conductor and a dissipation takes place (dissipative Josephson effect). The amplitude of the phase-coherent correction part of the conductance is discussed from the viewpoint of temperature, bias voltage, magnetic field, and phase-breaking length dependence. [S0163-1829(97)07741-2]

I. INTRODUCTION

There has been a great deal of interest in studying the transport properties of mesoscopic superconductor–normal-metal (S/N) structures with dimensions less than the phase-breaking length L_ϕ.[1–5] Presumably some experimental works were stimulated by theoretical papers.[6,7] In Ref. 7 the effect of Andreev reflections on the weak localization corrections to the conductance was analyzed. It was predicted that the conductance of this system oscillates with increasing the phase difference between superconductors, and the oscillation amplitude is of the order of e^2/h. The observed effect turned out to be much larger (more than one order of magnitude). It was established later that the observed phenomena are related to the proximity effect.[8–10] A condensate induced in the N film of an S/N system (see Fig. 1) due to the proximity effect leads to an increase in the N film conductance. There are two contributions of the condensate to the conductance;[11] first, the density of states (DOS) of the N film is decreased (this contribution diminishes the conductance); and second, there is another contribution of the condensate to the conductance which is similar, to some extent, to the Maki-Thompson term in the fluctuation paraconductivity. The last contribution equals the first one at zero energy (i.e., at zero voltage V and temperature T) and exceeds it at higher energies. Therefore the total change in the conductance due to the condensate is positive and depends on V and T in a nonmonotonous way.[11–14] It is interesting that the effect of the condensate on the conductance has a long-range character; that is, a change in the local conductivity of the N film (or a change in the electric field E) does not vanish at distances from the S/N interface much greater than the coherence length in the N film given by $\xi_N=\sqrt{\hbar D/2\pi k_B T}$ (here D is the diffusion constant in the N film and ξ_N is supposed to be shorter than L_ϕ). Long-range effects of this type were observed[5] and analyzed in theoretical works.[11,14,15] A particular manifestation of the long-range effects—a possibility to observe Shapiro steps in the absence of the dc Josephson effect in the system shown in Fig. 1(a)—was studied theoretically in a recent work of the authors.[16]

It was noted that the critical Josephson current I_{CJ} is determined by states with energies of the order of thermal ones: $\varepsilon \approx \pi k_B T$. The condensate amplitude $|F|$ for such energies decays with distance from the S/N interface as

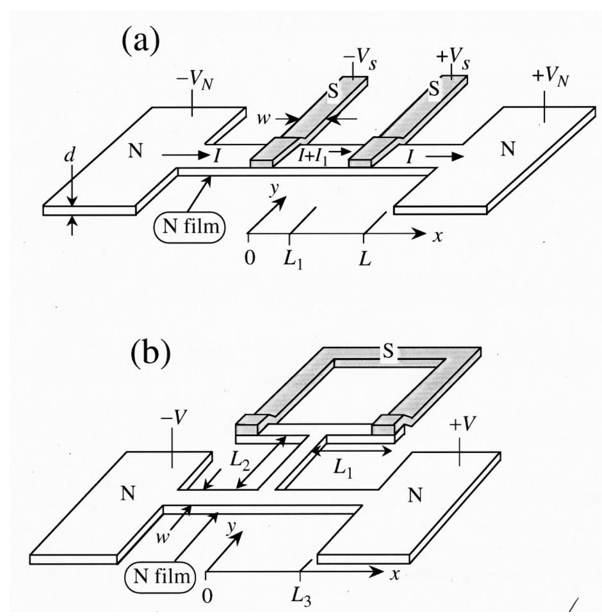

FIG. 1. Schematic top view of mesoscopic S/N/S junctions with a symmetric configuration. The hatched areas are the contact ones of a superconductor and a normal-metal film. (a) Two superconducting electrodes are formed on the normal-metal film and this film is terminated by normal reservoirs. (b) A normal-metal film is connected with an S/N loop in which magnetic flux is almost quantized.

$$|F|\propto|\exp(-k_\varepsilon|x-L_1|)|\propto\exp(-|x-L_1|/\xi_N),$$

where

$$k_\varepsilon=\sqrt{(-2i\varepsilon+\hbar\gamma)/\hbar D},$$

and $\varepsilon=i\pi k_B T$, where γ is the depairing rate in the N film which is related to L_ϕ: $L_\phi=\sqrt{D/\gamma}$ (assuming that $x>0$). Meanwhile, a condensate contribution to the conductance is caused not only by states with thermal energies, but also by states with low energies of the order of the Thouless energy $\varepsilon_L=\hbar D/L^2$. At such energies the condensate functions spread over the full length of the mesoscopic N film, i.e., over the length L which is supposed to be less than the phase-breaking length L_ϕ. The amplitude of the condensate functions at such energies decays as $\exp(-|x-L_1|/L)$. As shown in Ref. 16, a peculiarity in the N film conductance should appear at a voltage difference between superconductors V_S when the Josephson relation is fulfilled: $\hbar\omega=2eV_S$ (where ω is the frequency of the ac component of the bias voltage between superconductors). Long-range phase-coherent effects of a similar type should arise in the system in Fig. 1 if the conductance of the N film is measured as a function of the phase difference between superconductors which can be controlled, for example, by an applied magnetic field (in this case the superconductors should be connected). Under condition $2L_1\gg\xi_N$ the dc Josephson effect is negligible, but the conductance oscillations survive.

In this paper we present a theory of long-range phase-coherent effects in the transport properties of mesoscopic systems shown in Fig. 1(a). We calculate the temperature dependence of the Shapiro step and compare it with the corresponding dependence of the dc critical Josephson current. We also calculate the temperature and magnetic field dependence of the conductance for the system shown in Fig. 1(b). Finally, we show by a simple model that the Josephson effect may arise even if the Josephson coupling is exponentially small in Fig. 1(a). The effect appears only if an additional current I flows between two normal reservoirs. The critical current is proportional to the current I.

II. THEORY

As in Refs. 10, 11, and 16, we restrict ourselves to the dirty case ($l\ll\xi_N$, where l is the mean free path) and assume that the proximity effect is weak, that is, the amplitude of the condensate induced in the N film by the proximity effect is much smaller than the condensate amplitude in the superconductors. This assumption is valid if the S/N interface resistance exceeds the resistance of the N film and allows one to carry out analytical calculations for any specific geometry of the system. On the other hand, results obtained for the dependence of the conductance on different parameters (temperature T, bias voltage V, applied magnetic field H, etc.) remain qualitatively the same in case of a strong proximity effect when the condensate amplitude in the S and N films are comparable.

First we consider a stationary case when the phase difference between the superconductors is constant in time. The equation for the distribution function f describing the electric field and the current has the form[16]

$$\partial_x\{J_S+(\partial_x f)[1-m_-(x)]\}=-k_b^2 w J_1\cdot\delta(x\pm L_1). \quad (1)$$

Here

$$J_S(\varepsilon)=\frac{1}{4}\,\mathrm{Tr}\hat\sigma_z(\hat F^R\partial_x\hat F^R-\hat F^A\partial_x\hat F^A)\cdot f_0 \quad (2)$$

is the supercurrent per unit energy in the N film and $k_b^2=(2R_{b\square}\sigma d)^{-1}$, where $R_{b\square}$ is the S/N interface resistance per unit area, σ is the conductivity of the N film, d is the thickness of the N film, and w is the width of the S/N interface. The current

$$J_1=J_V+J_0,\quad J_V=-2(b_{\mathrm{sg}}+\nu\nu_s)f\equiv-G_V f \quad (3)$$

determines the charge transfer across the S/N interface per unit time. It consists of a dissipative current J_V and a nondissipative one J_0. Here, J_V is the sum of two terms; one is the subgap current (first term), and the other is the usual quasiparticle current which is proportional to the product of the densities of states ν and ν_S. The function b_{sg} determines the subgap conductance[18,19,21]

$$b_{\mathrm{sg}}=(1/8)\mathrm{Tr}(\hat F^R+\hat F^A)(\hat F_S^R+\hat F_S^A). \quad (4)$$

