Skyrmions in quantum Hall systems with realistic force laws

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We study the charged excitations of quantum Hall systems at integer filling fractions $\nu=2n+1$, for a force law that takes account of the finite width of the electron gas. For typical values of this width, in the limit of vanishing Zeeman energy we find that the low-energy excitations are skyrmions not only at $\nu=1$ but also at higher filling fractions. Our results lead to the prediction that, in typical samples, abrupt transitions to charged excitations with very large spins should be observable at filling fractions higher than $\nu=1$ if the Zeeman energy is reduced sufficiently. [S0163-1829(97)50704-1]

There has recently been a great deal of theoretical and experimental activity studying spin-related phenomena in two-dimensional electron systems in the quantum Hall regime.[1–6] In such systems, the combination of strong Landau quantization and extremely small Zeeman energy leaves the primary role in determining the ground-state spin correlations to the many-body interactions. Attention has been focused predominantly on the filling fraction $\nu=1$, at which interactions have particularly dramatic effects on the spin order. Whilst the ground state at this filling fraction is a simple ferromagnet[1,3] in which the electrons fill a single, spin-polarized Landau band, the low-energy charged excitations can have very unusual spin structures.[1]

At $\nu=1$, the picture that has emerged[1] is that, in the strong-field limit (when the cyclotron energy is large compared to all other energy scales), the form of the low-energy charged excitations depends strongly on the ratio of the Zeeman energy $Z=g\mu_B B$ to the exchange energy: this is proportional to $e^2/4\pi\epsilon\epsilon_0\ell$ for Coulomb interactions ($\ell\equiv\sqrt{\hbar/eB}$ is the magnetic length in a magnetic field B). For large Zeeman energies, the charged excitations involve the minimum number of spin reversals: they are the ''polarized quasiparticles,'' with sizes of order the magnetic length and spins of 1/2. In the opposite limit ($Z=0$), the charged excitations minimize the exchange energy by adopting the ''skyrmion'' configurations[1] of the underlying continuum ferromagnet, and have diverging spatial sizes and total spins. At intermediate values of the Zeeman energy, a compromise is reached in which the charged excitations have finite sizes and spins, which are large compared to the sizes and spins of the polarized quasiparticles;[2] following Ref. 2 we will refer to these large-spin particles as ''charged spin textures.'' The low-energy charged excitations are predicted to be charged spin textures if the Zeeman energy is less than $0.054(e^2/4\pi\epsilon\epsilon_0\ell)$,[7,1] which is typically the case in GaAs devices.[8] Experimental studies of such systems show that the charged excitations at $\nu=1$ do carry large spin,[4–6] in good agreement with current theory.

At higher odd filling fractions,[9,10] the low-energy properties in the strong-field limit depend only on the electrons in the uppermost Landau band. Consequently, such systems appear much like a system at $\nu=1$, differing only in the form of the single-particle states available to the electrons. As is the case at $\nu=1$, repulsive interactions lead to a ferromagnetic ground state for the uppermost band.[9] However, for vanishing Zeeman energy and for pure Coulomb interactions, the skyrmion excitations are found to be *higher* in energy than the polarized quasiparticles at filling fractions $\nu=3$, 5, and 7.[9,10] It is therefore believed that the low-energy charged excitations of quantum Hall systems at filling fractions higher than $\nu=1$ are polarized quasiparticles, even for vanishing Zeeman energy. The results of an experiment probing the spins of the charged excitations at $\nu=3$ and 5 are indeed consistent with spin-polarized excitations.[5]

Although the current experimental observations at $\nu=2n+1$ are in qualitative agreement with existing theory, there do remain important unanswered questions concerning the properties of the charged excitations. In particular, there has been no systematic study of the effects of the finite thickness of the electron layer on these particles (the one study[17] to have taken this into account considered only $\nu=1$). The purpose of the present work is to investigate how the above theoretical picture, established assuming *pure* Coulomb interactions, is affected when a more realistic force law is used. We find that *qualitative* differences arise: in the limit of vanishing Zeeman energy, skyrmions are found to be the low-energy excitations not only at $\nu=1$ but also at higher filling fractions. We will concentrate on the results of our calculations; details will be presented elsewhere.[11]

