ARTICLES

Electron-phonon interaction in disordered metal films: The resistivity and electron dephasing rate

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The temperature dependence of the resistance of films of Al, Be, and NbC with small values of the electron mean free path $l=1.5-10$ nm has been measured at 4.2–300 K. The resistance of all the films contains a T^2 contribution that is proportional to the residual resistance; this contribution has been attributed to the interference between the elastic electron scattering and the electron-phonon scattering. Fitting the data to the theory of the electron-phonon-impurity interference (M. Yu. Reiser and A. V. Sergeev, Zh. Eksp. Teor. Fiz. **92**, 224 (1987) [Sov. Phys. JETP **65**, 1291 (1987)]), we obtain constants of interaction of the electrons with transverse phonons, and estimate the contribution of this interaction to the electron dephasing rate in thin films of Au, Al, Be, Nb, and NbC. Our estimates are in a good agreement with the experimental data on the inelastic electron-phonon scattering in these films. This indicates that the interaction of electrons with transverse phonons controls the electron-phonon relaxation rate in thin-metal films over a broad temperature range.
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I. INTRODUCTION

The electron-phonon-scattering in impure metals is a long-standing problem. Both experimental and theoretical results, especially those which have been obtained for the last 15 years, indicate that the electron-phonon interaction in impure metals is modified significantly with respect to the ''clean'' case. As a result, the temperature dependences of both the inelastic scattering rate[1–7] and resistivity[8–10] for impure metals differ substantially from those for clean metals.

The electron-phonon interaction in disordered systems is due to two processes: the ''pure'' electron-phonon scattering (the only mechanism of the electron-phonon interaction in pure metals), and the inelastic electron scattering from *vibrating* impurities (defects, boundaries, etc.). The last mechanism, being combined with the *elastic* electron scattering and electron-phonon scattering, generates a variety of interference processes. Contributions of the inelastic electron-impurity scattering and the temperature-dependent component of elastic electron-impurity scattering to the resistivity have been considered in a number of theoretical papers.[11–14] These mechanisms result in a T^2 term in the resistance, whose magnitude is proportional to the residual resistance. However, for the interference phenomenon, it is important to take into account all the channels of electron scattering; neglecting some of them may result not only in a change of the magnitude of the effect, but also in the reversal of its sign.

The detailed analysis of the electron-phonon-impurity interference has been done by Reizer and Sergeev.[15] It has been shown, in contrast to the preceding theoretical works, that the contribution of the temperature-dependent component of the elastic electron-impurity scattering is *canceled out* by some processes of inelastic electron-impurity scattering. Another intriguing conclusion of Ref. 15 is that the interference contribution to the resistance is mainly due to the interaction with *transverse* phonons. Hence, the information on the interaction of electrons with transverse phonons can be obtained in the study of the interference correction to the resistance.

The T^2 dependence of the resistance has been observed for both ultraclean and disordered metals for a number of years (see Refs. 8–10). In the ''clean'' case this contribution is due to the electron-electron scattering in the presence of the umklapp processes; it is usually observed at temperatures $T<1$ K.[8] This contribution does not depend on the residual resistance; in disordered conductors, it is much smaller than the interference one. The T^2 contribution proportional to the resistivity was initially observed in alkali alloys at low temperatures ($T<2$ K).[8] It has been shown later that the temperature dependence of the resistance of ultrathin metal films is governed by the T^2 term over a broad temperature range: in Au films with an electron mean free path $l\sim 10$ nm—at $T=1-15$ K,[9] in more disordered Nb films with $l\sim 1$ nm—up to 120 K.[10] The proportionality of the T^2 contribution to the residual resistance demonstrates unambiguously that this term originates from the electron-phonon-impurity interference. The temperature dependence of the resistance of the films studied in Refs. 9 and 10 can be fitted very well with the sum of the Bloch-Grüneisen and interference contributions over a wide temperature range from liquid helium to room temperatures. Absolute values of

the T^2 contribution to the resistance of Au and Nb films, as well as the experimental data on alkali alloys,[8] are also in an agreement with the theoretical predictions.[15]

The elastic electron scattering from impurities, defects, and boundaries results in enhancement of the interaction of electrons with transverse phonons. It has been pointed out by Schmid[16] that the electron interaction with transverse phonons may play a significant role in the dephasing of the electron wave function in a wide temperature range where $q_T l \sim 1$ (q_T is the wave number of a thermal phonon). However, there have not been any reliable estimates of this effect, mostly because of the lack of experimental information on the constants of interaction with transverse phonons. As will be shown below, the experimental study of the interference correction to the resistivity allows one to evaluate the effective electron coupling with transverse phonons and, hence, calculate the inelastic electron-scattering time.

In this paper, we study the temperature and electron mean-free-path dependences of the resistance of thin Al, Be, and NbC films in which the electron and phonon parameters vary substantially. We separate the interference correction to the resistivity and estimate effective electron coupling with transverse phonons for these metals. For completeness, we compare the new data with the data for thin Au and Nb films.[9,10] Finally, we calculate the inelastic-electron-scattering rates due to the interaction with both transverse and longitudinal phonons, and compare the results with available experimental data.