In an SIS (here I denotes an insulator) Josephson junction, the so-called interference current is proportional to this function.[19] The nondissipative partial current through the S/N interface J_0 is equal to

$$\begin{aligned}J_0=&\mathrm{Tr}\hat\sigma_z\{2\,f_0(\hat F^R\hat F_S^R-\hat F^A\hat F_S^A)\\&+(f_{S0}-f_0)(\hat F^R+\hat F^A)(\hat F_S^R-\hat F_S^A)\}\\\equiv&J_{\mathrm{eq}}+J_{\mathrm{neq}}.\end{aligned} \quad (5)$$

The second term in Eq. (5), J_{neq}, differs from zero only in a nonequilibrium case when the distribution functions in S and N films are different. In the case of a weak barrier transmittance, $f_{S0}(\varepsilon)=\tanh(\varepsilon\beta)$, $\beta=1/(2k_BT)$ (the electric potential in superconductors is taken to be zero) and

$$f_0\cong[\tanh(\varepsilon+eV_N)\beta+\tanh(\varepsilon-eV_N)\beta]/2.$$

In the case $L_1\gg\xi_N$, the main contribution to the conductance of the N channel is caused by the second term on the left in Eq. (1). The function m_- is given by (see Refs. 11 and 12)

$$m_-=(1/8)\mathrm{Tr}(\hat F^R-\hat F^A)^2=(1/8)\mathrm{Tr}[(\hat F^R)^2+(\hat F^A)^2-2\hat F^R\hat F^A]. \quad (6)$$

The first two terms in Eq. (6) determine a change in the DOS of the N film due to the proximity effect. Indeed, the DOS in the N film equals

$$\nu=(1/4)\mathrm{Tr}\hat\sigma_Z(\hat G^R-\hat G^A)\cong1-(1/8)\mathrm{Tr}[(\hat F^R)^2+(\hat F^A)^2]. \quad (7)$$

In derivation of Eq. (7) we used the normalization condition[17] and assumed that $|\hat F^{R(A)}|$ are small. The last term in Eq. (6) is a function nonanalytical in both half-planes of the variable ε (the so-called anomalous function in Gor'kov and Eliashberg's terminology[20]). This term gives the main contribution to the conductance under condition $L_1\gg\xi_N$.

Equation (1) must be complemented by the boundary condition

$$f(\varepsilon,L)=F_N(\varepsilon)\equiv[\tanh(\varepsilon+eV_N)\beta-\tanh(\varepsilon-eV_N)\beta]/2. \tag{8}$$

Here we assumed that the function $f(\varepsilon,L)$ has an equilibrium form at the reservoirs corresponding to the electric potentials $\pm V_N$. Integrating Eq. (1), we obtain

$$J(\varepsilon)L(1+\langle m_-\rangle)=F_N(\varepsilon)-(k_b^2wL_1)J_{S/N}(1+\langle m_-\rangle_1). \tag{9}$$

Here $\langle m_-\rangle=L^{-1}\int_0^L dx\; m_-(\varepsilon,x)$ and $\langle m_-\rangle_1=L_1^{-1}\int_0^{L_1}dx\; m_-(\varepsilon,x)$. The constant $J_{S/N}$ is $J_{S/N}=J_{\rm neq}+J_V$. The terms J_S and $J_{\rm eq}\cdot(k_b^2w)$ cancel each other, and this can be checked using the explicit form of the functions $F^{R(A)}(\varepsilon,x)$ (see below).

The current I through the N channel is expressed as an integral over all energies from the partial current $J(\varepsilon)$ (Refs. 11, 19, and 21) by

$$I=d(\sigma/2e)\int d\varepsilon J(\varepsilon). \tag{10}$$

If we substitute $J(\varepsilon)$ from Eq. (9) into Eq. (10), we find the current-voltage $I(V)$ characteristic of the system. To do this, we must determine $\langle m_-\rangle$, i.e., we must find the spatial distribution of the functions $\hat{F}^{R(A)}(\varepsilon,x)$.

Knowing $\hat{F}^{R(A)}$ (see Appendix A), we can find $J(\varepsilon)$ from Eq. (9) and calculate the current I. We are interested in a contribution δS to the dimensionless conductance $S=R_L dI/d(2V_N)$ which depends on φ: $\delta S\propto\cos\varphi$, where $R_L=2L/\sigma d$ and φ is the phase difference between the superconductors. This contribution is caused by the term $\langle m_-\rangle$ and terms in $J_{S/N}$. One can show that if the condition

$$\varepsilon_L\equiv\hbar D/L^2\ll k_BT\ll\Delta \tag{11}$$

is fulfilled, the main contribution to δS is determined mainly by the so-called anomalous part, $\langle m_-\rangle_{\rm an}$, of $\langle m_-\rangle$. One can show that the last term on the right-hand side of Eq. (4) is small under the condition of Eq. (11). Therefore, we obtain from Eqs. (9) and (10)

$$\delta S=-(\beta/2)\int d\varepsilon F_N'(\varepsilon)\langle m_-\rangle. \tag{12}$$

Here $F_N'=[\cosh^{-2}(\varepsilon+eV_N)\beta+\cosh^{-2}(\varepsilon-eV_N)\beta]/2$, $\langle m_-\rangle=\langle m_-\rangle_{\rm reg}+\langle m_-\rangle_{\rm an}$, $\langle m_-\rangle_{\rm reg}=(1/8){\rm Tr}\langle(\hat{F}^R)^2+(\hat{F}^A)^2\rangle$, and $\langle m_-\rangle_{\rm an}=-(1/4){\rm Tr}\langle\hat{F}^R(\varepsilon,x)\hat{F}^A(\varepsilon,x)\rangle$. In the temperature regime satisfying Eq. (11), one can neglect the part $\langle m_-\rangle_{\rm reg}$. Using expression (A4) for $\hat{F}^{R(A)}(\varepsilon,x)$, one can easily find

$$\langle m_-\rangle_{\rm an}=-(r^2/8)\left|\frac{\Delta}{\sqrt{\Delta^2-(\varepsilon+i\Gamma_S)^2}}\right|^2|\theta|^{-2}\cos\varphi\Bigg[|\sinh\theta_2|^2\Bigg\{\frac{\sinh 2\theta_1'}{2\theta'}(|\cosh\theta|^{-2}-|\sinh\theta|^{-2})+\frac{\sin 2\theta_1''}{2\theta''}(|\cosh\theta|^{-2}+|\sinh\theta|^{-2})\Bigg\}+\left(\left|\frac{\cosh\theta_1}{\cosh\theta}\right|^2-\left|\frac{\sinh\theta_1}{\sinh\theta}\right|^2\right)\left(\frac{\sinh 2\theta_2'}{2\theta'}-\frac{\sin 2\theta_2''}{2\theta''}\right)\Bigg], \tag{13a}$$

$$\langle m_-\rangle_{\rm reg}=-(r^2/8){\rm Re}\,\frac{\Delta^2}{(\varepsilon+i\Gamma_S)^2-\Delta^2}\,\theta^{-2}\cos\varphi\Bigg[\sinh^2\theta_2\Bigg\{\frac{\sinh 2\theta_1}{2\theta}(\cosh^{-2}\theta-\sinh^{-2}\theta)+\frac{\theta_1}{\theta}(\cosh^{-2}\theta+\sinh^{-2}\theta)\Bigg\}+\frac{\sinh(2\theta_2)-2\theta_2}{2\theta}\left\{\frac{\cosh^2\theta_1}{\cosh^2\theta}-\frac{\sinh^2\theta_1}{\sinh^2\theta}\right\}\Bigg]. \tag{13b}$$

Here $r=w\rho L/(R_{b\square}d)$ is the ratio of the N channel and S/N interface resistances; this parameter is assumed to be small. And Γ_S is the damping rate in the superconductor. When $k_BT\ll\Delta$, the term $|\Delta/\sqrt{\Delta^2-(\varepsilon+i\Gamma_S)^2}|^2$ can be replaced by 1. The specific resistivity in the N film is ρ. The function θ is dependent on energy and related to k_ε: $\theta=\theta'+i\theta''=k_\varepsilon L$, $\theta_{1,2}=k_\varepsilon L_{1,2}$, $\theta_{1,2}=\theta_{1,2}'+i\theta_{1,2}''$.

