Long-range interaction of fluctuating vortices with a parallel surface in layered superconductors

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In extremely anisotropic layered superconductors randomly fluctuating vortex lines are attracted to a planar specimen surface by a long-range interaction decreasing as $\langle u^2\rangle/x_0^2$, where $x_0 \gg \lambda$ is the average vortex distance to the surface oriented perpendicular to the layers and $\langle u^2\rangle$ is the mean square vortex displacement, e.g., $\langle u^2\rangle \propto kT$ for thermal fluctuations. This long-range force exceeds the short-range exponential interaction $\propto \exp(-2x_0/\lambda)$ of straight vortices with their image vortex, where $\lambda=\lambda_{ab}$ is the penetration depth of supercurrents in the layers. The long-range attraction originates from the dipole-dipole interaction between each displaced pancake vortex and its image. It is analogous to the Casimir effect, which predicts an attraction between two closely spaced metal plates. The energy contribution of the additional stray field generated by the distorted vortex is calculated. For short-wavelength distortions this term decreases more rapidly with increasing surface distance x_0 and may be disregarded; for long-wavelength distortions the stray-field contribution is comparable to the energy of the dipole-dipole interaction, compensating it partly. [S0163-1829(97)04313-0]

I. INTRODUCTION

In the limit of vanishing Josephson coupling between the layers, high-T_c superconductors with layered structure may be described by the length $\lambda=\lambda_{ab}$ over which the supercurrents in the Cu-O-planes (ab planes) decay exponentially. In this limit of infinite anisotropy currents flowing perpendicular to the layers (along the uniaxial c axis) may be disregarded. Such superconductors may thus be modeled as a stack of superconducting sheets of zero thickness and distance s, which are isolated from each other. A magnetic field applied perpendicular to the layers then induces currents only in these layers, and vortex lines consist of a stack of two-dimensional vortex disks, also called ''pancake vortices''[1–3] or point vortices.

Since the layer spacing s is typically much smaller than λ, such a pancake stack in some respects behaves like a usual Abrikosov vortex line. In particular, in a pancake stack aligned along the c axis ($\hat{\mathbf{c}}\|\hat{\mathbf{z}}$), the in-plane components of the magnetic field of each pancake largely cancel, yielding the usual magnetic field of an Abrikosov vortex line, $\mathbf{B}(\mathbf{r})\approx\hat{\mathbf{z}}(\Phi_0/2\pi\lambda^2)K_0(r/\lambda)$, where $r=(x^2+y^2)^{1/2}$ and $K_0(x)$ is a modified Bessel function with the limits $K_0(x)\approx-\ln x$ for $x\ll 1$ and $K_0(x)\approx(\pi/2x)^{1/2}\exp(-x)$ for $x\gg 1$. The logarithmic infinity of this London approximation at $r\to 0$ is smeared over the finite radius $\approx\xi_{ab}$ of the vortex core. In high-T_c superconductors one has $s\ll\xi_{ab}\ll\lambda$. The interaction energy of two such straight vortices oriented along $\hat{\mathbf{c}}$ at a distance r_{ij} is per unit length $(\Phi_0^2/2\pi\lambda^2\mu_0)K_0(r_{ij}/\lambda)$; i.e., it is repulsive and decays exponentially for $r_{ij}\gg\lambda$.

A special case of the interaction of parallel vortices is the interaction between a vortex line at $x=x_0$, $y=0$ and its image line at $x=-x_0$, $y=0$ which is introduced to satisfy the boundary conditions at the planar surface $x=0$ of a superconductor filling the half-space $x\geq 0$. Since the image vortex has opposite orientation, this fictive vortex-antivortex interaction is attractive; i.e., a vortex parallel to a planar surface is attracted to this surface by an interaction $-(\Phi_0^2/2\pi\lambda^2\mu_0)K_0(2x_0/\lambda)$. This attraction competes with the repulsive force $\propto\exp(-x_0/\lambda)$ exerted by the surface-screening currents which push the vortex into the superconductor.

As shown recently,[4] the interaction of a vortex line or a pancake stack with a planar surface ceases to be of short-range exponential type when the vortex is not a perfectly straight line perpendicular to the Cu-O-layers. Namely, when the pancake stack fluctuates randomly (or regularly), its interaction with the surface decreases only algebraically as $1/x_0^2$ at large distances $x_0\gg\lambda$ from the surface. This long-range attraction of a fluctuating vortex line to the surface (or to its image) may be understood as follows.

Deplacing one pancake from its ideal position on the straight vortex line, say, from $(x_0,0,0)$ to $(x_0+u_x,u_y,0)$, is equivalent to adding a pancake at $(x_0+u_x,u_y,0)$ and an antipancake at $(x_0,0,0)$, which annihilates the pancake that was there (see Fig. 1). This pancake-antipancake pair interacts with its image by a dipole-dipole interaction which for $x_0\gg\lambda$ is attractive and of long range, decreasing as $1/x_0^2$. This interaction is isotropic in the sense that it depends only on the sum $u_x^2+u_y^2$. From this interpretation one sees that the attraction of a fluctuating pancake stack to the surface is analogous to the Casimir effect,[5] which describes the attraction of two closely spaced metal plates due to the zero-point fluctuations of the electromagnetic field in the gap.

