Phase diagram of a quantum Hall ferromagnet edge, spin-textured edges, and collective excitations

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We study the electric and magnetic properties of the edge of a two-dimensional electron gas in the presence of a magnetic field and at filling factor unity. We obtain the phase diagram of the system as a function of the smoothness of the confining potential and of the Zeeman energy. The existence of a spin-textured edge is proved as a function of these parameters. We also calculate the low-energy excitations of the spin-textured phase. We obtain that in addition to the classical edge magnetoplasmons, at small wave vectors, there is an almost dispersionless excitation, with a finite gap of energy at zero wave vector. This excitation is associated with internal excitations of the spin-textured edge phase. [S0163-1829(97)07140-3]

I. INTRODUCTION

There is great interest in understanding the properties of the edge states of a two-dimensional electron gas (2DEG) in the presence of a strong magnetic field B in the quantum Hall effect (QHE) regime. The edge states are important because they can control the magnetotransport of the 2DEG in a broad class of mesoscopic and macroscopic systems.[1] Also, under normal conditions the only gapless excitations in the QHE regime are edge excitations.[2]

Because of the screening properties of the 2DEG, the structure of the edge states changes when the smoothness of the edge confining potential $V_0(x)$ varies. For studying the edge state properties, we consider a semi-infinite x-y plane, with a straight edge parallel to the $\hat{\mathbf{y}}$ direction. In the case of sharp confinement potentials the electron density falls from the bulk value to zero in a distance of the order of the magnetic length $\ell=\sqrt{\hbar c/eB}$. For a sufficiently smooth confining potential, it has been proposed theoretically that the edge separates into incompressible and compressible regions.[3,4] This picture appears to be in reasonable agreement with recent experiments.[5] For intermediate smooth confining potentials, due to the exchange interactions it has been suggested that stripes of charge density corresponding to filling factor $\nu=1$ can be stabilized at the edge of the 2DEG.[6,7] This edge reconstruction can explain transport experiments in quantum dots[8]. For very smooth confining potentials, the formation of a Wigner crystal of holes at the edge of the 2DEG could happen.[9,10] Also, in the context of the mean-field theory of the composite fermions,[11] it has been proposed that for smooth confining potentials, the density distribution could show features related to the existence of fractional quantun Hall states.[12] In particular, in Ref. 12 it was obtained that as the confining potential becomes smoother, an incompressible region with filling factor 2/3 occurs at the edge. In this calculation there does not appear any signal of the existence of a stripe phase. This result is not in full agreement with exact diagonalization results,[7,13] which seem to indicate the existence of a charge reconstruction at the the edge very similar to the predicted by the Hartree-Fock calculations.[6,7] Probably, the reason for this discrepancy is that the mean-field treatment of the composite fermions does not describe appropiately the exchange interaction between electrons, which is the important physics behind the creation of stripes at the edge of the 2DEG at $\nu=1$.

When the spin degree of freedom is included in the theoretical models, effects such as spontaneous spin polarization[14] and spin textures[15–17] at the edges of quantum Hall systems have been predicted.

The low-energy collective excitations localized at the edge of the system have dispersion relations that depend strongly on the type of reconstruction at the edge. For a sharp edge, a magnetoplasmon with a dispersion of the form $q\ \ln q$ is expected.[18,19] Here $q\|\hat{\mathbf{y}}$ is the wave vector of the excitation. This is a classical result, which is obtained from solving the hydrodynamical equations for a sharp edge in the presence of a magnetic field. This dispersion relation can be also obtained from microscopic calculations.[20] For smooth compressible edges, in addition to the edge magnetoplasmon, the existence of new branches of acoustic excitations has been proposed[21] and measured.[22] The difference between the edge magnetoplasmon and the acoustic excitations originates from their charge distribution pattern; in the magnetoplasmon the charge varies monotonically across the edge, whereas the charge oscillates in the acoustic modes.

In this work we study the electrical and magnetical properties of the edge states of the 2DEG at $\nu=1$ as a function of the Zeeman coupling, $\tilde{g}=g\mu_B B$, and of the smoothness of the confining potential. In particular, we are interested in the existence and properties of a spin textured edge. Edge textures are configurations of the spin density that possess a topological charge density at the edge of the system. The unit spin field $\mathbf{n}(\mathbf{r})$ of the spin texture has the form[15]

$$n_x+in_y=\sqrt{1-f^2(x)}e^{i(G_s y+\theta)},\quad n_z=f(x),\tag{1}$$

where G_s is the wave vector of the spin-texture and θ is an arbitrary phase. In the spin-polarized bulk we have $f(x)=-1$. In the QHE regime the topological charge density coincides with the real charge density,[23,24] and, therefore, the charge density associated with the spin texture n has the expression

$$q(\vec{r}) = e\vec{n}\cdot(\partial_x\vec{n}\times\partial_y\vec{n})/4\pi. \qquad (2)$$

In the spin-textured edge case, Eq. (1), the charge density associated with the texture only depends on x and has the form $q=-e(G_s/4\pi)df/dx$. The system can develop a spin texture in order to modulate the charge density profile in the $\hat{\mathbf{x}}$ direction and, therefore, to screen the edge confinement. The lowering of the confinement energy competes with the cost in exchange and Zeeman energies. Therefore the spin-textured edge occurs only for smooth enough confinement potential and for small enough Zeeman coupling. Note that although the charge generated by the spin texture is invariant along the edge, the existence of the spin texture breaks translational invariance. However, the system is invariant under a combination of a translation along the edge and a spin rotation.[15]

The two main results of this work are the following.

(i) We obtain the phase diagram of the system as a function of the smoothness of the confining potential and of the Zeeman energy. By performing a full unrestricted Hartree-Fock (HF) calculation we obtain the range of parameters where a spin-textured edge can be expected. Given a confining potential, we obtain that the maximum value of $\tilde{g}$ where the spin texture exists is considerably smaller than the obtained in Ref. 15. This discrepancy occurs because Ref. 15 only considers the competition of the spin-textured state with the striped phase. However, we obtain that making V_0 smoother, the sharp edge becomes unstable to a smooth charge modulation in both the **x** and **y** directions before it becomes unstable to the stripe phase.

