Tunneling zero-bias anomaly in the quasiballistic regime

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We study the tunneling density of states (DOS) of the interacting electron gas beyond the diffusive limit. A strong correction to the DOS persists even at electron energies exceeding the inverse transport relaxation time, which could not be expected from the well-known Altshuler-Aronov-Lee (AAL) theory. This correction originates from the interference between the electron waves scattered by an impurity and by the Friedel oscillation this impurity creates. Account for such processes also revises the AAL formula for the DOS in the diffusive limit. [S0163-1829(97)09615-X]

Zero-bias anomaly in tunneling conductance is a generic phenomenon observed in a great variety of physical systems. One well-known type of zero-bias tunneling anomaly is that caused by the scattering of electrons on Kondo impurities positioned in the vicinity of the tunneling barrier.[1] However, the zero-bias anomaly was also observed[2] in systems containing no Kondo impurities at all. Namely, in experiments with disordered conductors the differential tunneling conductance is suppressed at small biases. This effect appears to be quite universal: it shows up in all dimensions, it is pronounced both in metals and semiconductors, and it depends only on the strength of disorder in the system. The zero-bias anomaly in differential conductance in the absence of Kondo impurities was puzzling for years until it was discovered[3,4] by Altshuler, Aronov, and Lee (AAL) that this phenomenon is due to the interaction between the itinerant electrons in a disordered conductor. Conductance of a point tunnel contact is proportional to the local tunneling density of electron states (DOS). AAL showed that the electron-electron interaction in the presence of disorder results in a negative correction to DOS, which is singular at the Fermi energy. In the case of tunneling into a two-dimensional conductor, the AAL result, $\delta\nu^{\mathrm{AAL}}$, for such a correction reads

$$\frac{\delta\nu^{\mathrm{AAL}}(\epsilon)}{\nu_0}=\frac{A}{E_F\tau}\ln(|\epsilon|\tau/\hbar). \tag{1}$$

Here E_F is the Fermi energy, τ is transport relaxation time, the parameter $A>0$ depends on details of the electron-electron interaction, $\nu_0=m/\pi\hbar^2$ is the free-electron density of states (m is the electron mass), and energy ϵ is measured from the Fermi level. Correction (1) is inversely proportional to τ and diverges if the electron energy approaches the Fermi level, $\epsilon\to 0$. The AAL theory assumes the diffusive motion of electrons, which constrains the electron energy to the interval $\epsilon<\hbar/\tau$. Clearly, in the case of a strong disorder, $E_F\tau\sim\hbar$, this condition is not restrictive. However, in cleaner samples (e.g., heterostructures with tunable density of two-dimensional electron gas[5]) the energy domain $\epsilon>\hbar/\tau$ becomes accessible, while the region of applicability of Eq. (1) shrinks.

The behavior of the density of states $\delta\nu(\epsilon)$ in the ballistic regime, $\epsilon\gtrsim\hbar/\tau$, has not been addressed in the literature to our knowledge. The main purpose of this paper is to show that interaction does lead to a significant correction to the DOS even for these large electron energies. This correction arises from the interference of scattering on an impurity and on the Friedel oscillation it creates. Account for such processes also revises the original AAL formula for the DOS in the diffusive limit, adding a nonsingular but large contribution to Eq. (1).

Electron density of states for energies larger than the inverse transport relaxation time is associated with the electron dynamics on a time scale shorter than τ. During such a small time an electron does not experience a large number of scattering events, i.e., the scattering on disorder potential can be treated in the lowest order of the perturbation theory, in contrast to the diffusive limit. This approximation accounts only for the trajectories of electrons that were scattered only on one impurity. We will show that the logarithmically divergent correction to the density of states appears in this approximation already.

