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Strong eigenfunction correlations near the Anderson-localization transition

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We study the overlap of two different eigenfunctions as compared with self-overlap in the framework of an infinite-dimensional version of the disordered tight-binding model. Despite a very sparse structure of the eigenstates in the vicinity of the Anderson transition, their mutual overlap is still found to be of the same order as self-overlap as long as the energy separation is smaller than a critical value. The latter fact explains the robustness of the Wigner-Dyson level statistics everywhere in the phase of extended states. The same picture is expected to hold for usual d-dimensional conductors, ensuring the s^{β} form of the level repulsion at a critical point. [S0163-1829(97)51724-3]

Recently, there has been considerable growth of interest in constructing a unified picture of wave functions and energy-level statistics for disordered conductors in the vicinity of Anderson metal-insulator transition.[1–12] Coming from the metallic phase, a typical wave function $\Psi_i(\mathbf{r})$ is extended and covers all the sample volume randomly, but uniformly. When the system approaches the point of the Anderson transition E_c, these extended eigenfunctions become less and less homogeneous in space, showing regions with larger and smaller amplitudes, and eventually forming a multifractal structure in the vicinity of E_c.

To characterize the degree of nonhomogeneity quantitatively, it is convenient to use the inverse participation ratio (IPR) $I(E)=\int d\mathbf{r}\ \alpha(\mathbf{r},E)$, where

$$\alpha(\mathbf{r},E)=\langle|\Psi_i(\mathbf{r})|^4\rangle_E\equiv\Delta\left\langle\sum_i\ |\Psi_i(\mathbf{r})|^4\delta(E-E_i)\right\rangle, \quad (1)$$

Δ is the mean level spacing and the angular brackets stand for the disorder averaging. For extended states this quantity is inversely proportional to the system volume: $I(E)=C(E)L^{-d}$, with L and d standing for the system size and spatial dimension, respectively. The coefficient C in this relation measures a fraction of the system volume where eigenfunction is appreciably nonzero. For random homogeneous states $C\sim 1$, whereas close to the mobility edge $E=E_c$ it becomes large and diverges like $C(E)\propto|E-E_c|^{-\mu}$, $\mu>0$,[13] signaling an increasing sparsity of eigenfunctions. Just at the mobility edge eigenfunctions occupy a vanishing fraction of the system volume and IPR scales like $I(E)\propto L^{-d+\eta}$, $\eta>0$. Such a behavior reflects fractal [actually, *multifractal* (Refs. 14, 2 and 4)] structure of critical eigenstates. At last, in the insulating phase any eigenstate is concentrated in a domain of finite extension ξ_l, and IPR stays finite in the limit of infinite system size $L\to\infty$.

This transparent picture serves as a basis for a qualitative understanding of spectral properties of disordered conductors. Indeed, as long as eigenstates are well extended they overlap substantially, and corresponding energy levels repel each other in the same way as do eigenvalues of large random matrices studied by Wigner and Dyson. As a result, the Wigner-Dyson (WD) statistics describes well the energy levels in a good metal.[15,16] In contrast, in the insulating phase, different eigenfunctions corresponding to levels close in energy are localized far apart from one another, and their overlap is negligible. This is the reason for the absence of correlations of energy levels in this regime—the so-called Poisson statistics.

However, close to the transition point such a reasoning should be used with caution. Naively one may expect that sparse (multifractal in the critical point) eigenstates fail to overlap, which would result in essential weakening of level correlations close to the mobility edge and vanishing level repulsion at $E=E_c$. However, a thorough investigation shows[1,3,5–9] that even at the mobility edge levels repel each other strongly, though the entire statistics is different from the WD one. One of the main purposes of the present paper is to resolve this apparent contradiction. We will show that critical eigenstates for nearby levels are strongly correlated, and overlap well in spite of their sparse structure.

