Magnetic flux noise in superconducting rings and disks close to the superconducting transition

S. A. L. Foulds, J. Smithyman, G. F. Cox, and C. M. Muirhead
University of Birmingham, Birmingham B15 2TT, United Kingdom

R. G. Humphreys
DRA, St Andrews Road, Malvern WR14 3PS, United Kingdom
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We report measurements of the magnetic flux noise in a series of superconducting rings and disks having a wide range of superconducting transition temperatures. In the absence of an applied magnetic field, the behavior of rings can be described in terms of a loop containing one or two superconducting weak links. We observe logarithmic flux creep in discrete flux quanta when a magnetic field is applied to such rings and then abruptly removed. For a Y-Ba-Cu-O ring and a complete disk, the behavior in the presence of an applied field is well described by a model in which the film displays a temperature-dependent resistivity. We offer an explanation for the observation that for a given superconductor and measurement geometry, the magnitude of the noise peak close to the superconducting transition T_c is similar for all specimens and we find that the magnitude of this peak is directly proportional to the transition temperature. [S0163-1829(97)03213-X]

I. INTRODUCTION

Magnetic noise has proved to be a powerful tool for investigating the motion of vortices in superconductors at very low flux densities. It also offers an advantage over transport measurements in that flux motion equivalent to voltages as low as attovolts may be detected. In particular, noise measurements can be used to study the motion of vortices back and forth between a small number of trapping sites where the voltage in a transport measurement would be indetectable. The magnetic field variations involved are sufficiently small that measurements are always made with a superconducting quantum interference device (SQUID), sometimes directly detecting the flux in a specimen and sometimes coupled via a flux transformer. Measurements have been reported on bulk samples,[1] thick[2,3] and thin films,[4–12] and single crystals.[4] Most of the reported work has been on $YBa_2Cu_3O_{7-\delta}$ (YBCO), while some has been on $Bi_2Sr_2CaCu_2O_{8+\delta}$ (BSCCO) (Refs. 4 and 5) and $Tl_2Ca_2Ba_2Cu_3O_x$ and $TlCa_2Ba_2Cu_3O_x$ (TCBCO).[5] The results of measurements on a large range of films, flakes, single crystals, and a flux transformer have been reviewed by Ferrari *et al.*[5]

Most of the reported work has involved a study of telegraph and $1/f$ noise, with some reports of $1/f^{1.5}$ noise.[6,7] Telegraph noise is usually attributed to the thermally activated jumping of flux quanta or flux bundles between one or more trapping sites and is normally observed very close to T_c. Such jumping is well described by a Lorentzian spectrum (Sec. III A). The magnitude of the jumps and the temperature dependence of the hopping frequency have been used to deduce the distance between the trapping sites and the heights of barriers, respectively.[5,8] $1/f$ noise has been attributed to a linear superposition of two-state hopping processes having a wide range of barrier heights between the energy wells. The frequency and temperature dependence of the noise has been used to deduce the dominant energy ranges involved.[4] Cooling the specimen in a magnetic field has been observed to cause an increase in the $1/f$ noise which is often linear in the cooling field and is, therefore, consistent with the above model.[9] The absence of a magnetic field dependence for the noise in a single crystal of YBCO has been interpreted as ruling out flux motion as the cause of the noise[1] and the $1/f$ noise has also been attributed to universal conductance fluctuations,[10] although both of these conclusions have been disputed.[5] Some work has also been reported on flux transformers,[11] but the geometry is then complicated by the presence of series crossovers in the pancake input coil of a SQUID. In very thin films of BSCCO (Ref. 6) and superlattices of YBCO and $PrBa_2Cu_3O_{7-\delta}$ (PBCO),[7] the noise has been shown to exhibit an extended $1/f^{1.5}$ dependence and this has been interpreted in terms of a diffusive motion of magnetic flux.

Despite extensive studies of the magnetic noise at temperatures well below T_c, the noise very close to T_c has been little studied in YBCO films, although very recently it has been used to probe the possible existence of a Kosterlitz-Thouless transition in the flux line lattice.[13] It has been also been noted that, for a given experimental arrangement, the amplitude of the noise peak is only weakly sample dependent,[5,8] although, to our knowledge, an explanation of this effect has not previously been given.

The aim of this paper is to report studies of the flux noise in a series of rings and disks close to their superconducting transition temperature and to offer explanations of the magnitude and temperature dependence of the noise. In so doing we aim to throw further light on some of the other observations referred to above.

II. EXPERIMENTAL METHOD

Specimens with a wide range of transition temperatures were studied in this work. These were YBCO, YBCO/PBCO superlattices, composite rings of YBCO and $Nd_{2-x}Ce_xCuO_{4-\delta}$ (NCCO), and niobium. All specimens, with the exception of the niobium ring F, were epitaxially grown thin films. Some specimens were produced by e-beam

TABLE I. Composition and parameters of specimens used in this work. The YBCO rings and disks were produced by e-beam evaporation (Ref. 14). The YBCO/PBCO disks were produced by pulsed laser deposition (Ref. 15) and consist of ten layers of YBCO, 3-nm-thick, separated by layers of PBCO. For disk I the PBCO was 6 nm thick and for disc J was 9 nm thick. The YBCO/NCCO specimen was also produced by pulsed laser deposition (Ref. 16) and consisted of two rings slightly greater than a semicircle with a small (~0.3 mm) overlap. The niobium ring was made by sputter deposition (Conductus, Inc).

