Time-differential perturbed-angular-correlation studies on ¹¹⁷In and ¹¹¹Cd in Li_{0.995}Cd_{0.005}NbO₃

Yoshitaka Ohkubo, Shin-ichi Uehara, and Yoichi Kawase

Research Reactor Institute, Kyoto University, Kumatori-cho, Sennan-gun, Osaka 590-04, Japan

Jin Nakamura, Takuya Okada, Shizuko Ambe, and Fumitoshi Ambe

The Institute of Physical and Chemical Research (RIKEN), Wako-shi, Saitama 351-01, Japan

Kichizo Asai

Department of Applied Physics and Chemistry, The University of Electro-Communications, Chofu-shi, Tokyo 182, Japan (Received 11 April 1997; revised manuscript received 16 July 1997)

The nuclear-electric-quadrupole interactions at ¹¹⁷In and ¹¹¹Cd nuclei arising from ¹¹⁷Cd and ^{111m}Cd, respectively, chemically introduced in Li_{0.995}Cd_{0.005}NbO₃ polycrystals were studied using the time-differential perturbed angular correlation of γ rays. The $|V_{zz}^{\text{lattice}}|$ values derived for ¹¹⁷In and ¹¹¹Cd coincided with each other, i.e., $3.4(1) \times 10^{20}$ V/m² and $3.4(5) \times 10^{20}$ V/m², respectively, meaning that the In and Cd ions are in the same lattice environment in the forms of In³⁺ and Cd²⁺. Although these probes are considered to occupy the Li sites, the above values of V_{zz}^{lattice} are substantially different from the previously reported value for ⁷Li, 0.7×10^{20} V/m², which is ascribable to the structural relaxation around the probe atoms. The temperature dependence of ω_Q of ¹¹⁷In indicates that the In ion vibrates anisotropically about its equilibrium position at high temperatures, as has been argued for the Li ion in LiNbO₃. [S0163-1829(97)04942-4]

Lithium niobate, LiNbO₃, is a ferroelectric with T_C = 1483 K.¹ Because of the similar sizes of Li⁺ (76 pm) and Nb⁵⁺ (64 pm),² this oxide while having the *ABO*₃ stoichiometry (*A* and *B* stand for metal elements) does not adopt the perovskite structure, but takes an ilmenite (FeTiO₃)-related structure.³ The oxide is an important nonlinear optic material. Its optical properties are influenced by doping the material with metal impurities.⁴ It is reported, for example, that In impurities increase the resistance of LiNbO₃ to optical damage.⁵ Knowledge of the lattice environment and the chemical state of dopants is essential for theoretical understanding of the impurity effects.

In the present study, we performed time-differential perturbed-angular-correlation (TDPAC) measurements on ¹¹⁷In arising from ¹¹⁷Cd and on ¹¹¹Cd arising from ¹¹¹mCd in polycrystalline samples of a chemical composition $Li_{0.995}Cd_{0.005}NbO_3$, both of whose parent nuclides were chemically introduced in order to obtain information about the valency of In and Cd. The assumption of purely ionic states of In and Cd, i.e., In^{3+} and Cd^{2+} , leads to the ratio of the electric quadrupole frequency for ¹¹⁷In³⁺ to that for ¹¹¹Cd²⁺ being 2.3.⁶ A deviation of the measured ratio from 2.3 may indicate a participation of the 5*s* and 5*p* electrons in the bonding.⁶

Figure 1 shows decay schemes of ¹¹⁷Cd \rightarrow ¹¹⁷In and ^{111m}Cd \rightarrow ¹¹¹Cd. The 749 keV excited state of ¹¹⁷In is populated by β^- decay of the parent ¹¹⁷Cd with a half-life $t_{1/2}$ = 2.49 h and decays to the 315 keV excited state through the 660 keV intermediate state having a spin $I=\frac{3}{2}$, $t_{1/2}$ = 53.6 ns, and an electric quadrupole moment Q=(-)0.59(1) b.⁷ Here and hereafter, the number in the parentheses represents the uncertainty in the least significant digit(s). The 396 keV excited state of ¹¹¹Cd, i.e., ^{111m}Cd with $t_{1/2}$ =48.54 min decays to the ground state through the 245 keV intermediate state having $I=\frac{5}{2}$, $t_{1/2}$ =85.0 ns, and

