

Resonance impurity states in a quantum well in the presence of electric and strong magnetic fields

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An analytical approach to the problem of an impurity electron (or hole) in a quantum well (QW), subject to electric and strong magnetic external fields both directed perpendicular to the heteroplanes, is developed. The impurity center is located at the edge of the QW. It is shown that the combined potential acting on the electron (or hole) resembles that of a double quantum well. One of the wells is formed by the Coulomb impurity potential and the QW boundary at which the center is located and the other well by the electric-field potential and the other boundary of the well. Analytical expressions for the energy levels are obtained. Our main interest is in the resonance when the levels associated with the two effective QW's anticross. The explicit dependences of the resonance splitting upon the width of the QW and on the magnitudes of the electric and magnetic fields are obtained. Estimates of the expected splittings are made using the usual parameters associated with GaAs QW's. [S0163-1829(97)03216-5]

I. INTRODUCTION

The problem of resonance devices based on impurity low-dimensional heterostructures has attracted considerable attention in recent years. The reason is that advances made in growth techniques and control by the development of molecular-beam epitaxy and metal organic chemical vapor deposition methods of manufacturing specific structures have increased considerably the amount of available experimental data. Similarly, it has become clear that a study of impurities in such systems is important because the optical and transport properties of devices made from these materials are strongly affected by the presence of shallow impurities.

Numerous experimental and theoretical papers have been written within the past decade concerning the effects of electric and magnetic fields acting on heterostructures containing impurities. Comprehensive summaries have been given in Santiago *et al.*¹ and Shi, Peeters, and Devresse² for example. Much of this work has been concerned with single quantum well (QW) structures. Greene and Bajaj^{3,4} and Greene and Lane⁵ studied the effects of magnetic fields by using a variational method. This method has been used also by Cen and Bajaj to study impurity states in symmetric⁶ and asymmetric⁷ QW's subject to parallel electric and magnetic fields directed perpendicular to the heteroplanes.

The majority of papers on this subject describe numerical calculations. Nevertheless, analytical methods of studying the effects of impurities in QW's subject to external fields are of much interest because they enable the basic physics of the problem to be kept clearly in view throughout the analysis. The authors have been involved in an analytical method which was devised originally for the problem of impurity states in a QW subject to a strong magnetic field.^{8,9} Effects on impurity states in a QW subject to parallel electric and strong magnetic fields directed perpendicular to the layers have been studied analytically^{10,11} by taking the QW to be

narrow compared to the radius of the impurity electron state.

In this paper, this analytical approach is extended to a QW system with the impurity at the edge of the QW. Although this system potentially has very interesting and useful properties, it does not appear to have been considered before. The combination of the lack of inversion symmetry in the electric field and the positioning of the impurity at the edge of the QW together generate a type of resonance structure in which the combined potential governing the electron states closely resembles that of a double quantum well. One of the wells is formed by the Coulomb impurity potential and the QW boundary at which the impurity is located; the other well is constructed from the electric-field potential and the other boundary of the QW. It is shown that such a system contains electron states which have a quasi-Coulomb character and are similar to size-quantized states. The study of such systems is important as properties such as tunneling times and optical transition strengths will be strongly influenced by such impurities.

The explicit dependencies of the impurity electron energies upon the magnitudes of the magnetic and electric fields and the width of the QW are obtained. Attention is focused on the resonance between the levels associated with the two effective wells so that tunneling of the electron through the barrier separating the effective wells becomes possible. It is shown that these levels anticross. Simple analytical expressions for the resonance splitting of the levels are derived. The energy gap between the resonance levels has a strong effect on the tunneling time and results in an alteration in the spatial distribution of the wave function. This in turn defines the kinetic and optical properties of such semiconductor structures in the presence of external fields.

II. GENERAL THEORY

The z axis is chosen to lie along the direction of the parallel uniform magnetic \mathbf{B} and electric \mathbf{E} fields that are

directed perpendicular to the heteroplanes and the QW is modeled by an infinite square well of width d . This analysis concentrates on the problem in which the impurity is positioned at the edge of the QW. In this case, the potential barrier caused by the electric field \mathbf{E} has the largest effect and has a major influence on the stability of the impurity states. The position of the impurity, which coincides with the left edge of the QW, is taken to be the point $z=0$. The other parameters relevant to the calculation are the impurity Bohr radius a_0 , the magnetic length a_B , and the dimensionless electric field s which is the electric field \mathbf{E} scaled relative to the impurity electric field. They are defined as usual by

$$a_0 = \frac{4\pi\epsilon\epsilon_0\hbar^2}{\mu e^2}, \quad a_B = \sqrt{\hbar/eB}, \quad s = \frac{\mathbf{E}}{\frac{1}{2}[e/4\pi\epsilon_0\epsilon a_0^2]},$$

where ϵ is the dielectric constant and μ and e are the effective mass and charge of the carrier, respectively. We assume further that the energy bands may be taken as spherically symmetric with a parabolic cross section and to be nondegenerate.