The overlap of two different eigenstates Ψ_i and Ψ_j corresponding to energy separation $|E_i-E_j|=\omega$ can be characterized by comparing the correlation function

$$\begin{aligned}\sigma(\mathbf{r},E,\omega)&=\langle|\Psi_i(\mathbf{r})|^2|\Psi_j(\mathbf{r})|^2\rangle|_{E,\omega}\\&\equiv\Delta^2R_2^{-1}(\omega)\times\left\langle\sum_{i,j}\ |\Psi_i(\mathbf{r})|^2|\Psi_j(\mathbf{r})|^2\right.\\&\left.\times\,\delta(E-E_i)\,\delta(E+\omega-E_j)\right\rangle \quad (2)\end{aligned}$$

at $\omega\neq 0$ with $\alpha(\boldsymbol{r},E)$, the latter function playing in such a context the role of the eigenfunction self-overlap. Here $R_2(\omega)$ denotes the two-level correlation function

$$R_2(\omega)=\Delta^2\left\langle \sum_{ij}\ \delta(E-E_i)\,\delta(E+\omega-E_j)\right\rangle . \qquad (3)$$

To study the function $\sigma(\boldsymbol{r},E,\omega)$ analytically, we consider an exactly solvable model of the Anderson transition—the so-called sparse random matrix (SRM) model.[17,18] This model deals with large $N\times N$ matrices (real symmetric or Hermitian), whose entries H_{ij} are independent random variables characterized by the probability distribution

$$\mathcal{P}(H_{ij})=(1-p/N)\,\delta(H_{ij})+(p/N)h(H_{ij}), \qquad (4)$$

where $h(z)=h(-z)$ is any even distribution function having finite second moment. The parameter $p>0$ has a meaning of mean nonzero elements per row. For $p>1$ the model describes a connected graph having locally a treelike structure with random connectivity. On a larger scale there exist large loops involving typically of the order of $\ln N$ sites. In the limit $N\to\infty$ the influence of loops is negligible, and the model belongs to the same universality class as disordered tight-binding model on the infinite tree (Bethe lattice). The latter model was thoroughly investigated by various methods,[19,20] and shown to possess an Anderson transition.

Additional interest in the structure of eigenfunctions on treelike structures is attributed to a recent paper.[21] There the problem of quasiparticle lifetime induced by Coulomb interaction in mesoscopic samples was mapped onto a disordered treelike tight-binding model in Fock space, which is similar to the Bethe lattice and SRM models and thus undergoes the localization transition. The fact that in the vicinity of the transition the corresponding extended eigenfunctions are extremely sparse made the authors of the paper[21] conclude that the level statistics differs from the WD one for such a regime. However, such a conclusion would be at variance with the results of an explicit calculation of the level-level correlation function performed in the framework of the SRM model,[17] where it was shown that $R_2(\omega)$ is given by the WD form everywhere in the region of delocalized states up to the transition point $E=E_c$.

It is necessary to mention that the SRM model has some considerable advantage when compared with the more conventional Bethe lattice model. That is, quantities like the IPR are not unambiguously defined in the Bethe lattice in the phase of extended states. Indeed, they require a consideration of a large but finite lattice to be well defined, and their limiting behavior may depend crucially on the boundary conditions imposed.[22] In contrast, all sites of the SRM model are essentially equivalent, and the model is free from boundary problems. The general expression for the IPR was derived in Ref. 18, and its critical behavior analyzed in Ref. 23.

Actually, the SRM model can be used to construct an effective mean-field theory of Anderson localization[23] valid at $d=\infty$.[24] Critical properties of such theories first discovered on the level of nonlinear σ models[25,26] turn out to be quite peculiar. In particular, the coefficient $C(E)$ diverges close to the transition point like $C(|E-E_c|\ll E_c)\propto\exp(\text{const}|E-E_c|^{-1/2})$,[23] in contrast to the expected power-law behavior for conventional d-dimensional systems. The origin of such a critical dependence was explained in Refs. 23 and 24, and stems from the fact that $C(E)$ is determined essentially by the ''correlation volume'' $V(\xi)$ (i.e., the number of sites at a distance smaller than correlation length ξ), which is exponentially large, $V(\xi)\propto\exp(\text{const}\,\xi)$, for treelike structures, whereas $V(\xi)\propto\xi^d$ for a d-dimensional lattice. Keeping this difference in mind, one can translate all the results obtained in the framework of $d=\infty$ models to their finite-dimensional counterparts.[24]