(ii) We study the dispersion relation of the collective excitations in a spin-textured edge phase. We obtain that in addition to magnetoplasmonlike excitations and the bulk spin-density waves, there exists a low-energy mode associated with internal excitations of this phase. This mode is almost dispersionless, and it has a finite energy at zero q. The existence of this gap is due to the finite width of the charge density produced by the spin texture in the $\hat{\mathbf{x}}$ direction.

This paper is organized as follows, in Sec. II we describe the Hartree-Fock approximation used for solving the microscopic Hamiltonian of the system. Section III is dedicated to the analysis of the phase diagram of the edge. In Sec. IV we compute the low-energy collective excitations of different phases existing at the edge of the 2DEG, in particular, the collective excitations in the spin-textured edge. We conclude with some possible experimental consequences and a brief summary.

II. MICROSCOPIC HAMILTONIAN AND HARTREE FOCK APPROXIMATION

We are interested in properties of the edge states of the $\nu=1$ quantum Hall state. In this regime we assume that the electron-electron interaction and Zeeman energies are much smaller than the Landau-level splitting, and we therefore restrict the orbital Hilbert space to the lowest Landau level. Since the confining potential only depends on the x coordinate, it is convenient to work in the Landau gauge, $\mathbf{A}=-Bx\hat{\mathbf{y}}$. The Hamiltonian of this system has the form (throughout this work we take ℓ as the unit of length and $e^2/\ell\epsilon$ as the unit of energy)

$$H=\sum_{k,\alpha}\left(V_0(k)+\frac{\tilde{g}}{2}\alpha\right)c^{\dagger}_{k,\alpha}c_{k,\alpha} + \frac{1}{2L_xL_y}\sum_{k,k',\vec{p},\alpha,\beta} v(p)e^{-p^2/2} \times e^{ip_x(k-k'+p_y)}c^{\dagger}_{k,\alpha}c^{\dagger}_{k',\beta}c_{k'+p_y,\beta}c_{k-p_y,\alpha}. \qquad (3)$$

Here $\alpha,\beta=+(\text{up})-(\text{down})$ are spin indices, L_x and L_y are the sample dimensions, $v(p)=2\pi e^2/\epsilon p$ is the two-dimensional Fourier transform of the Coulomb interaction, and $c^{\dagger}_{k,\alpha}$ creates an electron with spatial wave function $\psi_k(\mathbf{r})=(1/\sqrt{L_y\pi^{1/2}})\,e^{iky}e^{-(x-k)^2/2}$, and spin $\alpha/2$.

In order to change the smoothness of the edge continuosly we take V_0 as the potential created by a distribution of positive charge, $\rho_+(x)$, which falls linearly from its bulk value, $\rho_{\text{bulk}}=1/2\pi$, to zero, over a region of width W:

$$\rho_+(x)=\frac{\rho_{\text{bulk}}}{W}\begin{cases} W, & x<0\\ W-x, & 0<x<W\\ 0, & x>W.\end{cases} \qquad (4)$$

In this way the edge confining potential is smoother in direct proportion to W,

$$V_0(k)=2\int dx'\rho_+(x')\ln|k-x'|. \qquad (5)$$

In order to solve the Hamiltonian, Eq. (3), we make the Hartree-Fock pairing of the second-quantized operators,

$$H^{\text{HF}}=\sum_{k,\alpha}\left(V_0(k)+\frac{\tilde{g}}{2}\alpha\right)c^{\dagger}_{k,\alpha}c_{k,\alpha}+\sum_{k,k',q_y,\alpha,\beta}\tilde{v}(q_y,k-k') \times[c^{\dagger}_{k,\alpha}c_{k-q_y,\alpha}\langle c^{\dagger}_{k',\beta}c_{k'+q_y,\beta}\rangle - c^{\dagger}_{k,\alpha}c_{k'+q_y,\beta}\langle c^{\dagger}_{k',\beta}c_{k-q_y,\alpha}\rangle], \qquad (6)$$

with

$$\tilde{v}(q_y,k-k')=\frac{1}{L_xL_y}\sum_{q_x}v(\vec{q})e^{iq_x(k-q_y)}e^{-q^2/2}. \qquad (7)$$

The HF Hamiltonian is solved self-consistently allowing for the possibility of different broken translational symmetries in the ground state.[25,26] In particular, in this work we allow the system to modulate the charge and spin in both directions $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$. Because at $\nu=1$ the ground state of an infinite 2DEG is a Slater determinat, the HF approximation is a good approximation for describing the properties of the system at filling factors near unity. Therefore we believe the HF results can be trusted for intermediate confinement potential, $W<12$, and it should be necessary to be carefull in the interpretation of the HF results when the confinement potential is smooth enough such that correlated fractional quantum Hall states can occur.

To characterize the different solutions, it is very convenient to introduce the operators

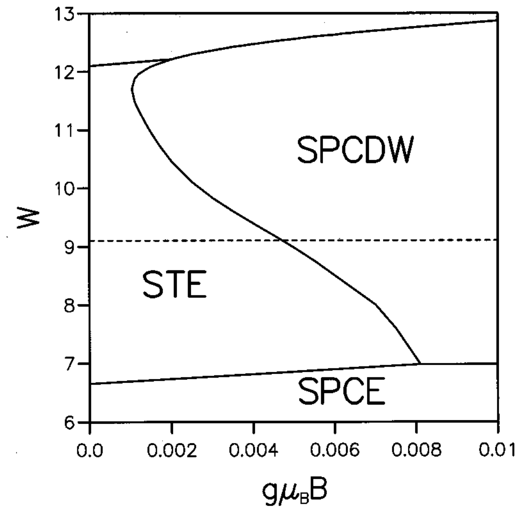

FIG. 1. Phase diagram, as a function of $\tilde{g}=g\mu_B B$ and W, of the edge of a 2DEG at $\nu=1$. The shadow region corresponds to the spin-textured and charge-density-wave phase. $\tilde{g}$ is in units of $e^2/\epsilon\ell$ and W in units of ℓ. The dashed line represents the value of W where the stripe phase has lower energy than the SPCE phase.