For the most of this work, we study the properties of the charged excitations at integer filling fractions $\nu=2n+1$ in the strong-field limit, for the force law (from now on we express energies in units of $e^2/4\pi\epsilon\epsilon_0\ell$ and lengths in units of the magnetic length ℓ)

$$V_w(r)\equiv\int\int_{-\infty}^{\infty}dz\,dz'\,\frac{e^{-(z^2+z'^2)/2w^2}}{2\pi w^2}\,\frac{1}{\sqrt{r^2+(z-z')^2}}. \tag{1}$$

This represents the interaction between electrons with Gaussian subband wave functions of width w. Although subband wave functions are not precisely of this form, the force law (1) captures the essential effect of the finite width of the electron layer: the softening of the repulsive interactions at short distances. Comparison with experimental systems may be achieved by equating w to the r.m.s. width of the subband charge density, which is typically of the order of the magnetic length.[12]

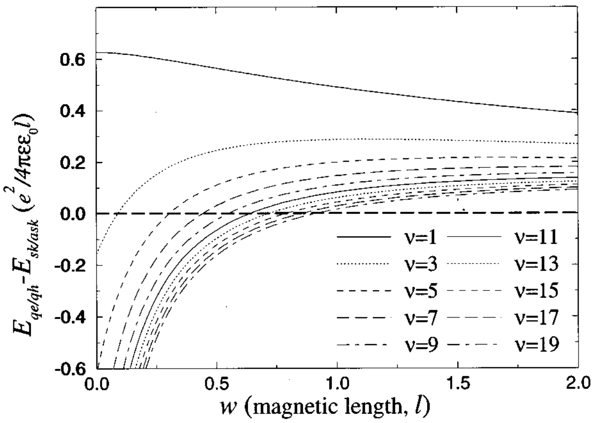

FIG. 1. Difference between the quasiparticle and skyrmion energy gaps for vanishing Zeeman energy at various filling fractions ν, as a function of the layer thickness w. A positive energy difference indicates that the skyrmion/antiskyrmion pair has a lower energy than the quasielectron/quasihole pair.

The main consequences of the finite width w at a filling fraction $\nu=2n+1$ may be understood by comparing the energy gap for the creation of a widely separated quasielectron/quasihole pair to that of a skyrmion/antiskyrmion pair, in the limit of vanishing Zeeman energy.[13] The quasiparticle gap is set by the exchange energy,[14] which may be written

$$E_{\mathrm{qe/qh}}=\frac{1}{2\pi}\int_0^\infty dq\,q\,\tilde{V}_w(q)[L_n(q^2/2)]^2e^{-q^2/2},\qquad(2)$$

where $\tilde{V}_w(q)=(2\pi/q)e^{q^2w^2}\,\mathrm{erfc}(qw)$ is the Fourier transform of the force law (1), and $L_n(z)$ is a Laguerre polynomial with degree n equal to the index of the uppermost Landau band. The skyrmion gap is set by the spin-wave stiffness,[14] and may be expressed as

$$E_{\mathrm{sk/ask}}=\frac{1}{2\pi}\int_0^\infty dq(q^3/2)\tilde{V}_w(q)[L_n(q^2/2)]^2e^{-q^2/2}.\qquad(3)$$

Figure 1 shows the difference between these two gaps as a function of the layer thickness w.

For Coulomb interactions $(w=0)$, the skyrmion gap is less than the quasiparticle gap only at $\nu=1$. It is just this comparison that was made by Wu and Sondhi,[10] and that lead them to conclude that, while skyrmions are the low-energy charged excitations at $\nu=1$ for vanishing Zeeman energy, this is *not* the case at higher odd filling fractions.

However, as can be seen from Fig. 1, only a very small width $(w\gtrsim 0.09\ell)$ is required before the skyrmions become lower in energy than the polarized quasiparticles at $\nu=3$. For a sample with $w=\ell$, skyrmions are the lower-energy excitations at all filling fractions up to and including $\nu=21$. The sensitivity of $E_{\mathrm{qe/qh}}-E_{\mathrm{sk/ask}}$ to changes in w is primarily due to the strong dependence of the skyrmion energy (3) on the short-range force law. That even a small value of w is sufficient to cause the skyrmions to be the lower-energy excitations is one of the principal observations of the present work.

