Magnetophonon resonance in the energy relaxation of electrons in a quantum well

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The magnetophonon resonance effect in the energy relaxation rate is studied theoretically for a quasi-two-dimensional electron gas in a semiconductor quantum well. An electron-temperature model is adopted to describe the coupled electron-phonon system. The energy relaxation time, derived from the energy relaxation rate, is found to display an oscillatory behavior as the magnetic-field strength changes, and reaches minima when the optical phonon frequency equals integer multiples of the electron cyclotron frequency. The theoretical results are compared with a recent experiment, and a qualitative agreement is found. [S0163-1829(97)11916-6]

Since the pioneering work of Gurevich and Firsov,[1] the magnetophonon resonance (MPR) effect has been extensively investigated, both experimentally and theoretically, in bulk semiconductors,[2] and more recently, in lower dimensional electronic structures.[3,4] This MPR effect is known to arise from the resonant coupling between electrons and optical phonons when the separation between two Landau levels approaches the phonon energy, and leads to oscillatory behavior in many transport properties, e.g., the magnetoresistivity.

Most previous studies have concentrated on the MPR effect in the momentum relaxation of charged carriers,[5–8] only a few have dealt with the MPR effect in the energy relaxation process.[9,10] In this paper, we report on a theoretical investigation of the MPR effect in the energy relaxation time (rate) of a two-dimensional electron gas (2DEG) formed in a semiconductor quantum well. Adopting an electron-temperature model,[12] the energy relaxation time, derived from the energy relaxation rate, is found to display an oscillatory behavior as the magnetic-field strength changes, and reaches minima when the optical phonon frequency becomes an integer multiple of the electron cyclotron frequency. The theoretical results are applied to a 2DEG in an InAs quantum well, and compared to a recent experiment.[11] A qualitative agreement is found.

In this paper, the electron-temperature model is used as the starting point. In this model, one assumes an electron-temperature T_e for the electron subsystem, and a lattice temperature T_l for the phonons.[12] This model only approximately describes a coupled electron-phonon system in a steady but nonequilibrium state, as the details of nonequilibrium is ignored. This model should be applicable in the present study, as the scattering time among electrons (for the electron subsystem to reach equilibrium) is typically much shorter than the scattering time between electrons and phonons.[12]

For clarity, the major approximations made in this paper are as follows. (i) We use an electron-temperature model. (ii) The electron-acoustic-phonon coupling is neglected, as it is less important when the electron temperatures are large.[10] (iii) For simplicity, only one subband is considered. (iv) The electrons are described with an effective mass, but the band nonparabolicity is included through a two-band Kane model.[11] (v) The electron-phonon interaction is treated within the first-order Born approximation, as the electron-phonon coupling is weak.[11] (vi) The many-particle effect (electron-electron interaction) is partly taken into account within the well-known random-phase approximation.[13] (vii) The electron is assumed to couple with the bulk optical phonon mode.[11] The coupling with other modes, e.g., interface mode, is neglected. (viii) The hot-phonon effect is neglected. Overall, we believe that these approximations are reasonable and sufficient, as a qualitative picture is sought.

Let us consider a 2DEG, in the presence of a perpendicular magnetic field, interacting with bulk optical phonons. When the coupled system is in a steady state, the electron energy relaxation rate can be calculated by evaluating the rate of increase of the number of phonons.[3,4,14] The electron energy relaxation time can be naturally derived from this rate. In the literature, there exist alternative approaches for calculating the energy relaxation time,[15–19] but we believe that the present one is more natural, simpler, and with a clear physical meaning. The energy relaxation rate per particle W can be written as[10,14]

$$W=\frac{\hbar\omega_{\mathrm{LO}}}{\tau_0}\left[n\left(\frac{\hbar\omega_{\mathrm{LO}}}{k_BT_l}\right)-n\left(\frac{\hbar\omega_{\mathrm{LO}}}{k_BT_e}\right)\right]P(T_e), \tag{1}$$

where $\tau_0^{-1}=(2\alpha\hbar\varepsilon_\infty/e^2)\omega_{\mathrm{LO}}^2(\hbar/2m^*\omega_{\mathrm{LO}})^{-1/2}$ is the time scale of the coupled electron-phonon system, and $P(T_e)=-(2\pi n_e)^{-1}\int_0^\infty dqq\,\mathrm{Im}\varepsilon^{-1}(q,\omega_{\mathrm{LO}})$. $n(x)=1/(e^x-1)$ is the Boson distribution function, ω_{LO} the optical phonon frequency, m^* the electron effective mass, α the Fröhlich coupling constant, and $\varepsilon(q,\omega)$ the dielectric function of the 2DEG at a given electron-temperature T_e. $\varepsilon(q,\omega)$ can be explicitly written as[20] $\varepsilon(q,\omega)=1-v(q)\Pi(q,\omega)$, where $v(q)=2\pi e^2f(q)/(\varepsilon_\infty q)$, $\Pi(q,\omega)=(m\omega_c/2\pi\hbar)\Sigma_{n,n'}C_{n,n'}(q)\pi_{n,n'}(\omega)$, $C_{n,n+l}(q)=[n!/(n+l)!]e^{-x}x^l[L_n^l(x)]^2$, $x=\hbar q^2/2m\omega_c$, and $\pi_{n,n'}(\omega)=2[n_F(\varepsilon_n)-n_F(\varepsilon_{n'})]/[\hbar\omega-(\varepsilon_{n'}-\varepsilon_n)+i\gamma]$. $f(q)$ is the