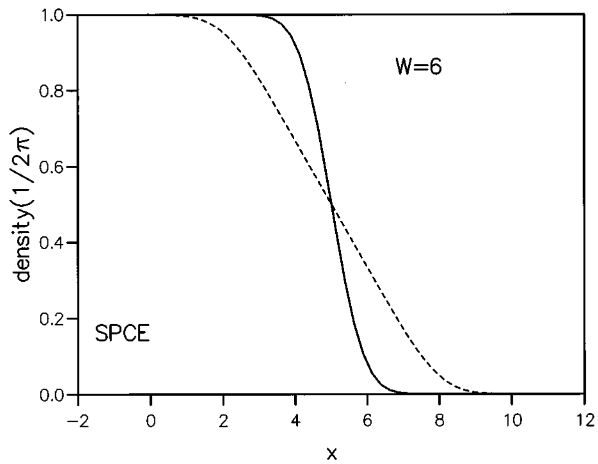

FIG. 2. Charge-density profile of a spin-polarized edge state as a function of x. The dashed line represents the background of positive charge created by the confining potential. In this figure $W=6$.

$$\rho_{\alpha,\beta}(\mathbf{q})=\frac{2\pi}{L_xL_y}\sum_k e^{-iq_x(k-q_y/2)}c^\dagger_{k,\alpha}c_{k+q_y,\beta} \tag{8}$$

which are related to the charge $n(\mathbf{q})$ and spin $\mathbf{S}(\mathbf{q})$ density operators through the relations

$$n(\vec{r})=\frac{1}{2\pi\ell^2}\sum_{\vec{q}\alpha}\langle\rho_{\alpha,\alpha}(\vec{q})\rangle e^{-q^2/4}e^{-i\vec{q}\vec{r}}, \tag{9}$$

and

$$S_z(\vec{r})=\frac{1}{4\pi\ell^2}\sum_{\vec{q}}[\langle\rho_{+,+}(\vec{q})\rangle-\langle\rho_{-,-}(\vec{q})\rangle]e^{-q^2/4}e^{-i\vec{q}\vec{r}}.$$

$$S_x(\vec{r})+iS_y(\vec{r})=\frac{1}{2\pi\ell^2}\sum_{\vec{q}}\langle\rho_{-,+}(\vec{q})\rangle e^{-q^2/4}e^{-i\vec{q}\vec{r}}. \tag{10}$$

By solving selfconsistently the Hartree-Fock equations we obtain the espectation values of the energy and of the different density operators.

III. PHASE DIAGRAM

The different solutions of the electric and magnetic edge structure can be characterized by the expectation values of the products $c^\dagger_{k,\alpha}c_{k',\beta}$ or by the expectation values of the operators $\rho_{\alpha,\beta}(\mathbf{q})$. In this work we find the following type of solutions (see Fig. 1).

A. Spin-polarized compact edge (SPCE)

In this state $\langle c^\dagger_{k,\alpha}c_{k',\beta}\rangle=\delta_{k,k'}\delta_{\alpha,\beta}\delta_{\alpha,-}$, and there is a maximum wave vector such that all states with smaller momentum are occupied. This maximum wave vector can be considered as the Fermi wave vector of the edge, k_F. In this solution the charge density falls from its bulk value $1/2\pi$ to zero in a distance of the order or the magnetic length, see Fig. 2, and the charge density is invariant along the edge. The spin-polarized compact edge is the sharpest edge possible. In this state the exchange energy gets its maximum possible value, and, therefore, it is the ground state for small values of W,[7] where the electrons cannot screen the confining potential.

B. Spin-polarized charge-density wave (SPCDW)

In this state only the majority-spin electronic states are occupied, i.e., $\langle\rho_{\alpha,\beta}(\mathbf{q})\rangle\propto\delta_{\alpha,\beta}\delta_{\alpha,-}$. In this class of solutions, the system modulates the charge along the $\hat{\mathbf{y}}$ direction, in order to screen the edge potential. In the spin-polarized QHE regime the system only can modulate smoothly the charge along the $\hat{\mathbf{x}}$ direction by modulating also the charge along the $\hat{\mathbf{y}}$ direction. At $\tilde{g}\rightarrow\infty$, the SPCDW state has lower energy than the SPCE state for $W>W_{\rm CDW}\simeq 7$. Since the phase transition between the SPCE and the SPCDW state is announced by the softening of the low-energy charge-density excitation of the SPCE (see below) this transition is a second-order phase transition.

For values of W near but bigger than $W_{\rm CDW}$, the SPCDW modulates very smoothly the charge-density profile across the edge, and forms a charge-density wave. This charge-density wave can be interpreted as a precursor of the Wigner crystal of holes.[9]

For stronger confining potentials, $W>8.5$, the density profile across the edge develops a modulation at the same time that the system modulates the charge density along the edge direction. We interpret this solution as the existence of an incipient Wigner crystal of holes on top of a stripe phase. In Figs. 3 and 4 we plot the charge density for a SPCDW edg coreesponding to $W=11$.