The outline of this paper is as follows. We briefly review the existing theoretical predictions in Sec. II. The details of the experiments and experimental data are present in Sec. III. The experimental results will be discussed in Sec. IV.

II. THEORY

The parameter that specifies the character of the electron-phonon interaction is the product of the electron mean free path l and the wave number of a thermal phonon q_T. The theory[15] has been developed for the ''clean'' limit $ql>1$; this requirement can be met in thin-metal films at temperatures $T>\hbar u/k_B l$. The correction to the resistivity $\Delta\rho_{\rm Int}$ due to the interference between the electron-phonon and electron-impurity scattering is described in this case by the expression[15]

$$\frac{\rho_{\rm Int}}{\rho_0}=\left[2\left(\frac{u_l}{u_t}\right)\beta_t+\left(\frac{\pi^2}{16}-1\right)\beta_l\right]\frac{2\pi^2 k_B^2}{3\varepsilon_F p_F u_l}\,T^2 \times\int_0^{\Theta_D/T}\left[\frac{2xe^x}{(e^x-1)^2}-\frac{2}{e^x-1}\right]x\,dx, \tag{1}$$

where β_l and β_t are the constants of interaction with longitudinal and transverse phonons, ε_F is the Fermi energy, p_F is the Fermi momentum, k_B is the Boltzmann constant, and u_l and u_t are the velocities of longitudinal and transverse phonons, correspondingly. At low temperatures ($T<\Theta_D/5$) the integral in Eq. (1) approaches $\pi^2/3$, and the dependence $\Delta\rho_{\rm Int}$ is simplified:

$$\frac{\Delta\rho_{\rm Int}}{\rho_0}=\left[2\frac{\beta_t}{u_t}+\left(\frac{\pi^2}{16}-1\right)\frac{\beta_l}{u_l}\right]\frac{2\pi^2}{3\varepsilon_F p_F}(k_B T)^2\equiv BT^2. \tag{2}$$

Equations (1) and (2) hold until the electron-impurity scattering remains the main scattering process ($\Delta\rho_{\rm Int}<\rho_0$).

The constants β_l and β_t describe coupling of electrons with thermal longitudinal and transverse phonons, correspondingly. In the jellium model (an isotropic Fermi surface and the Boom-Staver relation for the sound velocity) the coupling constants are simply related to one another:

$$\frac{\beta_t}{\beta_l}=\left(\frac{u_l}{u_t}\right)^2, \tag{3}$$

where $\beta_{l,t}=(\frac{2}{3}\varepsilon_F)^2\nu_0/2\rho_m u_{l,t}^2$, ρ_m is the metal density, ν_0 is the two-spin density of electron states (DOS).

The theory[15] predicts that the major contribution to $\Delta\rho_{\rm Int}$ stems from the interaction of electrons with *transverse* phonons [see Eq. (2), the term in the parentheses that is proportional to β_t]. It is noteworthy that electrons do not interact with transverse phonons in a simple isotropic model of a ''pure'' metal. In disordered metals, the electrons do interact with transverse phonons due to the electron scattering by ''vibrating'' impurities. This mechanism results in a positive contribution to the resistance. The contribution of longitudinal phonons, which is the sum of many channels of interaction, is negative due to the quantum nature of the interference.[15] The total correction $\Delta\rho_{\rm Int}$ is positive because an absolute value of the ''longitudinal'' contribution to $\Delta\rho_{\rm Int}$ does not exceed $\sim 2\,\%$ of the ''transverse'' contribution for a typical value of the ratio $u_l/u_t\sim 2-3$. Hence the prefactor B in Eq. (2) is determined mainly by the interaction with transverse phonons:

$$B\approx\frac{4\pi^2\beta_t}{3\varepsilon_F p_F u_t}\,k_B^2. \tag{4}$$

Altshuler[17] has considered the ''pure'' electron-phonon scattering in disordered systems and showed that the temperature dependence of the contribution $\Delta\rho_{e\text{-ph}}$ is similar to the temperature dependence of the resistance of a pure metal due to the electron-phonon scattering, the Bloch-Grüneisen law (see also Refs. 18 and 19):

$$\frac{\Delta\rho_{e\text{-ph}}}{\rho_0}=\frac{1}{2}\,\frac{\pi\beta_l\tau}{\hbar(p_F u_l)^4}\,(k_B T)^5\int_0^{\Theta_D/T}\frac{x^5 dx}{(e^x-1)(1-e^{-x})}, \tag{5}$$

where $\tau=l/\nu_F$ is the elastic transport time. Since the contribution of electron-phonon interaction $\Delta\rho_{e\text{-ph}}(T)$ does not depend on the residual resistance, the normalized term $\Delta\rho_{e\text{-ph}}/\rho_0$ should vary proportionally to the electron mean free path in the temperature range where the temperature dependence of the resistance is dominated by the electron-phonon scattering.