form factor for a quantum well, ε_∞ the dielectric constant, $n_F(x)$ the Fermi distribution function, $\omega_c = eB/m^*c$ the cyclotron frequency, and ε_n the energy of the Landau level.

In calculating the dielectric function, the random-phase approximation is employed, and a broadening parameter γ is inserted into the energy denominator in the first-order polarization function.[13] It must be pointed out that γ is not the width of a Landau level, but is rather a Lorentzian broadening of the δ function expressing the energy conservation. This γ originates from the scattering of electrons by phonons and impurities. For simplicity, in the present study, we have not attempted a self-consistent calculation[8] of γ.

The energy relaxation rate given in Eq. (1) is written explicitly with a factor $[n(\hbar\omega_{\rm LO}/k_BT_l) - n(\hbar\omega_{\rm LO}/k_BT_e)]$ to emphasize the underlying physics. If the electrons and phonons have the same temperature, then there is no net energy transfer between the two subsystems. Only when $T_e > T_l$, there is a net energy flow from the electron to the phonon system. This statement is correct even if the electron-phonon interaction is not treated as a perturbation. Writing the energy relaxation rate in the above form also helps us to see clearly the dependence of the relaxation rate on various system parameters. As we will show next, the dominant overall temperature dependence is contained in the factor $[n(\hbar\omega_{\rm LO}/k_BT_l) - n(\hbar\omega_{\rm LO}/k_BT_e)]$. Note that when $T_e > T_l$, the net energy transferred from the electrons to the phonons is assumed to dissipate away via some unspecified channels. The study of dissipation of phonons is, however, beyond the scope of this paper.

We have carried out detailed calculations of the electron energy relaxation rate (time) for a quantum well made of InAs, so that the results could be compared with a recent experiment.[11] First, let us investigate the overall feature of the energy relaxation rate. In Fig. 1, the function $P(T_e)$ versus the electron cyclotron frequency ω_c is shown for various electron densities n_e, electron temperature T_e, and broadening parameter γ, shown in the corresponding panels of the figure. The solid, dashed, and dotted curves are for the electron temperatures $T_e = 300$ K, 200 K, and 100 K, respectively. The quantum well width is 185 Å, and other parameters are taken for the material InAs:[11] band mass $m^*/m_e = 0.023$ without band nonparabolicity, phonon frequency $\hbar\omega_{\rm LO} = 30$ meV, electron-phonon coupling constant $\alpha = 0.052$, and $\tau_0 = 1.12\times10^{-13}$ s. The dependence of $P(T_e)$ on the quantum well width is not strong, thus it is not shown explicitly. As the conduction band of InAs is strongly nonparabolic, we have taken the band nonparabolicity into account via an energy dependent effective mass, the same approach as employed in Ref. 11.

In Fig. 1, the energy relaxation rate clearly shows an oscillatory behavior. When the phonon frequency (30 meV) is an integer multiple of the electron cyclotron frequency ω_c, the relaxation rate, or $P(T_e)$, exhibits a local maximum. This oscillatory behavior could be traced to the energy denominator in the dielectric function, arising from the energy conservation condition in the process of electron-phonon scattering. As the electron motion is completely quantized in a perpendicular magnetic field, the electron energy can only take discrete values, i.e., the Landau level energies. Thus, the