In Fig. 2 we plot the temperature dependence of $\delta S/r^2$ as a function of temperature k_BT/ε_L. Here δS was calculated for $D=54$ cm^2/sec, $L_1=0.5\;\mu$m, $L=1\;\mu$m, and $\gamma=V_N=0$. This value of D is typical for Ag or Cu as the N film[1,5] and provides $\xi_N=(81\text{ nm})/\sqrt{T}$, $\varepsilon_L=3.6\;\mu$V, and the mean free path l of about 40 nm. It is easily found that the system satisfies the condition L, $L_1\gg l$. As the superconductor, Nb with $\Delta=1.5$ meV was used. In Fig. 2, $\delta S_{\rm an}/r^2$ and $\delta S_{\rm reg}/r^2$ are also plotted, where $\delta S=\delta S_{\rm an}+\delta S_{\rm reg}$, and $\delta S_{\rm an}$ and $\delta S_{\rm reg}$ are the phase-dependent conductances which are due to the $\langle m_-\rangle_{\rm an}$ and $\langle m_-\rangle_{\rm reg}$ terms, respectively. It is clearly seen that $\delta S=0$ at $T=0$ and δS shows a reentrant behavior against temperature. This can be explained by follows. At $T=0$ the increase in the conductance due to Maki-Tompson-type fluctuation ($\delta S_{\rm an}$ term) is the same as the decrease due to the decrease in the density of states ($\delta S_{\rm reg}$ term). Here $\delta S_{\rm an}$ has a linear dependence of T^{-1} while $\delta S_{\rm reg}$ has an exponential dependence of T^{-1}. Therefore, δS has a maximum value around $T=\varepsilon_L/k_B$ [see also Fig. 3(a)] and at higher temperatures δS is almost the same as $\delta S_{\rm an}$.

The critical current $I_{\rm CJ}$ is easily found from Eqs. (2), (10), and (A4). Substituting expression (A4) into Eq. (2) and en-

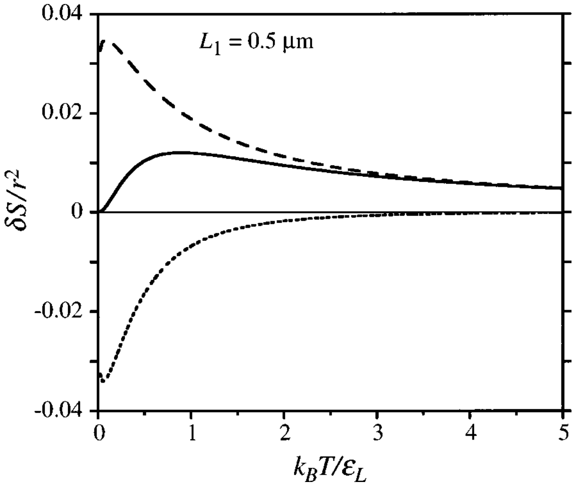

FIG. 2. Temperature dependences of the phase-coherent conductances: δS (solid line), δS_{an} (dashed line), and δS_{reg} (dotted line) for $L=1$ μm and $L_1=0.5$ μm.

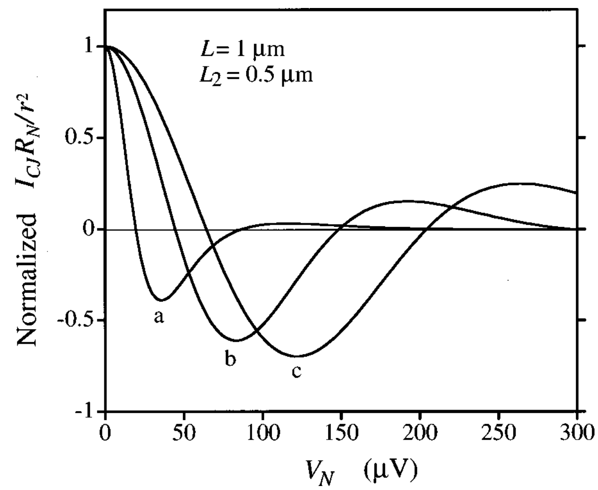

FIG. 4. Critical current $I_{\mathrm{CJ}}R_N/r^2$ as a function of the bias voltage V_N at $x=L_1$. The critical current is calculated for a, $T=0.1$ K; b, 0.5 K; and c, 1 K and is normalized by its value for $V_N=0$.

circling the integration contour in the upper half-plane for $\hat{F}^R$ and in the lower half-plane for $\hat{F}^A$, we obtain the supercurrent

$$I_S=I_{\mathrm{CJ}}\sin\varphi, \tag{14a}$$

where

$$I_{\mathrm{CJ}}R_L=(\pi k_BT)r^2\sum_{n=0}^{\infty}\ \mathrm{Re}\,\frac{\Delta^2}{\Delta^2+(\omega_n+ieV_N)^2}\,\frac{\sinh^2(\theta_nL_2/L)}{\theta_n\sinh(2\theta_n)}. \tag{14b}$$

Here $\theta_n=L[(2(\omega_n+ieV_N)+\hbar\gamma)/\hbar D]^{1/2}$ and $\omega_n=\pi k_BT(2n+1)$. By using this equation for the same values of D, L, and Δ as used for the calculation of δS and for $\gamma=V_N=0$, we can plot I_{CJ} as a function of temperature in Fig. 3(b). We also plot the temperature dependence of $\delta S/r^2$ as a function of temperature k_BT/ε_L for different L_1 (from 0.5 to 0.8 μm) and under the same condition for the calculation in Fig. 2. It can be seen from the figure that δS diminishes slowly with increasing temperature (as T^{-1} in the limit $2L\gg\xi_N$), whereas I_{CJ} decreases exponentially. This means that the phase coherency remains in the absence of the dc Josephson effect. The phenomenon is related, as noted above, to the contribution of low-energy states to the kinetic characteristics of the system. The phase coherency for such states with energies $\varepsilon\leqslant\varepsilon_L$ is spread over the full length of the mesoscopic system ($L<L_\phi$).

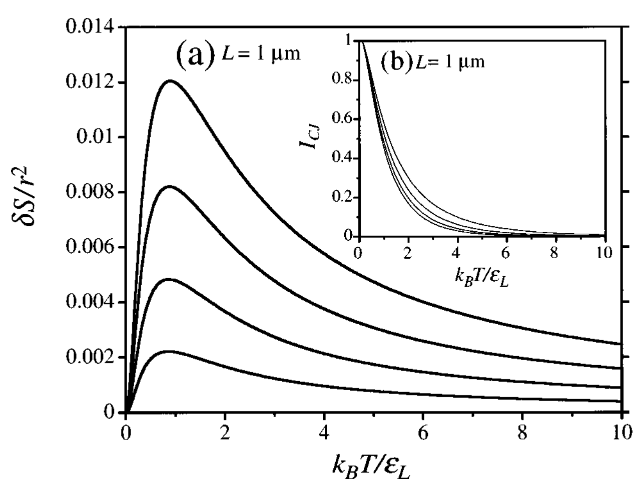

FIG. 3. Calculated amplitudes of (a) the correction to the conductance δS and (b) critical dc Josephson current I_{CJ} as a function of the normalized temperature k_BT/ε_L. They were calculated for $L_1=0.5$, 0.6, 0.7, and 0.8 μm (from top to bottom) for both δS and I_{CJ}.

We plot the dependence $I_{\mathrm{CJ}}(V)$ in Fig. 4 for different temperatures and nonzero values of V_N. One can see that I_C changes sign with increasing V_N. If condition (11) is fulfilled, I_{CJ} changes its sign at a characteristic voltage $eV\approx(\pi L/2L_1)\varepsilon_L\sqrt{k_BT/\varepsilon_L}$. One of the authors considered in an earlier study the case when distance L_1 was smaller than $\xi_N(\varepsilon)$ at $\varepsilon\approx\Delta$, i.e., $L_1<\sqrt{\hbar D/\Delta}$.[22]

In the calculation it is assumed that $L<L_\phi$. In the case where $L>L_\phi$, δS decreases as T increases, and the effect of L_ϕ can be discussed by taking into account γ in the calculation, assuming $L_\phi=\sqrt{D/\gamma}$. Figure 5 shows δS as a function of L_ϕ for the same values of D, L, and Δ as used for the previous calculation of δS and $V_N=0$. Here, δS decreases rapidly when L_ϕ becomes shorter than $2L=2$ μm. We turn now to a nonstationary case.