Q = +0.77(12) b.⁷ The intermediate state is split by the interaction of the electric quadrupole moment of the nuclei with an extranuclear electric-field gradient (EFG). Detection of the 90 keV (151 keV) γ rays with a detector sorts out a set of ¹¹⁷In (¹¹¹Cd) nuclei in the intermediate state with spin alignment against the direction of the detector from the source. Then, time-differential measurements of the 344 keV (245 keV) γ rays with another detector permits determination of the spin precession frequency of ¹¹⁷In (¹¹¹Cd) nuclei in the intermediate state and then determination of the splitting of the state. Thus, TDPAC of the 90 and 344 keV (151 and 245 keV) γ rays emitted in the successive γ transitions reveals the electric quadrupole interaction during the stay of ¹¹⁷In (¹¹¹Cd) nuclei in the intermediate state.

The parent nuclei ¹¹⁷Cd and ^{111m}Cd were separately obtained by irradiating enriched ¹¹⁶CdO and ¹¹⁰CdO, respectively, with thermal neutrons at Kyoto University Research Reactor Institute. The irradiated oxide was mixed with appropriate amounts of high-purity powders of Li₂CO₃ and Nb₂O₅ for a chemical composition of Li_{0.995}Cd_{0.005}NbO₃. The mixture was pressed into a pellet and heated in air at 1100 °C. The same process followed after the sample was repowdered and thereby Li_{0.995}Cd_{0.005}NbO3 was obtained. We tentatively use the formula Li_{0.995}Cd_{0.005}NbO₃ for our samples, use of which will be justified below. Loss of neither the metals nor the source nuclides was observed during these heating processes. The room-temperature x-ray-diffraction pattern of a sample without cadmium prepared by an identical heating process showed a single phase of LiNbO₃. TDPAC measurements were performed in the temperature region between 4.2 and 873 K for the samples containing ¹¹⁷Cd by means of a conventional fast-slow setup and four BaF₂ scintillation detectors. For the sample containing ^{111m}Cd, a TDPAC measurement was performed at 4.2 K

10 730



FIG. 1. Partial decay schemes of ${}^{117}Cd \rightarrow {}^{117}In$ and ${}^{111m}Cd \rightarrow {}^{111}Cd$.

using the same detection system. For the details of the experimental method, refer to Ref. 8.

Here, we give the expressions of directional anisotropy, $A_{22}G_{22}(t)$, for a unique static quadrupole interaction. The coefficient A_{22} depends only on the nuclear transitions. The perturbation factor $G_{22}(t)$ for an ensemble of randomly oriented microcrystals is a function of the electric quadrupole frequency ω_Q and the asymmetry parameter η of the EFG through the interaction Hamiltonian. The quantities ω_Q and η are defined as follows: $\hbar \omega_Q = -eQV_{zz}/[4I(2I-1)]$ and $\eta = (V_{xx} - V_{yy})/V_{zz}$. The asymmetry parameter η takes a value between 0 and 1. The perturbation factor $G_{22}(t)$ has the form

$$G_{22}(t) = [1 + 4\cos(\omega^{(0)}t)]/5, \qquad (1)$$

$$\omega^{(0)} = 6\omega_Q \left(1 + \frac{\eta^2}{3}\right)^{1/2}$$
 for the case of ¹¹⁷In, (2)

and

$$G_{22}(t) = \sum_{n=0}^{3} S_n(\eta) \cos(\omega_n t),$$
 (3)

$$\omega_0 = 0, \quad \omega_n = 6 \, \omega_Q C_n(\eta) \quad (n = 1, 2, 3)$$

for the case of ^{111m}Cd. (4)

The S_n and C_n in Eqs. (3) and (4) are numerically calculated for a given asymmetry parameter. It should be noted that for



FIG. 2. (a) TDPAC spectrum, $A_{22}G_{22}(t)$, of ¹¹⁷In (\leftarrow ¹¹⁷Cd) in Li_{0.995}Cd_{0.005}NbO₃ at 4.2 K with the solid curve showing the result of fitting with Eqs. (1) and (2) in the text, and (b) the corresponding spectrum of ¹¹¹Cd (\leftarrow ^{111m}Cd) with the solid curve showing the result of fitting with Eqs. (3) and (4) in the text.

the case of ¹¹⁷In, $G_{22}(t)$ contains only one frequency component and the values of ω_Q and η cannot be determined independently from the spectrum, whereas for the case of ¹¹¹Cd, they can be determined because the amplitudes S_n and the frequencies ω_n in Eq. (3) are modified in a characteristic way as a function of η .