To calculate the overlap function defined in Eq. (2) we follow Ref. 27 and use the identity relating $\alpha(r,E)$ and $\sigma(r,E,\omega)$ to advanced and retarded Green functions $G^{R,A}(r,E)=\Sigma_{i=1}^{N}[|\Psi_i(r)|^2/[E\pm i\,\eta-E_i)]$; $\eta\to 0^+$:

$$2\pi^2[\Delta^{-1}\alpha(r,E)\,\delta(\omega)+\Delta^{-2}\widetilde{R}_2(\omega)\sigma(r,E,\omega)] = \mathrm{Re}[\langle G^R(r,E)G^A(r,E+\omega)-G^R(r,E)G^R(r,E+\omega)\rangle], \qquad (5)$$

where $\widetilde{R}_2(\omega)$ is the nonsingular part of the level-level correlation function: $R_2(\omega)=\widetilde{R}_2(\omega)+\delta(\omega/\Delta)$. Let us consider for definiteness the ensemble of the real symmetric SRM, corresponding to systems with unbroken time-reversal invariance. For any site index $r=1,\ldots,N$, we introduce one eight-component supervector $\Phi^\dagger=(\Phi_R^\dagger,\Phi_A^\dagger)$ consisting of two four-component supervectors $\Phi_\sigma^\dagger=(\phi_{\sigma,b1},\phi_{\sigma,b2},\phi_{\sigma,f}^*,-\phi_{\sigma,f})$, where indices $\sigma=R,A$ and b,f are used to label advanced-retarded and boson-fermion subspaces, respectively. The ensemble-averaged products $\langle G^\sigma G^{\sigma'}\rangle$ for the RSM model in the limit $N\gg 1$ can be extracted from Ref. 17 and Appendix D of Ref. 23 and is given by

$$\langle G^\sigma(r,E)G^{\sigma'}(r,E+\omega)\rangle = (1-\tfrac{4}{3}\delta_{\sigma,\sigma'})\int DQ\langle\phi_{\sigma,b1}\phi_{\sigma,b1}\phi_{\sigma',b1}\phi_{\sigma',b1}\rangle_{g_T} \times\exp\left(\frac{i\pi\rho\omega N}{4}\,\mathrm{Str}\,Q\Lambda\right);$$

$$\langle\cdots\rangle_{g_T}=\int d\Phi(\cdots)\exp\left[\frac{i}{2}E\Phi^\dagger L\Phi+pg_T(\Phi)\right]. \qquad (6)$$

The function $g_T(\Phi)\equiv g_0(\Phi^\dagger T^\dagger T\Phi;\Phi^\dagger L\Phi)$ satisfies the integral equation

$$g_T(\Psi)=\langle[h_F(\Phi^\dagger L\Psi)-1]\rangle_{g_T}\ , \qquad (7)$$

where $h_F(t)=\int dz\ e^{-itz}h(z)$ is the Fourier transform of the distribution of nonzero elements of the SRM. The 8×8 supermatrices T satisfy the condition $T^\dagger LT=L$, where $L=\mathrm{diag}(1,1,1,1,-1,-1,1,1)$, and belong to a graded coset space whose explicit parametrization can be found in Refs. 15 and 28. The supermatrices Q are expressed in terms of T as $Q=T^{-1}\Lambda T$. At last, the matrix $\Lambda=\mathrm{diag}(1,1,1,1,-1,-1,-1,-1)$, and the density of states ρ is expressed in terms of the solution of the equation Eq. (7) as $\rho(E)=-2g_{0x}/(\pi B_2)$, where $B_2=\int dz\ h(z)z^2$, $g_{0x}=\partial g_0(x,y)/\partial x|_{x,y=0}$, $x=\Phi^\dagger\Phi$, and $y=\Phi^\dagger L\Phi$.