The measurement of the interference correction to the resistance allows one to estimate β_l and β_t. The same constants determine the inelastic electron-scattering rate due to the electron-phonon interaction. The inelastic-scattering rate due to the interaction with longitudinal phonons can be represented as (for details, see Refs. 16 and 20):

$$(\tau_{e\text{-ph},l})^{-1}=\tfrac{7}{2}\pi\zeta(3)\beta_l\,\frac{(k_B T)^3}{\hbar(p_F u_l)^2}\,F_l(q_l l), \tag{6}$$

where $q_l = k_B T/\hbar u_l$ is the wave number of a thermal longitudinal phonon. Function $F_l(z)$ can be expressed in terms of the Pippard function $\Phi_l(x)$ as follows:

$$F_l(z) = \frac{2}{7\zeta(3)} \int_0^{A_l} dx\ \Phi_l(xz)[N(x)+f(x)]x^2, \qquad (7)$$

where $A_l = k_B\Theta_D l/\hbar u_l z$, $N(x)$ and $f(x)$ are the Bose and Fermi functions, and

$$\Phi_l(x) = \frac{2}{\pi}\left(\frac{x\ \arctan(x)}{x-\arctan(x)} - \frac{3}{x}\right). \qquad (8)$$

At low temperatures ($T \ll \Theta_D$) the limits of the function $F_l(q_l l)$ are

$$F_l(q_l l) = \begin{cases} 1 & \text{if } l \gg \hbar u_l/k_B T \\ \dfrac{2\pi^3}{35\zeta(3)}\,(q_l l) & \text{if } l \ll \hbar u_l/k_B T. \end{cases} \qquad (9)$$

Interaction of electrons with transverse phonons in impure metals results in the inelastic electron-scattering rate:[16,20]

$$(\tau_{e\text{-ph},t})^{-1} = 3\pi^2\beta_t \frac{(k_B T)^2}{(p_F u_t)(p_F l)} F_t(q_t l), \qquad (10)$$

where

$$F_t(z) = \frac{4}{\pi^2} \int_0^{A_t} dx\ \Phi_t(xz)[N(x)+f(x)]x, \qquad (11)$$

$A_t = k_B\Theta_D l/\hbar u_t z$, and

$$\Phi_t(x) = \frac{2x^3+3x-3(x^2+1)\arctan(x)}{2x^3}. \qquad (12)$$

In the limiting cases $F_t(q_t l)$ is (at $T \ll \Theta_D$)

$$F_t(q_t l) = \begin{cases} 1 & \text{if } l \gg \hbar u_t/k_B T \\ \dfrac{\pi^2}{10}\,(q_t l)^2 & \text{if } l \ll \hbar u_t/k_B T. \end{cases} \qquad (13)$$

As we have already mentioned above, the interaction of electrons with transverse phonons gives the main contribution to the interference correction to the resistivity. Taking into account that the interaction constant β_t is usually an order of magnitude larger than β_l [see Eq. (3)], one may conclude that the interaction with transverse phonons also controls the inelastic electron-scattering rate over a wide temperature range, where $k_B T l/\hbar u_l$, $k_B T l/\hbar u_t \sim 1$. In Sec. IV C this statement will be supported by detailed calculations. In terms of the prefactor B [see Eq. (4)], the inelastic electron-scattering rate can be written as follows:

$$\tau_{e\text{-ph},t}^{-1} = \tfrac{9}{8} B T^2 F_t(q_t l)\,\tau^{-1}. \qquad (14)$$

We will use Eq. (14) in Sec. IV C for computing the inelastic electron-scattering rate from the resistivity data for Au, Al, Nb, Be, and NbC films.

III. EXPERIMENTAL RESULTS

A. Samples

Parameters of electron and phonon subsystems in our samples vary over a broad range; it is crucial for a comprehensive test of the theory. Aluminum, as well as previously studied gold,[9] is an example of a metal which can be well described within a free-electron approximation. The electron and phonon parameters of Al are well known; the mean free path of electrons can be accurately estimated from the measurements of the upper critical field below the temperature of the superconducting transition. The Bloch-Grüneisen theory, developed for an isotropic Fermi surface and a simple phonon spectrum, agrees reasonably well with the experimental values of the resistance of these metals at high temperatures ($\rho_{e\text{-ph}}^{\text{theor}}/\rho_{e\text{-ph}}^{\text{expt}} \approx 3$).[18] Beryllium is a metal with an anomalously high Debye temperature ($\Theta_D \approx 1000$ K); this might result in a substantial change of the temperature range where the interference term is the dominant temperature-dependent contribution to the resistance. Niobium carbide is a transition metal with a complex Fermi surface and strong electron-phonon interaction.