A. Nonstationary case

We consider again the system shown in Fig. 1(a) and calculate the conductance of the N film for the case when a dc and ac voltage $V_S(t)=V_0+V_\omega\cos(\omega t)$ is applied between the superconductors. As before, we find a correction to the N film conductance due to the proximity effect of the order of r^2 and we employ the same scheme of calculation as presented above. In order to obtain the equation for the distribution function, we need to write down an equation for Keldysh's function $\hat{G}$ taking into account temporal derivatives.[17] After averaging this equation over time and

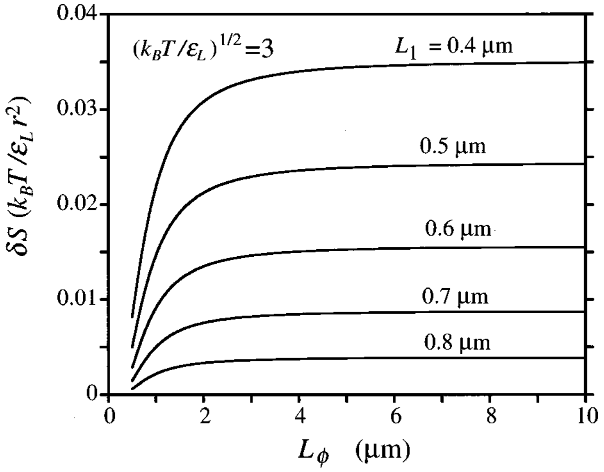

FIG. 5. Calculated δS as a function of the phase-breaking length L_ϕ for various values of L_1.

thickness, we arrive at Eq. (1) with the products F^RF^R, F^RF^A, etc., replaced by their time-averaged values $\overline{F^RF^R}, \overline{F^RF^A}$, etc. The equation for $\hat{F}^{R(A)}$ has the same form as Eq. (A1); the only difference is that the phase φ is time dependent: $\varphi(t)=2eV_0t/\hbar+(2eV_\omega/\hbar\omega)\sin(\omega t)+\varphi_0$. Repeating the calculations carried out in the previous section, we again obtain for δS the formulas (12) and (13) with $\overline{\cos\varphi}=J_1(2eV_\omega/\hbar\omega)\cos\varphi_0$ at $2eV_0=\hbar\omega$ and $\overline{\cos\varphi}=0$ otherwise, where J_1 is the Bessel function.[16] Therefore a Shapiro-like step should appear on the dependence of the N channel conductance as a function of V_0 if the Josephson relation is fulfilled: $V_0=\hbar\omega/2e$.[16,26] The step magnitude decreases with temperature as T^{-1}, whereas the dc Josephson critical current decays with T exponentially as shown in Fig. 3(b).

B. Conductance of the system in Fig. 1(b)

In this section we calculate the conductance of the system shown in Fig. 1(b) for a stationary case, i.e., the dependence of the conductance on the applied magnetic field. The only distinction from the case considered in Sec. II A is that we should take into account the conservation law for ''flows'' in each node[8]

$$\sum_i \mathbf{n}_i \boldsymbol{\partial}_i F^{R(A)}=0. \tag{15}$$

Here $\mathbf{n}_i$ is the unit vector coming out of the node. Then we obtain for $\hat{F}^{R(A)}$,

$$\hat{F}^{R(A)}(\varepsilon,x)=2\{r\hat{F}_S^{R(A)}(\varepsilon)/M^{R(A)}(\varepsilon)\}\sinh[k_\varepsilon^{R(A)}(L_3-x)], \tag{16}$$

here $0<x<L_3$, $r=(\rho L_3 w/R_{b\square}d)$ is the ratio of the N channel resistance with a length L_3 and a width w [see Fig. 1(b)] to the S/N interface resistances, and

$$M^{R(A)}(\varepsilon)=[\theta_3\{2\cosh\theta_3(\cosh\theta+\sinh\theta_2\cdot\sinh\theta_1) + \sinh\theta_3(\sinh\theta+\cosh\theta_2\cdot\sinh\theta_1)\}]^{R(A)}$$

where $\theta_{1,2,3}^{R(A)}=k_\varepsilon^{R(A)}L_{1,2,3}$ and $\theta=\theta_1+\theta_2$.

The correction to the conductance due to the proximity effect is determined again by Eq. (12) with

$$\langle m_-\rangle=(r^2/2)\{|M|^{-2}[\sinh(2\theta_3')/2\theta_3'-\sin(2\theta_3'')/2\theta_3''] - \mathrm{Re}(M)^{-2}[\sinh(2\theta_3)/2\theta_3-1]\}(1+\cos\varphi),$$

where M is defined above, $\theta_{1,2,3}=k_\varepsilon L_{1,2,3}$, $\theta=\theta'+i\theta''$, $k_\varepsilon=\sqrt{(2i\varepsilon+\hbar\gamma)/\hbar D}$, $\gamma=\gamma_0+Dk_H^2$, $k_H=2\pi Hw/\Phi_0$, and $\varphi=2\pi\Phi/\Phi_0$. Here Φ is the magnetic flux in the loop and Φ_0 is the magnetic flux quantum (see Appendix B).

We calculated the dependence of the correction to the conductance δS on temperature T, magnetic field H, and bias voltage V. In the calculation $D=120\ \mathrm{cm^2/sec}$, $L_1=L_2=L_3=0.5\ \mu$m and $w=0.15\ \mu$m. These values are the same as those measured experimentally for the Ag/Al junction.[23] It is also assumed that $\gamma_0/D=10^{12}\ \mathrm{m}^2$ which corresponds to $L_\phi=1\ \mu$m. In Fig. 6, δS for $H=0$ as a function of T is plotted with changing the bias voltage V. It is found that δS has a maximum as a function of temperature and the temperature where δS shows a maximum increases as V increases. In this junction ε_L is calculated to be about $32\ \mu$eV$=0.37$ K, and this value almost agrees with the temperature where δS shows a maximum. It is also found that δS has a behavior like T^{-1} as T increases and this temperature dependence is, of course, the same as that for δS discussed in previous sections.

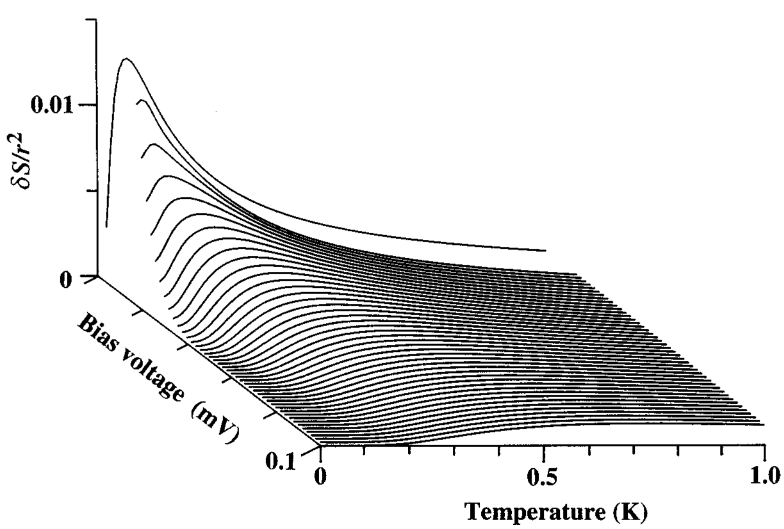

FIG. 6. Calculated $\delta S/r^2$ as a function of temperature with changing the bias voltage from 0 to 0.1 mV.