In view of the fact that small changes in the force law can have significant qualitative consequences, one might expect that the screening arising from Landau-level mixing (which we have excluded until now by considering the strong-field limit) could also be important. We have studied the effects of Landau-level mixing on the skyrmion (3) and quasiparticle (2) gaps at $\nu=2n+1$, within a random phase approximation (RPA) approach in which the interaction (1) is screened by a static wave-vector-dependent dielectric constant arising from the particle/hole excitations of the $2n+1$ filled Landau bands.[15] At filling fractions higher than $\nu=1$ we find that the values of the minimum subband widths above which skyrmions are lower in energy than polarized quasiparticles are increased by such screening. Thus, Landau-level mixing appears to favor the polarized quasiparticles over the skyrmions. Nevertheless, for the situation in which the cyclotron energy is equal to $e^2/4\pi\epsilon\epsilon_0\ell$ [$B=6.8$ T in GaAs (Ref. 8)], we find that when $w=\ell$ the skyrmions are the lower-energy excitations at all filling fractions up to and including $\nu=7$. While a more detailed study of Landau-level mixing is clearly desirable, the RPA results suggest that our qualitative conclusion is not affected: for a typical sample, skyrmions would have lower energies than polarized quasiparticles at filling fractions higher than $\nu=1$ if the Zeeman energy were sufficiently small.

We now turn to discuss the properties of the charged excitations at nonzero Zeeman energy. We return to considering the strong-field limit in which Landau-level mixing is negligible. The qualitative behavior that we find is not affected by the inclusion of a small amount of Landau-level mixing in the manner described above.

We have studied the charged excitations at finite Zeeman energy within the Hartree-Fock (HF) approximation introduced by Fertig *et al.*,[2] generalizing this approach to higher filling fractions $\nu=2n+1$, and to the force law (1). To restrict to a finite number of single-particle basis states (which is necessary for the numerical solution of the HF approach), we consider the electrons to lie on the surface of a sphere pierced by $2S$ flux quanta, for which the single-particle states in the Landau level with index n can have $\hat{\boldsymbol{z}}$-component of the angular momentum in the range $m=-S-n,\ -S-n+1,\ldots,S+n$.[16] The HF wave function for the positively charged excitation may then be written

$$|\mathrm{HF}\rangle\equiv\prod_{m=-S-n}^{S+n-1}(u_m c^\dagger_{nm\downarrow}+v_m c^\dagger_{nm+1\uparrow})|0\rangle,\qquad(4)$$

where $|0\rangle$ is the vacuum state, and $c^\dagger_{nm\sigma}$ creates an electron with spin σ in the state with Landau-level index n and angular momentum m. The HF procedure involves the minimization of the average energy with respect to the variational parameters $\{u_m,v_m\}$, subject to the constraint that the wave function is normalized. This leads to a set of self-consistent equations,[11] of the same form as those presented in Ref. 2, which may be iterated until convergence is achieved. The results that we will present are based on calculations with system sizes up to $2S=200$.

We will first discuss our results for the filling fraction $\nu=1$. Results for the case of Coulomb interactions $(w=0)$ are presented in Fig. 2. This figure shows the energy gap to the creation of a widely separated pair of particles with opposite charges, and the spin of one of these particles (due to an exact particle/hole symmetry, both particles have the

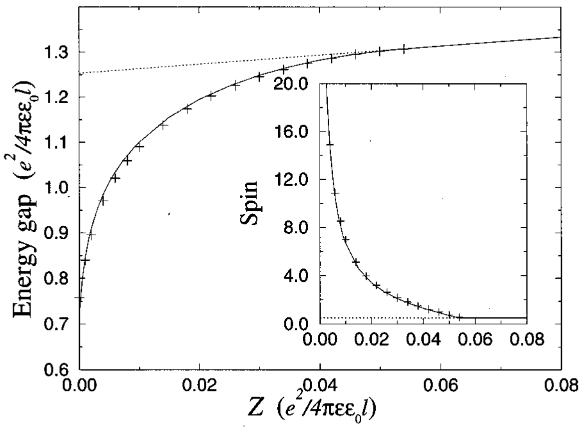

FIG. 2. Energy gap as a function of the Zeeman energy at $\nu=1$ and $w=0$, from Hartree-Fock calculations on a sphere with $2S=200$ (solid line), and $2S=100$ (crosses). The dotted line is the result for polarized quasiparticles. The spin of one charged excitation is shown in the inset.

same spin). We define the spin of an excitation to be the change in spin when it is introduced to the spin-polarized filled Landau band: the polarized quasiparticle therefore has spin 1/2.