There is an additional reason why superconducting films of Al and NbC [and, previously, Nb (Ref. 10)] have been chosen for these experiments. The contribution $\Delta\rho_{\text{Int}}$ is proportional to the electron mean free path only if all the other electron and phonon parameters remain the same, regardless of l. It has been verified for Nb films[4] and also for NbC in the course of this work that the residual resistance is inversely proportional to the electron-diffusion constant extracted from the upper critical field at $T < T_c$. These measurements allow us to conclude that the density of electron states ν_0 in these films does not depend on l within the limits of variation of the mean free path in our samples. We believe that this is also valid for films of nonsuperconducting metals like Be and Au, though we cannot measure l independently in these films. At the same time, the density of states in disordered thin films can differ substantially from that in bulk clean metals: the experimental value of ν_0 for Nb films is 1.6×10^{23} eV^{-1} cm^{-3},[4] whereas, for pure Nb $\nu_0 = 0.9\times10^{23}$ eV^{-1} cm^{-3}.[21]

A broad span of the mean-free-path values is crucial for unambiguous separation of the interference correction and the other temperature-dependent contributions to the resistivity. For Al and Be films the mean free path has been varied with the film thickness; the ion bombardment (ions of N^+ and Ar^+ of an energy of 200 keV and doses 40–260 μC) was exploited to decrease the mean free path in NbC films of a fixed thickness in a controllable manner.[22] For each metal, we will present the data for two films, which differ substantially in their values of l (see Table I).

The mean free path for Al was estimated from $\rho l = 9\times10^{-12}\ \Omega$ cm^2,[23] which agrees reasonably well with the DOS calculated from the electron heat capacity.[19] For beryllium we estimated the mean free path of electrons $l = 3(e^2\nu_0\rho_0 v_F)^{-1}$ using the value of ν_0 (calculated from the electron heat capacity) and the Fermi velocity v_F (from the free-electron model). There were several series of NbC samples; each series corresponded to a single film exposed to different doses of high-energy ions. Measurements of the upper critical field indicate that $\rho_0 \sim D^{-1}$ for each series,

TABLE I. Experimental and calculated parameters of samples.

Metal	Al		Be		NbC		Nb[a]		Au[b]	
Sample	1	2	1	2	1	2	1	2	1	2
d (nm)	22	13	45	68	20	20	14	10	34	12
l (nm)	17	8	11	8	4.3	3	2.8	1.2	28	12
ρ_a ($\mu\Omega$ cm)	5.5	12	13	18	68	98	14	32	3	7
T_{cr} (K)	35	45	76	87	45	65	38	120	11	14
T_0 (K)	1.1	3	6	8.4	8.8	13	4.4	11	0.3	0.7
Θ_D (K)	370		1000		370		275		170	
B (10^{-6} K^{-2})	1.1		0.3		1.5		15		2.7	
β_t	4.7		4.3		7.8		10		1.4	
β_t^{theor}	3.0		0.22		0.8		6.6		0.6	

[a]Data for Nb films from Ref. 10.
[b]Data for Au films from Ref. 9.

though there are large variations (up to several times) of the values of $\rho-\rho_0\approx\Delta\rho_{e\text{-ph}}$ at $T=300$ K for different films before the ion treatment. We believe that this is due to variations of the Debye temperature (370–500 K), which reflect the difference of the film stoichiometry. As a result, the values of $\rho_0 D\sim\nu_0^{-1}$ are different for each series, but within a single series the density of states is the same for all the samples. We will discuss in this paper the data obtained for the series of NbC samples with $\Theta_D=370$ K. The results for the other series are qualitatively the same.

B. Measurements

We have measured the temperature dependences of the resistance for Al, Be, and NbC films over the temperature range $T=4.2-300$ K. The total resistance of the samples was 10^4-10^6 Ω because of a meander-type patterning of the films; this enabled us to measure the resistance with an accuracy $\Delta\rho/\rho\approx10^{-6}$ at measuring currents small enough to avoid heating of the samples.

The temperature-dependent contribution to the resistance $\Delta\rho(T)$ is the difference between the total resistance ρ and the residual resistance ρ_0. The resistance in the region of a ''plateau'' just above the superconducting transition has been chosen as the residual resistance for Al and NbC films. For nonsuperconducting Be films ρ_0 was chosen as a minimum on the temperature dependence of the resistance: an increase of the resistance with decreasing temperature below ~10 K is due to the weak localization and interaction effects. A more accurate procedure of separation of the interference correction from other quantum corrections has been described for Au films.[9] This separation was necessary because the interference term in these rather clean films is the dominant one over a limited temperature range 2–10 K, and below 2 K the weak localization and interaction corrections become more important. In this paper we discuss the resistance of disordered films of Al, Be, and NbC in the temperature range $T=10-300$ K, where localization and interaction corrections are small, and the comprehensive separation of these corrections is not necessary. The dependences $\Delta\rho(T)/\rho_0$ for Al, Be, and NbC films are shown in Fig. 1; the error bars in Fig. 1 are mainly due to possible inaccuracy in determination of ρ_0.

The low-temperature dependences $\Delta\rho(T)/\rho_0$ are similar for all the samples: $\Delta\rho(T)$ is proportional to T^2, and the ratio $\Delta\rho(T)/\rho_0$ does not depend on the residual resistance. The values of the prefactor $B=\Delta\rho(T)/(\rho_0T^2)$, which determines the slope of the low-temperature dependences plotted in a double-logarithm scale, are listed for each metal in Table I. This table also includes the data for Au and Nb films from Refs. 9 and 10. At higher temperatures the dependences deviate from the ''T^2'' law; the larger the mean free path, the lower the temperature where this deviation is observed. For the most ''clean'' samples [such measurements have been done for NbC films[22] and for pure bulk Be (Ref. 24)] the temperature dependence of the resistance was close to the Bloch-Grüneisen law over the whole temperature range studied [see Fig. 1(c)].

