Electronic transport through one-dimensional magnetic superlattices

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We have investigated the quantum-mechanical transmission of an electron in narrow wires, in the presence of a spatially periodic magnetic field. The calculated conductance displays regular dips due to the formation of minigaps, and rapid oscillations due to electron transmission through the quasi-zero-dimensional states in the cavity regions between magnetic barriers. The periodic nature of the structure leads to a profile of quantization in conductance. The differences between the *electric* superlattice and *magnetic* superlattice are highlighted. [S0163-1829(97)08528-7]

In view of recent developments in microfabrication technology, the behavior of a two-dimensional electron gas (2DEG) under the influence of a nonhomogeneous magnetic field has become a rich subject for theoretical[1,2] and experimental[3–5] investigation. Nonuniform magnetic fields have been realized, for instance, by depositing lithographic patterned superconducting or ferromagnetic films on top of a heterojunction[3,4] or by applying a uniform magnetic field to a nonplanar 2DEG.[6] Weiss-type oscillations were observed in the magnetoresistance, which are closely related to similar oscillations induced by one-dimensional (1D) electrostatic potential modulation. On the theoretical side, the properties of a 2DEG in periodic magnetic fields,[7,8] and electronic transport under the influence of magnetic barriers,[9–12] random magnetic fields,[13,14] and a linearly varying magnetic field[15] have been investigated. It was shown that the ''magnetic Weiss oscillation'' is out of phase with the electric Weiss oscillation.[1] Electron tunneling through magnetic barriers is an inherently two-dimensional (2D) process, and the magnetic barriers possess wave-vector-dependent properties.[12] In a random-magnetic-field system, the localization length is not a monotonically decreasing function of magnetic-field randomness, in contrast to a random-potential system, in which it is.[14]

The purpose of this paper is to point out that the experimental study of a class of semiconductor nanostructures, the 1D *magnetic* superlattices, is now within reach, and to present theoretical predictions of their transport properties. The physics of lateral *magnetic* superlattices in an unbounded 2DEG is a topic of great current interest, and magnetotransport through spatially periodic magnetic fields has recently yielded very interesting results.[1–5] The lower dimensionality and high degree of quantum coherence in the 1D magnetic superlattices are features that will make the study of these systems rewarding. Another interesting point is that the motion of ballistic electrons in a periodic magnetic field is also believed to be closely related to the motion of composite fermions (CF) in a density modulated 2DEG in the fractional quantum Hall regime.[16] In the CF theory,[17,18] the effective magnetic field is a function of the local electron density and should be calculated self-consistently, in contrast to the case that we consider in this paper. Nevertheless, a better understanding of the electronic transport properties of a quantum wire in a periodic magnetic field might bring useful insights into CF theory.

A lateral superlattice defined by periodic *electric* potentials in the plane of a 2DEG has attracted much attention recently.[19–22] It has been shown that in such a lateral superlattice, minigaps with zero density of states, and minibands, may form. These lateral superlattice effects may be studied in the linear-response regime of small applied voltage by varying the Fermi energy E_F or the channel width. The index of a quantized conductance plateau has a one-to-one correspondence to the number of positive-velocity states in the energy band structure for the corresponding infinite modulated channel.[21,23] In this paper we investigate electronic transport in a 1D *magnetic* superlattice. It is found that the conductance shows periodic miniband and gap structures in the low plateaus, while aperiodic patterns caused by strong interchannel scattering appear in higher conductance plateaus, which is similar to the case of an *electric* superlattice. In contrast to the usual *electric* potential case, however, the form of the effective potential for magnetic barriers depends on the wave vector of the incident electron.[12] This unique feature should make the study of transmission through 1D *magnetic* superlattices very rewarding.