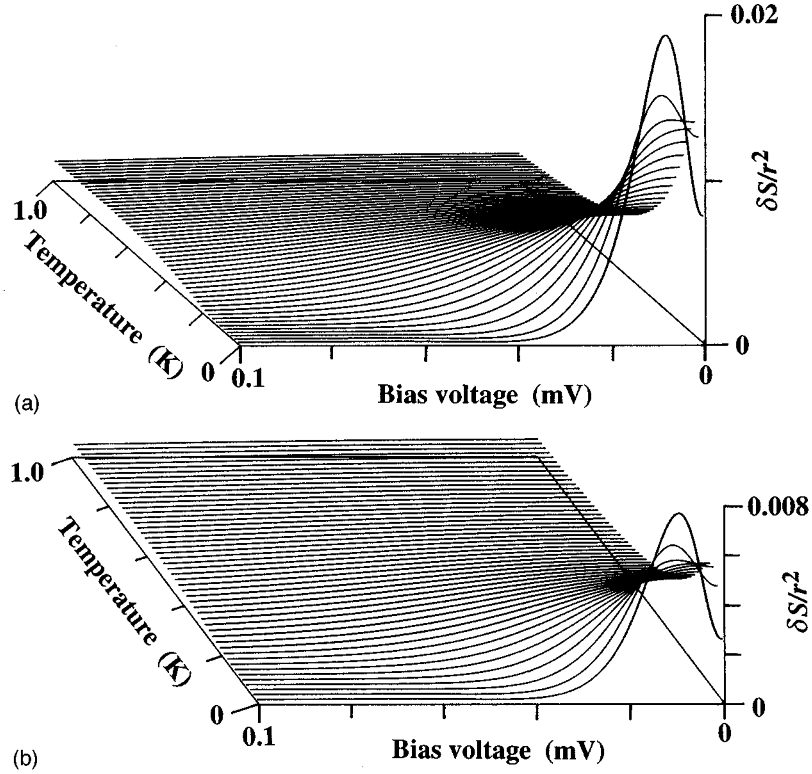

FIG. 7. Calculated $\delta S/r^2$ as a function of the bias voltage with changing temperature for (a) $H=0$ and (b) $H=15$ G.

The bias voltage dependence of δS is much stronger than the temperature dependence; δS as a function of V is shown in Fig. 7(a) for $H=0$ and (b) for $H=15$ G. When T is low, δS also shows a maximum as a function of V. These figures indicate that δS should be measured at a low bias voltage, since δS becomes extremely small at a high bias voltage.

Next, δS as a function of H is shown for various values of V in Fig. 8. Here, δS oscillates due to the interference caused by the magnetic field through the relation $\varphi=2\pi\Phi/\Phi_0$. The amplitude of δS corresponds to 26–260 e^2/h for the junction with normal resistances of 10–1Ω, assuming that $r=1$. This large oscillation has been confirmed experimentally for the junction with the structure shown in Fig. 1(b).[23]

Finally, the effect of the phase breaking length on δS was calculated. Figure 9 shows $\delta S-H$ curves for various values of γ_0/D at $T=0.3$ K. It was found that δS is greatly affected by the phase-breaking length.

III. JOSEPHSON EFFECTS IN THE ABSENCE OF THE JOSEPHSON COUPLING

In this section we consider a more general case than in Sec. II A. Namely, we assume dc and ac components for both currents I and I_1. It will be shown that even if the critical Josephson current I_{CJ} is exponentially small and can be neglected (negligible Josephson coupling), Josephson effects may arise in the system.[24] The dc and ac Josephson effects arise if the current I, besides the current I_1, flows in the system, and a dissipation takes place.

In order to make the analysis more understandable, a simplifying assumption is introduced. We suppose that the resistances of the N channel and the S/N interface differ slightly from their normal state values. It is valid, e.g., in a case of gapless superconductors, i.e., when the functions $\hat{F}_S^{R(A)}$ are small. This assumption is not essential, but allows one to greatly simplify the calculations and to make the physics of processes under consideration more transparent. Analyzing nonstationary processes, we admit an adiabatic approximation ($eV_{S,N}\ll\varepsilon_L$). Then, all formulas obtained previously remain valid. Integrating Eq. (1), we find

$$(\partial_x f)[1-m_-(x)]=\begin{cases} J+J_1-J_S, & 0<x<L_1, \\ J, & L_1<x<L. \end{cases} \tag{17}$$

Integrating Eq. (17) again and taking into account the boundary conditions of Eq. (8), we obtain for currents per unit energy

$$(d/\sigma)J=\frac{F_N\mathcal{R}_b+(F_N-F_S)\mathcal{R}_1}{\mathcal{R}_b\mathcal{R}+\mathcal{R}_1\mathcal{R}_2}, \tag{18a}$$

$$(d/\sigma)(J_1-J_S)=\frac{F_S\mathcal{R}_2+(F_S-F_N)\mathcal{R}_1}{\mathcal{R}_b\mathcal{R}+\mathcal{R}_1\mathcal{R}_2}. \tag{18b}$$

When obtaining Eqs. (18a) and (18b), we took into account finite values of potentials $\pm V_S$ at the superconductors (the potential at $x=0$ is taken to be zero). Therefore, instead of Eq. (3), one should write $J_V=G_V[F_S-f(L_1)]$, where the distribution function F_S in the superconductors is assumed to be in equilibrium. Its form is determined by Eq. (8) with V_N replaced by V_S. In the adiabatic approximation ($eV_{S,N}\ll\varepsilon_L<k_BT$), nonequilibrium component in J_0 can be neglected. Therefore, J_1 is determined by the expression

$$J_1=J_S+(d/\sigma)\mathcal{R}_b^{-1}[F_S-f(L_1)], \tag{19}$$

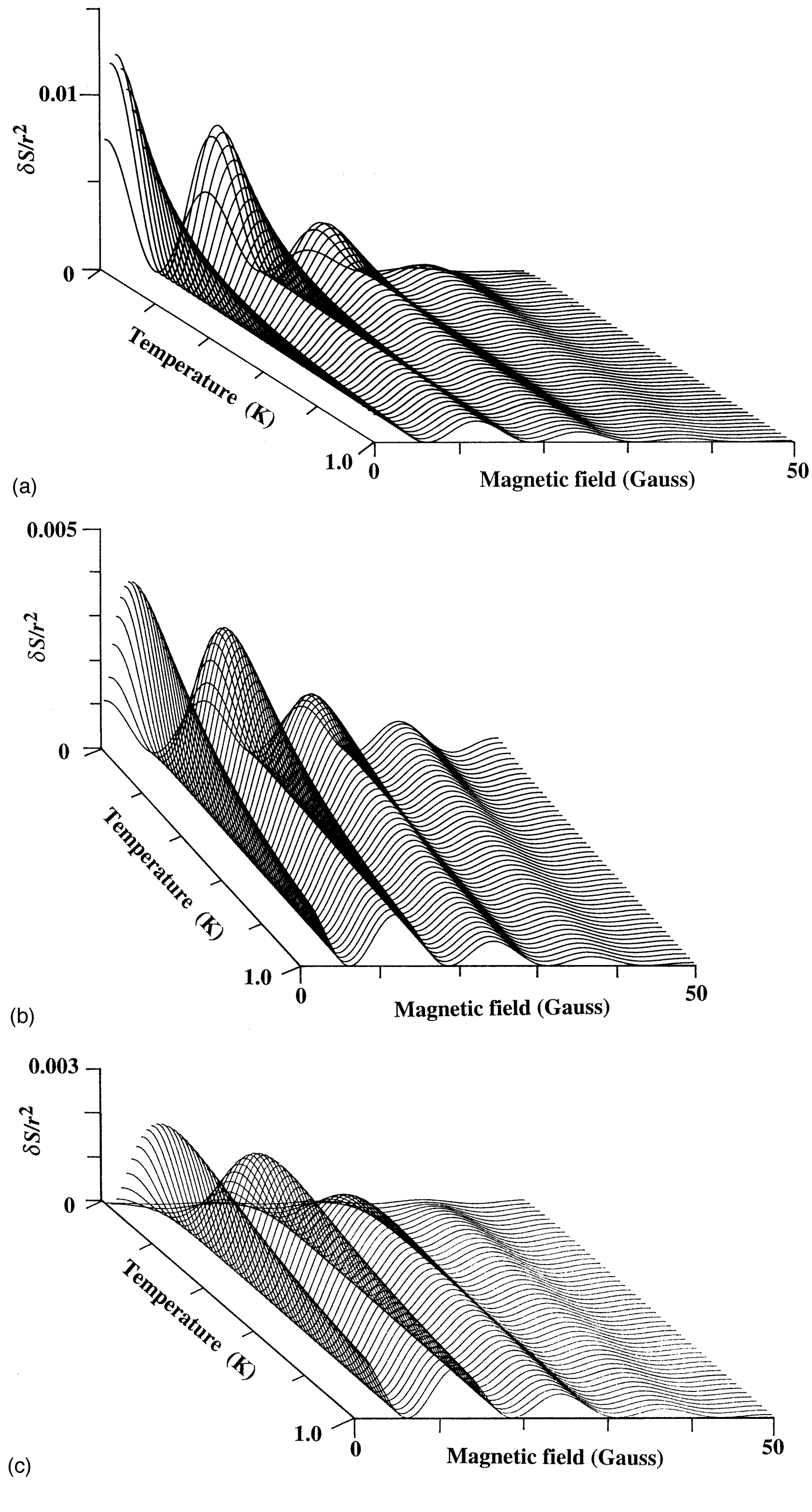

FIG. 8. Calculated $\delta S/r^2$ as a function of the applied magnetic fields with changing temperature for (a) the bias voltage $V=0$, (b) $V=0.028$ mV, and (c) $V=0.042$ mV.