Specimen	Material	o.d. (mm)	i.d. (mm)	Thickness (nm)	Comments
Ring *A*	YBCO	8	7.75	375	
Ring *B*	YBCO	8	7.75	375	Notch (25 μm left)
Ring *C*	YBCO	8	7.75	375	Notch (10 μm left)
Ring *D*	YBCO/NCCO	8	7.75	200	See caption
Ring *E*	YBCO/NCCO	8	6.0	200	See caption
Ring *F*	Nb	8	7.75	200	
Disk *G*	YBCO	8		300	Not annealed
Disk *H*	YBCO	8		300	Annealed
Disk *I*	YBCO/PBCO	8		84	See caption
Disk *J*	YBCO/PBCO	8		111	See caption

evaporation[14] and some by pulsed laser deposition.[15,16] The films were photolithographically patterned to form disks or annualr rings having an outer diameter of 8 mm. Ring specimens *B* and *C* had an additional notch patterned into them to locally reduce the track width to 25 and 10 μm, respectively. Rings $A-C$ and disk *H* were annealed after patterning. Details of all specimens are given in Table I.

The cryostat is described in more detail elsewhere.[17] Briefly, the specimen under test is held by a thin layer of silicon grease onto a silicon disk which is temperature controlled to an accuracy of ± 10 mK over the temperature range 4.2–100 K. The flux noise from the specimen is detected by a flux transformer close wound from ten turns of 50-μm Formex-insulated niobium wire on a 5-mm-diam, 30-mm-long tube made from a single layer of closely packed 120-μm enameled copper wires glued lengthways. The flux transformer was separated from the specimen by ~3 mm. The flux transformer is, in turn, connected to a commercial dc SQUID and can be raised above its superconducting transition to remove any trapped flux. A small magnetic field can be applied with a superconducting coil which is normally operated in persistent mode. The specimen stage is enclosed in two superconducting shields and suspended by three thin copper rods from a temperature-controlled ring. The temperature control heater and thermometer, the persistent mode switch, and a heater for the flux transformer are located outside the superconducting shields and make no observable contribution to the background noise of the system. The cryostat is mounted in conventional glass Dewars inside three nested μ-metal shields. The background field at the pickup coil was measured to be ~100 nT by rotating a separate pickup coil *in situ* at 4.2 K.

The experimental procedure varies according to the property measured.

(i) For noise measurements, the system is first cooled to 4.2 K. The specimen is then heated to a temperature above its superconducting transition, and the field coil and flux transformer heaters are operated for a few seconds to remove any circulating currents generated on cool down of the cryostat. The specimen is then stabilized at the desired temperature for about 10 min before a noise measurement is taken.

For measurements in a magnetic field, the field is trapped in persistent mode at a specimen temperature above the superconducting transition. We have found no subsequent dependence of the magnetic noise on the order in which different temperature readings are taken or upon the rate of change of temperature.

(ii) For flux expulsion measurements, an additional small field (~10 nT) is trapped in persistent mode at 4.2 K and the specimen is heated in stages until above T_c. The corresponding flux change in the pickup coil serves to provide a rough estimate of T_c and, for the annular rings, enables the magnitude of the flux jumps to be determined as explained in Sec. III A.

(iii) For M/H loops and flux creep measurements, the high sensitivity of the apparatus prevented us from making measurements at the flux quantum level unless the field coil was operated in persistent mode. Even while in persistent mode, however, we were able to make small changes in the magnetic field by passing a small current down the external leads, this current dividing in inverse proportion to the inductance of the field coil and the superconducting switch. We used this method to study both the flux creep and hysteresis loops of an annular ring as described in Sec. III B.

III. RESULTS AND DISCUSSION

We first consider the noise behavior of the three YBCO rings in zero applied magnetic field (Sec. III A) and then the flux creep behavior (Sec. III B). We examine the effect on the noise of applying a rang of larger magnetic fields up to 200 μT (Sec. III C). We describe the behavior of the two disk samples (Sec. III D) and finally consider five samples with lower transition temperatures (Sec. III E). Our most detailed experimental results were taken on YBCO ring A and YBCO disk *G*, and we will concentrate on the behavior of these specimens in our discussion.

A. Flux noise in YBCO rings in zero applied field

In Fig. 1, we show the results for the spectral density of the flux noise S_Φ in zero applied field and the response to a

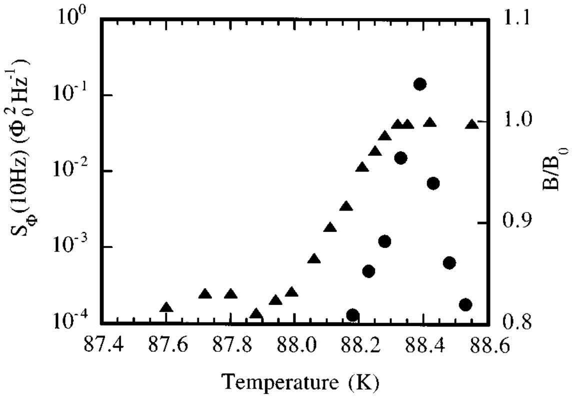

FIG. 1. Noise spectral density at 10 Hz vs temperature (●) and response to a 10-nT field applied at 4.2 K (▲) for the notched ring C. The response to the applied field has its midpoint 20 mK lower in temperature than the noise peak due to the 1-s settling time used in these measurements.

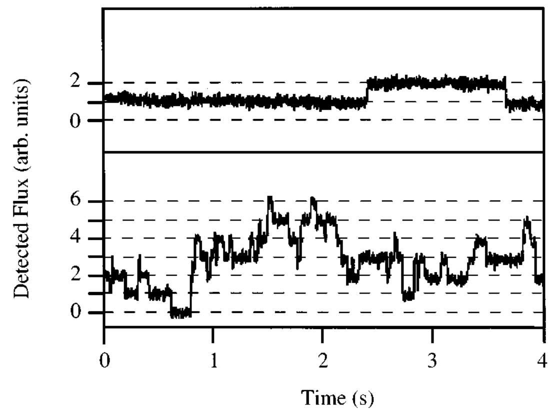

FIG. 3. Output voltage of SQUID electronics vs time for ring A. Results are shown for the same two closely spaced temperatures as in Fig. 2. The upper trace is for $T=86.00$ K. The lower trace is for $T=86.06$ K. The dotted lines are equally spaced and scaled to pass through the quantized voltage levels.

