

## Phonon-associated conductance through a quantum point contact

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By using an independent-boson model we study the electronic conductance through a quantum point contact in the presence of the electron-phonon interaction. We find that the phonon energy plays a crucial role in the quantum behavior of the conductance. [S0163-1829(97)06315-7]

Along with experimental improvements a better theoretical understanding of the fundamental characteristic of quantum transport has been developed.<sup>1</sup> Since Wees *et al.*<sup>2</sup> and Wharam *et al.*<sup>3</sup> discovered the phenomenon of conductance quantization of an electron moving through a short and narrow constriction between two wide electron-gas reservoirs, which is called a quantum point contact, there have been many theoretical investigations in this issue. Avishai and Band<sup>4</sup> presented the exact quantum-mechanical treatment of ballistic electron propagation through a quantum point contact. The calculated basic quantity is the transmission amplitude matrix  $t$ , from which the conductance is evaluated using the linear conductance formula. It shows that the conductance of a quantum constriction with width  $a$  and length  $L$  indeed approaches quantized values  $n(2e^2/h)$  as  $L \rightarrow \infty$ . For finite  $L$ , the conductance transition from the  $n$  to the  $(n+1)$  plateau is not abrupt but oscillatory. These oscillations are longitudinal wave resonances.

The quantum transport with dissipation and its correct theoretical treatment is an important issue. The unitarity condition (or the current conservation) leads to a feedback mechanism by which inelastic scattering processes change the probability of elastic scattering. This feedback mechanism is beyond the scope of simple perturbation theory.<sup>5</sup> In Ref. 6 Cai *et al.* presented an approach to calculate the one-dimensional (1D) electron tunneling with dissipation in an arbitrary barrier by using a solvable model for electron-phonon interaction. They indicated how the boundary conditions uniquely determine the transmitted current and the reflected current of an electron.

Until now to our knowledge the theoretical research only gave examples of phonon-resisted conductance<sup>7,8</sup> through a quantum wire or a quantum point contact. Meanwhile many examples of phonon-assisted tunneling in semiconductor superlattices,<sup>9,10</sup> quantum dots,<sup>11-14</sup> and resonant tunneling diode<sup>15</sup> and phonon-assisted resonant tunneling<sup>6,16,17</sup> through double-barrier devices are given. In this phenomenon photons or phonons open an additional transport channel in single-electron tunneling through the device.

In this paper we extend the approach of Cai *et al.* to study the effect of the electron-phonon interaction on conductance through a quantum constriction. Our results give a considerable modification to the quantized conductance of the device due to the electron-phonon interaction. We find that the high-energy LO phonon resists the motion of the electron, yet the low-energy phonon assists it and presents several anomalous behaviors.

Consider a noninteracting electron gas confined in a quantum point contact. We consider the quantum-mechanical motion of a single electron with effective  $m^*$  and the Fermi energy  $E$ . The quantum point contact is composed of two semi-infinite strips defined by  $(-\infty < x \leq 0, 0 \leq y \leq b)$  and  $(L \leq x < +\infty, 0 \leq y \leq b)$  and a finite narrower strip in between, defined by  $(0 < x < L, 0 \leq y \leq a)$  with  $a < b$ .

When an incident electron from the left strip (in region I,  $x < 0$ ) enters the narrow constriction (region II) and is scattered by the phonon field, it then arrives at the right strip (in region III,  $x > L$ ). Since we are interested in the phonon effect inside the narrow constriction, the Hamiltonian of the electron-phonon interaction has the form

$$H_{\text{int}} = \sum_{\mathbf{q}} [M(\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{R}-i\omega t}a_{\mathbf{q}} + \text{H.c.}] \Theta(x) \Theta(L-x), \quad (1)$$

