Beat-wave generation of plasmons in semiconductor plasmas

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It is shown that, in semiconductor plasmas, it is possible to generate large amplitude plasma waves by the beating of two laser beams with a frequency difference close to the plasma frequency. For narrow-gap semiconductors (for example n-type InSb), the system can simulate the physics underlying beat-wave generation in relativistic gaseous plasmas. [S0163-1829(97)10206-5]

Particle acceleration by large-amplitude longitudinal waves excited in an underdense plasma by a propagating laser pulse (wake-field accelerator), or by the beating of two laser beams with nearly equal frequencies (beat-wave accelerator), is an extremely interesting and far-reaching idea.[1] The efforts to translate this concept into reality, however, have to surmount two serious problems: (1) The creation of relativistic plasmas with $v_E^2/c^2 \sim 1$ (necessary for these processes) requires enormous field intensities in excess of 10^{16}–10^{18} W/cm^2, and (2) the plasma has to be extremely homogeneous.[2] These rather daunting requirements have made it difficult even to carry out exploratory experiments to test the proposed ideas.

Naturally, an alternative to standard plasma experiments, where these severe constraints could be mitigated, will be highly welcome. One could then begin initial experimentation to determine the validity of the theoretical framework, and shed light on the eventual feasibility of these ideas. Fortunately, such an alternative exists; it is provided by certain special plasmas to be found in the narrow-gap semiconductors.[3] It just so happens that in the two-band approximation of Kane's model dealing with narrow-gap semiconductors, the Hamiltonian of the conduction-band electrons mimics the relativistic form $H=(m_*^2c_*^2+c_*^2p^2)^{1/2}$. Here $c_*=(E_g/2m_*)^{1/2}$ plays the part of the speed of light, m_* is the effective mass of the electrons at the bottom of the conduction band, E_g is the width of the forbidden band, and p is the electron quasimomentum.

This formal similarity would just be an interesting curio, but for the fact that, in several materials, the characteristic velocity c_* that enters in the Kane dispersion law is much less than the speed of light (for example, $c_*\approx 3\times 10^{-3}c$ for InSb). Because of this, the jitter velocity of the electron fluid in the conduction band can become ''relativistic'' even when modest intensity electromagnetic fields are applied. A relativistic jitter is possible because the conduction band is partially empty, and as a result the electrons can become accelerated under the effect of an electric field. Nonparabolicity of the conduction band implies a nonlinear electron velocity-momentum dependence ($\mathbf{v}=\partial H/\partial \mathbf{p}$) which, in turn, leads to a nonlinear dependence of the current density on the electric field. This nonlinearity dominates the nonlinearity due to electron heating, provided the relevant wave frequencies are considerably higher than the effective collision frequency. Thus the dominant nonlinearity of this system exactly corresponds to the nonlinearity which lies at the foundation of the acceleration schemes mentioned earlier.

An expected consequence of this remarkable coincidence has been to use the methodologies of relativistic plasmas to develop a pseudorelativistic dynamics for the conduction electrons in order to delineate the optical properties of narrow-gap semiconductors.[4] The plasma physics of Kane-type semiconductors has also been actively investigated: in Ref. 5, different kinds of parametric excitations of density waves, and parametric amplification of electromagnetic (EM) waves are presented, while in Ref. 6, the authors explore the possibility of finding localized solitonic structures, in addition to studying the nonlinear self-interaction of EM waves in semiconductors.

In a recent publication,[7] we suggested that these semiconductor plasmas could become a veritable laboratory for testing the conceptual foundation of particle accelerators based on laser-plasma interaction. It was shown that an intense short laser pulse propagating through a semiconductor plasma generates a measurable longitudinal Langmuir wave in its wake (wake-field) even for very moderate laser powers. Whenever the laser pulse duration $T_L\sim\omega_e^{-1}$, the plasma frequency, wake-field excitation occurs. This process is naturally the analog of the expected laser wake-field excitation in the usual relativistic plasma. This mechanism for the nonlinear coupling between photons and plasmons is an efficient way to produce finite-amplitude plasma excitations in semiconductors with readily available technology.