10-nT applied field for ring C, as a function of temperature. We see that the excluded flux falls to zero and the flux noise peaks at a temperature close to the expected T_c for a YBCO thin film. We also find that the noise displays a rather accurate Lorentzian spectrum over the entire temperature range for which its amplitude is significantly above the white noise background of the SQUID. Examples of noise spectra for ring A are shown in Fig. 2. Observation of the time trace for ring A shows that this noise results from flux jumps between some five or so sharply defined levels (Fig. 3). In order to determine the size of these jumps in terms of flux in the ring, we have measured the sensitivity of the detection system to a small (10-nT) field applied both well above and well below T_c. The sensitivity of the pickup loop to a current flowing in a ring the same size as the specimen was also deduced from room-temperature mutual inductance measurements on copper coils of the same dimensions at a frequency of 1 kHz. From these measurements it is straightforward to deduce both the self-inductance of the specimen ring (15±3 nH) and the size of the flux jumps shown in Fig. 3. These were found to be in multiples of $(1.1\pm0.2)\Phi_0$. We note that this inductance is slightly smaller than the value expected for a ring of

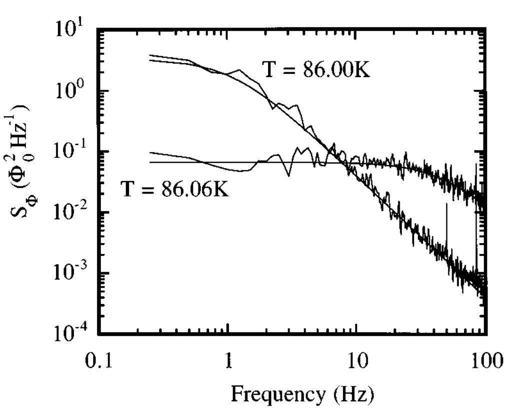

FIG. 2. Flux noise spectral density vs frequency for ring A at two different temperatures. The smooth curves are fits to Lorentzian spectra.

these dimensions for which we find $L\sim21$ nH. The average frequency of these flux jumps is seen to be very strongly dependent on temperature.

We can explain the observed behavior if we model the ring as a loop of superconductor close by a single Josephson junction. We should emphasize that any restriction which provides a barrier to flux motion into and out of the ring would suffice. It may be some form of localized junction or microbridge, or a portion of the ring may have a slightly lower T_c than the rest. We will, however, proceed with the model of a Josephson junction for calculational convenience.

The energy of a superconducting ring containing a Josephson junction is well known,[18]

$$U=\frac{(\Phi_i-\Phi_{\mathrm{ext}})^2}{2L}-\frac{I_c\Phi_0}{2\pi}\cos\left(\frac{2\pi\Phi_i}{\Phi_0}\right), \qquad (1)$$

where Φ_i is the flux in the ring, Φ_{ext} the applied flux, L the ring inductance, and I_c the critical current of the junction. For $\beta_L=2\pi LI_c/\Phi_0>1$, $U(\Phi_i)$ is multivalued and there are metastable minima in the energy. For $\beta_L\gg1$, which we will argue pertains to most of our experimental results, the minima in the energy are separated in Φ_i by an amount very close to Φ_0.

The barriers between the minima are surmounted by thermally activated jumping at a rate

$$\nu=\nu_a e^{-\Delta U^{\pm}/k_BT}, \qquad (2)$$

where ν_a is an attempt frequency. For $\Phi_{\mathrm{ext}}=0$, $\beta_L\gg1$, and $\Phi_i=n\Phi_0\ll LI_c$, it can easily be shown that the barriers between the minima are given by

$$\Delta U^{\pm}=\frac{I_c\Phi_0}{\pi}+\frac{\Phi_0^2}{8L}\pm n\,\frac{\Phi_0^2}{2L}, \qquad (3)$$

where the positive sign is for increasing Φ_i and the negative sign for decreasing Φ_i. Occupancy of higher-flux states becomes increasingly rare as expected from a Boltzman distribution.

For the parameters relevant to our experiment, only the lowest few energy states are occupied in the duration of a

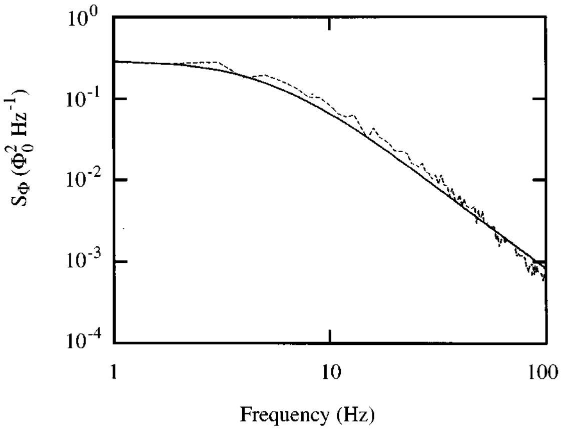

FIG. 4. Results of computer model for the noise spectral density resulting from thermally activated flux jumps in a ring having $L=17$ nH, $I_c=49$ μA, $T=86$ K, and an attempt frequency 10^{11} Hz. The solid line is a fit to a Lorentzian function.

measurement. We must emphasize that, in our experiment, the very rapid variation of ν with temperature results from the (very strong) temperature dependence of I_c close to T_c rather than the explicit dependence on temperature in Eq. (2).