For small Zeeman energies, the energy gap is less than that for polarized quasiparticles; in this regime, the lowest-energy HF state is a charged spin texture with a large spin. At large Zeeman energies, the lowest-energy HF state is the polarized quasiparticle ($|u_m|^2=1-|v_m|^2=0$), so the energy gap and spin become equal to those for the quasiparticles. Defining a critical Zeeman energy $Z_c^{\rm HF}$ by the value of Z at which the transition between the charged spin texture and the polarized quasiparticle occurs, we find $Z_c^{\rm HF}=0.054$.

Our qualitative results are very similar to those presented in Ref. 2, where the same situation ($\nu=1$, $w=0$) was studied in the disc geometry. However, there are quantitative differences: the energy gaps and spins that we find are consistently less than those of Ref. 2. These discrepancies may be due to larger finite-size effects on the disc than on the sphere. Finite-size effects are small in our calculations, as may be seen by comparing the results for $2S=100$ and 200 in Fig. 2.

We now introduce a nonzero width w. We find no changes in the qualitative behavior shown in Fig. 2. There are, however, quantitative corrections: at each (nonzero) Zeeman energy, the energy and spin of the charged spin texture are closer to those of the polarized quasiparticle. For example, at $Z=0.01$ the spin decreases from 6.7 for $w=0$ to 5.9 for $w=0.5\ell$, and 5.0 for $w=\ell$; at $Z=0.02$ there is a decrease from 3.4 ($w=0$) to 3.0 ($w=0.5\ell$), and 2.4 ($w=\ell$). The critical Zeeman energies are also reduced (see Table I). Similar qualitative behavior has been observed in exact-diagonalization studies of small systems.[17]

Much more interesting behavior occurs for the higher filling fractions. Figure 3 shows our results at $\nu=3$ and $w=\ell$. Although we do not present detailed results here, the same qualitative behavior is found for all higher filling fractions and values of w that we have studied ($3\leqslant\nu\leqslant7$, $w\leqslant3$), provided that for $Z=0$, w is sufficiently large that the skyrmion gap (3) is less than the quasiparticle gap (2).

TABLE I. Zeeman energies $Z_c^{\rm HF}$ (in units of $e^2/4\pi\epsilon\epsilon_0\ell$) at which the Hartree-Fock solution undergoes a transition from a charged spin texture (with average spin $S_c^{\rm HF}$) to a polarized quasiparticle. $Z_c^{(3/2)}$ is the Zeeman energy at which the polarized quasiparticle has the same energy as the spin-3/2 charged spin texture. N/T indicates that there is no transition.

	$\nu=1$		$\nu=3$		$\nu=5$	
w	$Z_c^{\rm HF}$ [$S_c^{\rm HF}$]	$Z_c^{(3/2)}$	$Z_c^{\rm HF}$ [$S_c^{\rm HF}$]	$Z_c^{(3/2)}$	$Z_c^{\rm HF}$ [$S_c^{\rm HF}$]	$Z_c^{(3/2)}$
0.0	.054 [.5]	.054	N/T	N/T	N/T	N/T
0.5	.048 [.5]	.049	.0018 [20.1]	N/T	.00012 [83]	N/T
1.0	.038 [.5]	.040	.0037 [8.1]	.0029	.0011 [25.0]	.00017
2.0	.026 [.5]	.027	.0044 [4.1]	.0043	.0018 [11.0]	.0014

Finite-size effects are larger in Fig. 3 than in Fig. 2, due to the much larger spins and spatial extents of the charged spin textures.

As is the case for $\nu=1$, with increasing Zeeman energy the spin of the charged spin-texture decreases and there is a critical value $Z_c^{\rm HF}$ above which the lowest-energy state is the polarized quasiparticle. However, in contrast to the continuous behavior found for $\nu=1$, for the higher filling fractions we observe a *discontinuous* transition at $Z_c^{\rm HF}$ from a charged spin texture with a large spin to the polarized quasiparticle. The average spin of the charged spin texture at the transition $S_c^{\rm HF}$ is given in Table I.