For larger values of W more exotic reconstructions can happen, however, we think that for very smooth confining potentials the HF approximation could be not appropiate and fractional quantum Hall states could happen.[11,12]

C. Stripe phase

In this state only the majority-spin electronic states are occupied, and the expectation values of the operator

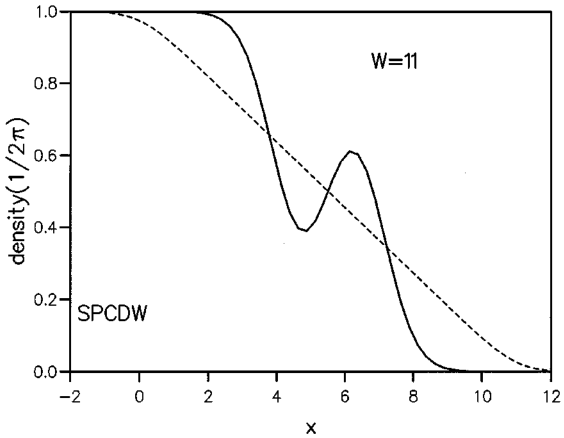

FIG. 3. Charge-density profile of a spin-polarized charge-density-wave edge state as a function of x. The charge density is averaged along the y direction. The dashed line represents the background of positive charge created by the confining potential. In this figure $W=11$.

$\langle c^{\dagger}_{k,-} c_{k',-}\rangle$ can be only 0 or 1. The occupation gets the value unity for wave vectors k satisfying the relation $k<k_1$, or the relation $k_2<k<k_3$, with $k_1<k_2<k_3$. In this solution we do not allow modulation of the charge-density along the edge direction. A charge-density profile corresponding to this solution is shown in Fig. 5 for the case of $W=11$. The stripe phase has lower energy than the SPCE for values of W larger than $W_c=9.1$, however, this phase does not appear in the phase diagram (Fig. 1) because we obtain that the stripe phase has always higher energy than the SPCDW. In fact, we obtain the the stripe phase is not a stable solution of the HF equation (see below).

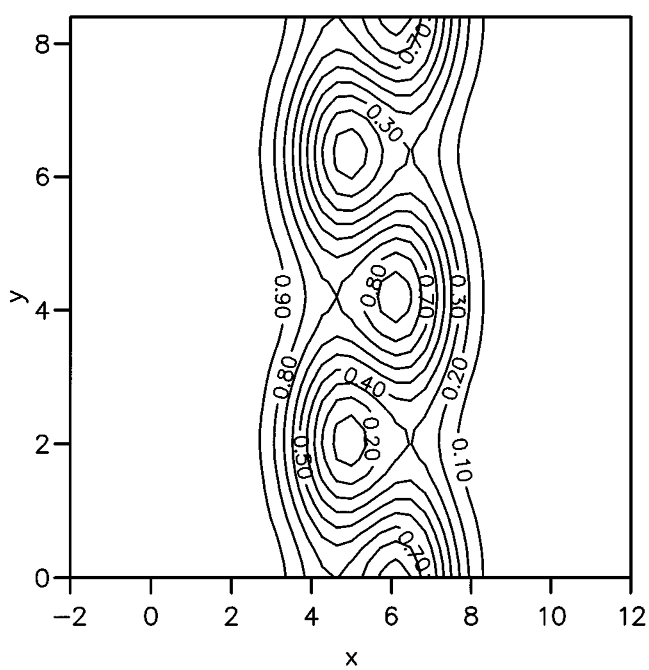

FIG. 4. Two-dimensional charge-density contour of a spin-polarized charge-density-wave edge state corresponding to $W=10$. This figure represents twice the unit cell in the y direction. The numbers are the value of the electron density in units of $1/2\pi$.

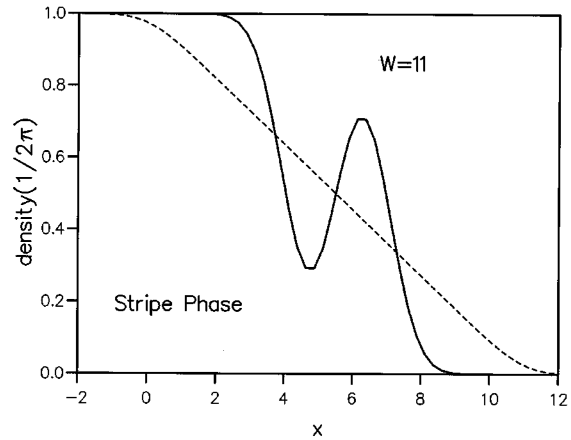

FIG. 5. Charge-density profile of a stripe phase state as a function of x. The dashed line represents the background of positive charge created by the confining potential. In this figure $W=11$.

D. Spin-textured edge (STE)

In this class of solutions the charge density and the z component of the spin density are not modulated along the $\hat{\mathbf{y}}$ direction, i.e., $\langle \rho_{\alpha,\alpha}(\mathbf{q})\rangle \propto \delta_{q_y,0}$. However, we find that the x and y components of the spin density varies along and across the edge, i.e., all the $\langle \rho_{\alpha,-\alpha}(\mathbf{q})\rangle$ can be different from zero. In the calculation we obtain that the operators $\rho_{\alpha,-\alpha}(\mathbf{q})$ are different from zero only for one wave vector of the form $\mathbf{q}=(0,G_s)$. Minimizing the energy with respect to G_s we obtain microscopically the periodicity of the spin texture. In this solution there is not higher harmonics of the spin texture because, in order to get a constant charge density along the edge, *only* one wave vector of the spin texture is possible. Using the relation between spin texture and charge density, Eq. (2), it can be proved easily that spin textures with a dependence on the coordenate y different than a *sine* implies charge modulation in the $\hat{\mathbf{y}}$ direction. In agreement with Ref. 15 we obtain that G_s increases linearly with W and with the Zeeman coupling $\tilde{g}$.