Jumping between two states having lifetimes τ_1 and τ_2 results in a Lorentzian noise spectrum[19]

$$S(f)=\frac{4A^2}{(\tau_1+\tau_2)}\frac{\tau^2}{1+(2\pi f)^2\tau^2}, \qquad (4)$$

where A is the amplitude of the jump, $1/\tau$ is an average jump rate given by $1/\tau=1/\tau_1+1/\tau_2$, and τ_1 and τ_2 depend on temperature. For the special case $\tau_1=\tau_2=\tau$, this becomes

$$S(f)=\frac{2A^2\tau}{1+(\pi f\tau)^2}. \qquad (5)$$

The spectrum is white for $f\ll 1/\tau\pi$ and falls as $1/f^2$ for $f\gg 1/\tau\pi$. As the temperature is varied therefore, there is a peak in the noise spectrum as the Lorentzian knee frequency f_0 (defined by $\pi f_0\tau=1$) passes through the measurement frequency.

We are not aware of any analytic result for the noise spectrum of a *multilevel* hopping process, particularly for the case where the τ values are Φ_i dependent. We will be making use of the Lorentzian form of our experimental noise spectra in the analysis which follows, and so it is of some importance to establish that the spectrum is indeed of this form. We have computer modeled the noise expected from a ring containing a weak link by allowing the flux to jump randomly between energy minima as described by Eq. (1) taking $\Phi_{\rm ext}=0$ and with the energy modulation amplitude chosen to provide a similar number of available minima ($\sim$5) as for the real specimen. The probability of a given jump is determined by Eq. (2), and a fast Fourier transform is used to obtain the noise spectrum. Here we have assumed an attempt frequency $\nu_a=10^{11}$ Hz, although the results are rather insensitive to the value of ν_a because of the dominant exponential dependence. A typical fit to the computed noise is shown in Fig. 4. This Lorentzian form is observed for both the computer simulations and in the data for all rings. We will henceforth assume that our noise data are correctly described by a Lorentzian function.

B. *M/H* loops and flux creep in YBCO rings

We have performed magnetization measurements on ring *A* in an applied magnetic field up to 200 nT (Ref. 20) as follows. First, the specimen was cooled in zero applied field to 80 K. A field having a triangular wave form was then applied and the specimen warmed until hysteresis in the SQUID output versus applied field was observed. Such hysteretic behavior occurs when the amplitude of the applied field is sufficient to cause the circulating current in the ring to exceed the critical current of the weak link.[21] While the applied field was in the falling part of the cycle and flux was leaving the specimen, the applied field was abruptly removed and the subsequent decay of flux observed (Fig. 5). The flux decays in discrete flux quanta with an overall time dependence which is logarithmic over two orders of magnitude in time. This is, to our knowledge, the only reported example of logarithmic flux decay at the flux quantum level.

We can make use of this decay curve to deduce the equivalent voltage-current relation that would be measured in a transport experiment. We concentrate here on the short-time scale behavior, where $\Phi_i \gg Lk_BT/\Phi_0$, so that backward jumping over the energy barriers is unimportant. We note first that the large number of steps observed in the flux decay is consistent with our earlier assertion that $\beta_L\gg 1$ in the range of our experiment. For $\beta_L\gg 1$ and for values of Φ_i not too

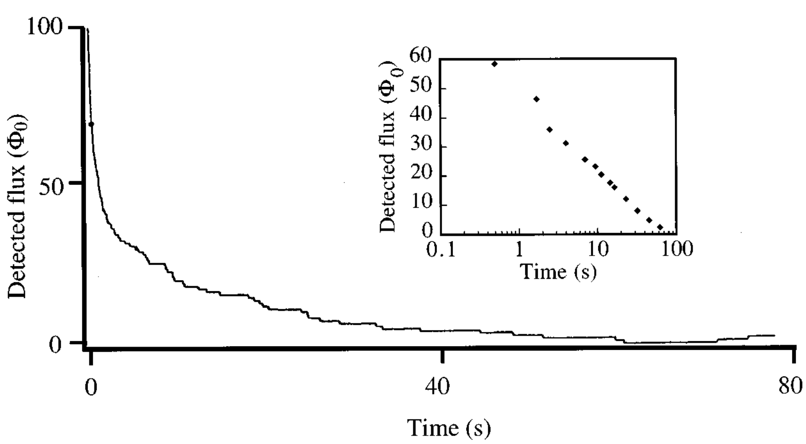

FIG. 5. Flux decay in quantum steps for ring *A* at 86 K after applying and removing a magnetic-field. Note the onset of backward jumping at long times. The inset is a plot against the logarithm of time derived from the main figure.

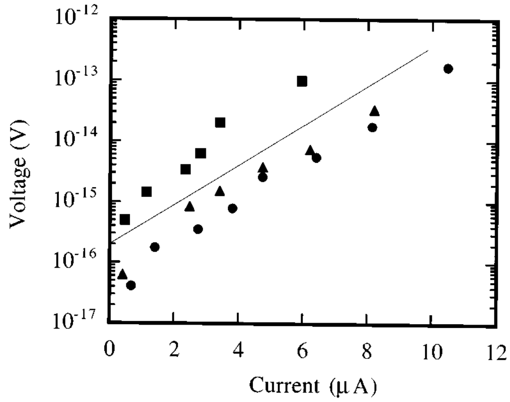

FIG. 6. Voltage-current characteristics for ring A deduced from flux decay data at three different temperatures 85.94 K (■), 85.80 K (▲), and 85.70 K (●). The solid line is from Eq. (6), assuming that the ring is closed by a single junction.

large ($\Phi_i = n\Phi_0 \ll LI_c$), the barrier against flux decay is given by Eq. (3), where we need only consider the negative sign.

