Alloy scattering limited longitudinal resistivity in n-type $Hg_{0.8}Cd_{0.2}Te$ in the extreme quantum limit: Screening effect

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A theoretical model has been developed to study the effect of magnetic-field-dependent and anisotropic screenings on the longitudinal resistivity in n-type $Hg_{0.8}Cd_{0.2}Te$ in the extreme quantum limit at low temperatures, taking into consideration different scattering mechanisms, viz. deformation potential acoustic phonon, ionized impurity, and alloy disorder. Other complexities, such as band nonparabolicity and nonequipartition of phonons, have been included in the calculation. The theoretically obtained values of the resistivity for magnetic-field-dependent and anisotropic screening have been compared with the classical case and also with the experimental results. The agreement with the experimental results improves when the effect of magnetic-field-dependent screening is taken into account. [S0163-1829(97)01824-9]

I. INTRODUCTION

Low-field magnetotransport in narrow-band-gap semiconductors at low temperature in the presence of a quantizing magnetic field has been investigated widely for a long period.[1–3] The magnetic field dependence of nonoscillatory transport parameters, such as mobility, thermoelectric power, etc., have been studied by many authors to understand the physics of scattering mechanisms and band structure in semiconductors at low temperatures.[4–6] In the presence of a quantizing magnetic field, the density of states (DOS) in semiconductors is quite different from their bulk DOS due to the formation of quantized Landau subbands.[1] Such DOS is responsible for oscillatory transport coefficients due to many occupied Landau subbands and strong magnetic-field dependence of the nonoscillatory transport parameters for the lowest Landau level occupation, i.e., in the extreme quantum limit (EQL) condition.

In the semiclassical model, the magnetic field cannot have any influence on the static screening, no matter how strong the field is. The reason is that within the scope of classical plasma electron dynamics, the magnetic field does not do any work and therefore the application of a magnetic field of arbitrary strength cannot provide any of the energy necessary for a redistribution of the shielding charges. From another point of view, the inability of the magnetic field to do work means that it cannot modify the energy spectrum because the Fermi-Dirac equilibrium distribution of the semiclassical model is not influenced by introducing a magnetic field. Magnetic-field effects only occur in conjunction with quantum effects arising from the quantum character of the free-carrier plasma. The quantum effects incorporate important changes in the electron energy spectrum due to the magnetic field (Landau quantization) and this results in substantial magnetic-field quantum effects that modify the static shielding law.[7–12] The occurrence of such magnetic-field quantum effects generally results in spatial anisotropy of the shielding law.[9] Such quantum effects, which may be of importance even in the absence of the field, result from the noncommutation of the free-carrier Hamiltonian with the potential of the scattering centers. The application of the magnetic field causes the electrons to redistribute themselves, resulting in a net increase in the screening length.

It has been found by many authors that the free-carrier screening is an important parameter that affects the transport coefficients in semiconductors.[7,8] The free-carrier screening for semiconductors is characterized by the Debye screening length, which is independent of magnetic field. Recently, Fortini[9] has worked out an elaborate theory for the screening parameters in the EQL, according to which the screening parameter is found to be magnetic-field dependent and anisotropic. In the past, the analysis of the mobility has been carried out assuming the magnetic-field-independent Debye (i.e., classical) screening. However, one should take the actual modification of the screening length, which would alter the magnetic field dependence of the mobility in the EQL. So, the magnetic-field-dependent and anisotropic screening should be incorporated in the calculation of the transport parameters to determine the correct magnetic-field dependence of the parameters.

In the present communication, a theoretical calculation of longitudinal mobility has been carried out in the EQL at low temperature by including the effect of magnetic-field-dependence screening. The other features included in the model are the inclusion of band nonparabolicity and nonequipartition of phonons. The model includes the low-temperature dominant scattering mechanisms, such as deformation potential acoustic phonon, ionized impurity, and alloy disorder while the effect of optical-phonon scattering is neglected since it contributes very little at low temperatures.[7]

However, the effect of screening can easily be incorporated for optical-phonon scattering in the present formalism in a straightforward manner as it can be done through the electron-phonon and electron-impurity coupling required to calculate the momentum relaxation time.

In the present analysis, the screening effect has been decoupled into two different parts: (i) magnetic-field dependence arising out of the DOS, and (ii) anisotropy. These two effects are individually treated and their effects on the mobility have been compared together with the classical case. Finally, the theoretical calculations for different screening are compared with the experimental results.[13]

II. THEORETICAL ANALYSIS

The $\vec{E}\cdot\vec{k}$ relation of a nonparabolic band semiconductor subjected to a large quantizing magnetic field along the z direction can be written as[6]

$$E=-\frac{E_g}{2}+\frac{E_g a_0}{2}+\frac{\hbar^2 k_z^2}{2m^* a_0}, \tag{1}$$

where E_g is the band gap, $\hbar$ is the Planck's constant divided by 2π, m^* is the band edge effective mass, k_z is the z component of electron wave vector k, and

$$a_0=\left[1+\frac{2\hbar\omega_c}{E_g}\left(1-|g|\frac{m^*}{2m_0}\right)\right]^{1/2} \tag{2}$$

measures the nonparabolicity.