The $A_{22}G_{22}(t)$ measured at 4.2 K of ¹¹⁷In and ¹¹¹Cd in Li_{0.995}Cd_{0.005}NbO₃ are shown in Figs. 2(a) and 2(b), respectively. The anisotropy is modulated in a manner characteristic of the electric quadrupole interaction in polycrystalline samples depending on the spin value of the intermediate nuclear state. They were analyzed with an expression assuming that the electric quadrupole frequency ω_Q has a Lorentzian distribution around the central value $\overline{\omega}$. The width of the Lorentzian distribution $\Delta \omega$ was taken to be 0 for the ¹¹⁷In time spectra at $T \leq 433$ K, and to be 3% of $\overline{\omega}$ for the ¹¹⁷In time spectra at $T \geq 573$ K and for the ¹¹¹Cd time spectrum at 4.2 K. We assume that the value of η for ¹¹⁷In is the same as that obtained for ¹¹¹Cd as assumed in Ref. 6. The solid curves in Fig. 2 are the results of the fitting.

In order to examine the site of 0.5 mol% Cd in LiNbO₃, we compare our result on ¹¹¹Cd with that obtained by Hauer et al.9 They studied the electric quadrupole interactions at ¹¹¹Cd (\leftarrow ¹¹¹In) in undoped LiNbO₃ single crystals and at ¹¹¹Cd (\leftarrow ^{111m}Cd) in undoped and 6 mol% Mg-doped LiNbO₃ single crystals after implantation of ¹¹¹In and ^{111m}Cd employing the TDPAC method. They observed two electric quadrupole frequencies that differ from each other by only about 5% in each of the above three systems, and each electric quadrupole frequency was the same in the three systems. Since the lattice site of 0.59 mol% In in 6 mol% Mgdoped LiNbO₃ had been determined by the PIXE (protoninduced x-ray emission)/channeling technique to be the Li site,¹⁰ it was concluded that both In and Cd occupy the Li site of undoped and Mg-doped LiNbO3.9 From Table II and Fig. 5 of Ref. 9, showing the room-temperature values of the quadrupole interaction at 111 Cd ($\leftarrow ^{111m}$ Cd) and the temperature variation of the two quadrupole frequencies of ¹¹¹Cd $(\leftarrow^{111}$ In), respectively, the weighted average of quadrupole frequency of ¹¹¹Cd (\leftarrow ^{111m}Cd) at 4.2 K is estimated to be

30.3(3) Mrad/s, which essentially coincides with our value of 29.7(2) Mrad/s. Our η value of 0.08 is considered to be eventually the same as the η value of the main component in Ref. 9, i.e., 0.11(2). This observation means that 0.5 mol% Cd occupy the Li site of LiNbO₃ and that the chemical formula Li_{0.995}Cd_{0.005}NbO₃ is justified.

Since the Li site has a three-fold symmetry axis, the EFG's at ¹¹⁷In and ¹¹¹Cd at the sites may provisionally be expected to be axially symmetric. Hauer et al. interpreted the obtained nonzero η values as being due to the influence of the considerable concentration of Nb vacancies in congruent $LiNbO_3$ ⁹ On the other hand, Catchen *et al.* interpreted the asymmetry of the EFG at 181 Ta (\leftarrow 181 Hf) at the Li site in stoichiometric LiNbO3 and LiTaO3, isostructural to LiNbO3 with $T_c = 938$ K, using an order-disorder model.¹¹ In both LiNbO₃ and LiTaO₃, it was observed that η decreases from the value of about 0.25 at room temperature to less than 0.1 above T_C . Catchen *et al.* considered that in the ferroelectric phase some of the Li ions occupy the interstitial sites (normally vacant metal-ion sites in oxygen octahedra) and break the axial symmetry of the Li site, whereas in the paraelectric phase equal occupancy of the normal and interstitial Li sites gives effectively zero asymmetry of the EFG. The observation by Hauer et al. of the two electric quadrupole interactions are consistent with this argument. As an additional cause of the nonzero η value, we consider that impurity probe ions with the electric charges larger than +1 are displaced from the normal Li position and deform the surroundings, as described later on.