When deriving Eq. (6), an evaluation of a functional integral by the saddle-point method has been employed; see details in Refs. 17 and 23. An accurate consideration shows that such a procedure is legitimate as long as (i) the matrix size N (playing in our model the role of the volume) is large

enough [much larger than the coefficient $C(E)$ determining the size dependence of IPR, see above]; and (ii) the energy difference ω is small enough [much smaller than $C^{-1}(E)$]. Though $C(E)$ is exponentially large near the transition point, it depends on the energy E only, so that when we keep E fixed and increase the system size N, the number of levels in the interval $C^{-1}(E)$ becomes arbitrarily large, since the level spacing scales as $1/N$.

Expanding both sides of Eq. (7) over Ψ, one can express $\langle \phi_{\sigma,b1}\phi_{\sigma,b1}\phi_{\sigma',b1}\phi_{\sigma',b1}\rangle_{g_T}$ in terms of the matrix Q as

$$\langle \phi_{\sigma,b1}\phi_{\sigma,b1}\phi_{\sigma,b1}\phi_{\sigma,b1}\rangle_{g_T} = \frac{4!}{B_4}\left[\frac{1}{2}g_{0,xx}Q^{\sigma\sigma}_{b_1b_1}Q^{\sigma\sigma}_{b_1b_1}+g_{0,xy}Q^{\sigma\sigma}_{b_1b_1}+g_{0,yy}\right] ,$$

$$\langle \phi_{R,b1}\phi_{R,b1}\phi_{A,b1}\phi_{A,b1}\rangle_{g_T} = -\frac{4}{B_4}[g_{0,xx}(Q^{RR}_{b_1b_1}Q^{AA}_{b_1b_1}+2Q^{RA}_{b_1b_1}Q^{AR}_{b_1b_1}) + g_{0,xy}(Q^{RR}_{b_1b_1}Q^{AA}_{b_1b_1})+g_{0,yy}] , \quad (8)$$

where $g_{0,xx}=\partial^2 g_0/\partial x^2|_{x,y=0}$, $g_{0,yy}=\partial^2 g_0/\partial y^2|_{x,y=0}$, $g_{0,xy}=\partial^2 g_0/\partial x\partial y|_{x,y=0}$, and $B_4=\int dz\ h(z)z^4$. This allows us to represent the right-hand side of Eq. (5) in the following form:

$$2\pi^2[\Delta^{-1}\alpha(r,E)\delta(\omega)+\Delta^{-2}\tilde{R}(\omega)\sigma(r,E,\omega)] = -\frac{4}{B_4}g_{0xx}\,\mathrm{Re}\langle(Q^{RR}_{b_1b_1}Q^{AA}_{b_1b_1}+2Q^{RA}_{b_1b_1}Q^{AR}_{b_1b_1} - \tfrac{1}{2}[Q^{RR}_{b_1b_1}Q^{RR}_{b_1b_1}+Q^{AA}_{b_1b_1}Q^{AA}_{b_1b_1}])\rangle_Q , \quad (9)$$

where

$$\langle\cdots\rangle_Q=\int dQ(\cdots)\exp\left(\frac{i\pi\rho\omega N}{4}\,\mathrm{Str}\,Q\Lambda\right).$$

The integrals over Q matrices are the standard ones,[15] yielding

$$\mathrm{Re}\langle Q^{RR}_{b_1b_1}Q^{AA}_{b_1b_1}\rangle_Q=1-2R_2^{(0)}(\omega/\Delta),$$

$$\langle Q^{RA}_{b_1b_1}Q^{AR}_{b_1b_1}\rangle_Q=-\frac{2i\Delta}{\pi(\omega+i0)}, \quad \langle Q^{RR}_{b_1b_1}Q^{RR}_{b_1b_1}\rangle_Q = \langle Q^{AA}_{b_1b_1}Q^{AA}_{b_1b_1}\rangle_Q=1 , \quad (10)$$

where $R_2^{(0)}(\omega/\Delta)$ is the level correlation function in the Gaussian orthogonal ensemble. Substituting this into Eq. (9), we finally find

$$\sigma(r,E,\omega)=\tfrac{1}{3}\,\alpha(r,E)=\frac{1}{N^2}\,\frac{4g_{0,xx}}{\pi^2\rho^2B_4}. \quad (11)$$