where $\mathcal{R}_b^{-1}=(w/R_{b\square})[\nu\nu_S+b_{sg}]$. The quantity $\mathcal{R}_b$ is an energy-dependent S/N interface resistance. In the considered case of small condensate functions $\hat{F}_S^{R(A)}$ and $\hat{F}^{R(A)}$, this dependence is weak ($\nu\approx$const, $\nu_S\approx$const, and $b_{sq}\ll 1$). The energy dependences of the resistances $\mathcal{R}_{1,2}=(L_{1,2}\rho/d)(1+\langle m_-\rangle_{1,2})$ and $\mathcal{R}=\mathcal{R}_1+\mathcal{R}_2$ are also weak because the correction due to the proximity effect $\langle m_-\rangle_{1,2}$ is small. Integrating Eqs. (18a) and (18b) and taking into account Eq. (10), we find the relations between the currents I and I_1, and the voltages $V_{S,N}$, in the adiabatic approximation

$$V_S=IR_1(\varphi)+I_1[R_b(\varphi)+R_1(\varphi)],$$

$$V_N=IR(\varphi)+I_1R_1(\varphi). \quad (20)$$

Here $R(\varphi)=R_1(\varphi)+R_2(\varphi)$; $R_{1,2}(\varphi)$ and $R_b(\varphi)$ are expressed through $\mathcal{R}_{1,2}$ and $\mathcal{R}_b$ as

$$R_{1,2}(\varphi)=\int_0^\infty d\varepsilon\cdot\beta\,\cosh^{-2}(\varepsilon\beta)\cdot\mathcal{R}_{1,2}\approx R_{1,2}-\delta R_{1,2}\cos\varphi, \quad (21a)$$

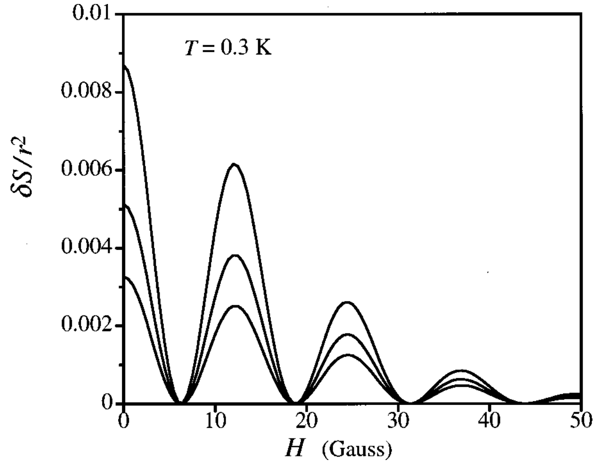

FIG. 9. Calculated $\delta S/r^2$ as a function of magnetic field for $\gamma_0/D=1.5\times10^{12}$, 1×10^{12}, and 5×10^{11} m^2 (from top to bottom). These values correspond to $L_\phi=1.4$, 1.0, and 0.8 μm, respectively.

$$R_b(\varphi)=\int_0^\infty d\varepsilon\cdot\beta\,\cosh^{-2}(\varepsilon\beta)\cdot\mathcal{R}_b\approx R_b-\delta R_b\cos\varphi. \tag{21b}$$

Equation (20) has an obvious physical meaning. The voltage V_S equals the sum of voltage drops across the region $(0, L_1)$ and the S/N interface. The voltage V_N equals the sum of voltage drops across the regions $(0, L_1)$ and (L_1, L). Each resistance depends on the phase difference in an explicit form. The small parts of the condensate contribution to the resistance $\delta R_{1,2}$ are proportional to the square of the condensate functions and are connected with $\langle m_-\rangle$ via the relation

$$\delta R_{1,2}=-R_{1,2}\beta\int_0^\infty d\varepsilon\cdot\cosh^{-2}(\varepsilon\beta)[\langle m_-(\varphi)\rangle_{1,2}-\langle m_-(\pi/2)\rangle_{1,2}]. \tag{22}$$

The averages $\langle m_-\rangle_{1,2}$ are defined in Eqs. (6) and (9). The distinctions of $R_{1,2}$ and R_b from their normal-state values are small and may be neglected.

The voltage V_S is related to the phase difference via the Josephson relation

$$2V_S=\hbar\partial_t\varphi/2e. \tag{23}$$

Excluding V_N from Eq. (20) and taking into account the relation of Eq. (23), we obtain for φ

$$\hbar\partial_t\varphi/4e+[(I+I_1)\delta R_1+I_1\delta R_b]\cos\varphi=(I+I_1)R_1+I_1R_b. \tag{24}$$

Equation (24) is analogous to the Josephson equation for a RSJ (resistively shunted junction). Integrating Eq. (24), we find the relation between the time-averaged voltage V_S and constant currents I and I_1, as

$$\overline{V_{S0}}=[\{(I+I_1)R_1+I_1R_b\}^2-\{(I+I_1)\delta R_1+I_1\delta R_b\}^2]^{1/2}. \tag{25}$$

The $I_1-\overline{V_{S0}}$ dependence for different values of I is shown in Fig. 10. It has the form of the $I-V$ curve for a RSJ with the critical current I_C which is equal, as follows from Eq. (25), to

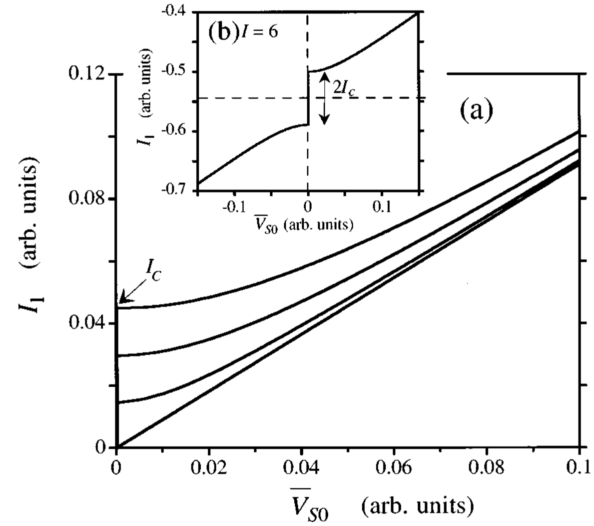

FIG. 10. Current-voltage $I_1-\overline{V_{S0}}$ characteristics of the S-N-S junction in Fig. 1(a). (a) $I_1-\overline{V_{S0}}$ characteristics for $I=0$, 2, 4, and 6 in arbitrary units (from bottom to top). Each curve is calculated for $R_1/R=0.1$, $\delta R_1/R_1=0.1$, $\delta R_b/R_b=0.01$ and $R_b=1\ \Omega$. Each curve is shifted by $IR_1/(R_1+R_b)$ in the current direction because the I-V curve shifts from $I_1=0$ by $-IR_1/(R_1+R_b)$ as shown in (b). (b) Real $I_1-\overline{V_{S0}}$ characteristics for $I=6$.

$$I_C=I\,\frac{|\delta R_1\cdot R_b-R_1\cdot\delta R_b|}{(R_b+R_1)^2}. \tag{26}$$

One can see that I_C is zero at $I=0$ and increases linearly with I unless the voltage drop $V_N\approx IR$ becomes comparable with k_BT/e (otherwise δR_1 decreases with V_N). The Joule heating must also be negligible. If the condition

$$2L_1>\xi_N(T) \tag{27}$$

is fulfilled, the main contribution to δR_1, as we mentioned before, is due to the anomalous term $\langle m_-\rangle_{1\text{an}}$ which decreases with T as T^{-1}. Meanwhile, the critical Josephson current I_{CJ} and δR_b are exponentially small. Indeed, if $k_BT<\gamma_S$ (γ_S is the spin-flip scattering rate in superconductors), $F_S^R\approx F_S^A\approx-i\Delta/\hbar\gamma_S$ and the integral in Eq. (21b) can be calculated by encircling integration with a contour in the upper (or lower) half-plane of ε. The interference part of b_{sg} [see Eq. (4)] decays with ε and the characteristic energy of this function is ε_{L1}. Therefore, $\delta R_b\sim\exp[-2L_1/\xi_N(T)]$. At $R_1\delta R_b\ll R_b\delta R_1$ and $R_b\gg R_1$, we obtain $I_C\approx I\delta R_1/R_b$.