where  $a_{\mathbf{q}}$  and  $a_{\mathbf{q}}^+$  are the phonon annihilation and creation operators, respectively,  $M(\mathbf{q})$  the electron-phonon scattering matrix, and  $\mathbf{R}$  the electron position. The electron-phonon interaction occurs only in the finite narrow constriction. In the following, we use a model that replaces  $e^{i\mathbf{q}\cdot\mathbf{R}}$  by 1 in  $H_{\text{int}}$ . The model is similar to the independent-boson model<sup>18</sup> used to describe some relaxation phenomena. Thus the Schrödinger equation for the electron moving in the narrow constriction is given by ( $\hbar = 1$ )

$$i \frac{\partial}{\partial t} \Psi^{\text{II}} = \left\{ -\frac{1}{2m^*} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + V_c(y) + B e^{-i\omega t} + B^\dagger e^{+i\omega t} \right\} \Psi^{\text{II}}, \quad (2)$$

where  $B = \sum_{\mathbf{q}} M(\mathbf{q}) a_{\mathbf{q}}$  and  $B^\dagger = \sum_{\mathbf{q}} M^*(\mathbf{q}) a_{\mathbf{q}}^\dagger$ ,  $V_c(y)$  is an infinite-square-well confining potential with width  $a$ . By neglecting the effect of the electron-phonon interaction on the phonon system, we assume the phonons are the independent bosons and in equilibrium with a heat bath, so that Eq. (2) can be separable in space and time. For an incident electron at energy  $E$  in the mode  $m$ , the general solution of Eq. (2) can be read as

$$\Psi_m^{\text{II}}(x, y, t) = \exp\left\{\frac{B}{\omega} e^{-i\omega t} - \frac{B^\dagger}{\omega} e^{i\omega t}\right\} \int \sum_i \{u_{im}(E') \times \phi_{i,E'}^{(+)}(x, y) + v_{im}(E') \phi_{i,E'}^{(-)}(x, y)\} e^{-iE't} dE', \quad (3)$$

in which  $\phi_{i,E'}^{(+)}(x, y)$  and  $\phi_{i,E'}^{(-)}(x, y)$  are independent eigenfunctions at eigenvalues  $E'$  without phonons. For the steady state of an injecting electron at energy  $E$  in the mode  $m$ , only components of  $\Psi_m^{\text{II}}(x, y, t)$  with energy  $E + n\omega$  ( $n=0, \pm 1, \pm 2, \dots$ ) exist. The wave function in region II is written as

$$\Psi_m^{\text{II}}(x, y, t) = \sum_n \Psi_{m,E+n\omega}^{\text{II}}(x, y) \exp[-i(E+n\omega)t], \quad (4)$$

with

$$\Psi_{m,E+n\omega}^{\text{II}}(x, y) = \sum_j \sum_k \frac{(B/\omega)^k (-B^\dagger/\omega)^{j-n+k}}{k! (j-n+k)!} \times \sum_{i=1}^j \{u_{im}(j) \phi_{i,j}^{(+)}(x, y) + v_{im}(j) \phi_{i,j}^{(-)}(x, y)\}, \quad (5)$$

where the index  $j$  corresponds to energy  $E + j\omega$  ( $j=0, \pm 1, \pm 2, \dots$ ),  $k \geq 0$ ,  $j-n+k \geq 0$ . Two independent eigenfunctions,  $\phi_{i,j}^{(+)}(x, y)$  and  $\phi_{i,j}^{(-)}(x, y)$ , are given by

$$\phi_{i,j}^{(\pm)}(x, y) = \sqrt{\frac{2}{a}} \sin\left(\frac{i\pi y}{a}\right) e^{\pm iq_i(j)x}, \quad (6)$$

where the wave numbers  $q_i(j)$  are

$$q_i(j) = \left[2m^* \left(E + j\omega + \frac{\Delta}{2}\right) - \left(\frac{i\pi}{a}\right)^2\right]^{1/2}. \quad (7)$$

In Eq. (7),  $i\pi/2$  is a constant energy shift due to the polaron effect. The magnitudes of  $u_{im}(n)$  and  $v_{im}(n)$  have of the order of  $|B|^{|n|}$ . With the condition that the electron-phonon interaction is weak, a cutoff of  $|n|$  up to  $N$  will assure that the transmitted and reflected currents is accurate up to  $|B|^{2N}$ . For the  $n$ th branch, the terms of Eq. (5) should be maintained to the order of  $|B|^{2N-|n|}$ . The wave function in region II can be obtained at channels  $E$  and  $E \pm \omega$  with accuracy up to  $|B|^2$ .