We now move to the discussion on the valency of In and Cd in LiNbO₃. We calculate the ratio of the electric quadrupole frequency of ¹¹⁷In to that of ¹¹¹Cd, $\omega_Q(^{117}\text{In})/\omega_Q(^{111}\text{Cd})$, assuming that In and Cd exist as purely ionic states, In³⁺ and Cd²⁺, and are in the same lattice environment. Employing the phenomenological model of Sternheimer, ¹² V_{zz} is expressed as $V_{zz} = (1 - \gamma_{\infty}) V_{zz}^{\text{lattice}}$ under the former assumption. Here, V_{zz}^{lattice} is due to the charges on the lattice ions surrounding the probe atom noncubic symmetrically. The quantity γ_{∞} is the Sternheimer antishielding factor representing the effect of the distortion of the closed-shell electron distributions in the probe atom. The latter assumption means that V_{zz}^{lattice} is the same for both probe nuclei. Then,

$$\frac{\omega_Q(^{117}\text{In})}{\omega_Q(^{111}\text{Cd})} = \frac{10}{3} \frac{Q(^{117}\text{In})}{Q(^{111}\text{Cd})} \frac{1 - \gamma_{\infty}(^{117}\text{In})}{1 - \gamma_{\infty}(^{111}\text{Cd})}.$$
 (5)

Using the calculated values of -29.3 as γ_{∞} (Cd) and -25.8 as γ_{∞} (In), ¹³ $|\omega_Q|^{117}$ In) $|\omega_Q|^{(111}$ Cd)| is evaluated to be 2.3, where the value of γ_{∞} (In) was taken to be the average value of γ_{∞} (Cd) = -29.3 and γ_{∞} (Sn) = -22.3.¹³ The observed ratio of 2.27(2) at 4.2 K agrees reasonably with the above calculated value of 2.3. Putting it in another way, the values of V_{zz}^{lattice} derived for ¹¹⁷In and ¹¹¹Cd by dividing the observed EFG's by the corresponding values of $1 - \gamma_{\infty}$ excellently coincide with each other, i.e., $3.4(1) \times 10^{20}$ V/m² and $3.4(5) \times 10^{20}$ V/m², respectively. We then consider that when ¹¹⁷Cd²⁺ decays to ¹¹⁷In³⁺, no observable structural relaxation takes place, In and Cd being in the same lattice environment, and that both In and Cd are predominantly ionic, i.e., they have the identical electronic configuration.



FIG. 3. Temperature dependence of the electric quadrupole frequency ω_Q of ¹¹⁷In in Li_{0.995}Cd_{0.005}NbO₃ (solid circles) together with those of ¹¹¹Cd (\leftarrow ¹¹¹In) (Ref. 9) (open circles) and of ⁷Li (Ref. 15) (solid line) in LiNbO₃. The dashed line represents the temperature dependence of V_{zz}^{lattice} at the Li site in LiNbO₃ calculated by the point-charge model with no anisotropic vibration effect (Refs. 9 and 15). The ω_Q of ⁷Li is scaled so as to be equal to our ¹¹⁷In data at 295 K.