Our model is a quantum wire of width W, a finite section of which is modulated along the channel by a 1D sinusoidal magnetic field of period a as depicted in Fig. 1. The spatial magnetic modulation can be written as

$$\mathbf{B}(x)=\begin{cases} 0\hat{\mathbf{z}}, & |x|>L/2 \\ B_m \sin\left[K\left(x+\dfrac{L}{2}\right)\right]\hat{\mathbf{z}}, & |x|\leqslant L/2, \end{cases} \tag{1}$$

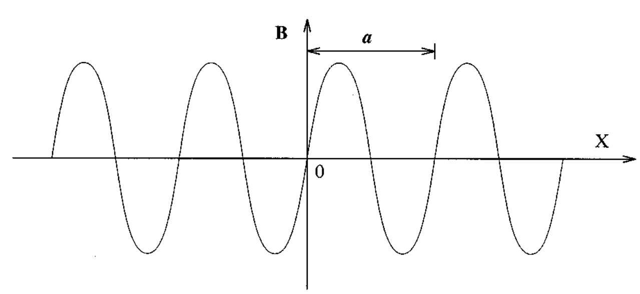

FIG. 1. The 1D sinusoidal magnetic field profile.

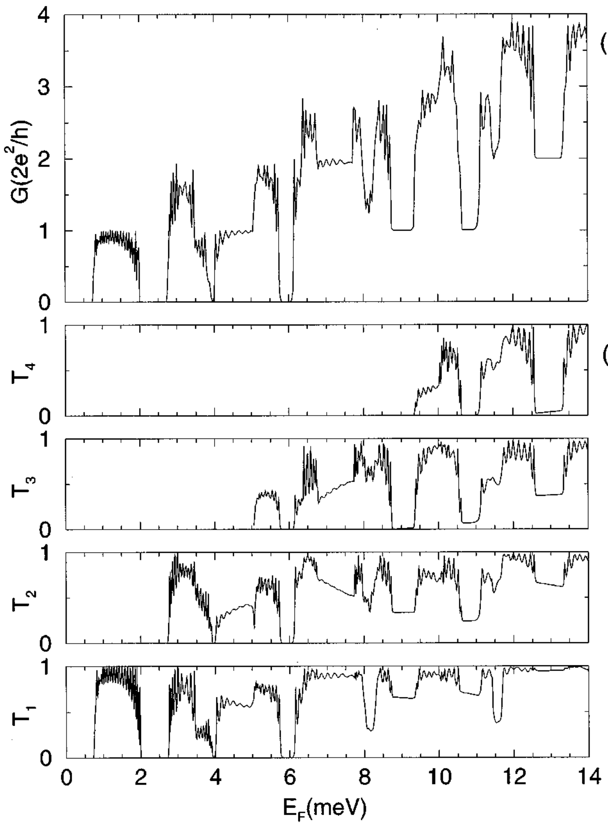

FIG. 2. (a) Conductance as a function of Fermi energy E_F for a finite period magnetic superlattice with 16 unit cells. $a=90$ nm, $W=100$ nm, and $B_m=1.5$ T. (b) Transmission coefficients of individual modes for the magnetic modulated quantum wire.