In the strong magnetic-field limit for which

$$a_B/a_0 \ll 1, \quad (2.1)$$

and in the effective-mass approximation, the longitudinal function $f^{(N,m)}(z)$ of the electron at a position $r(\boldsymbol{\rho}, z)$ for the case in which the electric field is directed along the negative z direction, satisfies the equation^{10,11}

$$\begin{aligned} -\frac{\hbar^2}{2\mu} \frac{d^2}{dz^2} f^{(N,m)}(z) + [V_{N,m}(z) - eEz] f^{(N,m)}(z) \\ = W_{N,m} f^{(N,m)}(z), \end{aligned} \quad (2.2)$$

with the boundary conditions

$$f^{(N,m)}(0) = 0, \quad f^{(N,m)}(d) = 0 \quad (2.3)$$

and with

$$V_{N,m}(z) = -\frac{e^2}{4\pi\epsilon_0\epsilon} \int \frac{|\chi_{\perp N,m}(\boldsymbol{\rho})|^2}{\sqrt{\rho^2 + z^2}} d\boldsymbol{\rho} \quad (2.4)$$

and $W_{N,m} = \mathcal{E} - \mathcal{E}_{\perp N,m}$ where \mathcal{E} is the energy of the impurity electron. In the above expressions, the function $\chi_{\perp N,m}(\boldsymbol{\rho})$ describes the transverse motion of the electron of energy $\mathcal{E}_{\perp N,m}$ in the magnetic field \mathbf{B} in the x - y plane. The Landau energies relative to the bottom of the conduction band are given by

$$\mathcal{E}_{\perp N,m} = \frac{\hbar e B}{2\mu} (2N + |m| + m + 1) \pm \mu_B B, \quad (2.5)$$

where μ_B is the effective Bohr magneton for the electron and the quantum numbers are N, m where $N = 0, 1, 2, \dots$ and $m = 0, \pm 1, \pm 2, \dots$.

In order to simplify the calculations, we consider only the ground transverse state for which $N = m = 0$ though the results obtained below will be valid qualitatively for any transverse state. The relevant transverse function has the form

$$\chi_{\perp 0,0}(\boldsymbol{\rho}) = \frac{1}{\sqrt{2\pi}a_B} \exp\left[-\frac{\rho^2}{4a_B^2}\right]. \quad (2.6)$$

Thus we may drop the subscripts from the variables related to the transverse motion and write V for $V_{0,0}$, f for $f^{(0,0)}$ and W for $W_{0,0}$. Note that Eq. (2.2) describes both Coulomb-type states ($W < 0$) and size-quantized states^{8,9} for which $W > 0$. It has been found that the levels of lowest energy have a quasi-Coulomb character under the condition $d \gg a_0$. These levels are affected strongly by external fields while the size-quantized levels depend only weakly upon the magnitudes of the fields and upon the impurity potential. Further, we consider the quasi-Coulomb states having energies $W < 0$.

In order to solve Eq. (2.2), it is convenient to introduce the notation

$$u = 2z/a_0\lambda, \quad g = 2\rho/a_0\lambda, \quad W_\lambda = -R/2\lambda^2,$$

where

$$R = e^2/4\pi\epsilon_0\epsilon a_0$$

is the impurity Rydberg constant. Equation (2.2) then becomes

$$\frac{d^2 f_\lambda(u)}{du^2} + \left(\lambda \langle 0 | (u^2 + g^2)^{-1/2} | 0 \rangle + \frac{\lambda^3}{8} su - \frac{1}{4} \right) f_\lambda(u) = 0, \quad (2.7)$$

where $\langle 0 | 0 \rangle$ is an average with respect to the function $\chi_{\perp 0,0}(\boldsymbol{\rho})$. The quantum number λ labels the states of the motion along the z axis. The transformation of coordinates affects the boundary conditions, which thus become

$$f_\lambda(0) = 0, \quad f_\lambda(2d/a_0\lambda) = 0. \quad (2.8)$$

The analysis of Eq. (2.7) will be based upon the Hasegawa-Howard method¹² together with a comparison equation.¹³

III. CALCULATIONS

Our approach to the problem is to consider the solutions to Eq. (2.7) for the three regions in turn and match them on the boundaries.