In the present paper we derive and discuss this long-range attraction of fluctuating vortices to the surface in detail and investigate how it is modified by accounting for the stray field. As is sometimes overlooked, the method of images allows to satisfy the boundary conditions for the magnetic

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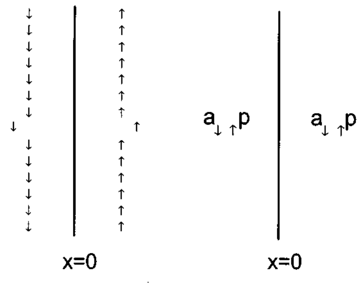

FIG. 1. Left: a distorted vortex line and its image composed of pancakes (↑) and antipancakes (↓). Right: the two dipoles generated by the displacement cause a long-range attraction between the distorted vortex and the surface (indicated by a vertical bold line).

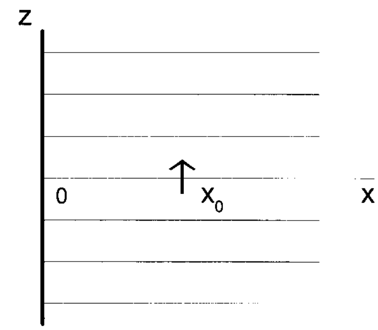

FIG. 2. A single pancake near the sample surface.

field at a planar surface only if the vortex or vortex lattice does not generate a perpendicular field component at the surface. This condition holds only if all vortices are straight and parallel to the surface, or if the vortex lattice is treated within continuum approximation and does not exhibit displacements perpendicular to the surface.[6] In the general case the correct solution of the Maxwell and London equations in the entire space is obtained by adding to the fields of the vortex and its image (antivortex) a *stray field* which is generated by the x component B_x of the vortex and antivortex fields at the surface. For example, if the component B_x is periodic with wave vector k_z, then the stray field decreases as $\exp(k_z x)$ into the vacuum ($x<0$) and as $\exp[-(k_z^2+\lambda^{-2})^{1/2}x]$ into the superconductor ($x\geqslant 0$).[6] The stray field of a single-pancake vortex has been calculated in Refs. 7 and 8. We will see that the contribution of the stray-field energy does not modify qualitatively the long-range attraction of fluctuating vortices to the surface.

The outline of our paper is as follows. In Sec. II we derive the flux and current densities of a single pancake in a superconducting half-space. The vortex-antivortex contribution to the interaction energy of a fluctuating vortex line with the surface is derived in Sec. III together with the self-energy of this line. The contribution of the stray-field energy is calculated in Sec. IV. These energy expressions are then applied in Sec. V to calculate the free energy and the surface attraction of a thermally fluctuating vortex. The results are summarized in Sec. IV.

II. MAGNETIC FIELD AND CURRENT DENSITY OF A POINT VORTEX IN A HALF-SPACE

We first derive the spatial distribution of the magnetic field and current density generated by a single pancake located at the point $(x_0,0,0)$ in the superconducting half-space $x\geqslant 0$ as shown in Fig. 2.[7,8]

Inside the superconductor one can present the current density and the magnetic field in the form $\mathbf{j}=\mathbf{j}^{va}+\mathbf{j}^{\rm str}$ and $\mathbf{B}=\mathbf{B}^{va}+\mathbf{B}^{\rm str}$, where the vortex-antivortex field $\mathbf{B}^{va}=\mathbf{B}^{v}+\mathbf{B}^{a}$ and current density $\mathbf{j}^{va}=\mathbf{j}^{v}+\mathbf{j}^{a}$ are generated by the pancake at $(x_0,0,0)$ and its image (antipancake) at $(-x_0,0,0)$. The stray field $\mathbf{B}^{\rm str}$ and the current density $\mathbf{j}^{\rm str}$ are required to satisfy the boundary conditions in the presence of a field component B_x perpendicular to the sample surface.

The y and z components of the field $\mathbf{B}^{va}$ and the x component of the current density $\mathbf{j}^{va}$ vanish on the sample surface by this image construction. The vector potentials $\mathbf{A}^{v,a}$ of the magnetic fields $\mathbf{B}^{v,a}=\mathrm{rot}\mathbf{A}^{v,a}$ of the pancake and its image satisfy the London equation

$$\mathbf{A}^{v,a}-\lambda^2\Delta\mathbf{A}^{v,a}=\pm\frac{s\Phi_0}{2\pi}\frac{\hat{\mathbf{z}}\times(\vec{\rho}\mp\vec{\rho}_p)}{|\vec{\rho}\mp\vec{\rho}_p|^2}\delta(z), \tag{1}$$

where $\vec{\rho}=(x,y,0)$, and $\vec{\rho}_p=(x_0,0,0)$. The solutions of this equation are

$$\mathbf{A}^{v,a}=\pm s\Phi_0\int\frac{d^3\mathbf{k}}{8\pi^3}\frac{i\vec{\kappa}\times\hat{\mathbf{z}}}{\kappa^2[1+k^2\lambda^2]}e^{i\mathbf{k}(\mathbf{r}\mp\vec{\rho}_p)}, \tag{2}$$

where we introduce $\mathbf{k}=(k_x,k_y,k_z)$, $\vec{\kappa}=(k_x,k_y)$, and $k=|\mathbf{k}|$, $\kappa=|\vec{\kappa}|$; below we write $q=k_y$, $p=k_z$.