The next step in the investigation of relativistic semiconductor plasmas is to explore if the beat-wave scheme for the generation of fast-moving large-amplitude longitudinal waves is also feasible. In this scheme, the plasma oscillation of frequency ω_e (the electron plasma frequency) is resonantly excited by the ponderomotive force of two propagating collinear laser beams with nearly equal frequencies ω_1 and ω_2 ($\omega_1\sim\omega_2$), such that $\omega_1-\omega_2\sim\omega_e\ll\omega_1,\omega_2$ (underdense plasma condition). Because the beat-wave generation

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of plasma waves is a resonant process, large-amplitude plasma waves can develop even for relatively weak lasers; the laser intensity requirement could be considerably less than what is necessary for the wake-field generation scheme. However, experimental observation of beat-wave excitation in normal gaseous plasmas is difficult because of the inherent inhomogeneities. Semiconductor plasmas, on the other hand, are rather homogeneous, and can be ideal for the experimental simulation of the generic beat-wave scheme. Since the nature of the induced longitudinal field will reflect important characteristics of the semiconductors, the experiment could easily lead to an exciting diagnostic as well. Although there are several materials with a nonlinear velocity-momentum relationship, we concentrate on semiconductors of the Kane type because they, in addition, provide us with a laboratory to simulate (with much smaller laser intensities) a relativistic plasma.

For studying high-frequency oscillations, the semiconductor plasma can be treated as an ideal electron fluid moving in a fixed ion lattice. The electrodynamics of this system can, then, be described by the set of electron fluid and Maxwell equations[4–7]

$$\nabla\times\mathbf{B}=\frac{\epsilon_0}{c}\frac{\partial\mathbf{E}}{\partial t}-\frac{4\pi e}{c}n\mathbf{v}, \tag{1}$$

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \tag{2}$$

$$\epsilon_0\nabla\cdot\mathbf{E}=4\pi e(n_0-n), \tag{3}$$

$$\frac{\partial n}{\partial t}+\nabla\cdot(n\mathbf{v})=0, \tag{4}$$

$$\frac{\partial\mathbf{p}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{p}=-e\mathbf{E}-\frac{e}{c}(\mathbf{v}\times\mathbf{B}), \tag{5}$$

where n is the electron concentration, ϵ_0 is the dielectric constant of the lattice, and $\mathbf{p}$ is the quasimomentum of the conduction-band electrons. This system is augmented by the nonlinear velocity-momentum relation

$$\mathbf{v}=\frac{\mathbf{p}}{m_*(1+p^2/m_*^2c_*^2)^{1/2}}. \tag{6}$$

Equation (6) is valid if $\omega\gg\nu$, where ω is the characteristic wave frequency and ν is the electron effective collision frequency. Using the condition for the absence of generalized vorticity,

$$\mathbf{B}=\frac{c}{e}\nabla\times\mathbf{p}, \tag{7}$$

and Eq. (6), Eq. (5) can be rewritten in the simplified form

$$\frac{\partial\mathbf{p}}{\partial t}+m_*c_*^2\nabla\gamma=-e\mathbf{E}, \tag{8}$$

where γ is the relativistic factor

$$\gamma=(1+p^2/m_*^2c_*^2)^{1/2}. \tag{9}$$

For simplicity we consider a one-dimensional problem, in which the laser radiation propagates along the z axis, and all physical quantities depend on only the space coordinate z and time t. Due to their natural confinement and homogeneity, the one-dimensional assumption is not very restrictive for solid-state plasmas. For this case, the transverse component of the momentum equation (8) can be written as

$$\frac{\partial\mathbf{p}_\perp}{\partial t}=-e\mathbf{E}_\perp, \tag{10}$$

which, on integration, yields $\mathbf{p}_\perp=(e/c)\mathbf{A}_\perp$, where $\mathbf{A}_\perp$ is the vector potential of the EM field. The longitudinal electron motion obeys

$$\frac{\partial p_z}{\partial t}+m_*c_*^2\frac{\partial}{\partial z}\left(1+\frac{(p_\perp^2+p_z^2)}{m_*^2c_*^2}\right)^{1/2}=e\frac{\partial\varphi}{\partial z}, \tag{11}$$

where φ is the scalar potential associated with the excited longitudinal field.