A. The left boundary of the QW (impurity center region)

Following the method developed in Ref. 8, under the condition

$$u \gg \langle 0 | g | 0 \rangle \sim 2a_B/a_0\lambda,$$

Eq. (2.7) transforms into the equation

$$\frac{d^2 f_\lambda(u)}{du^2} + \left(\frac{\lambda}{u} - \frac{1}{4} + \frac{\lambda^3}{8} su \right) f_\lambda(u) = 0. \quad (3.1)$$

In the zeroth approximation for which $s = 0$, we find that the two independent solutions to this equation are the Whittaker functions $W_{\lambda,1/2}$ and $M_{\lambda,1/2}$. The general solution is given by

$$f_\lambda(u) = A W_{\lambda,1/2}(u) + B M_{\lambda,1/2}(u), \quad (3.2)$$

where A and B are constants. In the region $u \ll 1$, an iteration method is performed by double integration of Eq. (2.7) using the trial functions satisfying the first boundary condition [from Eq. (2.8)],

$$f_{\lambda}^{(0)}(u) = \alpha u(u^2 + g^2)^{1/2} \ln[u + (u^2 + g^2)^{1/2}],$$

where α is a constant. A comparison of the coefficients is then made between the results of the integration taken for $u \gg \langle 0|g|0 \rangle$ and the standard expansion of the Whittaker functions involved in Eq. (3.2) for $u \ll 1$ (see, for example, Gradshteyn and Ryzhik¹⁴). When terms of the same order are equated, a set of two linear equations

$$A\Gamma^{-1}(-\lambda) + \frac{1}{4}\alpha\lambda\langle 0|g^2|0 \rangle = 0 \quad (3.3)$$

and

$$A\Gamma^{-1}(-\lambda)\varphi(\lambda) + B - \alpha\langle 0|g \ln|g|0 \rangle = 0 \quad (3.4)$$

result, where

$$\varphi(\lambda) = \psi(1-\lambda) + 1/2\lambda + 2C - 1 \quad (3.5)$$

and where $\Gamma(x)$ is the gamma function, $\psi(x)$ is the psi function (the logarithmic derivative of the gamma function), and C is the Euler constant ($= 0.577$).

In the next approximation, the effect of the electric field on the Coulomb energies W_{λ} can be calculated by the method of a comparison equation. (A detailed presentation of this method can be found in Ref. 13.) Following Ref. 13, the equation for the quantum number ν , which defines the impurity energy $W = -R(2\nu^2)^{-1}$, is given by

$$\int_0^{4\lambda} \left(\frac{\lambda}{u} - \frac{1}{4} \right)^{1/2} du = \int_0^{4\nu} \left(\frac{\nu}{u} + \frac{\nu^3}{8} su - \frac{1}{4} \right)^{1/2} du. \quad (3.6)$$

It follows from Eq. (3.6) that

$$\nu = \lambda \left[1 - \frac{3}{4}(s\lambda^4) \right]. \quad (3.7)$$

B. The region adjacent to the right-hand boundary of the QW

In this region, the effect of the electric field E overcomes the influence of the impurity center so that the comparison equation in place of Eq. (3.1) is the equation for the Airy functions Ai and Bi.¹⁵ The general solution to Eq. (3.1) can be written in the form

$$f_{\lambda}(u) = J \text{Ai}(-\eta) + K \text{Bi}(-\eta), \quad (3.8)$$

where J and K are constants. In the above expression, the following definitions have been made:

$$\eta(u)^{3/2} = \frac{3}{2} \int_{t_1}^u q(t) dt \quad \text{for } \eta > 0 \quad (3.9)$$

and

$$|\eta(u)|^{3/2} = -\frac{3}{2} \int_{t_1}^u |q(t)| dt \quad \text{for } \eta < 0, \quad (3.10)$$

where

$$q(t) = [\lambda/t - 1/4 + (\lambda^3/8)st]^{1/2} \quad (3.11)$$

and where t_1 is the greater root of the equation $q(t_1) = 0$.