The corresponding voltage is given by the rate of change of flux, i.e., $V = \nu_a \Phi_0 \exp(-\Delta U/k_B T)$ and the current I around the ring is $n\Phi_0/L$, from which it immediately follows that

$$\ln(V) = \ln(\nu_a \Phi_0) - \frac{1}{k_B T}\left(\frac{I_c \Phi_0}{\pi} + \frac{\Phi_0^2}{8L} - \frac{\Phi_0 I}{2}\right).$$

If the voltage V results from the jump of flux across m series junctions (i.e., the current flows through m parallel junctions) which are sufficiently strong to trap a flux quantum in the enclosed area, then[22,23] this equation becomes modified to

$$\ln(V) = \ln(\nu_a \Phi_0) - \frac{1}{m k_B T}\left(\frac{I_c \Phi_0}{\pi} + \frac{\Phi_0^2}{8L} - \frac{\Phi_0 I}{2}\right). \quad (6)$$

From the comparison between the derived V/I characteristics (Fig. 6) and Eq. (6), we see that there is quite good agreement for ring A if we take $m=1$. Ring A therefore behaves as a superconducting loop containing a single strong junction. The behavior of the notched ring B is very similar, but shows only half the slope in the ln V versus I plot, implying $m=2$. The behavior of the other notched ring C was qualitatively similar to ring B, but detailed flux creep measurements were not made on this specimen. The possible role of the notch in the different behavior of these rings clearly requires more detailed investigation.

C. Flux noise in YBCO rings in small applied magnetic fields

The effect of a 200-μT applied field on the noise of ring B is shown in Fig. 7. We see that the field has displaced the curve to lower temperature by some 30–40 mK consistent with a corresponding suppression in temperature of the $I_c(T)$ curve. The same effect is observed in the more heavily notched ring C, except that the suppression is less than half

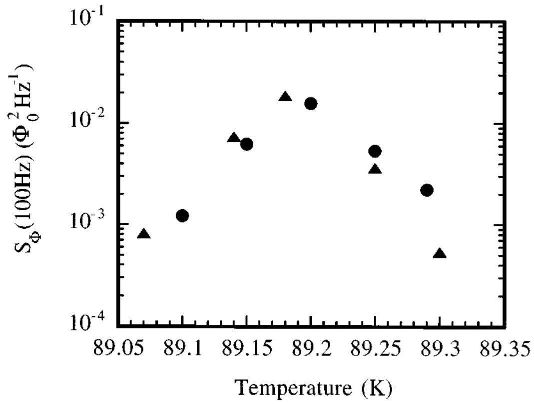

FIG. 7. Noise spectral density at 100 Hz vs temperature for ring B with the 25-μm constriction taken with applied magnetic fields 0 μT (●) and 200 μT (▲).

as large, not far outside our experimental error. In both rings the discrete jumps in the flux disappear, but the spectra remain accurately Lorentzian.

In the case of the un-notched ring A, the effect of an applied field was much stronger and we have therefore been able to make a more detailed study of the effect. The results are shown in Fig. 8. We note the following features.

(i) The magnitude of the noise peak is essentially the same for all fields.

(ii) The temperature of the noise peak drops monotonically with increasing field.

(iii) The sharp jumps in the time trace disappear, as with rings B and C, but the noise spectrum remains accurately Lorentzian.

(iv) The noise peak is broadened by a similar amount for all applied fields.

The behavior in an applied field is clearly different from the zero-applied-field case. The field *independence* of the peak height in S_Φ can be understood on rather general grounds, given the observed Lorentzian behavior. The integrated noise power over all frequencies must equal $k_B T/2$ by equipartition. By integration of Eq. (5), we see that this uniquely determines the noise power at $\pi f_0 \tau = 1$ and, hence, the magnitude of the noise peak as

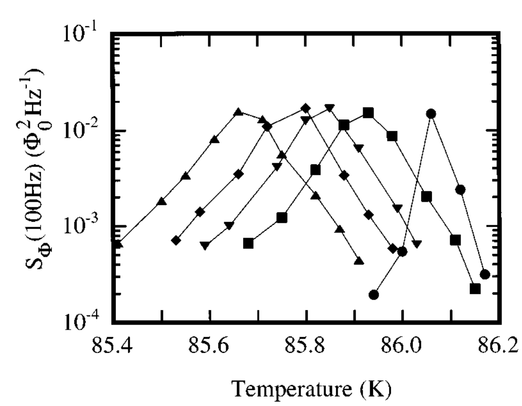

FIG. 8. Noise spectral density at 100 Hz vs temperature for ring A for a range of applied magnetic fields 0 μT (●), 2 μT (■), 10 μT (▼), 40 μT (◆), and 200 μT (▲). The lines are linear interpolations between the data points.

$$S_\Phi(f_0)=\frac{Lk_BT}{\pi f_0}. \tag{7}$$

Given the success of a simple junction model in explaining our zero-field data, we can ask whether the model can be used to explain the behavior in an applied field, the absence of any clearly defined jumps resulting, for example, from some form of flux flow damping of the junction. The clear change in the temperature dependence of the noise in an applied field is, however, difficult to explain in terms of a simple model of a ring containing a weak link. In the absence of any information on the size or nature of the link, we will therefore not pursue this approach further, but consider a totally different model for the behavior in an applied field, which we believe to be consistent with our observations.

Examination of the data in Fig. 8 shows that at a fixed temperature the noise power well below the peak is proportional to the applied field B. Similar behavior has also been observed in the $1/f$ flux noise power reported in thin films well below T_c.[9] Well above the peak, however, we find that the noise power scales as $\sim 1/B$. We also note that the bulk transition temperature would not be shifted by an observable amount for the fields used in this experiment.