It is evident from Eq. (11) that the laser radiation, represented by the $p_\perp^2$ term, always generates a longitudinal response. The details of the response will surely depend upon the nature of the inducing laser radiation. In order to investigate the resonant processes inspired by the beat-wave concept, we begin with two linearly polarized beams of frequencies ω_1 and ω_2, with $\omega_1-\omega_2\sim\omega_e\ll\omega_1,\omega_2$. The transverse component of the induced electron momentum can then be written as ($\mathbf{p}_\perp\sim A_\perp$)

$$\mathbf{p}_\perp=\hat{\mathbf{x}}p_1\exp(-\omega_1t+k_1z)+\hat{\mathbf{x}}p_2\exp(-\omega_2t+k_2z)+\text{ c.c.}, \tag{12}$$

where $k_{1,2}$ are the mean wave numbers associated with the laser beams, and the amplitudes p_1 and p_2 are related to the laser electric fields by $p_{1,2}=ieE_{1,2}/\omega_{1,2}$.

Note that the transverse waves satisfy the linear dispersion relation $\omega_{1,2}^2=\omega_{*e}^2+\epsilon_0^{-1}k_{1,2}^2c^2$, where $\omega_{*e}=(4\pi e^2n_0/\epsilon_0m_*)^{1/2}$ is the effective Langmuir frequency. If $\omega_{1,2}\gg\omega_{*e}$ then the phase velocity of the plasma wave $\omega_{*e}/(k_1-k_2)$ is equal to $v_g=c\epsilon_0^{-1/2}(1-\omega_{*e}^2/\omega_{1,2}^2)^{1/2}$, the group velocity of the laser beam. Since $c\epsilon_0^{-1/2}$ is the speed of light (approximately the phase velocity of the lasers) in the semiconductor, all relevant phase speeds are nearly equal. We can then safely assume that the resultant wave motion is a function of the single variable $\tau=t-z/v_g$, where $v_g\approx c\epsilon_0^{-1/2}$. The transparent (underdense) plasma ($\omega_{1,2}\gg\omega_{*e}$) assumption offers an additional simplification; we may assume that, during the interaction time of interest, the laser fields ($p_{1,2},E_{1,2}$) remain unchanged and can be presumed to be constant.[7]

Using Eqs. (3), (4), and (11), and choosing the initial conditions that there is no longitudinal motion at $\tau=0$, the scalar potential of excited longitudinal field φ is found to satisfy

$$\frac{d^2\phi}{d\tau^2}=\omega_{*e}^2\frac{u_g^2}{(1-u_g^2)}\left[\frac{u_g(1+\phi)}{[(1+\phi)^2-(1-u_g^2)(1+p_\perp^2)]^{1/2}}-1\right], \tag{13}$$

where we have introduced the dimensionless variables $\mathbf{p}=\mathbf{p}/m_*c_*$, $\phi=e\varphi/m_*c_*^2$, and $u_g\approx(c/c_*\epsilon_0^{1/2})$. Equation

(13) is very similar to the equation for the wake field φ driven by a single laser pulse.[7]

In order to demonstrate that φ of Eq. (13) can be resonantly driven, let us assume $p_\perp^2(p_\perp \sim A_\perp) \ll 1$. This, along with the fact that $u_g(\simeq c/c_* \epsilon^{1/2}, c \gg c_*) \gg 1$, converts Eq. (13) to $(\epsilon = u_g^{-2} \ll 1)$,

$$\frac{d^2\phi}{d\tau^2} + \omega_{*e}^2 \phi - \epsilon \frac{\omega_{*e}^2}{2}(3\phi^2 + \phi^3) = \omega_{*e}^2 \frac{p_\perp^2}{2}, \qquad (14)$$

an equation representing a driven nonlinear oscillator.

The driving force $\sim p_\perp^2$ [the right-hand side of Eq. (14)] contains a term with frequency $\Omega = \omega_1 - \omega_2$, and, if $\Omega \approx \omega_{*e}$, the plasma wave can be resonantly excited. The nonlinear terms on the left-hand side of Eq. (14) are due to the relativistic self nonlinearity of the longitudinal motion, and follow from the nonparabolicity of the conduction-band dispersion relation.