C. The intermediate region

Under the condition

$$s\lambda^4 \ll 1, \quad (3.12)$$

within the region defined by $u \gg 1$, and $(\lambda^3/8) su \ll \lambda/u$, the functions (3.2) and (3.8) can be matched and thus tunneling between the two wells of the effective potential occurs. Under the condition (3.12), the arguments of the Airy functions $\eta(u)$ given in Eqs. (3.9) and (3.10) and involved in Eq. (3.8) are calculated in an explicit form. A comparison of the expression¹⁴ given in Eq. (3.2) for $u \gg 1$ and that obtained from Eq. (3.8) for the asymptotic region $\eta < 0$ with $|\eta| \gg 1$ is then made.¹⁵ When terms of the same form are equated, a set of two linear equations

$$B\Gamma^{-1}(1-\lambda) - \frac{1}{2}J\pi^{-1/2}(\lambda s^{1/3})^{1/2} e^{-\Phi(\lambda)} = 0 \quad (3.13)$$

and

$$A - K\pi^{-1/2}(\lambda s^{1/3})^{1/2} e^{\Phi(\lambda)} = 0 \quad (3.14)$$

result where

$$\Phi(\lambda) = 2/[3s\lambda^3] + \lambda \ln(s\lambda^3/8). \quad (3.15)$$

On substituting the wave function (3.8) into the second boundary condition (2.8), we have

$$J \text{Ai}(-\eta_0) + K \text{Bi}(-\eta_0) = 0, \quad (3.16)$$

where $\eta_0 = \eta(2d/a_0\lambda)$. Using expressions (3.9) and (3.10), the parameter η_0 can be written in an explicit form

$$\eta_0 = p \left[1 - \frac{2s\lambda^4}{1-2s\lambda^4} x^{-3/2} (x^{1/2} - \tan^{-1} x^{1/2}) \right] \quad \text{for } x > 0, \quad (3.17)$$

$$\eta_0 = -|p| \left[1 + \frac{2s\lambda^4}{1-2s\lambda^4} y^{-3/2} \left(y^{1/2} + \frac{1}{2} \ln \frac{1-y^{1/2}}{1+y^{1/2}} \right) \right] \quad \text{for } x < 0, \quad (3.18)$$

where

$$p = \frac{ds^{1/3}x}{a_0(1+x)}, \quad x = \frac{s\lambda^2}{a_0(1-2s\lambda^4)} - 1 \quad \text{for } x > 0 \quad (3.19)$$

and where $y = -x$ for $x < 0$.

The set of linear algebraic equations (3.3), (3.4), (3.13), (3.14), and (3.16) are solved by the determinantal procedure, to give in turn the transcendental equation, which includes the tunneling, in the form

$$\text{Ai}(-\eta_0)\lambda^{-1}\Gamma^{-2}(-\lambda)[\varphi(\lambda) - 2|Q(\lambda)|] + \frac{1}{2}e^{-2\Phi(\lambda)}\text{Bi}(-\eta_0) = 0, \quad (3.20)$$

where the functions $\varphi(\lambda)$, $\Phi(\lambda)$, and the parameter η_0 are given by Eqs. (3.5), (3.15), and (3.17)–(3.19), respectively. The function $Q(\lambda)$ is given by

$$Q(\lambda) = \frac{2\langle 0|g \ln|g|0 \rangle}{\lambda\langle 0|g^2|0 \rangle} = \frac{\sqrt{\pi}}{2\gamma} \left(\ln \frac{\gamma}{\lambda} + 1 - \frac{1}{2} C \right) \quad \text{for } Q(\lambda) < 0, \quad (3.21)$$

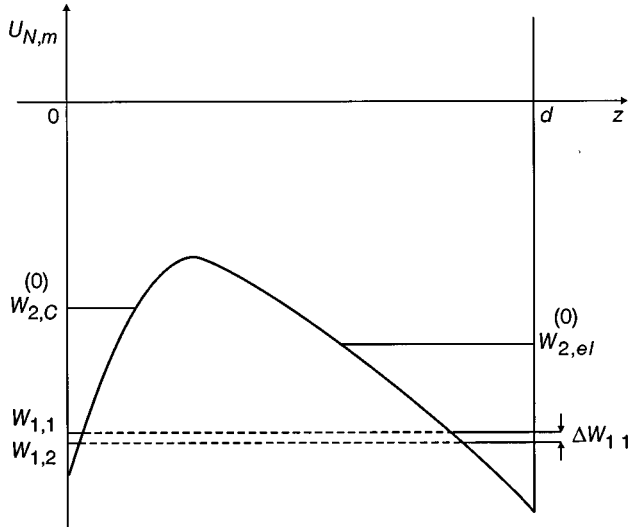


FIG. 1. Schematic form of the potential $U_{N,m}(z) = V_{N,m}(z) - eEz$, where $V_{N,m}$ is given by Eq. (2.4) and where $W_{2,C}^{(0)}$ [from Eq. (4.7)] and $W_{2,eI}^{(0)}$ [from Eq. (4.4)] are the first excited quasi-Coulomb and “electric” levels, respectively. The electric field E and the QW width d are chosen to provide resonance between the ground quasi-Coulomb and electric levels. $\Delta W_{1,1}$, given by Eq. (4.16), is the resonance splitting of these levels.

where $\gamma = 2^{1/2}a_B/a_0 \ll 1$. On solving Eq. (3.20), the quantum number λ can be found which in turn determines the impurity electron energy W_λ .