For computing the current (or conductance) with an accuracy up to  $|B|^2$ , we should keep the following terms:

$$\Psi_{m,E+\omega}^{\text{II}}(x, y) \approx \sum_{i=1}^J \left[ \frac{B}{\omega} f_{im}(0) + f_{im}(1) \right] \sqrt{\frac{2}{a}} \sin\left(\frac{i\pi y}{a}\right), \quad (8)$$

$$\Psi_{m,E}^{\text{II}}(x, y) \approx \sum_{i=1}^J \left[ \left(1 - \frac{BB^\dagger}{\omega^2}\right) f_{im}(0) + \frac{B}{\omega} f_{im}(1) - \frac{B^\dagger}{\omega} f_{im}(-1) \right] \sqrt{\frac{2}{a}} \sin\left(\frac{i\pi y}{a}\right),$$

$$\Psi_{m,E-\omega}^{\text{II}}(x, y) \approx \sum_{i=1}^J \left[ -\frac{B^\dagger}{\omega} f_{im}(0) + f_{im}(-1) \right] \times \sqrt{\frac{2}{a}} \sin\left(\frac{i\pi y}{a}\right),$$

with

$$f_{im}(n) = u_{im}(n) e^{iq_i(n)x} + v_{im}(n) e^{-iq_i(n)x}.$$

In region I and region III, the electrons are free to move in the  $x$  direction, but are confined in the  $y$  direction.

As an incoming initial electron in mode  $m$  enters the system from the left, the wave function in region I is read as

$$\Psi_m^{\text{I}}(x, y, t) = \sqrt{\frac{2}{b}} \sin\left(\frac{m\pi y}{b}\right) e^{ik_m(0)x - iEt} + \sum_{m'=1}^M \sqrt{\frac{2}{b}} \sin\left(\frac{m'\pi y}{b}\right) \times \sum_n R_{m'm}(n) e^{-ik_{m'}(n)x - i(E+n\omega)t}. \quad (9)$$

The transmitted wave exists in region III only and its wave function is read as

$$\Psi_m^{\text{III}}(x, y, t) = \sum_{m'=1}^M \sqrt{\frac{2}{b}} \sin\left(\frac{m'\pi y}{b}\right) \times \sum_n T_{m'm}(n) e^{ik_{m'}(n)x - i(E+n\omega)t}, \quad (10)$$

where the wave numbers  $k_m(n)$  are given by

$$k_m(n) = \left[2m^*(E+n\omega) - \left(\frac{m\pi}{b}\right)^2\right]^{1/2}. \quad (11)$$

Here  $R_{m'm}(n)$  and  $T_{m'm}(n)$  ( $m', m=1, 2, \dots, M$ ) are the reflection and transmission amplitudes due to a process with emission (or absorption) of  $n$  phonons, and are  $M$ -dimensional matrices. For the  $n$ th branch of the wave function corresponding to an electron with energy  $E+n\omega$ , the number  $M$  contains all conducting modes for which the wave number  $k_m(n)$  are real (i.e.,  $m < M_c$ ) and evanescent modes for which the wave numbers  $k_m(n)$  are imaginary (i.e.,  $M_c < m < M$ ). The number of evanescent modes is sufficient to guarantee convergence with desired accuracy. The role of evanescent modes in the narrow constriction is even more important. Therefore, we set  $J > a/\pi \sqrt{2m^*(E+\omega+\Delta/2)}$  in Eq. (8) and fix  $J$  so that convergence is assured.