IV. DISCUSSION

A. The interference correction to the resistance

The residual resistance ρ_0 for all the films is large; it exceeds the resistance contribution due to the electron-phonon interaction even at $T=300$ K. Deviations from the Matteisen rule in this range of ρ, or, in other words, any additional to ρ_0 and $\Delta\rho_{e\text{-ph}}$ contributions to the resistance are due to the interference between electron-phonon and electron-impurity scattering. Proportionality of the electron-phonon-impurity contribution to the residual resistance allows us to neglect other scattering processes (such as the electron-electron scattering), which could also, in principle, result in a quadratic temperature dependence of the resistance (for more detailed discussion, see Ref. 9).

All the data analyzed below have been obtained in the temperature range $T>T_0=\hbar u_t/k_Bl$ where the ''clean'' limit requirement $q_tl\geqslant1$ is satisfied and the phonon spectrum in the films can be considered three dimensional ($q_td>1$). The values of T_0 for each sample are listed in Table I. In Fig. 1 we compare the normalized temperature dependences of the resistance with the theoretical dependence comprising two terms: $\Delta\rho_{\text{Int}}/\rho_0$ [Eq. (1)] and $\Delta\rho_{e\text{-ph}}/\rho_0$ [Eq. (5)]. It is well known that $\Delta\rho_{e\text{-ph}}(T)$ calculated from the Bloch-Grüneisen theory using an isotropic Fermi surface and Debye phonon spectrum can differ substantially from the experimental values of $\Delta\rho_{e\text{-ph}}(T)$.[18,19] Taking this into ac-