We start with the most instructive case of the finite-range interaction potential, and we calculate the correction to the one-particle DOS in the quasiballistic limit due to a single short-range scatterer. Consider an impurity at the origin; its potential $U_{\mathrm{imp}}(\mathbf{r})$ induces a modulation of electron density around the impurity. In the Born approximation, one can find the oscillating correction, $\delta n(r)=n(r)-n_0$, to the electron density $n(\mathbf{r})=\Sigma_{\epsilon_l<0}|\psi_l(\mathbf{r})|^2$:

$$\delta n(r)=-\frac{\nu_0 g}{2\pi}\frac{\sin(2k_Fr)}{r^2}. \tag{2}$$

Here r is the distance from the impurity, k_F is the Fermi wave vector, $g=\int U_{\mathrm{imp}}(\mathbf{r})d\mathbf{r}$, and n_0 is the density of the electron gas in the absence of impurities. The single-electron wave function $\psi_l(\mathbf{r})$ satisfies the Schrödinger equation for noninteracting electrons, $\hat{H}_0\psi_l=(\epsilon+E_F)\psi_l$, where $\hat{H}_0=-(\hbar^2/2m)\nabla^2+U_{\mathrm{imp}}(\mathbf{r})$. The oscillating contribution, Eq. (2), is known as a Friedel oscillation.[6]

In the presence of interaction $V(\mathbf{r}-\mathbf{r}')$ between electrons, density oscillation (2) gives rise to an additional term in the Hamiltonian, $\hat{H}_{\mathrm{HF}}$. In the coordinate representation, $\hat{H}_{\mathrm{HF}}$ has the form (see, e.g., Ref. 7)

$$H_{\mathrm{HF}}(\mathbf{r}_1,\mathbf{r}_2)=V_H(\mathbf{r}_1)\delta(\mathbf{r}_1-\mathbf{r}_2)-V_F(\mathbf{r}_1,\mathbf{r}_2), \tag{3a}$$

$$V_H(\mathbf{r}) = \int V(\mathbf{r}-\mathbf{r}')\,\delta n(r')\,d\mathbf{r}', \tag{3b}$$

$$V_F(\mathbf{r}_1,\mathbf{r}_2) = V(\mathbf{r}_1-\mathbf{r}_2)\frac{\delta\rho(\mathbf{r}_1,\mathbf{r}_2)}{2}. \tag{3c}$$

Here V_H and V_F are the Hartree and the exchange (Fock) energies, respectively, $\delta\rho(\mathbf{r}_1,\mathbf{r}_2)$ is the perturbation of the density matrix $\rho(\mathbf{r}_1,\mathbf{r}_2)=\Sigma_{\epsilon_l<0}\psi_l^*(\mathbf{r}_2)\psi_l(\mathbf{r}_1)$ by the impurity. Only the electrons with the same spin participate in the exchange interaction, which is reflected by the factor 1/2 in Eq. (3c). The Hartree-Fock energy (3b), (3c) oscillates as a function of distance from the impurity in the same manner as $\delta n(r)$ does.

The local density of states, $\nu(\epsilon,\mathbf{r})=-(2/\pi)\mathrm{Im}G_\epsilon^R(\mathbf{r},\mathbf{r})$, is related to the retarded Green function of the electron $\hat{G}_\epsilon^R=[\epsilon-\hat{H}+i0]^{-1}$, where $\hat{H}=\hat{H}_0+\hat{H}_{\mathrm{HF}}$. Let us find now the correction to this Green's function, $\delta G_\epsilon^R(\mathbf{r},\mathbf{r})$, due to a coherent process, which includes a scattering on the impurity potential itself, and a scattering on the potential (3) formed by the Friedel oscillation. In the lowest-order perturbation theory approximation we get

$$\delta G_\epsilon^R(\mathbf{r},\mathbf{r}) = 2g\Big\{ G_\epsilon^R(\mathbf{r},0)\int G_\epsilon^R(0,\mathbf{r}_1)V_H(\mathbf{r}_1)G_\epsilon^R(\mathbf{r}_1,\mathbf{r})\,d\mathbf{r}_1 - G_\epsilon^R(\mathbf{r},0)\int G_\epsilon^R(0,\mathbf{r}_1)V_F(\mathbf{r}_1,\mathbf{r}_2) \times G_\epsilon^R(\mathbf{r}_2,\mathbf{r})\,d\mathbf{r}_1 d\mathbf{r}_2\Big\}. \tag{4}$$