In Fig. 6 is plotted the charge-density profile across the

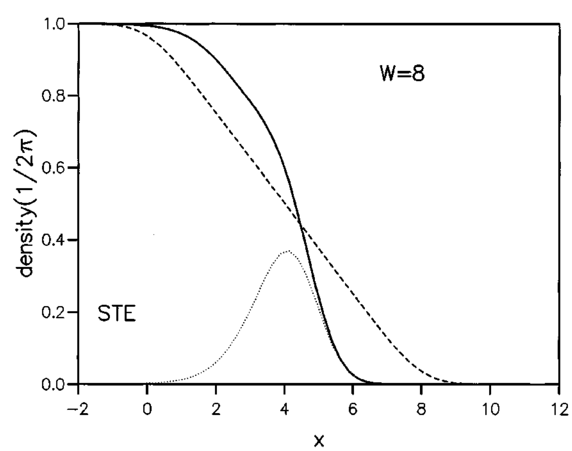

FIG. 6. Charge-density profile of a spin-textured phase state as a function of x. The dashed line represents the background of positive charge created by the confining potential. In this figure $W=8$ and $\tilde{g}=0$. The dotted line corresponds to the contribution of minority-spin electrons to the density profile.

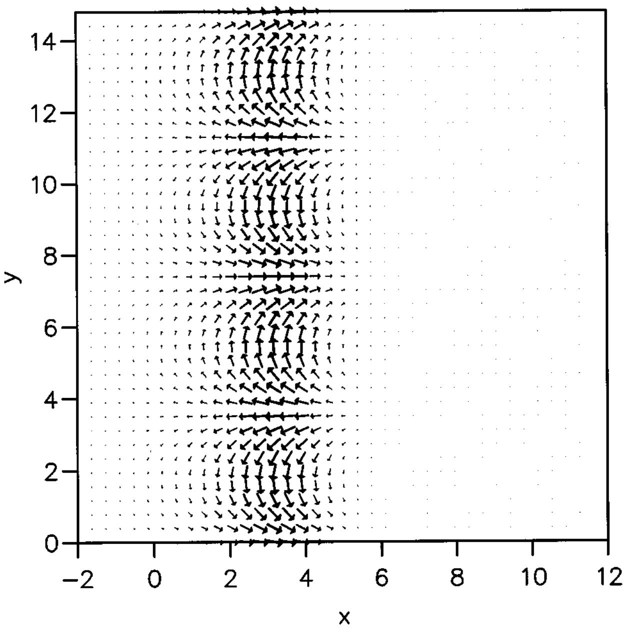

FIG. 7. Two-dimensional vector representation of the $x-y$ components of the spin density for a spin-textured edge. In this figure $W=8$ and $\tilde{g}=0$. This figure represents twice the unit cell in the y direction.

edge of a STE with $\tilde{g}=0$ and $W=8$. We also plot in Fig. 6 contribution of the minority-spin electrons to the charge density. For the same STE, we plot in Fig. 7 the projection of the spin density in the x-y plane. Since $\langle \rho_{\alpha,-\alpha}(0,G_s)\rangle$ is different from zero, the STE phase breaks the translational invariance along the edge and the spin rotational symmetry about the magnetic field. However, the STE is invariant under a symmetry composed of a translation along the edge and a spin rotation.[15] The states related with this symmetry correspond to the different values of θ in Eq. (1).

Since the phase transition between the SPCE and the STE state is announced by the softening of the low-energy spin-density excitation of the SPCE, see below, this transition is also a second-order phase transition. For $\tilde{g}=0$, the SPCE to STE transition occurs for confining potentials with $W>W_s=6.7$. This value of W_s increases with $\tilde{g}$, and for $\tilde{g}>\tilde{g}_c\simeq 0.008$ the system prefers to screen the edge potential by forming a SPCDW state rather than by creating a STE. This value of $\tilde{g}_c$ is about ten times smaller than the obtained by Karlhede *et al.*[15] This discrepancy occurs because in Ref. 15 the STE is assumed to compete only with the striped phase, and not with the SPCDW state.

It is important to note that the width of the charge and spin modulation in the $\hat{\mathbf{x}}$ direction is much larger than G_s. For example, in the case of $\tilde{g}=0$ and $W=8$, Fig. 6, the wave vector of the spin texture is, $G_s\approx 0.85$ and the width of the modulation is of the order of four magnetic lengths.

A final point to mention in this subsection is that since in the STE phase the only order parameters different from zero are $\langle \rho_{\alpha,-\alpha}(0,G_s)\rangle$ and $\langle \rho_{\alpha,\alpha}(q_x,0)\rangle$, it is possible to describe the STE ground state by a simple Slater determinant of the form

$$|\mathrm{STE}\rangle=\prod_k d_k^\dagger|0\rangle, \quad (11)$$

where

$$d_k^\dagger=u_k c_{k,-}^\dagger+v_k c_{k+G_s,+}^\dagger, \quad (12)$$

and u_k and v_k are obtained from the expectation values of the $\rho_{\alpha,\beta}$ operators, and verify the relation $u_k^2+v_k^2=1$. The set of eigenstates of the Hartree-Fock Hamiltonian orthogonal to the $d_k^\dagger$ states have the form

$$b_k^\dagger=-v_k c_{k,-}^\dagger+u_k c_{k+G_s,+}^\dagger. \quad (13)$$

Solutions of the form described by Eqs. (11) and (12) break translation invariance along the edge, as well as spin rotational symmetry. In the presence of Zeeman coupling the Hamiltonian, Eq. (3), has a spin rotational O(2) symmetry in the x-y plane. The STE phase breaks this symmetry. When the Zeeman coupling is zero, the STE phase spontaneously breaks all the rotation symmetries.

We have not found coexistence between the STE and SPCDW state. From our calculations, the line in Fig. 1 separating the STE from the SPCDW represents a first-order phase transition. This is consistent with the fact that these two states have different broken symmetries.

E. Spin-textured and charge-density-wave state

This is a fully broken symmetry ground state where the expectation values of all $\langle \rho_{\alpha,\beta}(\mathbf{q})\rangle$ are not zero. This state is a mixture of charge-density waves and spin textures and it is reached from both the STE and SPCDW states by making the edge confinement smoother. This phase corresponds to the shadow region in Fig. 1.