A ring of inductance L and containing a series resistance R has a flux noise spectral density

$$S_\Phi(f)=\frac{4k_BTRL^2}{R^2+(2\pi f)^2L^2}. \tag{8}$$

This expression has essentially the same Lorentzian form as that resulting from telegraph noise, only without the sharp jumps in the time trace. The noise peak occurs at $f_0=R/2\pi L$ and has the same value $S_\Phi(f_0)=Lk_BT/\pi f_0$ given by Eq. (7).

We emphasize that the noise at the peak is determined only by f_0, L, and T. The existence of the peak only requires R to change smoothly from a value less than to greater than $2\pi f_0 L$ as the temperature is changed.

Our model for the observed noise is therefore as follows.

In the presence of a magnetic field, the ring develops a resistance R, which results in current, and hence flux noise, spectra which are of Lorentzian form. *A priori* we make no assumption about the nature of this resistance. It may have its origin in flux creep between a series of potential barriers (since in the limit of very low current, where the probabilities of forward and backward jumps are very similar, this resistance tends to a constant value,[22] as required), or it may be a consequence of flux flow viscosity. Alternatively, it may be due to a fluctuation or percolation effect[24] in which the role played by the applied field is to change the magnitude of these effects rather than constitute a direct cause of the resistance.

In Fig. 9(a) we see that the resistance R, deduced from Eq. (8), has a quite different temperature dependence in the presence of an applied field and that this is field independent within the range measured. We know of no junction model which would explain this behavior. There is therefore a clear qualitative difference in the low- and zero-applied-field behavior. The determination of the conditions for equilibrium flux penetration into thin films is a nontrivial matter, depending as it does on the detailed geometry- and temperature-dependent penetration depth. This will be the subject of a future paper. We believe, however, that our smallest deliberately applied field of 2 μT is probably sufficient to cause equilibrium flux penetration into a 250-μm-wide track, whereas our background field of 0.1 μT is not.

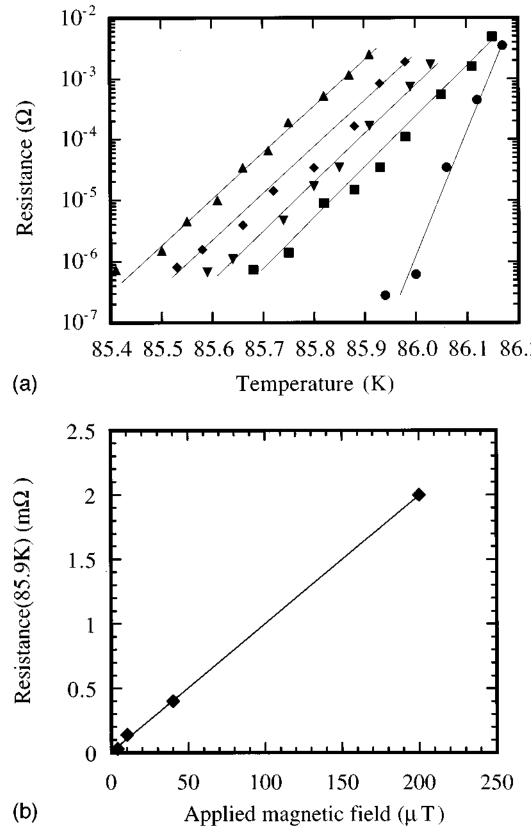

FIG. 9. (a) Resistance vs temperature for ring A deduced from the data of Fig. 8 and applied magnetic fields 0 μT (●), 2 μT (■), 10 μT (▼), 40 μT (♦), and 200 μT (▲). (b) Resistance of ring A vs applied magnetic field deduced at 85.9 K.

In Fig. 9(b) we plot the R versus applied field B at fixed temperature. We see a linear dependence on B over two orders of magnitude, consistent with a flux creep or viscous flow model and implying a flux density proportional to the applied field. The resistance goes to zero at $B=0$, within an experimental accuracy in B of ± 5 μT. This is consistent with observations of the field-dependent flux noise in continuous YBCO thin films at lower temperatures.[9] If we assume a uniform distribution of flux lines, the linearity in B up to 200 μT implies that the noise is unaffected by any flux-line–flux-line interactions at flux line separations down to ~ 3 μm.

We see from Fig. 9(a) that the dependence of $\ln R$ on temperature tends to the same linear relationship in all applied fields. This is again consistent with the low-current limit of a thermally activated flux creep process.[22] To proceed further we will pursue the specific model of flux creep and consider what can be deduced from our results in the context of this model.

Following Ref. 4, we use a form for the barrier of

$$U=U_0(1-[T/T_c]^4). \tag{9}$$

For T close to T_c, it follows that $U=4U_0(1-T/T_c)$. The resistivity therefore has a temperature dependence

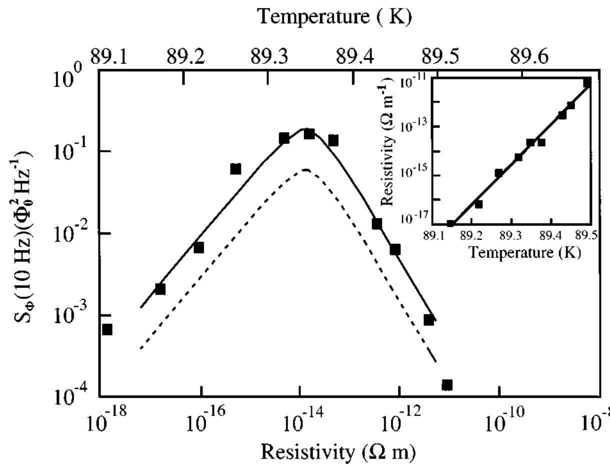

FIG. 10. The experimental data from disk G (■). The dotted line is the computed noise spectral density vs resistivity for an elipsoid of major axis 8 mm and minor axis 300 nm. The experimental and theoretical noise is expressed as an equivalent flux noise in a ring having the same parameters as ring A at the same distance from the flux transformer. The solid line is from the computed noise, scaled up by a factor of 2.8 to achieve a best fit. The horizontal (resistivity) axis has been scaled and shifted horizontally to fit the theoretical curve. The inset shows the resistivity vs temperature deduced from the main graph.