In Ref. 9, room-temperature V_{zz}^{lattice} values are compared among three TDPAC probes, ⁴⁴Sc (\leftarrow ⁴⁴Ti), ¹¹¹Cd $(\leftarrow^{111}In)$, and ^{181}Ta $(\leftarrow^{181}Hf)$ (Ref. 14) all occupying the Li site of LiNbQ₃. The $|V_{zz}^{\text{lattice}}|$ values are 2.7(3) $\times 10^{20} \text{ V/m}^2$, 3.1(5) $\times 10^{20} \text{ V/m}^2$, and 3.1(2) $\times 10^{20} \text{ V/m}^2$, respectively, for ⁴⁴Sc, ¹¹¹Cd, and ¹⁸¹Ta.⁹ The value for ¹¹⁷In obtained in the present study is $3.4(1) \times 10^{20}$ V/m². The uncertainties in the parentheses include both the uncertainties of the quadrupole frequencies and the electric quadrupole moments. These values of the V_{zz}^{lattice} 's are remarkably similar to each other. However, the $|V_{zz}^{\text{lattice}}|$ value for ⁷Li in LiNbO₃ at room temperature, 0.7×10^{20} V/m², calculated from the ⁷Li-NMR quadrupole coupling constant,¹⁵ is 4–5 times as small as those for the above TDPAC probes. The large difference seen in the V_{zz}^{lattice} value is not ascribable to the ionic size because the ionic radius of Li^+ at the octahedral site is 76 pm, which is among the ionic radii of Sc^{3+} (74.5 pm), Cd^{2+} (95 pm), In^{3+} (80 pm), and Ta^{5+} (64 pm).² In the TDPAC study of electric quadrupole interactions at the Ti site in TiO₂ using the ¹¹¹Cd (\leftarrow ¹¹¹In) and ¹⁸¹Ta (\leftarrow ¹⁸¹Hf) probes, where significant differences were found in the values of V_{zz} and η , and their temperature dependence, directional bonding effects on EFG were proposed to explain the differences.¹⁶ However, in the present case where all the involved atoms are ionic, a reasonable explanation would be that the TDPAC probe ions, all with the electric charges larger than +1, cause a structural relaxation: they deform the surroundings and are substantially displaced from the inherent Li position. This structural relaxation could also be a cause of the nonzero η , described earlier. As seen in the decay of ${}^{117}Cd^{2+}$ to ${}^{117}In^{3+}$, where no further observable structural relaxation takes place, the TDPAC probes with the electric charge +2 and with +3 are considered to be in an essentially identical lattice environment.

In Fig. 3 is plotted the $\omega_Q(T)/\omega_Q(0 \text{ K})$ value of ¹¹⁷In $(\leftarrow^{117}\text{Cd})$ vs temperature *T*, as solid circles. The electric

quadrupole frequency ω_Q of ¹¹⁷In increases by about 15% with increasing temperature in the range from 4.2 to 873 K. In the figure is also shown, as open circles, the $\omega_Q(T)/\omega_Q(0 \text{ K})$ for the weighted average of the two ω_Q 's of ¹¹¹Cd (\leftarrow ¹¹¹In) obtained by Hauer *et al.*⁹ Temperature dependence of ω_Q of ¹¹⁷In agrees with that of ¹¹¹Cd in the range between 4.2 and 295 K. This agreement supports the above stated consideration on the lattice environment and valency of In and Cd. The dashed line in the figure represents the $\omega_Q(T)/\omega_Q(0 \text{ K})$ calculated with the lattice parameters based on the point charge model.^{9,15} This calculation reproduces the temperature dependence of ω_Q for ¹¹⁷In only below room temperature. The ω_Q of ¹¹⁷In above room temperature clearly deviates upward from the dashed line. As discussed below, we consider that there is another cause of increasing ω_Q of ¹¹⁷In.

The ⁷Li quadrupole coupling constant, eQV_{zz}/h , for LiNbO₃ between 297 and 953 K increases linearly as the temperature increases from 297 to 953 K.¹⁵ The solid line in Fig. 3 represents the scaled temperature dependence of the ⁷Li coupling constant. Obviously, the large gradient of the line cannot be explained by the changes of the lattice parameters. Halstead interpreted this large linear increase of the quadrupole coupling constant as being due to the anisotropic vibration of Li. We consider that the upward deviation of ω_Q of ¹¹⁷In from the dashed line in Fig. 3 is also due to the anisotropic vibration of In. There is a large difference between the rates of change for ⁷Li and ¹¹⁷In as seen in Fig. 3. It implies that the effect of the anisotropic vibration is much smaller for In. We can qualitatively understand this difference to be due to the fact that the mass of In is much larger