If we neglect the nonlinear terms in Eq. (14) (i.e., $\epsilon \to 0$), the resonant mode will have secular growth. *The relativistic nonlinearity provides a saturation mechanism* without which the model will be quite unphysical. It must be remarked here that any substantial departure from parabolicity will lead to a nonlinearity which will provide the desired saturation. Although the details can be quite different for different systems, the same basic physical mechanisms will be operational, and the essence of the following calculation will be valid for a variety of semiconductors.

By employing simple analytical tools, we can extract much information from Eq. (14). Following standard procedure, let us seek a solution of the form[8]

$$\phi = \phi_0(\tau)\exp(-i\Omega\tau) + \text{ c.c.}, \qquad (15)$$

where $\phi_0(\tau)$ is a slow time-dependent amplitude. Clearly Eq. (15) can be an approximate solution only when we concentrate only on the resonant terms. After simple manipulations, the resonant terms lead to the leading-order equation

$$i\frac{d\phi_0}{d\tau} + \delta\phi_0 + \beta|\phi_0|^2\phi_0 = -\lambda, \qquad (16)$$

where $\delta = \Omega - \omega_{*e}$ is the frequency mismatch, $\beta = \frac{3}{4}\epsilon\omega_{*e}$, and $\lambda = \frac{1}{2}\omega_{*e} p_1 p_2^*$. To proceed further, we let $\phi_0 = A\exp(i\psi)$, and obtain two coupled first order equations for A and ψ.[8] Using the obvious initial condition $A(0) = 0$, and eliminating ψ, we find

$$\left(\frac{dA}{d\tau}\right)^2 = \lambda^2 - \left(\frac{\beta}{4}A^3 + \frac{\delta}{2}A\right)^2, \qquad (17)$$

from which, among other things, the steady-state amplitude $(dA/d\tau = 0)$ can be easily inferred. For perfect phase matching $(\delta = 0)$, the saturation amplitude is

$$A_{\max} = \left(\frac{4\lambda}{\beta}\right)^{1/3} = \left(\frac{8}{3\epsilon}|p_1||p_2|\right)^{1/3}, \qquad (18)$$

and the initial growth is linear in time $A = \lambda\tau = 0.5\omega_{*e}|p_1||p_2|\tau$. It is also straightforward to estimate the saturation time

$$\tau_{\text{sat}} \approx 2.8\omega_{*e}^{-1}\epsilon^{-1/3}(|p_1||p_2|)^{-2/3}. \qquad (19)$$

For a finite mismatch $(\delta \neq 0)$, the saturation amplitude A_δ can be obtained solving the cubic [right-hand side of Eq. (17)],

$$A_\delta^3 + \frac{2\delta}{\beta}A_\delta - \frac{4\lambda}{\beta} = 0. \qquad (20)$$

Notice that due to the nonlinear term $\beta|\varphi_0|^2$ in Eq. (16), the maximum amplitude does not correspond to $\delta = 0$. Analysis of Eq. (20) shows that the maximum value of $A_{\delta\max} = 1.6A_{\max}$ corresponds to the frequency mismatch

$$\delta = -1.3\omega_{*e}\epsilon^{1/3}(|p_1||p_2|)^{2/3}. \qquad (21)$$

Approximate analytical formulas (18)–(21) can help us gauge the efficiency of the beat-wave mechanism. Let us take the n-InSb plasma for which the required parameters are $T = 77$ K, $m_* = m_e/74$, $\epsilon_0 = 16$, and $c_* = c/253$. For the CO_2 laser beams with respective vacuum wave lengths $\lambda_1 = 10.81$ μm and $\lambda_2 = 10.6$ μm, the resonant Langmuir frequency is $\omega_{*e} = 3.5 \times 10^{12}$ corresponding to a plasma density $n = 8 \times 10^{14}$ cm^{-3}, and plasmon wave length $\lambda_e = 0.13$ mm.

For these parameters, the electron collision frequency $\nu \sim 10^{-2}\omega_{*e}$ is negligible. Notice that when the collision frequency approaches the plasma frequency, serious damping of the wave field can occur. However, for a variety of semiconductors at sufficiently low densities the deleterious effects of collisions can be avoided. Appropriate conditions for each semiconductor will have to be individually worked out.

