PHYSICAL REVIEW B VOLUME 55, NUMBER 24 15 JUNE 1997-II

Periodic models for thin optimal absorbers of electromagnetic radiation

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(Received 29 April 1997)

We study a system composed of two thin crossed lamellar gratings, with lamellae composed of a Drude metal. We show that such a system can yield striking behavior for wavelengths of electromagnetic radiation much longer than the grating period, with reflectance, transmittance, and absorptance being independent of wavelength. We thus present periodic models that can duplicate the behavior of thin critical metallic films while having a much simpler and more practical geometry than they. [S0163-1829(97)52524-0]

Thin metallic films can readily be made of metals like gold and silver that display very striking optical behavior: the reflectance, transmittance, and absorptance of the films are practically independent of wavelength over a spectral range from the end of the visible to the far infrared.[1–7] The structure of such metallic films is complicated, and has been investigated in detail.[1–7] It has been argued that the complicated structure has in fact a fractal nature, and that this is an essential feature of models that can successfully explain wavelength independent optical behavior over such a wide spectral range.

One interesting recent study[8] has presented a model for the wavelength independent behavior, based on a generalized form of Ohm's law, with Ohmic parameters replacing the complex dielectric constants of the thin film constituents. As well, a dynamic effective medium approximation is involved.

In order to see whether the linkage between fractal geometry and wavelength independent optical behavior is indeed necessary, we have investigated a simple periodic structure that is amenable to a perturbation analysis of a rigorous electromagnetic scattering formulation.[9,10] The structure, shown in Fig. 1, consists of two identical lamellar gratings, with orthogonal periodicity axes, placed one behind the other with a separation S. Each lamellar grating has segments composed of alternately a dielectric with constant equal to that of the background material ($\varepsilon_1=1$) and a metal whose complex dielectric constant is conveniently represented as a power law:

$$\varepsilon_2 = i\tilde{\varepsilon}k^{-p}, \tag{1}$$

where $\tilde{\varepsilon}$ is a constant of proportionality that characterizes the long wavelength behavior of the metal in question, and $k=2\pi/\lambda$ is the wave number of the radiation impinging normally on the crossed grating. Such a power law accurately models the behavior of metals like gold, silver, and aluminum in the midinfrared and beyond. In fact, such metals have the exponent p close to unity and the imaginary part of the dielectric constant much larger than the real part; they are called Drude metals.

A rigorous modal method has been elaborated to deal with the diffraction by lamellar gratings.[11,12] The method relies on solving a transcendental equation to find the propagation constant μ of the modes along the z axis:

$$\cos(\beta_1 g_1)\cos(\beta_2 g_2) - \frac{1}{2}\left(f\frac{\beta_1}{\beta_2} + \frac{1}{f}\frac{\beta_2}{\beta_1}\right) \times \sin(\beta_1 g_1)\sin(\beta_2 g_2) = 1, \tag{2}$$

where $\beta_i^2 = k^2\varepsilon_i - \mu^2$ $(i=1,2)$ describes the spatial variations of the modes along the x axis for (say) grating 1 in the dielectric and metallic regions, respectively. The quantity f is polarization dependent: for the case where the incident electric field is along the y axis ($E_\parallel$ polarization), $f=1$, while for the other principal polarization ($H_\parallel$), $f=\varepsilon_2/\varepsilon_1$. Expression (2) is in fact a generalization to electromagnetic theory of the fundamental equation of the well-known Kronig-Penney model[13] of solid state physics.