- ¹K. Nassau and H. J. Levinstein, Appl. Phys. Lett. 7, 69 (1965).
- ²R. D. Shannon, Acta Crystallogr., Sect. A: Cryst. Phys., Diffr., Theor. Gen. Crystallogr. **32**, 751 (1976).
- ³S. C. Abrahams and P. Marsh, Acta Crystallogr., Sect. B: Struct. Sci. 42, 61 (1986).
- ⁴R. C. Alferness, Science **234**, 825 (1986).
- ⁵T. R. Volk and N. M. Rubinina, Ferroelectr. Lett. Sect. **14**, 37 (1992).
- ⁶R. S. Raghavan, P. Raghavan, and J. M. Friedt, Phys. Rev. Lett. **30**, 10 (1973).
- ⁷R. B. Firestone, in *Table of Isotopes*, 8th ed., edited by V. S. Shirley (Wiley, New York, 1996), Vol. I.
- ⁸Y. Ohkubo, Y. Kobayashi, K. Asai, T. Okada, and F. Ambe, Phys. Rev. B **47**, 11 954 (1993); Y. Yanagida, J. Nakamura, K. Asai, N. Yamada, Y. Ohkubo, S. Ambe, T. Okada, F. Ambe, S. Uehara, and Y. Kawase, J. Phys. Soc. Jpn. **64**, 4739 (1995).
- ⁹B. Hauer, R. Vianden, J. G. Marques, N. P. Barradas, J. G. Correia, A. A. Melo, J. C. Soares, F. Agulló-López, and E. Diéguez, Phys. Rev. B **51**, 6208 (1995).

than that of Li and the ionic bond between In^{3+} and O^{2-} is stronger than that between Li⁺ and O^{2-} . In the case of ¹⁸¹Ta (\leftarrow ¹⁸¹Hf) at the Li site of LiNbO₃, the quadrupole frequency increases by only a few percent in the temperature range from 295 to 1100 K,¹¹ different from the present case of ¹¹⁷In. The mass of Ta is larger than that of In and, moreover, the ionic bond between Ta⁵⁺ and O²⁻ is stronger than that between In³⁺ and O²⁻, resulting in essentially no anisotropic vibration effect. Finally, we note that the ω_Q of ¹⁸¹Ta (\leftarrow ¹⁸¹Hf) at the Li site of LiTaO₃ increases by about 11% as temperature increases from 295 K to about T_c (=938 K).¹¹ For a better understanding of the observed temperature dependence, the present study should be extended to measurement of ω_Q of ¹¹⁷In and ¹¹¹Cd in LiNbO₃ and LiTaO₃ at high temperatures.

Well-defined static electric quadrupole interactions were observed at ¹¹⁷In at the Li site in LiNbO₃ employing the TDPAC method. From the comparison of the quadrupole frequency ω_Q of ¹¹⁷In with that of ¹¹¹Cd, both measured at 4.2 K, it is concluded that ¹¹⁷In at the Li site in LiNbO₃ is predominantly ionic in nature. It was observed that the ω_Q of ¹¹⁷In increases about 15% with increasing temperature in the range from 4.2 to 873 K, which is not explained only in terms of the temperature dependence of the lattice constants. The steep increase above room temperature is most probably ascribed to an increase of anisotropic vibration of ¹¹⁷In³⁺ ions.

The authors wish to express their thanks to Y. Iimura of the Institute of Physical and Chemical Research (RIKEN) for the x-ray powder-pattern measurement.

- ¹⁰L. Kovács, L. Rebouta, J. C. Soares, M. F. da Silva, M. Hage-Ali, J. P. Stoquert, P. Siffert, C. Zaldo, Zs. Szaller, and K. Polgár, Mater. Sci. Eng. B 9, 505 (1991).
- ¹¹G. L. Catchen and D. M. Spaar, Phys. Rev. B 44, 12 137 (1991);
 G. L. Catchen, J. M. Adams, and T. M. Rearick, *ibid.* 46, 2743 (1992).
- ¹²R. M. Sternheimer, Phys. Rev. **130**, 1423 (1963).
- ¹³F. D. Feiock and W. R. Johnson, Phys. Rev. 187, 39 (1969).
- ¹⁴We note that Ta normally occupies the Nb site of LiNbO₃ existing as Ta⁵⁺. Since Hf occupies the Li site of LiNbO₃, ¹⁸¹Ta arising from ¹⁸¹Hf is considered to remain at the Li site. See L. Rebouta, P. J. M. Smulders, D. O. Boerma, F. Agulló-López, M. F. da Silva, and J. C. Soares, Phys. Rev. B 48, 3600 (1993); L. Rebouta, J. C. Soares, M. F. da Silva, J. A. Sanz-Garcia, E. Diéguez, and F. Agulló-López, Nucl. Instrum. Methods Phys. Res. B 45, 495 (1990), and Ref. 11.
- ¹⁵T. K. Halstead, J. Chem. Phys. 53, 3427 (1970).
- ¹⁶J. M. Adams and G. L. Catchen, Phys. Rev. B **50**, 1264 (1994).