IV. RESULTS

The combined potential generated by the quasi-Coulomb field of the impurity (2.4), the effects of the electric field E , and the boundaries of the QW is shown in Fig. 1. The main object of this approach is to investigate resonances between the quasi-Coulomb states formed by the impurity potential and the left-hand boundary of the QW and the states formed by the uniform electric field E and the right-hand boundary of the QW. Equation (3.20) enables the resonance process to be kept clearly in view. The last term in the left-hand part of this equation describes the tunneling of an electron from the impurity well towards the triangular well close to the right boundary through the potential barrier which has a power $\Phi \gg 1$.

A. The zeroth approximation

At this stage, the term mentioned above may be neglected. As a result, Eq. (3.20) decomposes into two independent equations

$$\text{Ai}(-\eta_0) = 0 \quad (4.1)$$

and

$$\varphi(\lambda) - 2|Q(\lambda)| = 0, \quad (4.2)$$

representing the two effective wells. Then Eq. (4.1) describes the ground and nearest so-called electric levels in the triangular well adjacent to the right-hand boundary at $z = d$. The solution to Eq. (4.1) has the form¹⁵

$$\eta_0 = \alpha_k, \quad k = 1, 2, 3, \dots, \quad (4.3)$$

where $\alpha_1 = 2.34$, $\alpha_2 = 4.09$, $\alpha_3 = 5.52$, $\alpha_4 = 6.79, \dots$, for example. On substituting the expression (3.17) in (4.3), the quantum numbers λ_k and the electric energy levels $W_k^{(0)}$ can be written in the explicit form

$$2 \frac{W_k^{(0)}}{R} = -1/\lambda_k^2 = -\frac{1}{2} \left(\frac{d}{a_0} s - \alpha_k s^{2/3} \right) - \left[\frac{1}{4} \left(\frac{d}{a_0} s - \alpha_k s^{2/3} \right)^2 + \frac{8}{3} s \right]^{1/2}. \quad (4.4)$$

Equation (4.2) describes the ground and nearest impurity levels in the quasi-Coulomb well adjacent to the left-hand boundary at $z = 0$. The solution to Eq. (4.2) then has the form⁸

$$\lambda_n = n + \delta_n, \quad n = 1, 2, 3, \dots, \quad (4.5)$$

where the quantum defect δ_n ($\ll 1$) in an unbounded semiconductor is given by

$$\delta_n = [2|Q(n)| - \psi(n) - 2C + 1 - 1/(2n)]^{-1}. \quad (4.6)$$

As pointed out above, in order to take into account the effect of the electric field E on the quasi-Coulomb levels, the quantum number λ_n in Eq. (4.5) should be replaced by the quantum number ν_n given in Eq. (3.7), so that

$$2 \frac{W_n^{(0)}}{R} = -\frac{1}{\nu_n^2} = -\frac{1}{\lambda_n^2} - \frac{3}{2} s \lambda_n^2. \quad (4.7)$$

Thus, in the zeroth approximation, the system of the energy levels is the sum of two independent series of energies. The first series is formed by the electric levels $W_k^{(0)}$ as given by Eq. (4.4). The second series $W_n^{(0)}$ are the quasi-Coulomb levels shifted towards low energies by the electric field E . The electron having an energy $W_k^{(0)}$ is localized within the triangular well close to the right boundary of the QW while the electron having the energy $W_n^{(0)}$ is localized within the impurity well close to the left-hand boundary of the QW. For a sufficiently weak electric field E , the group of the electric levels has a higher energy than the quasi-Coulomb group, so that the relevant states are not in resonance.

If the electric field increases in magnitude, both groups of levels move toward lower energies. It follows from Eqs. (4.4) and (4.7) that, under condition $d/a_0 \gg 1$, the shift ($\sim -sd/a_0$) of the electric levels exceeds the shift ($\sim -3n^2s/2$) of the quasi-Coulomb levels. As a result, these two groups of levels can become equal to one another. Under the condition $W_n^{(0)} = W_k^{(0)}$, the relevant quasi-Coulomb and electric levels appear to be in resonance. On using expressions (4.4) and (4.7), this condition becomes

$$1/\lambda_n^2 = ds/a_0 - \alpha_k s^{2/3} - \frac{1}{3} s \lambda_n^2, \quad (4.8)$$

where λ_n and α_k are defined by Eqs. (4.5) and (4.3), respectively.