In order to calculate the transmission and reflection matrices  $T(M \times M)$  and  $R(M \times M)$  and the unknown matrices

$U(J \times M)$  and  $V(J \times M)$ , we should match the wave function and its derivative with respect to  $x$  at  $x=0$  and  $L$ . It is useful to define the matrices

$$I_{[M \times M]} = \{\delta_{mm'}\}, \quad K(n)_{[M \times M]} = \{k_m(n) \delta_{mm'}\},$$

$$Q(j)_{[J \times J]} = \{q_i(j) \delta_{ii'}\}, \quad (12)$$

and the matrix  $A_{[M \times J]}$  of overlap integrals

$$A_{mi} = \frac{2}{\sqrt{ab}} \int_0^a \sin\left(\frac{m\pi y}{b}\right) \sin\left(\frac{i\pi y}{a}\right) dy. \quad (13)$$

We obtain the following equations for the  $n$ th branch with energy  $E + n\omega$ :

$$I\delta_{n0} + R(n) = \sum_j f(j-n)A[U(j) + V(j)], \quad (14)$$

$$K(n)[\delta_{n0} - R(n)] = \sum_j f(j-n)AQ(j)[U(j) + V(j)],$$

$$T(n) = \sum_j f(j-n)A[e^{iQ(j)L}U(j) + e^{-iQ(j)L}V(j)],$$

$$K(n)T(n) = \sum_j f(j-n)AQ(j)[e^{iQ(j)L}U(j) - e^{-iQ(j)L}V(j)],$$

with

$$f(j-n) = \sum_k \frac{(B/\omega)^k}{k!} \frac{(-B^\dagger/\omega)^{j-n+k}}{(j-n+k)!}. \quad (15)$$

These equations consist of four coupled matrix equations through which components of the wave function in different energy branches couple with each other due to phonon emission and absorption. To express the current conservation and the Landauer conductance formula of an incident electron in propagating mode  $m$ , the flux normalized transmission and reflection amplitudes are defined as

$$r_{m'm}(n) = [k_{m'}(n)/k_m(0)]^{1/2} R_{m'm}(n), \quad (16)$$

$$t_{m'm}(n) = [k_{m'}(n)/k_m(0)]^{1/2} T_{m'm}(n).$$

We can solve Eq. (14) to determine first  $U(n)$  and  $V(n)$ , then  $T(n)$  and  $R(n)$  ( $n=0, \pm 1, \pm 2, \dots$ ) by using serial substitutions. As the zero-order approximation of the phonon operator, i.e., by neglecting the electron-phonon interaction, the following matrix equation for the unknown  $U(0)$  and  $V(0)$  can be derived from Eq. (14):

$$\begin{pmatrix} A^T K(0)A + Q(0), & A^T K(0)A - Q(0) \\ [A^T K(0)A - Q(0)]e^{iQ(0)L}, & [A^T K(0)A + Q(0)]e^{-iQ(0)L} \end{pmatrix} \begin{pmatrix} U(0) \\ V(0) \end{pmatrix} = \begin{pmatrix} 2A^T K(0) \\ 0 \end{pmatrix}, \quad (17)$$

where  $A^T$  is the transpose of  $A$ . The matrices  $T^{[0]}(0)$  and  $R^{[0]}(0)$  are given in terms of  $U(0)$  and  $V(0)$ ,

$$T^{[0]}(0) = A[e^{iQ(0)L}U(0) + e^{-iQ(0)L}V(0)],$$

$$R^{[0]}(0) = A[U(0) + V(0)] - I. \quad (18)$$

Then, electronic conductance  $G_0$  in the absence of the electron-phonon interaction is obtained from the multichannel Landauer formula

$$G_0 = \frac{2e^2}{h} T_r[\langle t^{[0]}(0)t^{[0]\dagger}(0) \rangle]. \quad (19)$$

In this case, unitarity relations for the reflection and transmission amplitudes are maintained within high accuracy,

$$\sum_{m'=1}^M (\langle |t_{m'm}^{[0]}(0)|^2 \rangle + \langle |r_{m'm}^{[0]}(0)|^2 \rangle) = 1, \quad (20)$$

with

$$m = 1, 2, \dots, M,$$

where  $\langle \rangle$  means the average over the phonon assemble.