The Green function $G_\epsilon^R(\mathbf{r},\mathbf{r}')$ for a free electron at large distances, $k_F|\mathbf{r}-\mathbf{r}'|\gg 1$, and small energies, $\epsilon\ll E_F$, is

$$G_\epsilon^R(\mathbf{r},\mathbf{r}') = \frac{me^{i\pi/4}}{\hbar^2\sqrt{2\pi k_F|\mathbf{r}-\mathbf{r}'|}}e^{i(k_F+\epsilon/\hbar v_F)|\mathbf{r}-\mathbf{r}'|} \tag{5}$$

in two dimensions; ϵ is measured from the Fermi energy.

Below we will be interested in the density of states averaged over the spatial scales much larger than the Fermi wavelength $\lambda_F\equiv 2\pi/k_F$. Therefore, we should retain only those corrections, $\delta\nu(\epsilon,\mathbf{r})$, that are smooth functions of $\mathbf{r}$. Let us show now, using the Hartree contribution as an example, that Eq. (4) indeed yields such a correction. This contribution corresponds to the following process. The electron starts motion at the point $\mathbf{r}$, then experiences two scatterings, first on the impurity potential in the origin and second on the potential formed by the Friedel oscillation in point $\mathbf{r}_1$, and finally returns to point $\mathbf{r}$; see Fig. 1. Motion along this closed contour is represented in Eq. (4) by the product $G_\epsilon^R(\mathbf{r},0)G_\epsilon^R(0,\mathbf{r}_1)G_\epsilon^R(\mathbf{r}_1,\mathbf{r})\propto\exp[i\phi(\mathbf{r},\mathbf{r}_1)]$, where

$$\phi(\mathbf{r},\mathbf{r}_1) = (r+r_1+|\mathbf{r}_1-\mathbf{r}|)(k_F+\epsilon/\hbar v_F) \tag{6}$$

is the geometric phase acquired by the electron. There is another strongly oscillating factor in the integrand of Eq. (4)—the scattering potential $V_H(\mathbf{r}_1)\propto\sin(2k_Fr_1)$. Obviously, the result of integration is determined by the domain in space where the total phase of the integrand, $\phi(\mathbf{r},\mathbf{r}_1)-2k_Fr_1$, is a slow function of $\mathbf{r}_1$. The corresponding electron trajectories are those close to the straight line; see trajectory A in Fig. 1.

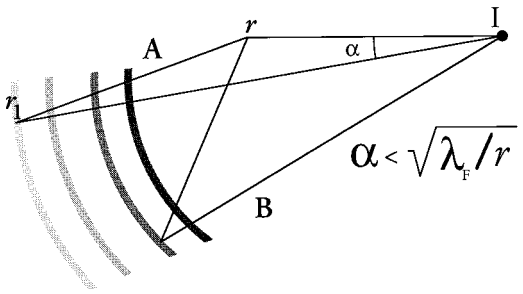

FIG. 1. Two typical trajectories (A, B) of an electron scattered by an impurity (I) and by the corresponding Friedel oscillation (concentric arcs). The correction $\delta\nu(\mathbf{r})$ is dominated by the trajectories of the type A, for which the electron is almost scattered back at I and $\mathbf{r}_1$.

At $r_1>r$, Eq. (6) yields the total phase of the integrand $2(\epsilon/\hbar v_F)r_1$. Remarkably, this phase does not depend on $\mathbf{r}$. As a result, the Hartree correction to the Green function, Eq. (4), becomes a *nonoscillating* function of r. Similar arguments can be applied to the evaluation of the exchange correction to the Green function. The resulting expression for the interaction correction to the local DOS is

$$\delta\nu(\epsilon,\mathbf{r}) \approx -\frac{[V(0)-2V(2k_F)]\nu_0^4 g^2}{8k_F^2 r^2} \tag{7}$$

if the distance from the impurity lies within the interval $\max\{\lambda_F,d\}\lesssim r\lesssim\hbar v_F/\epsilon$, drops rapidly ($\propto 1/r^3$) at $r\gtrsim\hbar v_F/\epsilon$, and saturates at $r\lesssim\max\{\lambda_F,d\}$. Here d is the characteristic spatial scale of the interaction potential, and $V(0)$ and $V(2k_F)$ are the Fourier components of the interaction potential appearing from the exchange and the Hartree terms, respectively.