On solving Eq. (4.8), the magnitudes of the electric field at resonance s_{nk} can be found. These define the cases in which the n th quasi-Coulomb level and the k th electric level are in resonance. The dependencies of the quasi-Coulomb

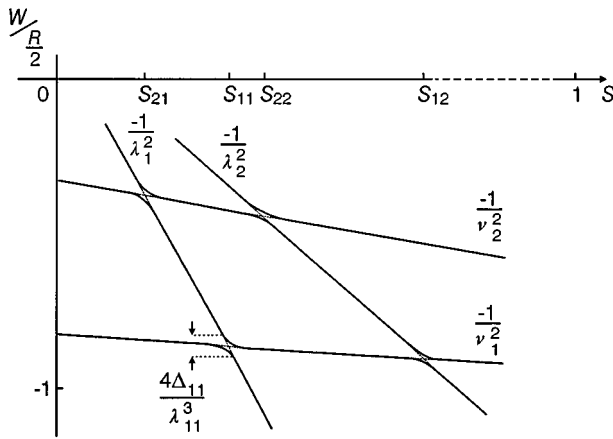


FIG. 2. Dimensionless energy $2W/R$ versus the dimensionless electric field s (solid lines, arbitrary units). The dashed lines display the independent ground (ν_1, λ_1) and first excited (ν_2, λ_2) quasi-Coulomb (4.7) and electric (4.4) levels. The relevant resonance fields s_{nk} and the resonance splitting of the ground levels ($n=k=1$), where Δ_{nk} is defined by Eq. (4.11), are indicated.

energy levels $W_n^{(0)}$ in Eq. (4.7) and the electric energy levels $W_k^{(0)}$ in Eq. (4.4) upon the magnitude of the electric field s are displayed in Fig. 2.

B. The first approximation

In the first approximation, the last term on the left-hand part of Eq. (3.20) is taken into account. Note that this Eq. (3.20) satisfies the limiting case of zero electric field. Setting $E=0$ in Eq. (3.20) and using the asymptotic expressions for Airy functions $\text{Ai}(-\eta_0)$ and $\text{Bi}(-\eta_0)$ for large values of $|\eta_0| \gg 1$ with $\eta_0 < 0$, the quantum numbers λ_n can be obtained in the form

$$\lambda_n = n + \frac{1}{2} \delta_n (1 + \{1 + (4n/[n!^2 \delta_n]) (2d/a_0 n)^{2n} \times \exp(-2d/a_0 n)\}^{1/2}), \quad (4.9)$$

where δ_n is defined by Eq. (4.6). The expression (4.9) coincides completely with that obtained for the quantum numbers of the diamagnetic impurity center positioned at the edge of the QW in the absence of the electric field.⁸

We expand the Airy functions $\text{Ai}(-\eta_0)$ and $\text{Bi}(-\eta_0)$ in Eq. (3.20) and the function $\varphi(\lambda)$ in the power series in $(\lambda - \lambda_k)$ and $(\lambda - \lambda_n)$, respectively, where λ_k is given by Eqs. (4.3) and (4.4) and λ_n by Eqs. (4.5) and (4.6). Also we use explicit expressions for $\eta_0(\lambda)$, $\varphi(\lambda)$, and $\mathcal{Q}(\lambda)$ from Eqs. (3.17), (3.5), and (3.21), respectively, and the result

$$\lambda^{-1} \Gamma^{-2}(-\lambda_n) = n^{-1} (n!)^2 \delta_n^2.$$

On substituting the expansions obtained into Eq. (3.20) and using Eqs. (4.1) and (4.2), we arrive at a quadratic equation for the quantum number λ . The roots of this equation can be written in the form

$$\lambda^{(1,2)} = \frac{1}{2} (\lambda_n + \lambda_k) \pm [\frac{1}{4} (\lambda_n - \lambda_k)^2 + \Delta_{nk}^2(s)]^{1/2}, \quad (4.10)$$

where

$$\Delta_{nk}(s) = \beta_{nk}^{1/2} s^{1/3} e^{-\Phi(s)} \quad (4.11)$$

and where

$$\beta_{nk} = - \frac{\text{Bi}(-\alpha_k) n \lambda_k^3}{4 \text{Ai}'(-\alpha_k) n!^2}.$$