The coefficient $\frac{1}{3}$ in Eq. (11) corresponds to the case of unbroken time-reversal symmetry (orthogonal ensemble). For the unitary ensemble (broken time reversal symmetry) the same consideration yields the coefficient $\frac{1}{2}$ instead, so that the general relation reads

$$\sigma(r,E,\omega)=\frac{\beta}{\beta+2}\,\alpha(r,E) , \quad (12)$$

where β is the conventional symmetry parameter equal to $\beta=1$ (2) for the orthogonal (unitary) ensembles. This relation between the overlap of two different eigenfunctions $\sigma(r,E,\omega)$ and self-overlap $\alpha(r,E)$ constitutes the main result of the present publication. It is valid *everywhere* in the phase of extended eigenstates, up to the mobility edge $E=E_c$, provided the number of sites (the system volume) exceeds the correlation volume. In particular, it is valid in the critical region $|E-E_c|\ll E_c$, where a typical eigenfunction is very sparse and self-overlap (hence the IPR) grows like $\exp(\,\mathrm{const}|E-E_c|^{-1/2})$.[23]

Equation (12) implies the following structure of eigenfunctions within an energy interval $\delta E=\omega<C^{-1}(E)$. Each eigenstate can be represented as a product $\Psi_i(r)=\psi_i(r)\Phi_E(r)$. The function $\Phi_E(r)$ is an eigenfunction envelope of ''bumps and dips'' which is smooth on a microscopic scale comparable with the lattice constant. It is the same for all eigenstates around energy E, reflects underlying gross (multifractal) spatial structure, and governs the divergence of self-overlap at the critical point. In contrast, $\psi_i(r)$ is Gaussian white-noise component fluctuating in space on the scale of the lattice constant. It fills in the ''smooth'' component $\Phi_E(r)$ in an individual way for each eigenfunction, but is not critical, i.e., is not sensitive to the vicinity of the Anderson transition. These Gaussian fluctuations are responsible for the factor $\beta/(\beta+2)$ (which is the same as in the corresponding Gaussian ensemble) in Eq. (12).

As was already mentioned, this picture is valid in the energy window $\delta E\sim C^{-1}(E)$ around the energy E, the number of levels in this window being large as $\delta E/\Delta\sim NC^{-1}(E)\gg 1$ in the thermodynamic limit $N\to\infty$. These states form a kind of Gaussian ensemble on a spatially nonuniform (multifractal for $E\to E_c$) background $\Phi_E(r)$. Since the eigenfunction correlations are described by formula (12), which has exactly the same form as in the Gaussian ensemble, it is not surprising that the level statistics has the WD form everywhere in the extended phase.[17]

We believe on physical grounds that the same picture should hold for a conventional d-dimensional conductor. First of all, the general mechanism of the transition is the same in $d<\infty$ and $d=\infty$ models. Furthermore, the sparsity (multifractality) of eigenstates near the transition point takes its extreme form for $d=\infty$ models,[24] so that, since strong correlations (12) take place at $d=\infty$ it would be very surprising if they do not hold at finite d as well. Finally, Eq. (12) was proven by an explicit calculation in the weak-localization regime,[27] where $\sigma(\mathbf{r},E,\omega)=[\beta/(\beta+2)]\,\alpha(\mathbf{r},E)=V^{-2}[1+\Pi(\mathbf{r},\mathbf{r})]$, with V being the system volume and $\Pi(\mathbf{r},\mathbf{r})$ the diffusion propagator.