$\rho \propto \exp(-4U_0/k_BT)$ so that $\ln\rho$ is linear in T for a small range of T as observed.

We deduce a value of U_0 of 3 eV, which is one to two orders of magnitude larger than that deduced from flux noise at lower temperatures[4] or flux creep[22,25] data in YBCO. Application of the same formula to our zero-applied-field data implies a value for U_0 of 7 eV. We note that the particular power $(T/T_c)^n$ in Eq. (9) only enters the calculated energy barrier as a factor n.

In summary, therefore, our model is that in zero applied field the behavior of our ring specimens is well explained by a superconducting loop closed by a single weak link, which we have taken to be a Josephson junction, but that in small applied fields, the behavior is dominated by an effective resistance which has its origin in thermally activated flux creep. More work is needed to understand the transition region which occurs between 0.1 and 2 μT.

D. Flux noise in YBCO disks

We concentrate first on the behavior of disk G which was the only YBCO sample which was *not* annealed after patterning. The noise power at 10 Hz in zero applied field is shown in Fig. 10, together with theoretical curves explained below. (Cooling this sample in a field of 200 μT caused a shift of the curve to lower temperature by only about 50 mK.) The noise spectrum shows a clear Lorentzian behavior at and above the peak. The peak corresponds, as with the rings, to the knee in the Lorentzian spectrum passing through the measurement frequency. Below the peak, the frequency dependence is $\sim 1/f^{1.7-1.8}$, slightly less strong than the $1/f^2$ for a Lorentzian. There is no evidence of flux jumping in the time trace at any temperature.

The absence of quantized flux jumps is perhaps not surprising, since there is no defined area into which they can jump. The fact that the noise is Lorentzian at the peak and that the magnitude of the noise peak is similar to that of the rings is less immediately obvious.

The Lorentzian behavior again suggests an explanation in terms of an effective resistance. The analysis is, however, a little more complex due to the lack of a simply defined ring of calculable inductance and cross sectional area, and because of the rather close values of the disk and pickup coil radii. We have therefore proceeded in a manner similar to that described by Clem[26] for determining the flux noise from *normal* infinite metal sheets, but extended to lower resistivities and modified to accommodate our finite geometry:

Briefly, the flux noise can be obtained by calculating the real part of the complex impedance Z which would be observed if the flux transformer were opened and the impedance between the two leads measured. The fluctuation dissipation theorem implies that the current noise in the *unbroken* flux transformer will be that due to the Johnson voltage noise from the real part of the impedance, $\mathrm{Re}\{Z\}$, in series with the total circuit impedance, i.e., $S_I = 4k_BT\,\mathrm{Re}\{Z\}/|Z|^2$.

We model the YBCO disk as an ellipsoid having major axis equal to the disk diameter and minor axis equal to the film thickness and having a temperature dependent resistivity ρ. We have taken the flux transformer to be a single ring of radius equal to the real pickup coil and at a separation from the film of 3 mm. Again, we will make no *a priori* assumptions about the origin of this resistance. $\mathrm{Re}\{Z\}$ can be computed by a straightforward extension of the method used in Ref. 27 for calculating the complex susceptibility of ellipsoidal normal metal particles. We take $|Z|$, with little error, to be that arising from the inductance of the pickup loop in series with the input coil to the SQUID. In Fig. 10 we show the experimental noise data for disk G. Here we have used our measured values of the self-inductance of a ring and its mutual inductance to the pickup coil to express the noise as the equivalent flux noise in a ring having the same parameters as ring A. The result of the above computation, expressed in the same way, is shown by the dotted line. Both the computed and experimental data show a power spectrum which is white on the high-temperature side of the transition, tending to $1/f^{1.75}$ on the low-temperature side. We see, however, that the computed noise peaks at a value below that observed experimentally, but that by scaling our theoretical curve up in magnitude by a factor of 2.7, we obtain rather a good fit to the experimental data. The fact that this fit is achieved with a linear choice of temperature scale shows that the experimental resistivity is an exponential function of temperature, again consistent with thermally activated flux creep, although we recall that there was very little field dependence to the observed noise. In the inset to Fig. 10, we show the resistivity versus temperature deduced from the fit. We note that the magnitude of the flux noise predicted by our model is rather sensitive to the separation of the pickup loop from the film, and that we have also approximated the ten-turn pickup loop by a single turn. The agreement with the experimental data is not unreasonable considering the simplicity of our model. We conclude therefore that the behavior of disk G is quite well explained by treating the disk as a material with a temperature-dependent resistivity, although further measurement and computation will be needed to determine whether the observed discrepancy in the *magnitude* of the noise is significant. It is worth emphasizing that the fluctuation dissipation theorem places a *minimum* value on the noise only. Again using Eq. (9), we find an energy barrier $\sim$6.5 eV, a factor $\sim$2 larger than that in ring A. We also see that a single activation energy again describes the behavior throughout the whole of the transition region.

We finally note that, in predicting a change from white to $1/f^{1.75}$ behavior as the temperature falls below the noise peak, our resistivity model shows a small but significant difference in behavior compared to models based on the study of a random walk in two dimensions,[28] which change from white to $1/f^{1.5}$. It also differs from the Lorentzian behavior associated with telegraph noise, which changes from white to $1/f^2$. A detailed comparison of the various models will be the subject of a future paper.