In order to consider the effect of the electron-phonon interaction on the conductance of a quantum constriction we study the phonon-associated transmission in which the incident electron absorbs (or emits) one phonon due to electron-phonon inelastic scattering. Substituting the solution of Eq. (17),  $U(0)$  and  $V(0)$ , into Eq. (14) for the branch with energy  $E + \omega$ , we can obtain the matrix equation for the unknown matrices  $U(1)$  and  $V(1)$ :

$$MT(1,1) \begin{pmatrix} U(1) \\ V(1) \end{pmatrix} = -\frac{B}{\omega} MT(1,0) \begin{pmatrix} U(0) \\ V(0) \end{pmatrix}, \quad (21)$$

where a supermatrix  $MT(\alpha, \beta)$  has been defined:

$$MT(\alpha, \beta) \equiv \begin{pmatrix} A^T K(\alpha) A + Q(\beta) & A^T K(\alpha) A - Q(\beta) \\ [A^T K(\alpha) A - Q(\beta)] e^{iQ(\beta)L} & [A^T K(\alpha) A + Q(\beta)] e^{-iQ(\beta)L} \end{pmatrix}. \quad (22)$$

Because the  $U(+1)$  and  $V(+1)$  are of the order of  $|B|$ , they can be written as

$$\begin{aligned} U(1) &= -\frac{B}{\omega} U'(1), \\ V(1) &= -\frac{B}{\omega} V'(1). \end{aligned} \quad (23)$$

Here  $U'(+1)$  and  $V'(+1)$  are the numerical matrices, which obey the matrix equation

$$MT(1,1) \begin{pmatrix} U'(1) \\ V'(1) \end{pmatrix} = MT(1,0) \begin{pmatrix} U(0) \\ V(0) \end{pmatrix}. \quad (24)$$

The transmission and the reflection matrices that the incident electron absorbs one-phonon by inelastic scattering,  $T^{[1]}(1)$  and  $R^{[1]}(1)$ , are given by

$$\begin{aligned} T^{[1]}(1) &= \frac{B}{\omega} A [e^{iQ(0)L} U(0) + e^{-iQ(0)L} V(0) - e^{iQ(1)L} U'(1) \\ &\quad - e^{-iQ(1)L} V'(1)], \end{aligned} \quad (25)$$

$$R^{[1]}(1) = \frac{B}{\omega} A [U(0) + V(0) - U'(1) - V'(1)].$$

For the incident electron at  $E$  emitted one phonon, we can obtain the matrix equation in a similar way,

$$MT(-1,-1) \begin{pmatrix} U'(-1) \\ V'(-1) \end{pmatrix} = MT(-1,0) \begin{pmatrix} U(0) \\ V(0) \end{pmatrix}, \quad (26)$$

where both  $U'(-1)$  and  $V'(-1)$  are the unknown numerical matrices, and they are related to the unknown coefficients matrices,  $U(-1)$  and  $V(-1)$

$$U(-1) = \frac{B^\dagger}{\omega} U'(-1), \quad (27)$$

$$V(-1) = \frac{B^\dagger}{\omega} V'(-1).$$

The corresponding transmission and reflection matrices,  $T^{[1]}(-1)$  and  $R^{[1]}(-1)$ , are given as

$$\begin{aligned} T^{[1]}(-1) &= -\frac{B^\dagger}{\omega} A [e^{iQ(0)L} U(0) + e^{-iQ(0)L} V(0) \\ &\quad - e^{iQ(-1)L} U'(-1) - e^{-iQ(-1)L} V'(-1)], \end{aligned} \quad (28)$$

$$R^{[1]}(-1) = -\frac{B^\dagger}{\omega} A [U(0) + V(0) - U'(-1) - V'(-1)].$$

The electronic conductance due to one-phonon absorbed or emitted,  $G_p(\pm 1)$ , is given in terms of the flux normalized transmission matrices

$$G_p(\pm 1) = \frac{2e^2}{h} T_r[\langle t(\pm 1) t^\dagger(\pm 1) \rangle], \quad (29)$$