In particular, we want to mention here the state that appears when, starting from a STE phase, we make W larger. We find that for very smooth confining potentials higher harmonics of the spin texture appear. In that case the edge is described better by an anti-Skyrmion crystal than by a simple spin texture.[27] However for the values of W where this solution occurs, we expect the correlation effects becoming important, and probably for these large values of W the HF approximation could be not appropiate for describing this system.

IV. COLLECTIVE EXCITATIONS

In this section we study the low-energy collective excitations occuring at the edge of the 2DEG at filling factor unity. We first analyze the collective excitations of the SPCE state, showing that the phase transitions to the SPCDW edge or the STE are second order. We also discuss the instability of the stripe phase against modulation of the charge density along the edge direction.

In the second part of this section we study the collective excitations of the spin-textured edge, and, in particular, the collective mode associated with internal excitations of the STE.

A. Collective excitation of the SPCE

This state has two types of collective excitations: charge-density excitations (CDE's) and spin-density excitations (SDE's). The CDE's only involve changes in the charge den-

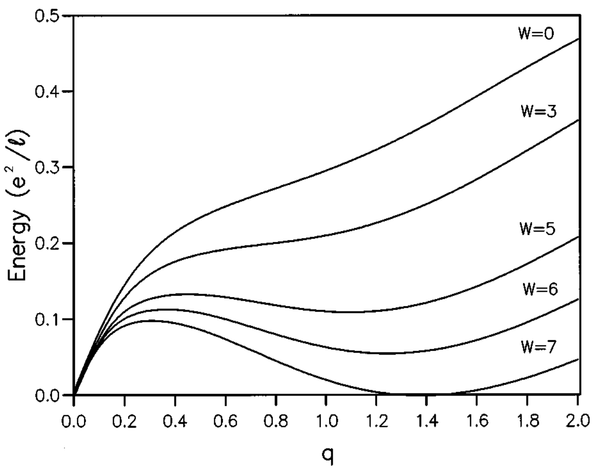

FIG. 8. Energy as a function of the momentum q of the lowest-energy charge-density excitations of a SPCE, for different values of W.

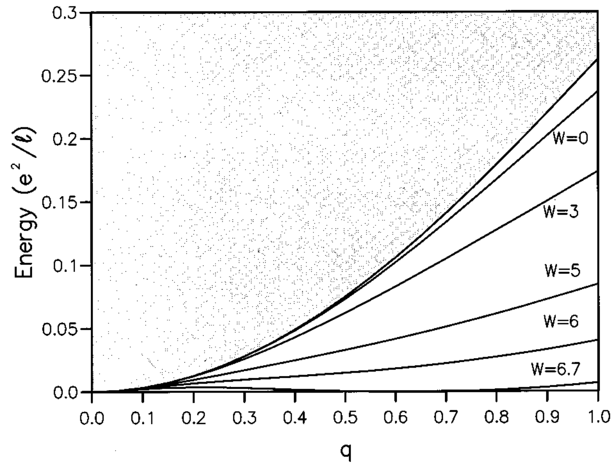

FIG. 9. Energy as a function of the momentum q of the low-energy spin-density excitations of a SPCE, for different values of W.

sity whereas the SDE's also involves changes in the spin density. Due to the symmetry of the SPCE ground state the collective excitations can be labeled by a quantum number q, which corresponds to the momentum in the $\hat{\mathbf{y}}$ direction of the excitations.

1. Charge-density excitations

The elementary CDE's of this spin-polarized state are constructed by allowing all conserving spin particle-hole excitations of momentum q to couple, i.e., the CDE's of the SPCE can be written in the form

$$|\psi_q^i\rangle=\sum_k a_k^i(q)c_{k+q,-}^\dagger c_{k,-}|\mathrm{SPCE}\rangle. \qquad (14)$$

In this equation $|\mathrm{SPCE}\rangle$ repesents the SPCE ground-state wave function, which is a Slater determinant formed with all the majority spin one electron wave functions of momentum smaller than k_F. Therefore, since the spin is conserved, the sum over momenta k is restricted to the region $k_F-q<k<k_F$. The coefficients $a_k^i(q)$ are obatined by minimizing the energy of the excitations, $\hbar\omega^i(q)=\langle\psi_q^i|H|\psi_q^i\rangle$, with the condition $\Sigma_k|a_k^i(q)|^2=1$. This procedure reduces finally to the diagonalization of the matrix

$$\langle\mathrm{SPCE}|c_{k',-}^\dagger c_{k'+q,-}Hc_{k+q,-}^\dagger c_{k,-}|\mathrm{SPCE}\rangle, \qquad (15)$$

the matrix elements of which are computed using the Wick theorem. This method of calculation is equivalent to the time-dependent Hartree-Fock approximation.[20]

In Fig. 8 we plot the lowest-energy CDE of a SPCE for different values of the confining potential. In agreement with the semiclassical results,[18] all the curves disperses as $q\ \ln q$ at small wave vectors. As the W becomes larger, i.e., the confinement potential smoother, the lowest-energy CDE develops a minimum in the dispersion at wave vectors around $q\sim1.4$. This minimum becomes a soft mode at $W\sim7$, indicating the existence of an instability in the system. This value of W coincides with W_{CDE} and, therefore, we conclude that the transition SPCE to SPCDW edge is a second-order phase transition driven by this instability.