Equation (4.10) describes the effect of anticrossing between the energy levels which are derived from the state originally in different parts of the effective potential. It follows from Eq. (4.10) that, if the electric field s and the resonance value s_{nk} are far apart so that

$$\frac{1}{4} (\lambda_n - \lambda_k)^2 \gg \Delta_{nk}^2(s), \quad (4.12)$$

then the quantum numbers are close to those obtained in the zeroth approximation, namely,

$$\lambda^{(1)} \cong \lambda_n, \quad \lambda^{(2)} \cong \lambda_k. \quad (4.13)$$

In the case of resonance for which

$$s = s_{nk}, \quad \lambda_n = \lambda_k \equiv \lambda_{nk}, \quad (4.14)$$

we have

$$\lambda^{(1,2)} = \lambda_{nk} \pm \Delta_{nk}(s_{nk}). \quad (4.15)$$

As pointed out above, the quantum number λ_n in Eqs. (4.13)–(4.15) should be replaced by ν_n [Eq. (3.7)]. At resonance, the differences between the quantum numbers $\lambda^{(1)}$ and $\lambda^{(2)}$ and the associated energies $W_{\lambda^{(1)}} - W_{\lambda^{(2)}} = \Delta W_{nk}$ are given by

$$\lambda^{(1)} - \lambda^{(2)} = 2\Delta_{nk}(s_{nk}), \quad \Delta W_{nk} = 2R\Delta_{nk}(s_{nk})/n^3. \quad (4.16)$$

Thus if the resonance between the quasi-Coulomb levels (4.7) and the electric levels (4.4) occurs, any crossing arising in the zeroth approximation turns into anticrossing in the next approximation.

V. DISCUSSION

The approach described above enables the basic physics connected with the electron states to be kept in view throughout the analysis. As the effect of the magnetic field and the width of the QW on the impurity energy levels had been studied in detail previously,^{3–5,8} we have concentrated here on the influence of the electric field E on the electron states. If the applied electric field s and resonance fields s_{nk} are widely separated, the system of energy levels is the sum of independent quasi-Coulomb (4.7) and electric (4.4) levels. The wave function is concentrated within either the impurity well or triangular well close to the left- or right-hand boundary of the QW, respectively. In the case of resonance for which $s \cong s_{nk}$, the n th quasi-Coulomb and k th electric states become very close in energy. The relevant gap is defined by Eq. (4.16) and resonance tunneling between the impurity and triangular wells is possible and a drastic redistribution of the wave function occurs. The wave functions related to the components of the energy doublet attain the twin-peaks configuration. Calculations of the wave functions call for specific consideration. The pattern of the energy lev-

els versus the magnitude of the electric field is shown in Fig. 2.

Under the condition $a_0/d \ll 1$, an approximate solution to Eq. (4.8) is given by

$$s_{nk} \cong 1/\lambda_n^2(a_0/d) + \alpha_k/\lambda_n^{4/3}(a_0/d)^{5/3} + \frac{1}{3}(a_0/d)^2. \quad (5.1)$$

It follows from Eq. (5.1) that, for a fixed index of the quasi-Coulomb level n , the resonance field s_{nk} increases as a function of the index of the electric level k . Meanwhile, for a fixed index k , the resonance field s_{nk} decreases with increasing index n . Also, the expression (5.1) enables the dependence of the resonance field on the width of the QW d to be determined. The wider the QW, the smaller the resonance field s_{nk} . In turn, it is clear from Eqs. (4.11) and (3.15) that the resonance gap Δ_{nk} rises as the resonance field s_{nk} increases. From the approach described above, we obtain the dependence of the binding energy $|W_\lambda|$ of the impurity electron on the magnitude of the magnetic and electric fields. It follows from Eqs. (4.7), (4.6), and (3.21) that, if the magnetic or electric field increases in magnitude, the binding energy increases in each case.

A comparison of our analytical results with those obtained by numerical methods would be desirable at this point. Cen and Bajaj⁶ have developed a variational method for the calculation of the binding energy of the hydrogenlike impurity electron in “dielectric” quantum wells for which the dielectric constant of the barrier material is much less than that of the well material for the cases when both the magnetic and electric fields are taken to be perpendicular to the heteroplanes. As pointed out in Ref. 6, a difference between the values of the dielectric constants has little effect allowing a qualitative comparison between the results to be made. In contrast to our calculation, the coordinate $z=0$ in Ref. 6 was taken to be at the central point of the QW. Thus, in order to make a comparison, the binding energy calculated in Ref. 6 should be increased by an amount $\frac{1}{2}eEd$. Taking this factor into account, good qualitative agreement is found between our calculations of the binding energy and the experimental data shown in Fig. 6 of Ref. 6.