If the condition of Eq. (27) is violated, the Josephson critical current I_{CJ} should be taken into account and the total critical current I_C^* is not zero at any current I

$$I_C^*=\sqrt{I_C^2+I_{\text{CJ}}^2}. \tag{28}$$

This relation together with Eq. (26) determines the critical current for any ratio between ξ_N and $2L_1$.

Let us finally consider the problem concerning ordinary Shapiro steps, i.e., concerning peculiarities on the I_1-V_S curve in the presence of ac components of the currents

$$I(t)=I+I_\Omega\cos\Omega t,$$
$$I_1(t)=I_1+I_{1\Omega}\cos\Omega t. \tag{29}$$

Substituting these expressions into Eq. (24), we can easily find the height of the first Shapiro step δI_1 on a part of the $I_1 - V_S$ curve where it differs from Ohm's law only slightly

$$\delta I_1 \cdot (R_b + R_1) = \left| \frac{R_1 \delta R_b - R_b \delta R_1}{I R_1 + I_1 (R_b + R_1)} \right| \begin{cases} |I_{1\Omega} I|, & I_\Omega = 0, \\ |I_\Omega I_1|, & I_{1\Omega} = 0. \end{cases} \tag{30}$$

It follows from Eq. (30) that, in the absence of $I(t)$, the Shapiro step is absent if the condition of Eq. (27) is satisfied and the Josephson critical current I_{CJ} is negligible. However, when either the dc or ac component of the current $I(t)$ is not zero, the Shapiro step arises on the $I_1 - \overline{V_S}$ characteristics. One can easily find the form of the $I_1 - V_S$ curve near the Shapiro step. Considering large enough currents I_1, we obtain for V_S near the Shapiro step

$$\overline{V_S} = V_\Omega + [\{\overline{V_{S0}} - V_\Omega\}^2 - \{I_{1\Omega} I_C (R_b + R_1)^2 / 2\overline{V_{S0}}\}^2]^{1/2}. \tag{31}$$

Here $V_\Omega = \hbar\Omega/4e$, and $\overline{V_{S0}}$ and I_C are determined by Eqs. (25) and (26), respectively. Therefore, the form of the $I_1 - \overline{V_S}$ characteristics is similar to a corresponding dependence for a RSJ.[25]

In our previous work[16] we analyzed the possibility to observe ac phase-coherent phenomena measuring Shapiro steps on the $I_1 - \overline{V_S}$ curve (we assumed that $I_\Omega = 0$). Using Eq. (20), it is easy to calculate the height of the Shapiro step as

$$\delta\overline{V_N} = (I_{1\Omega} I / \overline{V_{S0}}) |\delta R (R_b + R_1) - \delta R_1 \cdot R|. \tag{32}$$

In the considered case of a large S/N interface resistance R_b, the main contribution to δV_N is due to the first term in Eq. (32). The value of δR was calculated for the case of superconductors with a nonzero energy gap.[16]

The magnitude of the critical current I_C is determined by the interference part of the condensate contribution to the N channel resistance δR_1 which can be found from Eq. (22). Using the functions $\hat{F}^{R(A)}$ in Eq. (A3), we find for the interference part

$$\langle m_- \rangle_{1\text{int}} = \langle m_-(\varphi) \rangle_1 - \langle m_-(\pi/2) \rangle_1,$$

$$\langle m_- \rangle_{1\text{int}} = -(r^2/8)(|F_S|^2 |\sinh\theta_2/\theta|^2 \{|P_y|^2 [\sinh(2\theta_1')/2\theta_1' + \sin(2\theta_1'')/2\theta_1''] - |P_x|^2 [\sinh(2\theta_1')/2\theta_1' - \sin(2\theta_1'')/2\theta_1'']\} + \mathrm{Re} F_S^2 \sinh^2\theta_2/\theta^2 \{P_y^2 [\sinh(2\theta_1)/2\theta_1 + 1] - P_x^2 [\sinh(2\theta_1)/2\theta_1 - 1]\}). \tag{33}$$

Figure 11 shows $\delta R_1 / R_1 r^2$ as a function of temperature $k_B T/\varepsilon_L$ for different L_1 (from 0.5 to 0.8 μm) and $L = 1\ \mu$m. Similar to the case considered in Sec. II, the magnitude of δR_1 decreases with T as T^{-1} in the order of magnitude $\delta R_1 \approx R_1 (r^2/8) \varepsilon_{L1}/k_B T$ (if $\Delta \cong \hbar\gamma_S$) provided that condition (27) is fulfilled.

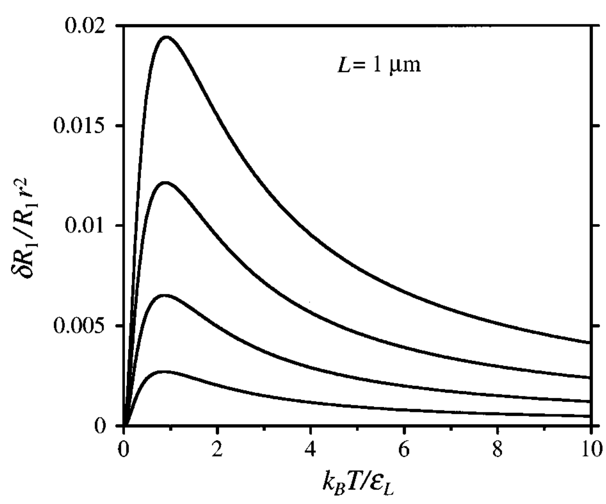

FIG. 11. Temperature dependence of the phase-dependent correction δR_1 to the resistance R_1. The normalized value $\delta R_1 / R_1 r^2$ is plotted for $L_1 = 0.5$, 0.6, 0.7 and 0.8 μm (from top to bottom).

IV. CONCLUSIONS

In the dirty limit we have calculated the correction δS to the conductance of the N channel due to the proximity effect. The proximity effect is assumed to be weak, that is, the condensate functions induced in the N film are small; this case is realized if the S/N resistance exceeds the N channel resistance. We have shown that the correction δS oscillates with increasing the phase difference between superconductors even in the case when the dc Josephson effect is negligible and the supercurrent in the S/N/S junction is absent.

The amplitude of δS oscillations decreases with T as T^{-1}, whereas the magnitude of the Josephson critical current diminishes exponentially; $I_{CJ} \approx \exp[-2L_1/\xi_N(T)]$ at $2L_1 > \xi_N(T)$. The phase coherency is related to the fact that the main contribution to δS is due to the states with low energies $\varepsilon \approx \varepsilon_L = \hbar D/L^2$. The condensate functions at such energies spread over the full length of the mesoscopic system: $|F^{R(A)}| \sim |\exp(-k_\varepsilon^{R(A)} x)| \sim \exp(-x/L)$, where $k_\varepsilon^{R(A)} \approx \sqrt{\mp 2i\varepsilon/\hbar D}$. The contribution to the critical current I_C is caused by thermal energies $\varepsilon \sim k_B T$ at which the condensate functions decay exponentially over the coherence length. If dc (V_0) and ac ($V_\omega \cos\omega t$) voltages are applied between superconductors a Shapirolike step appears on the dependence $\delta S(V_0)$ at $V_0 = \hbar\omega/2e$. The amplitude of this step decreases with T as T^{-1}.