In order to find the averaged density of states, one should sum up contributions of the type given by Eq. (7) from all the impurities and then average over point $\mathbf{r}$ where the correction is measured. Introducing the concentration of impurities n_i and using $\hbar/\tau=2\pi\nu_0 n_i g^2$, we arrive at the following expression for the interaction correction to the averaged DOS in the quasiballistic ($\epsilon\gg\hbar/\tau$) limit:

$$\frac{\delta\nu(\epsilon)}{\nu_0}\equiv-\frac{\langle\delta\nu(\epsilon,\mathbf{r})\rangle}{\nu_0} = \frac{[V(0)-2V(2k_F)]\nu_0\hbar}{4\pi E_F\tau}\ln\left|\frac{\epsilon}{\Delta}\right|, \tag{8}$$

with $\Delta=\min\{E_F,\hbar v_F/d\}$.

In principle, the correction may be of any sign depending on the relation between $V(0)$ and $V(2k_F)$. However, in any realistic system the interelectron interaction is sufficiently smooth, $d\gtrsim\lambda_F$, and $V(0)\gg V(2k_F)$. Therefore, in the following we will concentrate on the exchange contribution, which dominates in the correction to the DOS.

The derivation of Eq. (8) is valid for energies ϵ exceeding $\hbar/\tau$, which is the high-energy cutoff in the AAL theory; see Eq. (1). The quasiballistic formula (8) at the boundary of the region of its applicability, $\epsilon\sim\hbar/\tau$, does not match AAL's result (1). The reason for this mismatch is the choice of the high-energy cutoff. Physically, AAL cutoff means taking into account only that part of the Friedel oscillation formed by the electron states within a narrow energy strip,

$-\hbar/\tau \lesssim \varepsilon < 0$, below the Fermi level. This cutoff was dictated by the range of applicability of the diffusion approximation for the electron dynamics AAL used.[3,4] On the other hand, our analysis leading to Eq. (8) shows that electron states within a much wider strip, $-\Delta \lesssim \varepsilon < 0$, are important for the correction. As it turns out, this wider strip is important for the calculation of the DOS at $\epsilon \lesssim \hbar/\tau$ as well. To show this and to remedy the mismatch, below we calculate the DOS without using the diffusion approximation.

We are interested in the spatially averaged density of states, which makes it possible to use the standard[6] diagrammatic techniques. The correction to the averaged one-particle density of states has the form

$$\delta\nu(\epsilon,T) = -\frac{2}{\pi}\mathrm{Im}\int \frac{d\mathbf{p}}{(2\pi)^2}\,\delta G(i\epsilon_n \to \epsilon + i0, \mathbf{p}), \qquad (9)$$

where ϵ_n is the fermionic Matsubara frequency, T is the temperature. (For brevity we omit the Planck constant in all the intermediate formulas.) We will calculate the correction to the electron propagator, $\delta G(i\epsilon_n, \mathbf{p})$, to first order in the screened electron-electron interaction $V_{\mathrm{sc}}(i\Omega_l, \mathbf{Q})$. In the metallic regime ($E_F\tau \gg 1$) the exchange contribution to the propagator is

$$\delta G(i\epsilon_n,\mathbf{p}) = -[G(i\epsilon_n,\mathbf{p})]^2 T\sum_{\Omega_l}\int \frac{d\mathbf{Q}}{(2\pi)^2}\theta(\epsilon_n(\Omega_l-\epsilon_n)) \times[\Gamma(i\Omega_l,\mathbf{Q})]^2 V_{\mathrm{sc}}(i\Omega_l,\mathbf{Q}) \times G(i\epsilon_n-\Omega_l,\mathbf{p}-\mathbf{Q}). \qquad (10)$$