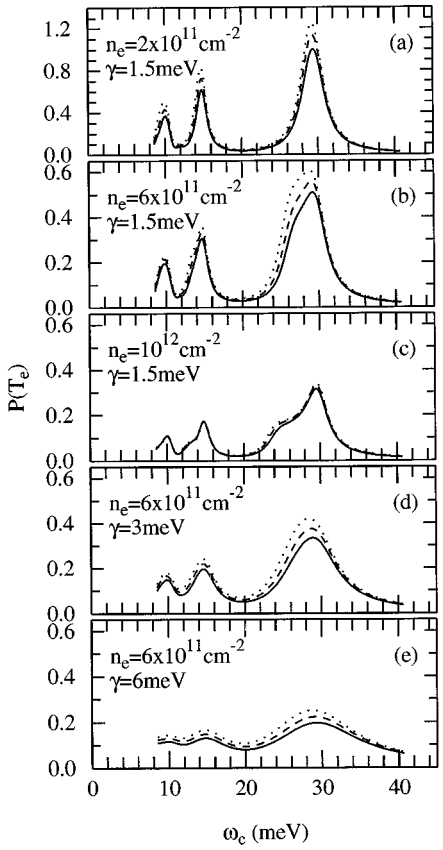

FIG. 1. The function $P(T_e)$ versus the electron cyclotron frequency ω_c is shown for different electron densities and broadening parameters which are shown in the corresponding panels. The solid, dashed, and dotted curves are for the electron temperature $T_e = 300$ K, 200 K, and 100 K, respectively. The quantum well width is 185 Å, and other parameters are taken for an InAs quantum well with a lattice temperature of 4.2 K.

electron-phonon coupling becomes resonant, when the phonon energy equals the energy difference between the initial and final electron states.

From Fig. 1, one notices that the temperature dependence of $P(T_e)$ can be detected more easily for lower electron densities. This is because, in the temperature range studied here, the lower the electron density, the more sensitive the dependence of the electron distribution is on the electron density. As the electron density increases [see Fig. 1(c)], there is a shoulder structure appearing about $\omega_c = 25$ meV. This feature is found to originate from the detailed q dependence of the dielectric function of the 2DEG, and disappears for a larger broadening parameter.

A strong dependence of $P(T_e)$ on the broadening parameter γ is evident in Fig. 1. As γ becomes larger, the peak amplitude of the energy relaxation rate is reduced. As the peak value of the imaginary part of the dielectric function is roughly proportional to the inverse of γ, one sees that the peak amplitude of the relaxation rate, or $P(T_e)$, is almost linearly proportional to γ.

From the electron energy relaxation rate we find, in a rather natural way, the electron energy relaxation time τ, by writing the relaxation rate per particle W as

$$W = \frac{\hbar\omega_{\rm LO}}{\tau}. \tag{2}$$

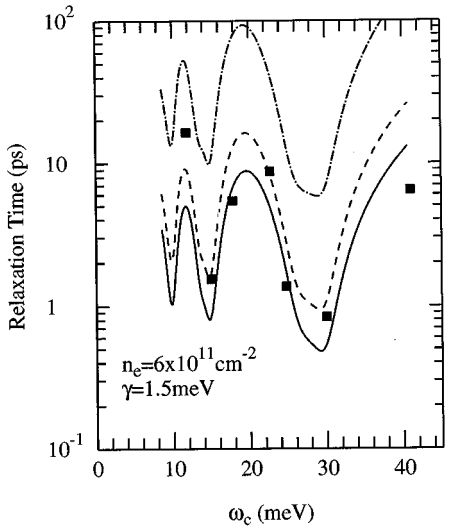

FIG. 2. The calculated energy relaxation time τ is displayed as a function of the electron cyclotron frequency ω_c, for an InAs quantum well with well width 185 Å. The solid, dashed, and dash-dotted curves are for the electron temperature $T_e = 300$ K, 200 K, and 100 K, respectively. The solid squares are the experimental data of Vaughan *et al.*.

This relaxation time τ can be interpreted as an averaged electron Landau level lifetime. In Fig. 2, the calculated energy relaxation time τ is displayed as a function of the electron cyclotron frequency ω_c, for an InAs quantum well with width 185 Å. The solid, dashed, and dash-dotted curves are for the electron temperatures $T_e = 300$ K, 200 K, and 100 K, respectively. The solid squares are the experimental data of Vaughan *et al.*[11] The electron density is taken from the experiment. The electron temperature T_e and broadening parameter γ are, however, treated as fitting parameters. The experimental data were extracted from the measured electron cyclotron transmission intensity versus incident laser power, via the solution of a group of rate equations.[11]

In the electron temperature model, the electron distribution among the Landau levels is determined by T_e. In the experiment, this distribution is adjusted by varying the incident laser power intensity.[11] Unfortunately, Ref. 11 does not provide information about the electron distribution from which one could extract an effective temperature. It is clear that the experimental data fall near the curves with $T_e = 200$ K and $T_e = 300$ K, which are reasonable values. The lattice temperature is kept at $T_l = 4.2$ K. Thus the present theoretical calculation is in qualitative agreement with the experiment. At lower electron temperatures, the relaxation time increases rapidly. From Fig. 2, one sees that the most important parameter is the electron temperature; γ influences the relaxation time in a less sensitive way. A picture qualitatively similar to Fig. 2 could be obtained if γ is increased to 3 meV (not shown here).

In summary, the MPR effect in the energy relaxation time (rate) is studied for a 2DEG in a quantum well. Adopting an electron-temperature model, the energy relaxation time (rate) is found to display oscillatory behavior as a function of the magnetic field. The theoretical results are in qualitative agreement with a recent experiment.

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