in which  $t(\pm 1) = t^{[1]}(\pm 1)$  with accuracy up to  $|B|$ . In the calculation we assume that phonons are in equilibrium at temperature  $T$ , so  $\langle B^\dagger B \rangle = \sum_q |M(q)|^2 n_q$  and  $\langle B B^\dagger \rangle = \sum_q |M(q)|^2 (1 + n_q)$ , with the phonon number  $n_q = [\exp(\omega/k_B T) - 1]^{-1}$ . In order to ensure current conservation, which leads to a feedback effect of inelastic scattering processes on the probability of elastic scattering, we shall calculate  $T(0)$  with accuracy up to  $|B|^2$  (one-phonon process). Substituting back  $U(\pm n)$  and  $V(\pm n)$  ( $n=0, \pm 1$ ), into Eq. (14) we get the expression of  $T^{[2]}(0)$

$$\begin{aligned} T^{[2]}(0) &= \frac{B B^\dagger}{\omega^2} A [e^{iQ(-1)L} U'(-1) + e^{-iQ(-1)L} V'(-1)] \\ &\quad - \frac{B B^\dagger}{\omega^2} A [e^{iQ(0)L} U(0) + e^{-iQ(0)L} V(0)] \\ &\quad + \frac{B^\dagger B}{\omega^2} A [e^{iQ(1)L} U'(1) + e^{-iQ(1)L} V'(1)], \end{aligned} \quad (30)$$

and

$$T(0) = T^{[0]}(0) + T^{[2]}(0). \quad (31)$$

The transmission and reflection currents of a incident electron in propagating mode  $m$  at  $E$  are obtained from the transmission and the reflection matrices of branches with energy  $E$  and  $E \pm \omega$ .

The conductance of the elastic channel related to the feedback effect,  $G_p(0)$ , is given by

$$G_p(0) = \frac{2e^2}{h} \sum_{m=1}^{M_c} \sum_{m'=1}^{M_c} \langle t_{m'm}(0) t_{m'm}^\dagger(0) \rangle, \quad (32)$$

where the sum in Eq. (32) runs over only the propagating modes of the wire. The total conductance in the presence of the electron-phonon interaction is

$$G_T = G_p(0) + G_p(1) + G_p(-1). \quad (33)$$

In the presence of the electron-phonon interaction the unitarity relations for the reflection and transmission amplitudes have much more complicated forms than Eq. (20) due to the mixture between components of the wave function in different energy channels.

Figure 1 presents the influence of the electron-phonon interaction on conductance quantization. The conductance  $G$  (in units of  $2e^2/h$ ) is plotted as functions of the Fermi energy  $E_F$  (in units of meV) of the incident electron in the quantum point contact at temperature  $T = 30$  K with structure