Here $G(i\epsilon_n,\mathbf{p}) = [i\epsilon_n - \xi_p + (i/2\tau)\mathrm{sgn}\epsilon_n]^{-1}$ is the electron Green's function in the dirty conductor, Ω_l is the bosonic Matsubara frequency, and Γ is the standardly defined (see, e.g., Ref. 6, p. 634) vertex function calculated in the ladder[4,6] approximation. As long as we are developing theory applicable for any relation between electron energy and $\hbar/\tau$, we cannot use the usual diffusion form for the vertex function. The formula valid for an arbitrary momentum $\mathbf{Q}$ and energy Ω_l transfer is

$$\Gamma(i\Omega_l,\mathbf{Q}) = \left(1 - \frac{1/\tau}{\sqrt{(|\Omega_l|+1/\tau)^2 + (v_F Q)^2}}\right)^{-1}. \qquad (11)$$

Note that in the limit $\Omega_l, v_F Q \ll \tau^{-1}$, Eq. (11) reduces to the standard diffusion[4] expression. On the other hand, the one-scatterering case studied in the first part of the paper corresponds to the quasiballistic limit, $\Omega_l, v_F Q \gg \tau^{-1}$, of Eq. (11). In this limit one should expand Eq. (11) up to the first order in $1/\tau$.

Calculation of the interaction correction to the DOS consists now of substitution of Eq. (10) into Eq. (9) and straightforward integration with account for Eq. (11).

The case of a finite-range electron-electron interaction is especially simple because we can replace $V_{\mathrm{sc}}(i\Omega_l,\mathbf{Q})$ in Eq. (10) by the Fourier component of the unscreened interaction potential $V(Q)$. In this case the correction coincides with the exchange term in formula (8). It means that the formula for the exchange correction to the DOS,

$$\frac{\delta\nu(\epsilon)}{\nu_0} = \frac{V(0)\nu_0\hbar}{4\pi E_F\tau}\ln\left|\frac{\epsilon}{\Delta}\right|, \qquad (12)$$

is valid for the energies both larger and smaller than $\hbar/\tau$.

For the long-range Coulomb interaction, screening should be taken into account, $V_{\mathrm{sc}}(i\Omega_l,\mathbf{Q}) = V(Q)/[1+V(Q)\Pi(i\Omega_l,Q)]$. Here the polarization operator in the random phase[6] approximation is

$$\Pi(i\Omega_l,\mathbf{Q}) = \nu_0\left(1 - \frac{\Gamma(i\Omega_l,Q)|\Omega_l|}{\sqrt{(|\Omega_l|+1/\tau)^2+(v_F Q)^2}}\right). \qquad (13)$$

Straightforward evaluation of Eqs. (9) and (10) with account of Eqs. (11) and (13) yields

$$\frac{\delta\nu(\epsilon,T)}{\nu_0} = -\frac{1}{8\pi E_F\tau}\int_{\epsilon}^{\Delta}\frac{d\Omega}{\Omega}\ln\left[\frac{\Delta^2}{\Omega\sqrt{\Omega^2+(1/\tau)^2}}\right], \qquad (14)$$

where $a_B = \hbar^2\kappa/me^2$ is the Bohr radius, κ is the dielectric constant, $\overline{\epsilon} \equiv \max\{|\epsilon|, T\}$, and the cutoff energy Δ is given now by $\Delta = \hbar v_F/a_B$.

The correction to the one-particle DOS, Eq. (14), is our main quantitative result. The diffusive and the quasiballistic asymptotic behavior of the correction is easily found.

In the diffusive limit, $\epsilon \ll \hbar/\tau$, the exchange correction to the one-particle DOS has the form

$$\frac{\delta\nu(\overline{\epsilon})}{\nu_0} = -\frac{\hbar}{16\pi E_F\tau}\left\{\ln\left(\frac{\overline{\epsilon}a_B^4}{\hbar D^2\tau}\right)\ln(\overline{\epsilon}\tau/\hbar) + 2[\ln(\tau\Delta/\hbar)]^2\right\}, \qquad (15)$$

where $D = v_F^2\tau/2$ is the diffusion coefficient. The first term of the sum in Eq. (15) is the result of the Altshuler-Aronov-Lee theory.[3] The second, new, term is not singular. This part of the correction represents the contribution of electrons deep in the Fermi sea, with energies below the ''$\hbar/\tau$ strip.''