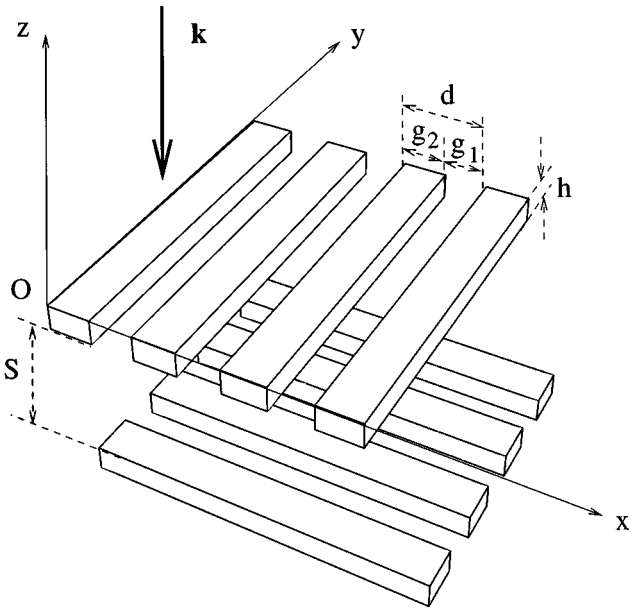

FIG. 1. The geometry of a crossed lamellar grating.

The diffraction properties of lamellar gratings, when the incident radiation has a wavelength much in excess of the grating period d, are dominated by the dispersion behavior of the lowest or acoustic mode (see, for example Ref. 14). An asymptotic analysis of Eq. (2) shows that the $E_\parallel$ dispersion relation is to leading order:

$$\mu^2 = k^2 \varepsilon_o, \quad \text{where} \quad \varepsilon_o = \sum_i g_i' \varepsilon_i. \tag{3}$$

Here, ε_o is the ordinary dielectric constant of the grating, where $g_i' = g_i/d$, and the sum runs over the two species making up the grating. In fact, this equation generalizes naturally to an n species lamellar grating.

For $H_\parallel$ polarization, the dispersion equation is

$$\mu^2 = k^2 \varepsilon_x, \quad \text{with} \quad \varepsilon_x^{-1} = \sum_i g_i'/\varepsilon_i, \tag{4}$$

where ε_x is the extraordinary dielectric constant of the grating. Equations (3) and (4) suggest the lamellar grating 1 can be replaced by a uniaxial crystal at long wavelengths.[9,15–17] Given the dielectric constant ε_2 has magnitude far larger than unity, we have

$$\varepsilon_x \approx 1/g_1' \quad \text{and} \quad \varepsilon_o \approx i g_2' \tilde{\varepsilon} k^{-p}. \tag{5}$$

In consequence, μ for the case of the acoustic band in $H_\parallel$ polarization is always a linear function of k, while for $E_\parallel$ polarization the acoustic band varies in form according to the spectral index p, in a way governed by the equation

$$\mu_o = k^{1-p/2} \sqrt{i g_2' \tilde{\varepsilon}}. \tag{6}$$

Note that, as p increases, the acoustic band becomes increasingly steep near $k=0$, until, when $p=2$ the acoustic band is lost entirely as the lowest band tends to a nonzero value at $k=0$. In fact, for all values of $p \geqslant 2$, the lowest branch of the dispersion relation approaches that appropriate to a perfectly conducting lamellar grating, so that the results of the diffraction by lamellar gratings with $p \geqslant 2$ are for long wavelengths identical to those of perfectly conducting gratings in $H_\parallel$ polarization.

In Fig. 2 we display the dispersion diagram for the seven lowest bands resulting from the solution of Eq. (2) for a lamellar grating with alternating air and silver elements. Note that the values of $|\mu_o|$ have been scaled by $K = 2\pi/d$, so that as k tends to zero, all the higher bands tend to integer values, while the acoustic band is well represented by estimate (6).

Given the dispersion relation for the lowest mode and the structure of the corresponding modal fields, we can solve the diffraction problem by an individual grating to leading order. We find for $E_\parallel$ polarization that the reflection and transmission coefficients are, respectively,

$$r_E \approx \frac{-Y_E/2}{1+Y_E/2}, \quad t_E \approx \frac{1}{1+Y_E/2}. \tag{7}$$

For the case of $H_\parallel$ polarization, the corresponding results are

$$r_H \approx \frac{ikh+Y_H/2}{1+Y_H/2}, \quad t_H \approx \frac{1}{1+Y_H/2}, \tag{8}$$