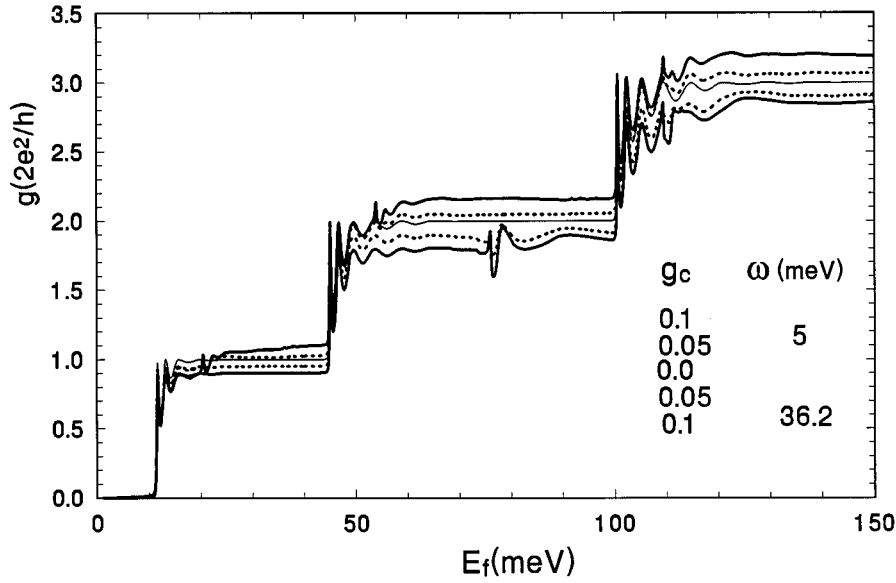


FIG. 1. Conductance  $G$  (in units of  $2e^2/h$ ) as functions of the Fermi energy  $E_F$  (in units of meV) of the incident electron at temperature 30 K with  $b=1000$  Å,  $a=300$  Å,  $L=2000$  Å. The solid (dotted) curves is for  $g \equiv \sum_q [|M(q)|/\omega]^2 = 0.1(0.05)$ . The upper two curves are for  $\omega=5$  meV, the low two curves are for 36 meV.

parameters  $b=1000$  Å,  $a=300$  Å, and  $L=2000$  Å. In the figure there are five curves. The solid (dotted) curves are for the electron-phonon coupling constant  $g \equiv \sum_q [|M(q)|/\omega]^2 = 0.1(0.05)$ . We can find the top two curves are for low-energy phonon,  $\omega=5$  meV. The bottom two curves are for high-energy LO phonon, 36 meV, which is the LO phonon in AsGa. The middle thin curve is for the case without the electron-phonon interaction. It is obvious that the interaction between the electron and LO phonon resists the conductance in AsGa, but the low-energy phonon may assist the conductance.

Figure 2 displays the conductance as functions of width  $a$  (in units of Å) of the quantum constriction at temperature  $T=30$  K, Fermi energy  $E_F=65$  meV, and the electron-phonon coupling constant  $g_c=0.1$  with  $b=1000$  Å. The solid (dotted) curves are for  $L=2000$  Å (500 Å). We can also find the high-energy LO phonon, 36 meV, causes the lowest step of conductance, which is degenerate for different lengths  $L=2000$  and 500 Å. Contrarily, a long sample may have large conductance steps for the low-energy phonon,  $\omega=5$  meV.

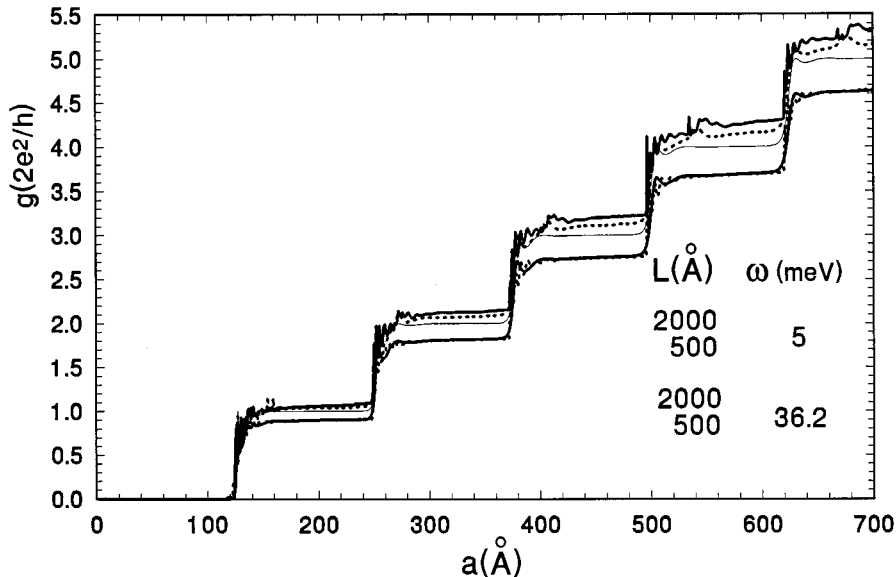


FIG. 2. Conductance as functions of the width  $a$  (in units of Å) at temperature  $T=30$  K, Fermi energy  $E_F=65$  meV, the electron-phonon interaction  $g_c=0.1$ , and  $b=1000$  Å. The solid (dotted) curve is for  $L=2000$  Å (500 Å). The thin solid curve is for the case without the electron-phonon interaction. The sample length has no effect on the conductance for high-energy LO phonon.