Replacing $C(E)$ by the d-dimensional correlation volume $\sim\xi^d$, we conclude that, for E close to E_c, Eq.(12) should be valid for $\omega<\Delta_\xi$, where $\Delta_\xi\propto 1/\xi^d$ is the level spacing in the correlation volume. For larger ω, $\sigma(\mathbf{r},E,\omega)$ is expected to decrease as $\omega^{-\eta/d}$ according to the scaling arguments,[2,4,12] so that we find $\sigma(\mathbf{r},E,\omega)/\alpha(\mathbf{r},E)\sim(\omega/\Delta_\xi)^{-\eta/d}$, up to a numerical coefficient of order of unity. Again, for any value of the energy E in the delocalized phase, taking the system size

L large enough, $L \gg \xi$, we have a large number of levels $\delta E/\Delta \sim \Delta_\xi/\Delta \propto (L/\xi)^d$ in the energy window δE, where Eq. (12) holds, so that the level correlation will be of the WD form.

Finally, let us consider what happens when we go from the critical regime (ξ large, but $L \gg \xi$) to the critical point ($\xi \gg L$). For this purpose, let us keep the system size L fixed and change the energy toward E_c, so that ξ increases. When ξ is comparable to the system size, $\xi \sim L$, we have $\Delta_\xi \sim \Delta$. This is the border of applicability of the above consideration. Correspondingly, we find

$$\sigma(\boldsymbol{r},E,\omega)/\alpha(\boldsymbol{r},E) \sim 1, \quad \omega < \Delta, \tag{13}$$

and $\sigma(\boldsymbol{r},E,\omega)/\alpha(\boldsymbol{r},E) \sim (\omega/\Delta)^{-\eta/d}$ for $\omega > \Delta$. When E further approaches E_c, the correlation length $\xi \gg L$ becomes irrelevant, so that these results will hold in the critical point ($\xi = \infty$). Of course, Eq. (13) is not sufficient to ensure the WD statistics in the critical point, since there is only of order of one level within its validity range $\delta E \sim \Delta$. Indeed, the numerical simulations show that the level statistics on the mobility edge is different from the WD one.[1,3,5–7]

However, Eq. (13) allows us to make an important conclusion concerning the behavior of $R_2(\omega)$ at small $\omega < \Delta$, or, what is essentially the same, the behavior of the nearest-neighbor spacing distribution $P(s)$, $s = \omega/\Delta$, at $s < 1$. For this purpose, it is enough to consider only two neighboring levels. Let their energy difference be $\omega_0 \sim \Delta$. Let us now perturb the system by a random potential $V(\boldsymbol{r})$ with $\langle V(\boldsymbol{r}) \rangle = 0$ and $\langle V(\boldsymbol{r}) V(\boldsymbol{r}') \rangle = \Gamma \delta(\boldsymbol{r} - \boldsymbol{r}')$. For the two-level system this reduces to a 2×2 matrix $\{V_{ij}\}, i,j = 1,2$, with elements $V_{ij} = \int d^d \boldsymbol{r}\ V(\boldsymbol{r}) \Psi_i^*(\boldsymbol{r}) \Psi_j(\boldsymbol{r})$. The crucial point is that the variances of the diagonal and off-diagonal matrix elements are, according to Eq. (13), equal to each other up to a factor of order of unity:

$$\langle V_{11}^2 \rangle / \langle |V_{12}^2| \rangle = \sigma(\boldsymbol{r},E,\omega)/\alpha(\boldsymbol{r},E) \sim 1\,. \tag{14}$$

The distance between the perturbed levels is given by $\omega = [(V_{11} - V_{22} + \omega_0)^2 + |V_{12}|^2]^{1/2}$. Choosing the amplitude of the potential in such a way that the typical energy shift $V_{11} \sim \Delta$, and using Eq. (14) and the standard symmetry consideration, we immediately conclude that in the critical point $P(s) \simeq c_\beta s^\beta$ for $s \ll 1$ with a coefficient c_β of order of unity, in agreement with the numerical findings.[3,5,7]

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