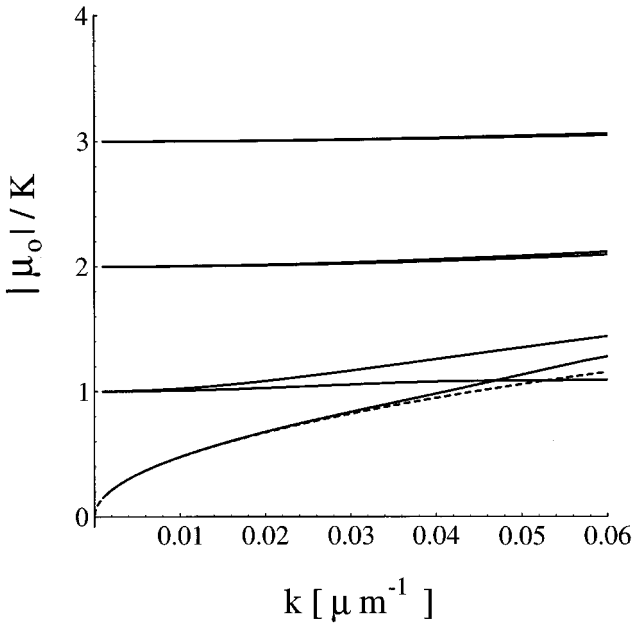

FIG. 2. The dispersion curves for $E_\parallel$ polarization, for a single lamellar grating with $d=0.5$ μm and with silver elements of width $g_2=0.375$ μm. The dashed curve represents the acoustic band estimated from Eq. (6); the dispersion characteristics of this band control the long wavelength properties of the grating. The bands converging to $|\mu_0|/K=1,2,3$ as k approaches zero correspond to nonpropagating orders in the grating.

where h is the thickness of the grating (see Fig. 1). The grating admittances[18] for the two polarizations are given by

$$Y_E \approx -ikh\varepsilon_o, \quad Y_H \approx -ikh(1+\varepsilon_x). \tag{9}$$

Note that, using Eq. (5), Y_H is of order k, as is r_H, while t_H is, to leading order, unity.

It is interesting that Eqs. (7)–(9) resemble the expressions (3.6) of Sarychev *et al.*[8] However, our derivation is more general, being based on a power law variation for the complex dielectric constant, while Sarychev *et al.*[8] express their answer in terms of the long wavelength conductivity σ_m.

The analysis of the energy properties of the composite structure of Fig. 1 requires the use of a coherent ray tracing argument, in which the diffraction properties of each grating are characterized by scattering matrices. In the case of long wavelengths, much greater than either the period of the grating or the separation S, evanescent coupling can be neglected and the problem can be solved using only the specular or undiffracted orders (in both reflection and transmission) whose phase change across the gap is negligible. We can derive expressions for the transverse reflected and transmitted fields, $\boldsymbol{r}$ and $\boldsymbol{t}$, in terms of the corresponding components of the incident field vector $\boldsymbol{I}$. For example

$$\boldsymbol{t} = \boldsymbol{\rho}^{-1} \boldsymbol{T} \boldsymbol{\rho} (\boldsymbol{I} - \boldsymbol{R} \boldsymbol{\rho}^{-1} \boldsymbol{R} \boldsymbol{\rho})^{-1} \boldsymbol{T} \boldsymbol{I}, \tag{10}$$

where $\boldsymbol{R} = \mathrm{diag}(\mathrm{r_E}, \mathrm{r_H})$ and $\boldsymbol{T} = \mathrm{diag}(\mathrm{t_E}, \mathrm{t_H})$ denote the reflection and transmission scattering matrices, and where $\boldsymbol{\rho}$ is an orthogonal matrix that performs the field rotation needed to be able to apply Eqs. (7)–(9) when dealing with fields incident on the second grating (which is rotated by 90° with