For deformation potential acoustic-phonon scattering the electron-phonon interaction matrix element can be written as

$$C_i|f_i(q)|^2=C_{\mathrm{ac}}q/\phi(q), \tag{3}$$

where $C_{\mathrm{ac}}=E_1^2\hbar/2\rho u_{\mathrm{ac}}$ and $\phi(q)=[1+(q_s^2/q^2)g(q_\perp,q_z)]^2$ is used for screening, E_1 is the acoustic-phonon deformation potential constant, and ρ is the mass density. $\phi(q)$ represents the dielectric function, which includes the anisotropic and magnetic-field-dependent contributions due to magnetic quantization, q_s is the classical inverse screening length, and is equal to $(ne^2/\epsilon_0\epsilon_s k_B T_L)^{1/2}$, where n is the electron concentration, ϵ_s is the dielectric constant, ϵ_0 is the free-space permittivity, k_B is the Boltzmann constant, and T_L is the lattice temperature. The function $g(q_\perp,q_z)$ is defined as[9]

$$g(q_\perp,q_z)=\exp(-1/2l^2q_\perp^2)I\left(\frac{4\hbar^2q_z^2}{8m^*a_0k_BT_L}\right). \tag{4}$$

Substituting $u=l^2q_\perp^2/2$ and $v=\hbar^2q_z^2/8m^*a_0k_BT_L$, we can write the above function as

$$g(q_\perp,q_z)=\exp(-u)I(4v), \tag{5}$$

where $I(x)=\Sigma_{n=0}^{\infty}(-x)^n/2^n(n+1)!!$

Now the relaxation time for the acoustic-phonon scattering via deformation potential can be written as[14,15]

$$1/\tau_{\mathrm{ac}}=\frac{(m^*a_0)^{1/2}C_{\mathrm{ac}}}{4\sqrt{\pi}\hbar^2l^3}f_{\mathrm{ac}}\left(\frac{N_R}{\sqrt{E+\hbar\omega_0}}+\frac{N_R+1}{\sqrt{E-\hbar\omega_0}}\right), \tag{6}$$

where N_R is the equilibrium phonon occupation number, $l=(\hbar/eB)^{1/2}$, B being the magnetic field, $\omega_0=eB/m^*$, and $f_{\mathrm{ac}}=(1+\beta)^{2.5}(1+\beta+\alpha_s)^{-2}$, where $\beta=l^2q_z^2/v$ and $\alpha_s=q_s^2l^2\exp(-u)I(4v)$, q_s being the classical inverse screening length.

For anisotropic screening, the screening parameter can be written, from the analytical expression of Fortini,[9] as

$$\alpha_s=q_s^2l^2\left(1-\frac{q_s^2l^2}{2}\right). \tag{7}$$

Similarly the expressions for the relaxation time for the ionized impurity and alloy disorder scatterings can be written as[14,15]

$$1/\tau_{\mathrm{imp}}=\frac{C_{\mathrm{imp}}l^2}{4\pi(e_d+4E)}(2m^*a_0)^{-1/2}\frac{1}{\sqrt{E}} \tag{8}$$

and

$$1/\tau_{\mathrm{alloy}}=\frac{C_{\mathrm{alloy}}}{2\pi\hbar^2l^2}\left(\frac{1+\beta}{1+\beta+\alpha_s}\right)^2(2m^*a_0)^{1/2}\frac{1}{\sqrt{E}}, \tag{9}$$

where, $e_d=\hbar^2n_0e^2\exp(-1/2l^2q_\perp{}^2)I(\xi_0q_z{}^2)/2m^*a_0\epsilon_s\epsilon_0k_BT_L$.

In the above expressions, $C_{\mathrm{imp}}=N_Ie^4/(\epsilon_s\epsilon_0)^2$ and $C_{\mathrm{alloy}}=3\pi^2x(1-x)E_0^2/16N_A$, where N_I is the ionized impurity concentration, N_A is the number of atoms per unit volume, x is the mole fraction, and E_0 is the alloy scattering potential.

III. RESULTS AND DISCUSSION

The longitudinal resistivity for n-type $Hg_{0.8}Cd_{0.2}Te$ has been calculated for different screenings in the EQL at 4.2 K for $n=5\times10^{20}\ \mathrm{m}^{-3}$. The theoretical results have been compared with the experimental work of Nimtz and Gebhardt.[13] These authors clearly indicate that the Wigner crystallization is observed in $Hg_{0.8}Cd_{0.2}Te$ only at temperatures lower than 4.2 K and it behaves as a nondegenerate free-electron gas at 4.2 K. This is also suggested by other investigators who observed Wigner crystallization at very low temperatures, in the millikelvin region.[16,17] The theoretical model has been used to analyze the experimental results in n-HgCdTe assuming nondegenerate statistics. The assumption is justified by the experimental data in n-type HgCdTe. Furthermore, the application of high magnetic field reduces the Fermi energy. In EQL, where all carriers are on the lowest Landau level and the Fermi energy is inversely proportional to the square of the magnetic field,[1] this causes the electron gas to become nondegenerate in high magnetic fields. The numerical values of the parameters used for the present calculations are the following:[7,18] $E_1=9.0$ eV, $E_0=1.9$ eV, $N_I=1\times10^{20}\ \mathrm{m}^{-3}$, $E_g=70.0$ meV, $\epsilon_s=18.0$, $N_A=1\times10^{27}\ \mathrm{m}^{-3}$, $u_{\mathrm{ac}}=30173.0\ \mathrm{ms}^{-1}$, $\rho=7654.0\ \mathrm{kg\,m}^{-3}$, $g=90.0$, and $m^*=0.0065m_0$.