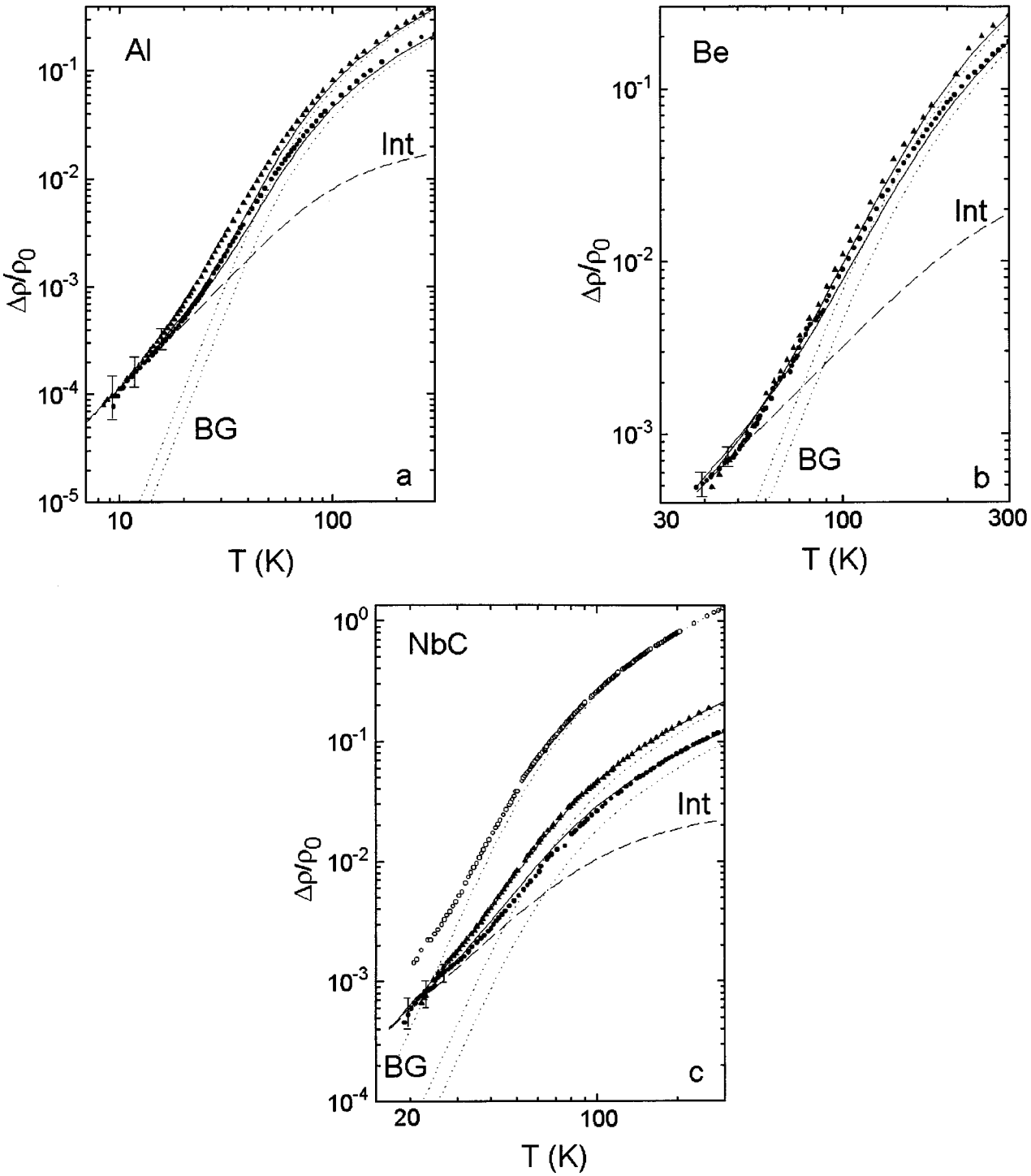

FIG. 1. Temperature dependences of the resistivity $\Delta\rho(T)/\rho_0$ for (a) Al films (▲, $l=16.5$ nm; ●, $l=8$ nm), (b) Be films (▲, $l=11$ nm; ●, $l=8$ nm), and (c) NbC films (○, $l=13$ nm; ▲, $l=4.3$ nm; ●, $l=3$ nm). The dashed curves are the electron-phonon-impurity interference term, the dotted curves are the Bloch-Grüneisen term, and the solid curves are the sum of $\rho_{\mathrm{Int}}+\rho_{e\mathrm{ph}}$.

count, we have used the following method for separation of the two contributions [this method was used also for the films of Au (Ref. 9) and Nb (Ref. 10)]. The temperature dependence of the resistance at low temperatures $\Delta\rho\propto T^2$ has been attributed to the temperature dependence of the interference term. Indeed, one can securely neglect the Bloch-Grüneisen term at low temperatures because of its much stronger temperature dependence $\Delta\rho_{e\text{-ph}}\propto T^5$. The low-temperature dependence $\Delta\rho_{\mathrm{Int}}(T)/\rho_0$ has been extrapolated to $T=300$ K according to Eq. (1) (dashed curves in Fig. 1). The difference between the experimental value of $\Delta\rho(T)/\rho_0$ and the estimated value of $\Delta\rho_{\mathrm{Int}}(300\ \mathrm{K})/\rho_0$ has been identified with the Bloch-Grüneisen term, $\Delta\rho_{e\text{-ph}}$. It is noteworthy that thus obtained $\Delta\rho_{e\text{-ph}}(300\ \mathrm{K})$ coincides with the resistivity of a ''pure'' metal at room temperature. The dotted curves in Fig. 1 show the Bloch-Grüneisen contribution calculated over the whole temperature range according to Eq. (5). The solid curves represent the sum of the Bloch-

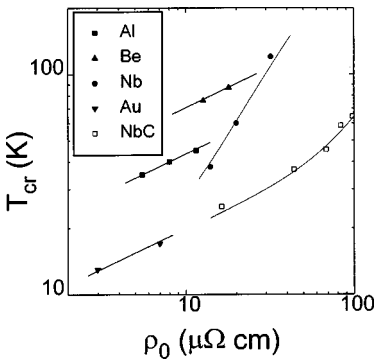

FIG. 2. The crossover temperature T_{cr} vs the residual resistivity for films of different metals. The data for Au and Nb films are taken from Refs. 9 and 10. Solid lines are the calculated dependences $T_{cr}(\rho_0)$, fitted to the crossover temperature of one of the samples of each metal.

Grüneisen and interference contributions; a good agreement of the experimental $\Delta\rho(T)/\rho_0$ dependences with the theoretical ones has been observed for all the films studied.

The interference and Bloch-Grüneisen terms become equal at a crossover temperature T_{cr}, which depends on the mean free path [compare Eqs. (1) and (5)]. For films with small values of l, the interference contribution is substantial even at the room temperature. On the contrary, for rather ''clean'' films the effects of the electron-phonon-impurity interference become unobservable: the temperature dependence of the resistance of clean NbC films with $l>10$ nm can be well described by the Bloch-Grüneisen term at all temperatures $T>T_c$.

The crossover temperature should satisfy the condition $\rho_0 T_{cr}^3 = \text{const}$ if the crossover occurs at low temperatures, where simple power-law dependences hold for $\Delta\rho_{\text{Int}}$ and $\Delta\rho_{e\text{-ph}}$. For NbC film 2 and for Nb film 2, T_{cr} is too high to use simple low-temperature asymptotics for the Bloch-Grüneisen and interference contributions (for the NbC film $T_{cr}\approx 0.2\Theta_D$, for the Nb film $T_{cr}\approx 0.5\Theta_D$). Numerical calculations on the basis of Eqs. (1) and (5) have been made in this case. The crossover temperature as a function of the resistance is shown for all the films studied in Fig. 2. The calculated dependences $T_{cr}(\rho)$, fitted to the experimental value of T_{cr} for one of the films of each metal, are shown in Fig. 2 by solid lines. The temperature of the crossover between the interference and Bloch-Grüneisen terms is well described by the theory.