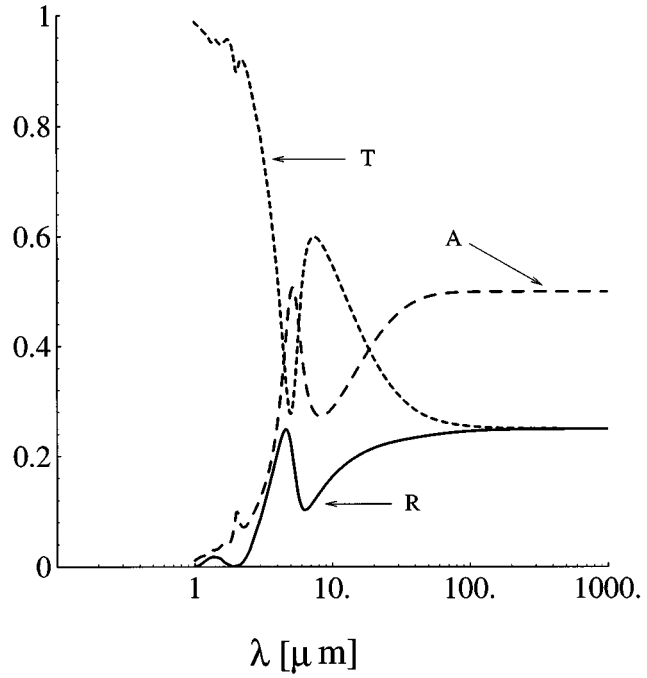

FIG. 3. The reflectance R, transmittance T, and absorptance A, for normally incident radiation, as a function of wavelength (λ), for the structure of Fig. 1 with $d=0.5$ μm, silver elements of width $g_2=0.375$ μm and thickness $h=0.560$ nm, and separation $S=2d$.

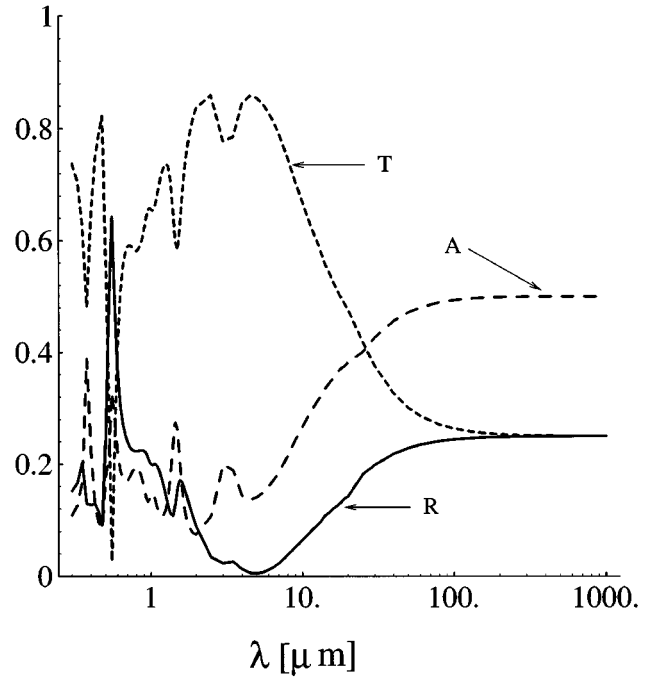

FIG. 4. The reflectance R, transmittance T, and absorptance A, for normally incident radiation, as a function of wavelength λ, for a monolayer of aluminum spheres with radius 0.1580 μm and period 0.5 μm along both x and y axes.

respect to the first). This then yields the specular order reflection and transmission coefficients for the crossed lamellar grating:

$$r_{12} \approx r_E + \frac{r_H t_E^2}{1 - r_E r_H} \approx r_E \approx t_E - 1, \tag{11}$$

$$t_{12} \approx \frac{t_E t_H}{1 - r_E r_H} \approx t_E, \tag{12}$$

with both r_{12} and t_{12}, to leading order, being independent of the polarization of the incident radiation.