Recently, Cao *et al.*¹⁶ have calculated variationally the excitonic states in a superlattice coupled with an enlarged quantum well in the presence of an electric field. For the resonance electric field, the energy levels in the quantum well and Stark levels in the superlattice were shown to anticross. Moreover, the pattern of the energy levels given in Ref. 16 correlate well with that shown in Fig. 2. Thus it may be safely suggested that the anticrossing of the resonance levels caused by the electric field is similar for various types of low-dimensional structures.

The transition energy from the ground impurity state ($W_1^{(0)}$) to the first excited state ($W_2^{(0)}$) is of interest both experimentally and theoretically.^{6,17} On using Eqs. (4.7), (4.9), (4.6), and (3.21), the expression for the transition energy $W_2^{(0)} - W_1^{(0)}$ can be written in an explicit form. In order to simplify the result, a sufficiently wide QW and a sufficiently strong magnetic field are necessary. In the logarithmic approximation ($\gamma \ll 1$, $|\ln \gamma| \gg 1$), and under the condition $\exp(-d/a_0)$ less than $\gamma |\ln \gamma|^{-1}$, we obtain

$$\begin{aligned} \frac{W_2^{(0)} - W_1^{(0)}}{\frac{1}{2}R} = & \frac{1}{4} \left[3 - \frac{7\gamma}{\sqrt{\pi}|\ln \gamma|} + \frac{1}{2} \left(\frac{d}{a_0} \right)^4 \exp\left(-\frac{d}{a_0}\right) \right] \\ & - \frac{3}{2} s \left[3 + \frac{2\gamma}{\sqrt{\pi}|\ln \gamma|} + 2 \left(\frac{d}{a_0} \right)^4 \exp\left(-\frac{d}{a_0}\right) \right]. \end{aligned} \quad (5.2)$$

From Eq. (5.2), it follows that, if the electric field increases in magnitude and if the width of the QW increases, the transition energy decreases in each case. These results are in general agreement with experimental data¹⁷ in which the transition energy has been measured experimentally but for zero magnetic field. However, if the magnetic field increases in magnitude, the transition energy (5.2) was found to increase in agreement with the numerical results presented in Ref. 6.

On considering possible experiments, estimates of suitable values for the parameters for the GaAs QW ($\mu = 0.067m_0$, $\epsilon = 12.5$, $a_0 = 98.7 \text{ \AA}$) for a well for which the width $d \gg a_0$ must be made. For a strong magnetic field ($B = 40 \text{ T}$, $a_B/a_0 = 0.4$) and a weak electric field ($E_{11} = 3.8 \times 10^4 \text{ V m}^{-1}$, $s_{11} = 0.066$), the resonance splitting ΔW_{nk} of the ground quasi-Coulomb and “electric” levels ($k=n=1$, $\beta_{11} = 0.21$, $\lambda_1 = 1.28$) may be found from Eq. (4.16) such that $\Delta W_{11} = 1.2 \text{ meV}$. Also, a redshift of the ground impurity level may be obtained from Eq. (4.7) with the result that $\Delta W_1^{(0)} = 1 \text{ meV}$. These values are those typically found in experiments. The chosen electric field E_{11} causes the penetration through the potential barrier to be relatively weak. This in turn leads to the result that an extremely wide QW width of $d = 1500 \text{ \AA}$ is needed to demonstrate this effect. When this value for E_{11} is exceeded, the penetration increases and the above method of solving Eq. (3.20) becomes inappropriate. However, clearly in the presence of a stronger electric field ($s \leq 1$), the effect of the resonance splitting holds for QW’s of standard periods $d \sim (3-5)a_0$. In this case, a numerical approach should be used.

VI. CONCLUSIONS

In summary, we have developed an analytical approach for the problem of an electron (hole) captured by an impurity center positioned at the edge of a QW in the presence of strong magnetic and electric fields directed perpendicular to the heteroplanes. It has been shown that the combined potential is similar to that of a double quantum well as depicted in Fig. 1. For specific values of the electric field, resonance between the levels associated with different wells occurs and tunneling between the wells becomes possible. The relevant resonance energy levels are found to anticross. The analytical dependencies of the resonance splitting of the levels upon the width of the well and on the magnitudes of the electric and magnetic fields are obtained. This in turn defines the tunneling time of the electron and the oscillator strength of optical transitions in semiconductor structures with impurity quantum wells in the presence of magnetic and electric fields. Such a geometry does not appear to have been considered before but it is expected to be of interest and useful on account of its influence on the electrical transport and optical properties of devices constructed in this way.

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