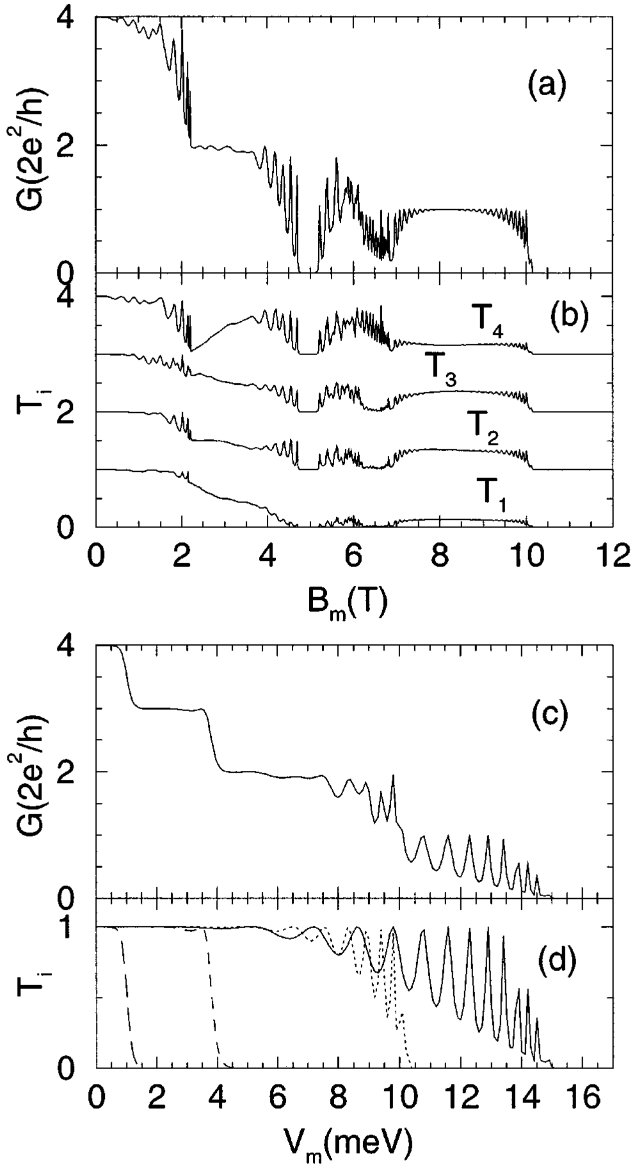

FIG. 3. (a) The conductance as a function of the *magnetic* modulation amplitude B_m for a quantum wire with 16 unit cells. $a=90$ nm, $W=100$ nm, and $E_F=12$ meV. (b) Transmission coefficients T_i of individual modes for the *magnetic* superlattice. The curves are offset for clarity. (c) The conductance as a function of the *electric* modulation amplitude V_m for a finite period *electric* superlattice. The other parameters are the same as (a). (d) Transmission coefficients T_i of individual modes for the *electric* superlattice. The solid, dotted, dashed, and long-dashed curves correspond to $i=1$, 2, 3, and 4, respectively.

where $K=2\pi/a$, $L=Na$, and $N=1,2,\ldots$ is the number of unit cells. The magnetic field is taken to point along the z direction, normal to the plane of the wire. The quantum wire consists of a finite periodic magnetic superlattice and two straight leads; i.e., we do not bother about the details of how electrons are injected into the wire or emitted from it. For simplicity we assume hard walls and zero potential inside the wire. We consider the process where electrons enter through the left lead, scatter inside the magnetic superlattice region, and then reflect back or transmit to the right lead.

We ignore inelastic scattering throughout the device. Spin is accounted for by twofold degeneracy in the Landauer formula[24–26] and is ignored otherwise throughout the calculations. In the effective-mass approximation, the Hamiltonian describing such a system is

$$H=\frac{(\mathbf{P}+e\mathbf{A})^2}{2m^*}+V_c(y), \tag{2}$$

with

$$V_c(y)=\begin{cases}0, & |y|<W/2\\ \infty, & |y|\geqslant W/2,\end{cases} \tag{3}$$

where m^* is the effective mass of an electron, for which we take the value $m^*=0.067m_0$, appropriate for the GaAs layer. The vector potential $\mathbf{A}$ of the magnetic modulation field $\mathbf{B}(x)$ is chosen to be

$$\mathbf{A}=\begin{cases}\left(0,\quad -\dfrac{B_m}{K},\qquad\qquad 0\right), & |x|>L/2\\[2ex] \left(0,\quad -\dfrac{B_m}{K}\cos\left[K\left(x+\dfrac{L}{2}\right)\right],\quad 0\right), & |x|\leqslant L/2.\end{cases} \tag{4}$$

In our calculations a numerical algorithm based on the finite element method was used. Essentially, we discretize the modulated region into a fine mesh on which the Schrödinger equation is solved. In this study, we have discretized