B. Electron-phonon interaction constants

Taking into account that the interaction of electrons with transverse phonons gives the major contribution to the interference correction, we can estimate the constant of interaction with transverse phonons from measurements of the T^2 term in the resistivity [see Eqs. (2) and (4)]. The electron and phonon parameters, which have been used for estimating the interaction constants from the resistance data, are listed in Table II. The smearing of the electron states due to the elastic scattering ($\hbar/\tau \leqslant 0.1$ eV) is much smaller than a typical energy scale of variations of the density of states in all these metals. Therefore, the electron-impurity scattering should not affect the DOS and Fermi energy. For Al, Be, Nb, and Au we have used the values of ε_F for clean metals,[25] for NbC we used the value of ε_F from Ref. 26. We have also used the Fermi velocity values averaged across a Fermi surface (v_F for Be was calculated from ε_F in accord with the free-electron model).

The experimental values of β_t are compared with the theoretical estimates β_t^{theor} [Eq. (3)] in Table I. For Al, Nb, and Au, β_t^{theor} is $\sim(1.5-2)$ times smaller than β_t. This can be considered as a reasonable agreement, if we take into account a simple jellium model which has been used (a spherical Fermi surface and Debye phonon spectrum). An order-of-magnitude difference between β_t and β_t^{theor} in NbC films can be attributed to a poorly defined stoichiometry of the films, and, therefore, a large uncertainty in the electron and phonon parameters. The jellium model is hardly applicable for beryllium, a metal with a very complex Fermi surface[25] and anomalously small electron density of states. It might explain why the experimental values of β_t are by a factor ~ 20 greater than β_t^{theor}. Another possible reason for such a big discrepancy might be a decrease of sound velocities in thin Be films (compared to anomalously high u_t and u_l in bulk Be) (see, f.i., Ref. 16).

Our experiment shows that an accurate calculation of β_t requires knowledge of ''real'' phonon and electron parameters. However, similar combinations of the electron and phonon parameters enter both the equation for the interfer-

TABLE II. Parameters of the metals.

Metal	ε_F (eV)	p_F (10^{-19} g cm s^{-1})	v_F (cm/s) $\times 10^7$	u_l (10^5 cm/s)	u_t (10^5 cm/s)	ν_0 (10^{22} eV^{-1} cm^{-3})	ρ (g/cm^3)
Al	12	1.8[a]	13[b]	6.3[a]	3.1[a]	1.6	2.7
Be	12	2.2[a]	22[a]	13[c]	9[c]	0.45	1.84
NbC	11	1.6	2[d]	9[e]	4.4[e]	2.6	7.7
Nb	5.3	1.2[a]	2.7[f]	5.1[a]	1.7[a]	16	8.4
Au	5.5	1.3[a]	14[g]	3.2[a]	1.2[a]	1.5	19.3

[a]Reference 25.
[b]Reference 28.
[c]W. C. Overton, J. Chem. Phys. **18**, 113 (1950).
[d]Reference 22.
[e]W. Weber, Phys. Rev. B **8**, 5082 (1973).
[f]Reference 27.
[g]Reference 9.

ence correction to the resistivity [Eq. (2)] and for the inelastic-scattering rate due to transverse phonons [Eq. (10)]. This allows us to use the resistivity data for an accurate estimate of the electron-phonon inelastic-scattering rate [see Eq. (14)], provided the elastic scattering time is known from the experiment. As it will be shown below, our estimates of $\tau_{e\text{-ph}}^{-1}$ agree well with the available experimental data.

C. Electron-phonon relaxation times

As we have discussed in Sec. II, interaction of electrons with transverse phonons controls both the temperature-dependent part of the resistivity and the electron energy relaxation in a metal with a relatively small value of the electron mean free path. The resistivity measurements allow us to calculate the energy relaxation rate $\tau_{e\text{-ph}}^{-1}$ in thin films of the investigated metals. We will compare the calculations of the electron-phonon relaxation rate with available thin-film experimental data on $\tau_{e\text{-ph}}^{-1}$. These data have been obtained from the weak localization experiments, electron heating experiments, and the study of nonequilibrium phenomena in superconductors (for extensive reviews, see Refs. 1, 2, and 20).

The inelastic-scattering rate due to interaction with transverse phonons can be expressed for a disordered metal in terms of the elastic-scattering rate τ^{-1} and the interference correction to the resistivity $\Delta\rho_{\text{Int}}/\rho_0 = BT^2$ [Eq. (14)]. We evaluated $\tau^{-1} = \nu_F/l$ from the published experimental data, and then calculated the temperature dependences of $\tau_{e\text{-ph},t}^{-1}$ (Fig. 3, the dotted lines). For comparison, the temperature dependences of the longitudinal-phonon contribution to $\tau_{e\text{-ph},l}^{-1}$ [Eqs. (6)–(9)] are also shown in Fig. 3 with the dashed lines.