From Eqs. (5)–(9)

$$Y_E = g_2' h \tilde{\varepsilon} k^{1-p}, \tag{13}$$

so that r_{12} and t_{12} are independent of wavelength for $\lambda \gg d$, if and only if $p=1$. Hence, wavelength-independent reflectance, transmittance, and absorptance are achieved by incorporating Drude metals into structured systems. If $p=1$, we find the reflectance R and transmittance T are

$$R = |Q|^2 T, \quad T = |1+Q|^{-2}, \tag{14}$$

where $Q = g_2' h \tilde{\varepsilon}/2$, while the absorptance $A = 1 - R - T$. We maximize the absorptance if the quantity $\tilde{\varepsilon}$ is real, and we find the optimal value occurs when $Q=1$, in which case $A = \frac{1}{2}$, $T = R = \frac{1}{4}$. This is in fact the highest possible absorptance that can be achieved by a structure much thinner than the free space wavelength λ. Sarychev *et al.*[8] have commented on this as the optimal value, but in their system it was achieved only for area fractions of metal near 50%. The system of Fig. 1 can be tuned to achieve the optimum for any value of $g_2' = g_2/d$ by varying h for a given $\tilde{\varepsilon}$ (or metal).

Experimental studies of the silver and gold films tend to give wavelength-independent absorptance values around 0.40, and reflectances and transmittances both around 0.30 for all wavelengths larger than approximately 1 μm, and for metal area fractions in the range 60–70 %. One experimental study[6] of a thin silver film used the measured reflectances and transmittances to deduce effective refractive index values as a function of wavelength. In keeping with Eq. (6) for $p=1$, both the real and imaginary parts of the refractive index were proportional to $\sqrt{\lambda}$, with the imaginary part being slightly higher than the real part (in keeping with absorptance values less than the optimal 0.50).

In Fig. 3 we display theoretical curves calculated for a particular case of the structure of Fig. 1. A rigorous diffraction formulation[10] was used to calculate the properties of a single lamellar grating, and then the coherent ray tracing method leading to Eqs. (11) and (12) was used to obtain the reflectance, transmittance, and absorptance for unpolarized light of the crossed lamellar grating. The thickness of the grating was obtained by setting the quantity Q equal to unity. The absorptance in Fig. 3 settles in to its long wavelength limit of 0.50 for $\lambda > 40$ μm.

In order to show the variety of periodic structures that may exhibit wavelength-independent behavior, we have used a rigorous diffraction formulation for biperiodic gratings[19,20] to study the diffraction by thin structures in aluminum. Both a monolayer of spheres placed in square array and a square array of air holes in an aluminum sheet have been modelled. We have used the actual optical constants for aluminum[21] in the wavelength range up to 30 μm, and a Drude fit for the data between 20 and 30 μm as a means of extrapolation beyond the tabulated range. For each structure, we have been able to find a value of radius giving $A=0.50$, $R=T=0.25$,

independent of wavelength. For example, in Fig. 4 we show reflectance, transmittance, and absorptance as a function of wavelength for a monolayer of spheres with radius 0.1580 μm. Both reflectance and transmittance vary strongly in the resonance region around the grid period and up to around 5 μm, before settling in to their long wavelength values of 0.25. We have also shown that for an angle of incidence of 60° the optimal radius becomes somewhat polarization dependent (0.1534 μm for TE waves, and 0.1632 μm for TM waves).

Note that our results show that, once periodic systems attain values of R and T near 0.25, they remain thereafter near these values. By contrast, the model of Sarychev *et al.*[8] gives a plateau region in which R and T are effectively constant, after which R decreases towards zero while T increases towards unity.

The studies reported here have shown the variety of thin periodic systems that can yield wavelength-independent reflectance, transmittance, and absorptance, provided they are composed of Drude metals. Periodic systems are not only easier to model rigorously than random systems, but they are also capable of providing reproducible and nonfragile designs for possible applications.

One interesting difference between the results for periodic systems presented here and experimental results of disordered systems is that the latter revert to wavelength-independent behavior much more rapidly than the former. Typically, thin silver and gold films show this behavior from around 1 μm on, whereas the regular systems we have investigated do not exhibit it until around 40 μm. Further theoretical and experimental studies will be needed to elucidate the reasons for this, and we are preparing a detailed account of them.

Two of the authors (N.A.N. and G.H.D.) have been supported by Australian Research Council Grants, from the DEETYA. This body also provided computing facilities. The support of the Science Foundation for Physics within the University of Sydney is also acknowledged.

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