Using a simple model, in which the condensate functions $\hat{F}^{R(A)}$ and $\hat{F}_S^{R(A)}$ as well as the distinction of all resistances from their values in the normal state are supposed to be small, we have also considered the behavior of the S/N/S system shown in Fig. 1(a) at different currents I and I_1. It has been shown that the Josephson effects may arise even at negligible Josephson coupling ($I_{CJ} \sim \exp[-2L_1/\xi_N(T)] \ll 1$). The effective critical current I_C is zero at $I=0$ and increases with increasing I while I remains smaller than $k_B T/eR$. The conditions for observing Shapiro steps have also been found.

ACKNOWLEDGMENTS

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APPENDIX A

In this appendix we present expressions for the condensate functions $\hat{F}^{R(A)}(\varepsilon,x)$. Using the expressions given here, one can easily calculate $\langle m_- \rangle_{1,2}$ [see Eqs. (6) and (9)]. In the considered case of small $\hat{F}^{R(A)}(\varepsilon,x)$ these functions obey the linear equation (see Refs.10 and 11)

$$\partial^2_{xx}\hat{F}^{R(A)} - (k_\varepsilon^{R(A)})^2\hat{F}^{R(A)} = -2(k_b^2 w)\,\delta(x \pm L_1)(\hat{F}_S^{R(A)} - \hat{F}^{R(A)}). \quad (A1)$$

Here $k_\varepsilon^{R(A)} = \sqrt{(\mp 2i\varepsilon + \hbar\gamma)/\hbar D}$ and the functions $\hat{F}_S^{R(A)}$ are defined in Eq. (2). The boundary conditions to Eq. (11) are

$$\hat{F}^{R(A)}(\varepsilon, \pm L) = 0. \quad (A2)$$

In the presence of a magnetic field, the vector potential should be taken into account (see Appendix B).

The solution of Eq. (A1) has the form [for brevity we omit indices $R(A)$]

$$\hat{F}(\varepsilon,x) = F_x(\varepsilon,x)\, i\hat{\sigma}_x \sin(\varphi/2) + F_y(\varepsilon,x)\, i\hat{\sigma}_y \cos(\varphi/2), \quad (A3)$$

where

$$F_x(\varepsilon,x) = \frac{r}{\theta} F_S \begin{cases} P_x \sinh\theta_2 \sinh(k_\varepsilon x), & 0<x<L_1, \\ P_x \sinh\theta_1 \sinh k_\varepsilon(L-x), & L_1<x<L, \end{cases} \quad (A4a)$$

$$F_y(\varepsilon,x) = \frac{r}{\theta} F_S \begin{cases} P_y \sinh\theta_2 \cosh(k_\varepsilon x), & 0<x<L_1, \\ P_y \cosh\theta_1 \sinh k_\varepsilon(L-x), & L_1<x<L. \end{cases} \quad (A4b)$$

Here $r = \rho L w / R_{b\square} d$ is the ratio of the N channel and S/N interface resistances, $\theta = \theta_1 + \theta_2$, $\theta = k_\varepsilon L$, $\theta_{1,2} = k_\varepsilon L_{1,2}$, $L_2 = L - L_1$, $k_\varepsilon = \sqrt{(2i\varepsilon + \hbar\gamma)/\hbar D}$, $P_x = [\sinh\theta + (r/\theta)\sinh\theta_1 \cdot \sinh\theta_2]^{-1}$, and $P_y = [\cosh\theta + (r/\theta)\cosh\theta_1 \cdot \sinh\theta_2]^{-1}$, where φ is the phase difference between the superconductors. In the case of ordinary superconductors with an energy gap, we have

$$F_S^{R(A)} = \Delta/\xi_\varepsilon^{R(A)} \quad \text{and} \quad \xi_\varepsilon^{R(A)} = \sqrt{(\varepsilon \pm i\Gamma_S)^2 - \Delta^2}. \quad (A5)$$

Then Eq. (A1) is valid under condition $r \ll 1$. In case of gapless superconductors

$$F_S^{R(A)} = \pm \frac{\Delta}{\varepsilon \pm i\gamma_S}, \quad (A6)$$

where γ_S is the spin-flip scattering rate which is assumed to be large as compared to Δ. In this case of small $F_S^{R(A)}$, Eq. (A4) is valid for arbitrary r.

APPENDIX B

Here we establish a relationship between the phase difference φ and the magnetic field H. In the presence of the magnetic field, one has to take into account vector potential **A** in Eq. (A1). It looks like

$$[\partial_x + (2ie/c\hbar)\mathbf{A}]^2 \hat{F}_H^{R(A)} - (k_\varepsilon^{R(A)})^2 \hat{F}_H^{R(A)} = -2(k_b^2 w)\,\delta(x \pm L_1)\hat{F}_{S,H}^{R(A)}. \quad (B1)$$

Making the transformation of the Green's functions

$$\hat{F}_H^{R(A)} = S_H \hat{F}^{R(A)} S_H^+, \quad (B2)$$

we exclude **A** from Eq. (A1). The transformation matrix S_H has the form

$$\hat{S}_H = \cos\chi_H + i\hat{\sigma}_z \sin\chi_H,$$

where

$$\chi_H = -(e/c\hbar)\int_0^x \mathbf{A}\, d\mathbf{l}.$$

The functions $\hat{F}_S^{R(A)}$ contain the phase χ. The total gauge-invariant phase $\tilde{\chi}$ is

$$\tilde{\chi}(x) = \chi + \chi_H. \quad (B3)$$

In the N film [see Fig. 1(a)] we have

$$\tilde{\chi}(\pm L_1) = \chi(\pm L_1) - (e/c\hbar)\int_{0(\mathrm{N})}^{\pm L_1} \mathbf{A}\, d\mathbf{l}. \quad (B4)$$

In the presence of a magnetic field the phase difference $\tilde{\varphi} = \tilde{\chi}(L_1) - \tilde{\chi}(-L_1)$ is

$$\tilde{\varphi} = \varphi - (e/c\hbar)\int_{-L_1(\mathrm{N})}^{L_1} \mathbf{A}\, d\mathbf{l}. \quad (B5)$$

The index N means that the integration is performed over the N region. The condensate momentum in the superconducting part is equal to

$$\mathbf{P}_S \equiv \partial_x \tilde{\chi} = \partial_x \chi - (e/c\hbar)\mathbf{A}. \quad (B6)$$

Integrating Eq. (B6) over the superconducting part, we obtain

$$\int_{(S)} \mathbf{P}_S d\mathbf{e} = \varphi - 2\pi n - (e/c\hbar)\int_{-L_{1(S)}}^{L_1} \mathbf{A}\, d\mathbf{l}. \quad (B7)$$

Excluding φ from Eqs. (B5) and (B7), we find the sought for relationship between $\tilde{\varphi}$ and H as

$$\tilde{\varphi}=2\pi(\Phi/\Phi_0+n)-\int_{(S)}\mathbf{P}_S d\mathbf{l}. \tag{B8}$$

Here Φ is the magnetic flux through the S/N/S system, $\Phi_0=hc/2e$ is the flux quantum, and n is an integer. The integral in Eq. (B8) is zero if either the magnetic flux does not penetrate into the superconducting strips or the critical current in the S/N/S system is very small. If the magnetic field penetrates into the S strips uniformly, we obtain

$$\int_{(S)}\mathbf{P}_S d\mathbf{l}=P_S L_S=2eI_S L_S/\pi\sigma\Delta=(2eL_S/\pi\sigma\Delta)I_C\sin\tilde{\varphi}. \tag{B9}$$

Here L_S is the length of the superconducting part. Here we used the formula for the supercurrent at low temperatures $(k_B T\ll\Delta)$: $I_S=(\pi/2)\sigma\Delta P_S/e$. The factor in front of $\sin\varphi$ is small, if I_c is small enough. For example, in case $L_1\gg\xi_N$ we have from Eq. (14b)

$$I_C(2eL_S/\pi\sigma\Delta)\cong(r^2L_S/2\xi_N)\exp(-2L_1/\xi_N). \tag{B10}$$

That is, the factor in Eq. (B9) is small because $r\ll1$ and $2L_1>\xi_N$ by assumption.

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