2. Spin-density excitations

The elementary SDE's of this spin-polarized state are constructed by allowing all spin-flip particle-hole excitations of momentum q to couple. The SDE's of the SPCE can be written in the form

$$|\psi_q^i\rangle=\sum_k a_k^i(q)c_{k+q,+}^\dagger c_{k,-}|\mathrm{SPCE}\rangle. \qquad (16)$$

Since the electron-hole pairs do not conserve spin, and the ground state is spin polarized, there is not restriction on the values of k in Eq. (16). The coefficients $a_k^i(q)$ and the energy of the excitations are obtained in the same way as in the CDE's case, which for the SDE's implies diagonalization of the matrix

$$\langle\mathrm{SPCE}|c_{k',-}^\dagger c_{k'+q,+}Hc_{k+q,+}^\dagger c_{k,-}|\mathrm{SPCE}\rangle. \qquad (17)$$

In Fig. 9 we plot the low-energy SDE's of a SPCE for different values of the confining potential. For all values of W, the SDE expectrum contains a continuum of excitations starting from an energy $\tilde{g}+4\pi\rho_s q^2$, where ρ_s is the spin stiffness of the 2DEG at $\nu=1$. These excitations extend over all the system and they are the well-known bulk spin-density waves.[28] In the SPCE phase the total spin and the third component of the spin are good quantum numbers. The ground state has the maximum value of the total spin and in absence of Zeeman coupling the ferromagnetic state is degenerated in all the possible values of the z component of the total spin. The spin-density wave is the Goldstone mode associated with the broken symmetry occuring when the z direction is privileged and the z component of the spin is taken as the order parameter of the ferromagnetic phase.

In addition to the spin-density waves, the SDE expectrum also contains a W-dependent branch, with lower energy than the bulk spin waves and which is spatially localized at the edge of the 2DEG. As the confinement becomes smoother, this edge localized SDE develops a minimum in energy a finite wave vector. This mode becomes soft for $W>6.7$, indicating the existence of a spin instability in the system. This value of W coincides with W_S and, therefore, we conclude

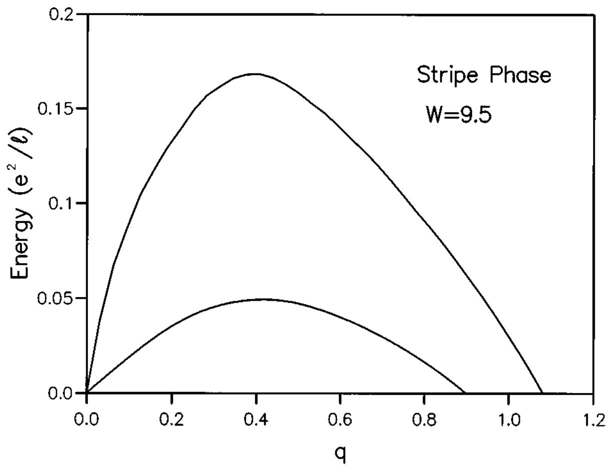

FIG. 10. Energy as a function of the momentum q of the two lowest-energy charge-density excitations of a stripe edge. In this figure $W=9.5$.

that the transition SPCE to STE is a second-order phase transition driven by this spin wave instability.

B. Charge-density excitations of the stripe phase

As in the SPCE the collective CDE's are constructed by the linear combination of electron-hole pairs of the form

$$|\psi_q^i\rangle=\sum_k a_k^i(q)c_{k+q,-}^{\dagger}c_{k,-}|\mathrm{SP}\rangle, \tag{18}$$

where $|\mathrm{SP}\rangle$ represents the stripe phase wave function. The stripe phase is a spin-polarized state characterized by three Fermi wave vectors: k_1, k_2, and k_3 and for small q the sum over momenta k is restricted to the regions $k_1-q<k<k_1$ and $k_2-q<k<k_2$. The existence of these two regions made possible that the charge distribution of the CDE oscillates across the edge, making possible the appearance of an acoustic mode.

In Fig. 10 we show the two lowest-energy excitations of a stripe phase corresponding to $W=9.5$. The lowest-energy one varies linearly with q and it is the acoustic mode. All the other modes disperses as classical magnetoplasmons $q\ \ln q$. It is important to note in Fig. 10 that the lower-energy modes become negative at finite q, announcing the instability of the stripe phase. This result is independent of the value of W, and confirms the results show in the phase diagram that the stripe phase is not a stable solution of the system.

C. Collective excitations of spin-textured edge

In the STE phase the low-energy collective excitation can be written as a linear combination of the electron-hole pairs of the form. $d_{k+q}^{\dagger}d_k$ and $b_{k+q}^{\dagger}d_k$. From the form of the d and b operators, Eqs. (12) and (13), the collective excitations have the general expression

$$|\psi_q^i\rangle=\sum_{k,\alpha,\alpha'} a_{k,\alpha,\alpha'}^i(q)c_{k+g_\alpha+q,\alpha}^{\dagger}c_{k+g_{\alpha'},\alpha'}|\mathrm{STE}\rangle, \tag{19}$$

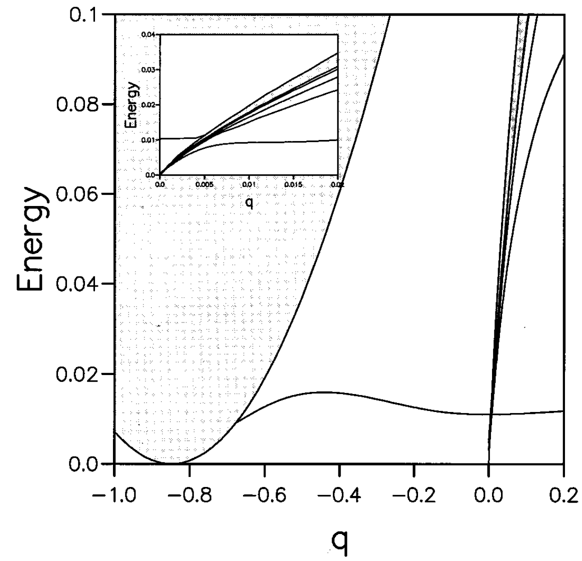

FIG. 11. Low-energy collective excitations of the spin-textured edge phase. The results corrrespond to $\tilde{g}=g\mu_B B=0$ and $W=8$. The energy is in units of $e^2/\epsilon\ell$ and the wave vector q in units of ℓ^{-1}.

where $g_-=0$ and $g_+=G_s$. The coefficients $a_{k,\alpha,\alpha'}^i$ and the energy of the excitations are obtained in the same way as in the previous cases, which in the STE case implies the diagonalization of the matrix

$$\langle\mathrm{STE}|c_{k'+g_{\beta'},\beta'}^{\dagger}c_{k'+g_\beta+q,\beta}Hc_{k+g_\alpha+q,\alpha}^{\dagger}c_{k+g_{\alpha'},\alpha'}|\mathrm{STE}\rangle. \tag{20}$$

Due to the existence of the spin texture, the collective excitations are a mixture of spin- and charge-density excitations. Also note that because the STE is invariant under a translation along the edge plus a spin rotation,[15] any electron spin flip is accompanied by a change of the electron wave vector in $\pm G_s$.