Measurements were made on the annealed YBCO disk H. This disk also displays a Lorentzian spectrum at and above the noise peak and has a very similar peak magnitude to the unannealed disk. The behavior below the peak is somewhat different in that, although still having an exponential temperature dependence, it displays a clear $1/f$ spectrum, rather than the $1/f^{1.75}$ of the unannealed disk. It has been shown by Ref. 29 that a process causing random fluctuations which involves thermal activation over a uniform distribution of barrier heights results in $1/f$ noise, albeit with a linear, rather than exponential, temperature dependence, and this theory has been adapted to flux motion in order to explain the $1/f$ noise in thin YBCO films well below T_c.[4,5] As with the unannealed disk there is only a very small effect due to a 200-μT applied field, which simply pushes the curve down in temperature by $\sim$50 mK. We have no detailed explanation for the differences in the behavior of annealed and unannealed disks at the present time. There is also insufficient information on specimen preparation given in the literature to enable us to compare our results with those of other authors, but the role of annealing in a number of properties is certainly a subject which warrants further study.

E. Effect of temperature on magnitude of the noise peak

An important prediction of both our thermally activated flux jump model for rings and our resistivity models for both rings and disks is that, for a given geometry, the magnitude of the noise peak at a given frequency will be proportional to temperature. We have therefore made noise measurements on a series of rings and disks having a range of transition temperatures, but all having the same outer diameter. In Fig. 11 we show the magnitude of the noise peak for all our specimens. The dotted line is a best fit to the ring samples, from which we are able use Eq. (7) to deduce a typical ring inductance of 17 nH, which is, within our experimental error, the same as that deduced experimentally in Sec. III A. We see that, with the exception of one of the superlattice samples (disk I), the data are in broad agreement with the above predictions. The behavior of ring samples $D-F$ is very similar to that of the YBCO rings $A-C$, in that the noise is Lorentzian over the whole temperature range and in zero applied field is associated with flux quantum jumps. We should also stress that *all* our disk specimens display an approximately Lorentzian spectrum at and above the noise peak. An important conclusion of our work is that a model based on resistance provides a natural explanation of this fact

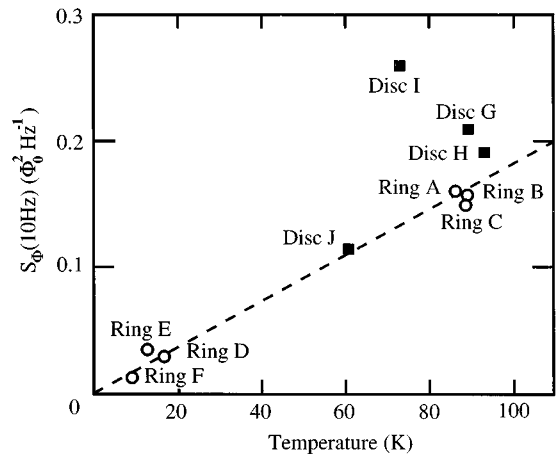

FIG. 11. Noise spectral density at 10 Hz for all specimens: disks (■) and rings (○). The dotted line is from Eq. (7) for $L=17$ nH.

and hence of the observation that at a given temperature the noise peaks at the same value for all specimens. This provides a possible explanation of the specimen independence of the noise peak magnitude noted in Refs. 5 and 12, although their measurement system was rather different to ours.

We turn finally to an alternative and rather tentative model for the behavior of the disk specimens. As we have noted, our model based on resistivity requires an upward scaling of the computed data in order to achieve a quantitative fit and it is not clear, without more detailed computation and experimental measurement, whether this difference can be accommodated. Indeed, we might expect the noise of the disks to be somewhat smaller than for the rings on the rather general ground that the disk acts as an inductor whose scale is set by the size of the pickup loop rather than the outer diameter of the disk. The observed magnitude of the noise peaks for the disks is, if anything, slightly larger than that for the rings. We also note that the values for the energy barriers deduced from Eq. (9) are, within a factor of $\sim$2, the same for all specimens, other than for the two lower-temperature rings D and E, for which it is an order of magnitude smaller, and the niobium ring F, for which we have insufficient data. References 5 and 8 also found $U_0\sim$5 eV for *telegraph* noise close to the transition in a continuous YBCO thin film. These observations all point to a common origin for the noise in disks and rings, and we tentatively suggest that the noise observed in our disks may also be due to the motion of single flux quanta into and out of a region not too far from the disk center, this motion being sufficiently damped so as to smooth out any sharp jumps on the time trace. Verification of this explanation will require a detailed consideration of the energy barriers and the response function of the pickup loop; the necessary theory has yet to be developed.

IV. CONCLUSIONS

We have reported the flux noise in a number of rings and disks having a large range of superconducting transition temperatures. The magnetic noise in rings close to the transition results from flux jumps between a few sharply defined levels. We have shown that these jumps consist of single flux quanta and that the behavior is well explained by thermal activation between the lowest few metastable states of a superconducting loop closed by a single Josephson junction. Detailed studies of the YBCO ring specimens show that in the presence of a small applied magnetic field, the sharp quantum jumps disappear, but the spectra remain accurately Lorentzian and have the same magnitude at the noise peak. This behavior is then well explained by a model in which the ring is treated as a fixed inductance having a temperature-dependent resistance directly proportional to the applied field. The same model is in quite good quantitative agreement with the temperature dependence of the noise in an unannealed YBCO disk. However, in an annealed YBCO disk, the frequency dependence of the noise deviates from our model below the noise peak. We offer an explanation for the observation that all specimens in a given experimental configuration and with the same transition temperature display approximately the same value for the magnitude of the noise peak.

The behavior of a series of other rings having a wide range of superconducting transition temperatures is also found to be in good quantitative agreement with this model. We find that our YBCO samples have behavior which is consistent with thermal activation energies in the range 3–7 eV. Finally, we suggest a possible alternative explanation for the noise in disks in terms of the heavily damped motion of single flux quanta.

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