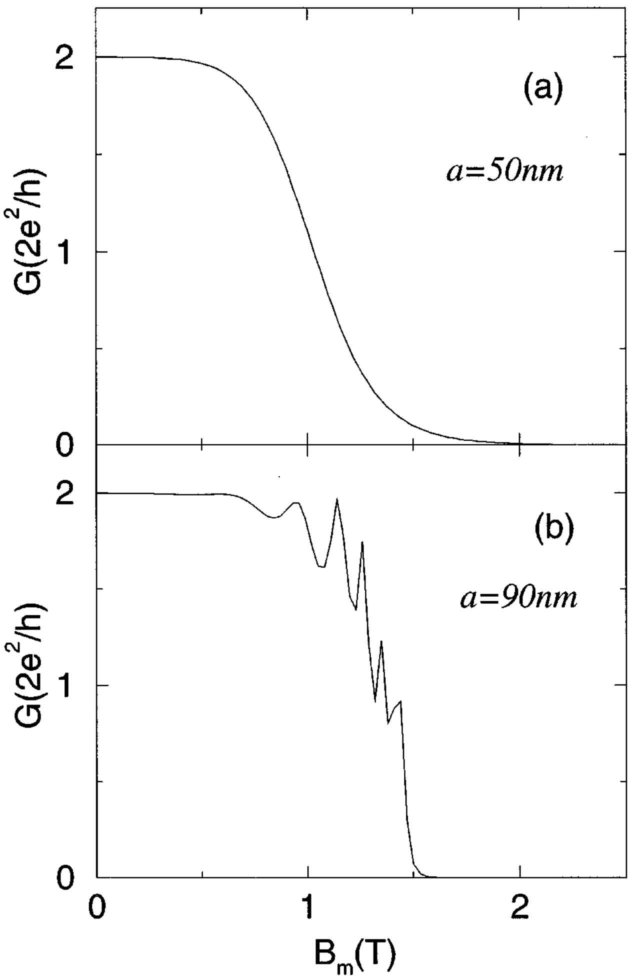

FIG. 4. Conductance as a function of the magnetic modulation amplitude B_m for $W=100$ nm, $N=16$, and $E_F=4$ meV. (a) and (b) correspond to $a=50$ and 90 nm, respectively.

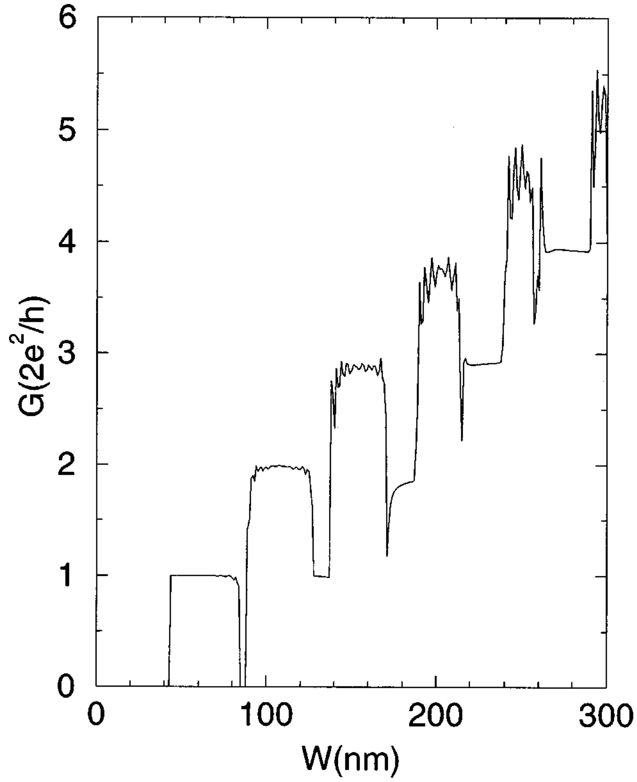

FIG. 5. Conductance as a function of the wire width W for $B_m=0.5$ T, $N=16$, $a=90$ nm, and $E_F=3$ meV.

the modulated region using 16 371 nodal points which gave good convergence for the numerical data in this paper. The wave functions in the leads were calculated separately. The wave functions and their spatial derivatives are then matched at the boundaries between the leads and modulated region, after which the transmission and reflection coefficients may be extracted. The details of this numerical scheme can be found in Refs. 27 and 28.

In Fig. 2(a) we show that the conductance displays regular dips due to the formation of minigaps, and rapid oscillations due to electron transmission through the coupled quasi-zero-dimensional states in the cavity regions between the magnetic barriers. Each group of conductance oscillations evolves into a continuous miniband in the limit of an infinitely long superlattice. The basic features of the formation of minibands and gaps are observable for a 1D magnetic superlattice with even a few periods. Some narrow minibands and gaps, however, reveal themselves only for a rather long modulated wire. The formation of minibands and minigaps in a finite lateral surface superlattice has been reported using split-gate structures, in which the 1D lateral surface superlattice is realized by a periodic modulation of constriction width instead of the magnetic modulation.[29]

In contrast to the electric modulated case,[19–21] the number of oscillations in the first conduction plateau has no simple direct correspondence to the number of unit cells. This result shows that the effective potential of the magnetic barriers for electron motion in the wire is complicated.[7] The oscillations in higher plateaus are more irregular because of the strong coupling between modes. The conductance, plotted in Fig. 2(a), shows a profile of quantization in units of $2e^2/h$ due to the periodic nature of the structure. Unlike the usual quantization of a quantum point contact, however, the conductance does not increase monotonically but rather steps up and down between quantized levels. The conductance quantization is related to the band structure for the corresponding infinite system.[23] However, none of the individual modal transmission coefficients in Fig. 2(b) shows quantization by itself.[30] See, for example, the region near 9 meV. The quantization occurs as the various modes are mixed by the periodic barriers.