Localization experiments provide information on the electron dephasing rate τ_{φ}^{-1}, which is determined by the electron-electron and electron-phonon scattering:

$$\tau_{\varphi}^{-1} = \tau_{e\text{-}e}^{-1} + \tau_{e\text{-ph}}^{-1}. \tag{15}$$

The experimental data for Nb, Au, and Al films from Refs. 27–29 [Figs. 3(a), 3(c), and 3(d)] represent the electron-phonon relaxation rate that is the difference between the dephasing rate and the electron-electron scattering rate [Eq. (15)]. From the information on $\tau_{e\text{-ph}}^{-1}$, we choose the high-temperature data ($T > 10$ K) because at lower temperatures, where the phonon wavelength becomes larger than the film thickness, the phonon spectrum and the electron-phonon interaction can be substantially modified in comparison with the bulk metals.[4,6] The modification of the phonon spectrum in thin films is not universal, it depends on the material as well as on the film morphology. This question is out of the scope of the present paper, and we will consider below only the temperature range $T > 10$ K, where the modification of the phonon spectrum can be neglected.

The electron-phonon scattering rate in NbC films has been measured in Ref. 22 near the critical temperature of the films (12–14 K), where the film resistance is very sensitive to the electron heating. The cooling rate of an electron subsystem driven from equilibrium by a modulated electromagnetic radiation has been directly measured in the same films as we used in the present work.

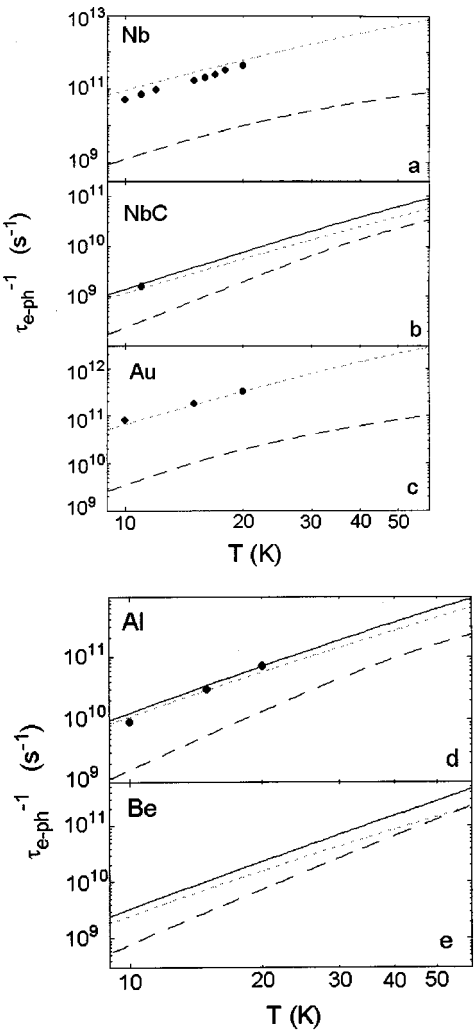

FIG. 3. Inelastic electron-scattering rate calculated from the resistivity data. The dotted curves are the contribution of transverse phonons ($\tau_{e\text{-ph}}^{-1}$), the dashed curves are the contribution of longitudinal phonons ($\tau_{e\text{-ph}}^{-1}$), the solid curves are the total rate. Dots (●) are experimental data for Nb films (Ref. 27), NbC films (Ref. 22), Au films (Ref. 29), and Al films (Ref. 28).

To the best of our knowledge, there is no information on the inelastic electron-phonon scattering in thin Be films. Localization experiments are not efficient in this respect, for the phase breaking in such films should be governed by the electron-electron interaction over the temperature range available for the localization measurements.

Figures 3(a)–3(b) demonstrate that the ''transverse'' contribution to the inelastic-scattering rate, calculated from the data on the interference correction to the resistance, is in excellent agreement with the experimental data on $\tau_{e\text{-ph}}^{-1}$.[22,27–29] The electron-phonon scattering rate in thin-metal films is dominated by the interaction with transverse phonons over a wide temperature range. Even for Be films, where the prefactor B is several times smaller than in the other metals, the contribution of longitudinal phonons should become significant only at temperatures $T > 60$ K.

V. CONCLUSION

It has been shown that the T^2 term in the temperature dependence of the resistivity of thin Al, Be, and NbC films is

proportional to the residual resistivity and therefore this contribution is unambiguously attributed to the interference between the electron-phonon and the elastic electron scattering. The T^2 term has an interference nature even in Be films, where one could expect a relatively large T^2 contribution due to the electron-electron scattering. The temperature dependence of the resistance is governed by the interference term at relatively low temperatures; over a broader temperature interval (up to room temperature) the resistance of all the films is well described by the sum of the interference and Bloch-Grüneisen terms. The crossover temperature at which these two contributions are of the same magnitude crucially depends on the electron mean free path and on the ratio of the longitudinal and transverse sound velocities: the larger the ratio u_l/u_t, the broader the temperature interval where the resistance is dominated by the interference contribution.

In impure metals the interaction of electrons with transverse phonons plays a key role in both the interference correction to the resistance and the inelastic electron-phonon scattering. By fitting experimental data to the theory,[15] the constants of interaction of the electrons with transverse phonons have been determined. Using the interaction constants, we have calculated the contributions of longitudinal and transverse phonons to the inelastic electron-phonon scattering rate in thin films of Au, Al, Be, Nb, and NbC. Our calculations are in a good agreement with available experimental data; this demonstrates that the interaction of electrons with transverse phonons controls the electron energy relaxation in thin-metal films over a broad temperature range.

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