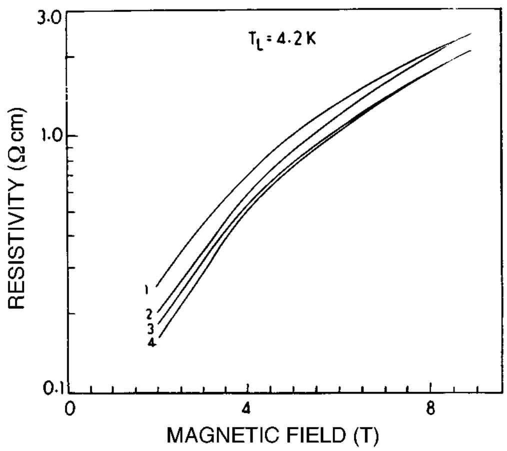

FIG. 1. Variation of the longitudinal resistivity in n-type $Hg_{0.8}Cd_{0.2}Te$ in the extreme quantum limit at 4.2 K with magnetic field for different screening effects. Curves 1, 3, and 4 denote the variations due to quantum screening, anisotropic screening, and classical screening, respectively, while curve 2 represents the experimental results.

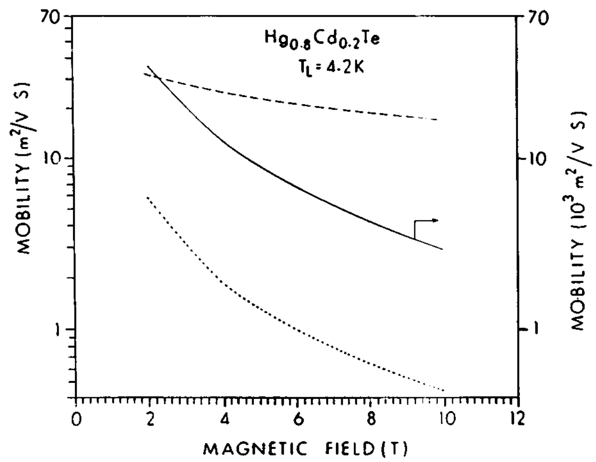

FIG. 2. Variation of the theoretical longitudinal mobility in n-type $Hg_{0.8}Cd_{0.2}Te$ in the extreme quantum limit as a function of magnetic field at a lattice temperature of 4.2 K for $n = 5\times10^{20}$ m^{-3}. It also shows the magnetic field variation of the longitudinal mobility for the acoustic phonon (solid), the ionized impurity (dashed), and the alloy disorder (dotted) scatterings.

The variation of the theoretical longitudinal resistivity with magnetic field is shown in Fig. 1 for various screenings together with the experimental results. It is seen from the figure that mobility decreases with magnetic field in the EQL. This is due to variation of the DOS for the lowest Landau level, which ultimately affects the mobility through the scattering rate.[1] It can be seen from the figure that the inclusion of the magnetic-field-dependent screening lowers the mobility and so enhances the resistivity compared to the classical screening. However, the effect of anisotropic screening due to magnetic field lies in between the magnetic-field-dependent screening and classical screening. The inclusion of magnetic-field-dependent and anisotropic screenings in the scattering matrix increases the scattering rate and hence decreases the relaxation time of carriers compared to the classical screening. Hence mobility due to the former decreases compared to the latter.

Figure 2 shows how the various scattering mechanisms contribute to the mobility in EQL. It is seen from the curve that the inclusion of alloy disorder scattering reduces the mobility significantly compared to ionized impurity or acoustic-phonon scattering. When the alloy disorder scattering is included, because of the enhanced scattering rate, the mobility decreases. The effect of alloy scattering is more prominent in determining the mobility of an alloy semiconductor such as n-type $Hg_xCd_{1-x}Te$. Such an observation has been confirmed in explaining low field mobility in the presence of high magnetic field.[14,15]

The present calculation of the mobility indicates that the effect of screening has an important role in explaining the magnetic-field dependence of the transport parameters. The magnetic-field-dependent screening is found to be more significant compared to the classical screening in the sense that its inclusion in the theoretical calculations brings it closer to the experimental results. However, the anisotropic screening is also an important parameter but not as significant compared to the magnetic-field-dependent screening but its inclusion definitely improves the theoretical results compared to the classical case. Furthermore, it can be seen that the anisotropic screening disappears at higher magnetic fields and it merges to the classical case [as also seen from Eq. (7)]. The agreement between the experimental and theoretical results with magnetic-field-dependent screening improves at higher magnetic fields due to strong confinement effect caused by the high magnetic field in the EQL.

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