We note that the distinction between the continuous transition at $\nu=1$ (in which the spin falls smoothly to 1/2 at $Z=Z_c^{\rm HF}$) and the first-order transition at higher filling fractions (in which the spin jumps from $S_c^{\rm HF}\neq1/2$ to 1/2 at $Z=Z_c^{\rm HF}$) is an artifact of the HF approximation. In an exact treatment, these transitions must *both* be discontinuous, since both result from level crossings in which there are changes in the spin quantum number. [The wave function (4) is not an eigenstate of spin, so the *average* spin of the HF groundstate can vary continuously.] Nevertheless, the HF results do indicate that the nature of the level crossings differ qualitatively at $\nu=1$ and at higher filling fractions.

At $\nu=1$, the continuous reduction in spin suggests that the transition to the polarized quasiparticle occurs from the smallest possible charged spin texture, which has spin 3/2. This is confirmed by the good agreement (see Table I) be-

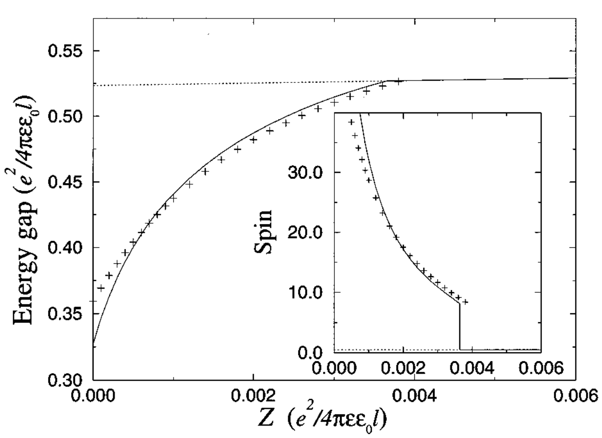

FIG. 3. Same as Fig. 2 for $\nu=3$ and $w=\ell$.

tween Z_c^{HF} and the value $Z_c^{(3/2)}$ at which the spin-3/2 charged spin texture has the same energy as the polarized quasiparticle (we obtain the values $Z_c^{(3/2)}$ from exact-diagonalization studies on the sphere[11]). The slight deviations can be accounted for by the fact that the HF approach is not exact.

At higher filling fractions, the HF results indicate that the transition occurs from a charged spin-texture with a spin much larger than 3/2. Indeed, we find that Z_c^{HF} is consistently *larger than* $Z_c^{(3/2)}$. Thus, when $Z=Z_c^{(3/2)}$ (such that the polarized quasiparticle and the spin-3/2 charged spin-texture have equal energies), the HF results show that there exists a lower-energy charged spin texture, with a spin larger than 3/2. This confirms that the transition involves a change of spin that is larger than unity.

We now turn to discuss the experimental implications of our results.

For $\nu=1$, the reduction of finite-size effects and the introduction of the thickness w change the quantitative results of Fertig *et al.*[2] At any given (nonzero) value of the Zeeman energy, the spin of the charged spin texture is reduced. This brings the HF results into better agreement with experimental measurements.[4–6]

For higher filling fractions, we find that, in typical samples, charged spin-textures would be lower in energy than the polarized quasiparticles if the Zeeman energy were sufficiently small. However, the critical values below which the Zeeman energy must lie in order that these particles appear are very small: they are about 1/10 of the critical Zeeman energies at $\nu=1$ (see Table I). This condition is not typically achieved in GaAs devices. In particular, the range of Zeeman energy studied in the experiments of Schmeller *et al.*[5] ($Z \gtrsim 0.0075$) lies above the critical Zeeman energies that we predict for their sample ($w \simeq 0.5\ell$) at $\nu=3$ and 5: the observation of spin-polarized charged excitations under these conditions[5] is therefore consistent with our results.

It may be difficult to bring the Zeeman energy below the critical values we predict by simply reducing the magnetic field and carrier density, owing to the increasing effects of disorder. To observe charged spin textures at filling fractions higher than $\nu=1$, it may be necessary to reduce the Zeeman energy by other means. This can best be done through the application of external pressure, which can be used to tune the Zeeman energy to zero.[18] Our results lead to the prediction that, as the Zeeman energy of a typical sample is reduced from its value at ambient pressure, the low-energy charged excitations at filling fractions higher than $\nu=1$ should undergo abrupt transitions from polarized quasiparticles to charged spin textures with very large spins.

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