Figure 3 shows the temperature behavior of the conductance. The quantum steps of the conductance are plotted as functions of the Fermi energy for different temperatures, and phonons in different energies. Similar to Fig. 2 the LO phonon, 36 meV, causes the lowest step of conductance, which is degenerate for different temperatures  $T=40$  and 10 K. As to the case of the low-energy phonon,  $\omega=5$  meV, the high temperature (40 K) causes large conductance steps (the solid thick curve).

This is similar to the situation in barrier-well quantum structures,<sup>6,9-17</sup> where the additional transport channels assist the electron motion through the structures, in which the transverse confinement in the quantum point structures may open transverse channels in the electron transport that assist the conductance. Our calculation results show that the crucial point here is the suitable phonon energy (for example, in our model  $\omega=5$  meV) for the enhancement in the conductance. Meanwhile several quantum behaviors also can be seen under the low-energy phonon condition. On the contrary if the condition is not met, the phonon-electron interaction will resist the conductance through a quantum region.<sup>7,8</sup>

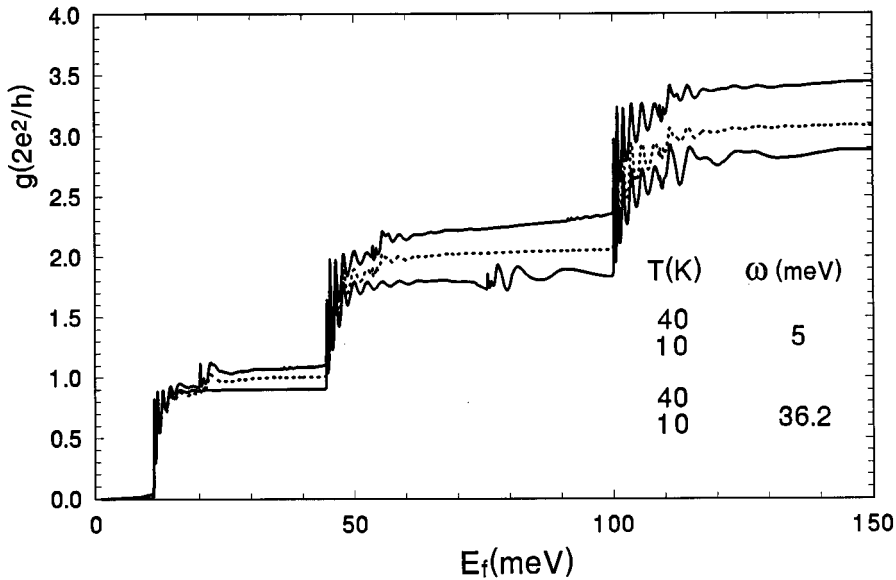


FIG. 3. The temperature behavior of the conductance with  $b=1000 \text{ \AA}$ ,  $a=1000 \text{ \AA}$ ,  $L=2000 \text{ \AA}$ , and  $g_c=0.10$ . The solid (dotted) curve is for  $T=40 \text{ K}$  ( $10 \text{ K}$ ). The temperature has no effect on the conductance for high-energy LO phonon.

In summary, an independent-boson model is used to calculate the electronic conductance through a quantum point contact in the presence of the electron-phonon interaction. We have found that the effect of the electron-phonon interaction on the conductance depends crucially on the energy of the phonon. The high-energy LO phonon resists the motion of the electron; the sample length and the temperature have almost no effect on the ballistic transmission. On the contrary, the low-energy phonon, for example,  $\omega=5 \text{ meV}$ , assists the motion of the electron; the big electron-phonon in-

teraction, long sample, and high temperature ( $40 \text{ K}$ ) may cause large conductance steps.

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