In the quasiballistic limit, $\epsilon \gg \hbar/\tau$, exchange correction to the one-particle DOS is

$$\frac{\delta\nu(\overline{\epsilon})}{\nu_0} = -\frac{\hbar}{8\pi E_F\tau}[\ln(\overline{\epsilon}/\Delta)]^2. \qquad (16)$$

The leading term in the energy dependence of the correction is $\propto(\ln\overline{\epsilon})^2$ at any relation between $\overline{\epsilon}$ and τ. In the crossover region, the correction is given by

$$\frac{\delta\nu(\overline{\epsilon})}{\nu_0} = -\frac{\hbar}{8\pi E_F\tau}\{[\ln(\overline{\epsilon}/\Delta)]^2 + f(\overline{\epsilon}\tau)\}. \qquad (17)$$

Here $f(x) = (-1/2)[\ln(x)]^2 + \mathrm{Li}_2(-x^2)$, where $\mathrm{Li}_2(x)$ is a second-order polylogarithm function.[8] Asymptotes of the function $f(x)$, $f(x\to\infty) = 0$ and $f(x\to 0) = -(1/2)[\ln(x)]^2$, enable one to obtain from Eq. (17) the limits (16) and (15), respectively.

The density of states Eq. (14) describes adequately the electron tunneling without the lateral momentum conservation, such as tunneling through an inhomogeneous barrier. However, the electron-electron interaction affects the tunneling through a homogeneous barrier as well. We will consider below tunneling between two identical quantum wells, assuming (in accordance with the experiments[5]) the lateral momentum conservation in the course of tunneling. We will

find that the interaction correction to the conductance also has a logarithmic zero-bias anomaly.

In the absence of disorder and of the electron-electron interaction, the conservation of in-plane momentum implies that an electron can tunnel only at zero bias. Disorder alone smears this singularity, leading to the I-V characteristic[9]

$$I_0(V)=G_0\frac{eV(\hbar/\tau_s)^2}{(eV)^2+(\hbar/\tau_s)^2}, \tag{18}$$

with the width given by the inverse quantum lifetime of electrons in the wells, $1/\tau_s$. Here V is the bias applied to the contact, and G_0 is the zero-bias conductance.

Electron-electron interaction adds a singular at zero bias, negative logarithmic correction to the tunneling current:

$$\frac{\delta I(V)}{I_0(V)}\approx\frac{\hbar}{\pi E_F\tau}\ln(d/a_B)\ln(\overline{eV}/\Delta). \tag{19}$$

Here $\overline{eV}\equiv\max\{eV,T\}$ is assumed to satisfy[10] the conditions $\overline{eV}\ll\hbar/\tau_s$, $v_F/\sqrt{a_B d}$, and d is the separation between wells. The upper cutoff for the correction is $\Delta=\hbar v_F/a_B$, and by no means $\hbar/\tau$. In the absence of interaction, Eq. (18) would lead to a peak in the differential conductance dI/dV at zero bias. The negative diverging correction (19) splits this peak into two. The separation between the maxima of these two subpeaks is

$$eV_{\rm sp}=\frac{\hbar}{\tau_s}\sqrt{\frac{\ln(d/a_B)}{8\pi E_F\tau/\hbar}}. \tag{20}$$

The subpeaks should be resolved at sufficiently low temperatures, $T\lesssim eV_{\rm sp}$. An estimate for $V_{\rm sp}$ for the data of Turner *et al.* [5] gives $V_{\rm sp}\approx 0.05$ mV. It is important that Eqs. (19) and (20) are valid at any relation between eV, $eV_{\rm sp}$, and the energy $\hbar/\tau$.

In summary, we studied the tunneling density of states of interacting two-dimensional electron gas beyond the diffusive limit. A significant interaction-induced suppression of the density of states persists at electron energies even larger than the inverse transport relaxation time, which could not be expected from the well-known Altshuler-Aronov-Lee theory.[3] The AAL formula for the density of states at low energies is also revised, and an additional nonsingular, however large, contribution was found.

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