In Fig. 11 we plot the lowest-energy collective excitations for the case $\tilde{g}=0$ and $W=8$. The expectrum consists basically of two parts: (i) a continuum of excitations starting from an energy $\tilde{g}+4\pi\rho_s(q-G_s)^2$, and (ii) a set of discrete branches around $q=0$. The former excitations extend over all the system and far from the edge they evolve into the bulk spin-density waves. The dispersion relation of the spin-density waves starts at $q=-G_s$, because in the STE phase the electron-hole pairs involving a majority spin flip have the form [see Eq. (19)] $c_{k+G_s+q,+}^{\dagger}c_{k,-}$. In the absence of Zeeman coupling these excitations become gapless. This branch of excitations corresponds to the Goldstone mode associated with one of the broken symmetries occuring in the STE phase: spin rotation around an axis of the x-y plane. On the other hand, the low-energy excitations starting at $q=0$ are localized at the edge of the system and they correspond to edge excitations of the STE phase.

We describe now the character of the edge excitations of the STE phase. At small wave vectors, all but one of the low-energy excitations are gapless at $q=0$, and have a dispersion of the form $q\ \ln q$. The analysis of the coefficcients

$a^i_{k,\alpha,\alpha'}$ of these gapless excitations reveals that they are localized at the edge of the system in a region of thickness of the order of the magnetic length. These excitations correspond to the classical edge magnetoplasmons[18,20] of the system. The difference with the edge magnetoplasmon of the spin-polarized compact edge is that in the STE the spin and charge excitations are mixed.

As mentioned above, in addition to the magnetoplasmon there is a low-energy excitation that is practically dispersionless at small wave vectors and that has a finite gap at $q=0$. This excitation anticrosses with the edge magnetoplasmon, see inset of Fig. 11. It is localized at the edge of the system but with a thickness equal to the spatial width of the charge modulation in the $\hat{\mathbf{x}}$ direction. In the case of $W=8$ and $\tilde{g}=0$, this thickness is around four magnetic lengths, see Fig. 6. This thickness is rather independent on the wave vector of the excitation, and therefore this mode is almost dispersionless in q. The coefficcients $a^i_{k,\alpha,\alpha'}$ corresponding to this mode show that this excitation is one in which the transverse component of the spin polarization becomes time dependent. Also, there is a motion of the charge density associated with the spin texture across the edge.

This mode corresponds to an internal excitation of the STE phase. In a classical calculation, at $q=0$, this mode should be gapless and it should be the Goldstone mode associated with spin rotations in the x-y plane. However, a finite gap appears in the Hartree-Fock calculation. We think the reason is the following: this excitation changes the x and y components of the spin at the same time that it changes the momentum along the y direction. The change in the momentum in the y direction modifies the charge-density modulation, which exits in the x direction because of the spin texture. This modulation of the charge density is confined at the edge of the system. The external, exchange, and Hartree potentials confine the charge modulation against motion towards the outer part of the edge, whereas the exchange and Hartree potentials confine the charge against motion towards the inner part of the 2DEG. This excitation of the STE rotates the projection of the spin-density in the x-y plane and also moves the charge-density modulation across the edge. Since the motion of the charge density across the edge is confined, quantum mechanically an energy gap does appear in this mode even at $q=0$. We think that the rotational mode of the spin texture is coupled with a breathing mode of the charge across the edge.[26]

Because of the external potential at positive large values of q, the energy of this excitation increases. For negative values of q we find that this excitation is damped into the spin density wave region at wave vectors of the order of $-G_s$.

V. EXPERIMENTAL CONSEQUENCES

From the phase diagram, Fig. 1, the maximum value of the Zeeman energy for the existence of the STE phase is $\tilde{g}=0.008$ This is a rather small value but it can be reached in GaAs systems by applying hydrostatic pressure.[29] In order to get a STE phase a smooth confining potential is also necessary. It is possible to tune the edge potential to the appropriate value by applying gate bias to the edge of the 2DEG.[30]

The existence of the STE affects the spin polarization of the 2DEG edge at $\nu=1$. By using a local NMR probe[31] the polarization of an edge as a function of the Zeeman coupling or of the strength of the confinement potential can be studied. As in the case of Skyrmions[24] a variation of the spin polarization with these parameters should probe the existence of the STE phase.[32]

As discussed above, in the STE phase there exists a low-energy excitation with a finite gap at $q=0$. The detection of this mode propagating along the edge of a 2DEG should be a probe of the existence of the STE phase. It could be possible to detect the existence of this mode by time-resolved magnetotransport experiments[5] or by measuring the transmission of electromagnetic waves.[19]

VI. SUMMARY

In summary, we have studied the electronic and magnetic structure of the edge of a 2DEG in the $\nu=1$ QHE regime. We have obtained the phase diagram of the system as a function of the Zeeman coupling and the smoothness of the confinement potential. We obtain the range of parameters where a spin-textured edge phase is expected. We have also studied the collective excitations of this phase. We have found the existence of a low-energy gapfull collective excitation associated with the symmetry of the spin-textured phase.

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