Nonmonotonic conductance quantization for varying magnetic modulation amplitude B_m is shown in Fig. 3(a). The conductance steps down by two units of $2e^2/h$ and up by one unit, and then down to zero as B_m is increased. Transmission coefficients T_i of individual modes for the magnetic superlattice are shown in Fig. 3(b); the curves are vertically offset for clarity. Although the total conductance, which is the sum of the modal transmissions, is essentially quantized, the individual transmissions, plotted in Fig. 3(b), have complicated features as a function of B_m.

For comparison, we calculated the conductance for an *electric* superlattice with amplitude V_m and sinusoidal barriers. The parameters of the structure are the same as that in Fig. 3(a). There is no mode mixing in such a constant width 1D *electric* superlattice, in contrast to the magnetic one, in which there is. The conductance for a 1D *electric* superlattice steps down monotonically as the traveling modes in the leads are blocked by the potential barriers with increasing electric modulation amplitude V_m, as shown in Figs. 3(c) and 3(d). No two-unit drop is seen for increasing electric modulation amplitude V_m in Fig. 3(c). Clearly, the unique features of the wave-vector-dependent effective potential for the magnetic barriers are critical for the two-unit drop. In our calculations, the two-unit drop in conductance with increasing magnetic field is predominant but not universal; occasionally we find cases with a one-unit drop. The reason for this is that the effective potential of the magnetic barriers for electron motion along the quantum wire depends on the mode of the incident electron in the leads.[7]

Figures 4(a) and 4(b) show the calculated conductance as a function of B_m for $a=50$ and 90 nm, respectively. For the shorter a, the conductance steps down by two units of $2e^2/h$ smoothly. For the longer a, the conductance exhibits oscillations before the modes are magnetically depleted. The peak values of the conductance oscillations in Fig. 4(b) should be an integer multiple of $2e^2/h$. Some of the peaks fail to reach a quantized value due to the limited number of data points. The peaks are very narrow and therefore require more extensive computing. We find that the oscillations become more prominent as a becomes longer. It is thus suggested that the formation of an effective loop of edge states is essential for the conductance oscillations as discussed by Takagaki[10] and Yoshioka.[31] One is then dealing with an Aharonov-Bohm-type interference.

Finally, we show the results of our calculations for the conductance as a function of the width W of the quantum wire, in Fig. 5. Again we see conductance dips associated with the formation of minigaps, and rapid oscillations reflecting electron transmission through the coupled quasi-zero-dimensional states in the cavity regions between the barriers. It is interesting to note that the rapid oscillations are suppressed for $W<80$ nm in Fig. 5. This can be easily understood, because in this case, the magnetic length $l_B=36.3$ nm at $B_m=0.5$ T is close to half the width of the wire W. Therefore, the effects of the magnetic barriers are not significant, but effects of a periodic magnetic field emerge as the width of the wire increases and becomes much larger than l_B. The oscillations become more pronounced for increasing W as shown in Fig. 5.

In conclusion, we have investigated electronic transport in 1D magnetic superlattices. It is shown that many features of these systems differ from those of the electrostatically modulated ones. We find that unlike the electric superlattice, the number of oscillations in the first conduction plateau has no simple direct correspondence to the number of unit cells N. The conductance of 1D magnetic superlattices does not decrease monotonically with increasing B_m but rather steps up and down between quantized levels, sometimes going to zero. The effects of magnetic modulation on the conductance are pronounced when the width of the quantum wire W increases at a fixed B_m. The unique features of magnetic modulation and low dimensionality make the 1D magnetic superlattices different from the electric ones or the 2D magnetic ones, so we believe they should be of great interest.

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