Assuming that the two laser beams have equal intensities with an electric field $E_{1,2} = 10^5$ V/cm, or equivalently, $p_\perp^2 \approx 0.4$, the maximum longitudinal field can grow to $E_l \approx 120$ V/cm in a total time $\tau_{\text{sat}} \approx 80\omega_{*e}^{-1}$. Note that the saturation time [Eq. (19)] is larger for smaller amplitudes of laser radiation. When the time for reaching saturation is greater than the collision period, the collisional effects must be included. This could be achieved by formally adding the term $\nu\partial\phi/\partial\tau$ on the left-hand side of Eq. (14). We do not consider this effect here, it will trivially reduce the amplitude of the longitudinal field, in addition to introducing bistability and hysteresis.

We have thus demonstrated that strong, easily measurable longitudinal waves can be excited by two near-frequency laser beams propagating in an n-type InSb plasma. The character of the excited wave, of course, depends not only upon the defining parameters of the system, but also on the details of the beat-wave mechanism. One of the very important aspects of the proposed beat-wave and similar schemes is that the relativistic nonlinearities will provide a saturation mechanism. This general hypothesis, among others, can be readily tested in a cheap, readily available, relativistic, and highly uniform semiconductor plasma. Experimentation of this type could answer several preliminary questions as to the eventual viability of these schemes, and provide a basic diagnostic for the nonlinear optical properties of solid state systems as well.

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[1] T. Tajima and J. M. Dawson, Phys. Rev. Lett. **43**, 267 (1979); P. Sprangle, E. Esary, A. Ting, and G. Joyce, Appl. Phys. Lett. **53**, 2146 (1988); J.M. Dawson, Phys. Scr. **T52**, 7 (1994).
[2] R. J. Noble, Phys. Rev. A **32**, 460 (1985); W. Horton and T. Tajima, *ibid.* **31**, 3937 (1985); L. M. Gorbunov and V. I. Kirsanov, Sov. Phys. JETP **69**, 329 (1989); G. Matthieussent, Phys. Scr. **T50**, 51 (1994).
[3] O. Kane, J. Phys. Chem. Solid **1**, 249 (1957).
[4] N. Tzoar and J. I. Gersten, Phys. Rev. Lett. **26**, 1634 (1971); Phys. Rev. B **4**, 3540 (1971).
[5] P. A. Wolff and G. A. Pearson, Phys. Rev. Lett. **17**, 1015 (1966); J. I. Gersten and N. Tzoar, *ibid.* **27**, 1650 (1971); F. G. Bass and V. A. Pogrebnyak, Fz. Tverd. Tela (Leningrad) **14**, 1766 (1972) [Sov. Phys. Solid State **14**, 1518 (1972)]; L. A. Ostrovskii and V. G. Yakhno, *ibid.* **15**, 427 (1973); [*ibid.* **15**, 306 (1973)]; Ya. I. Kishenko and N. Ya. Kotsarenko, *ibid.* **18**, 3295 (1976) [*ibid.* **18**, 1920 (1976)]; V. S. Paverman and N. A. Papuashvili, Fiz. Tekh. Polprovodn. **19**, 1386 (1985) [Sov. Phys. Semicond. **19**, 852 (1985)].
[6] K. A. Gorshkov, V. A. Kozlov, and L. A. Ostrovskii, Zh. Éksp. Teor. Fiz. **65**, 189 (1973) [Sov. Phys. JETP **38**, 93 (1974)]; F. G. Bass, L. B. Vatova, and Yu. G. Gurevich, Fiz. Tverd. Tela (Leningrad) **15**, 3053 (1973) [Sov. Phys. Solid State **15**, 2033 (1974)]; F. G. Bass and S. I. Khankina, Fiz. Tekh. Tela Poluprovodn. **18**, 353 (1984) [Sov. Phys. Semicond. **18**, 220 (1984)].
[7] V. I. Berezhiani and S. M. Mahajan, Phys. Rev. Lett. **73**, 1837 (1994).
[8] M. N. Rosenbluth and C. S. Liu, Phys. Rev. Lett. **29**, 701 (1972); C. M. Tang, P. Sprangle, and R. N. Sudan, Phys. Fluids **28**, 1974 (1985); R. Bingam, R. A. Cairns, and R. G. Evans, Plasma Phys. Control. Fusion **28**, 1735 (1986).