To determine the stray field $\mathbf{B}^{\rm str}$ *inside* the superconductor we introduce the phase φ of the superconducting order parameter and the vector potential $\mathbf{A}^{\rm str}$ for the field $\mathbf{B}^{\rm str}=\mathrm{rot}\mathbf{A}^{\rm str}$. In the London gauge $\mathrm{div}\mathbf{A}^{\rm str}=0$, $A_z^{\rm str}=0$ the equations for $\mathbf{A}^{\rm str}$ and φ take the form

$$\mathbf{A}^{\rm str}-\lambda^2\Delta\mathbf{A}^{\rm str}=\frac{\Phi_0}{2\pi}\nabla_2\varphi, \tag{3}$$

$$\Delta_2\varphi=0, \tag{4}$$

where ∇_2 is the two-dimensional gradient in the xy plane and $\Delta_2=\partial^2/\partial x^2+\partial^2/\partial y^2$. The boundary condition for Eqs. (3) and (4) is $j_x(0,y,z)=0$. Outside the superconductor the stray field $\mathbf{B}^{\rm str}$ is a potential field, i.e., $\mathbf{B}^{\rm str}=\nabla\psi$, where the potential $\psi(x,y,z)$ satisfies the Laplace equation

$$\Delta\psi=0. \tag{5}$$

The complete solution $\mathbf{B}$ is thus given by $\mathbf{B}^{va}+\mathbf{B}^{\rm str}$ inside and by $\mathbf{B}^{\rm str}$ outside the superconductor.

Introducing the Fourier transform

$$f(x,y,z)=\frac{1}{4\pi^2}\int\tilde{f}(x,q,p)e^{iqy+ipz}dq\,dp, \tag{6}$$

we present the solution of Eqs. (1)–(4) for the Fourier transforms of the vector potential $\tilde{\mathbf{A}}^{\rm str}(x,q,p)$ and the current density $\tilde{\mathbf{j}}^{\rm str}(x,q,p)$ inside the sample in the form

$$\tilde{A}_x^{\rm str}=-\frac{\Phi_0}{2\pi}\frac{q\varphi(q,p)}{1+p^2\lambda^2}[e^{-|q|x}+p^2\lambda^2e^{-\gamma x}], \tag{7}$$

$$\tilde{A}_y^{\rm str} = \frac{i}{q}\frac{d\tilde{A}_x^{\rm str}}{dx}, \tag{8}$$

$$\tilde{j}_x^{\rm str} = \frac{\Phi_0}{2\pi\mu_0}\frac{qp^2\varphi(q,p)}{1+p^2\lambda^2}[e^{-\gamma x} - e^{-|q|x}], \tag{9}$$

$$\tilde{j}_y^{\rm str} = -i\frac{\Phi_0}{2\pi\mu_0}\frac{|q|p^2\varphi(q,p)}{1+p^2\lambda^2}\left[\frac{\gamma}{|q|}e^{-\gamma x} - e^{-|q|x}\right], \tag{10}$$

where

$$\varphi(q,p) = -\frac{2\pi i s[e^{-|q|x_0} - e^{-\gamma x_0}]}{|q| + p^2\lambda^2\gamma + (1+p^2\lambda^2)\sqrt{q^2+p^2}} \tag{11}$$

and $\gamma = \sqrt{\lambda^{-2} + q^2 + p^2}$.

In this paper we consider the long-range interaction of a fluctuating stack of pancakes with the sample surface. We will treat the case when $x, x_0 \gg \lambda$ and therefore $\gamma x_0 \gg 1$. It follows then from Eqs. (9) and (10) that the Fourier transforms $\tilde{j}_x^{\rm str}$ and $\tilde{j}_y^{\rm str}$ depend on $x + x_0$ and are equal to

$$\tilde{j}_x^{\rm str} \approx \frac{is\Phi_0 qp^2}{\mu_0(1+p^2\lambda^2)}\frac{e^{-|q|(x+x_0)}}{|q| + \gamma p^2\lambda^2 + \sqrt{q^2+p^2}(1+p^2\lambda^2)}, \tag{12}$$

$$\tilde{j}_y^{\rm str} \approx \frac{s\Phi_0|q|p^2}{\mu_0(1+p^2\lambda^2)}\frac{e^{-|q|(x+x_0)}}{|q| + \gamma p^2\lambda^2 + \sqrt{q^2+p^2}(1+p^2\lambda^2)}. \tag{13}$$

Knowing the fields $\mathbf{B}^{va}$ and $\mathbf{B}^{\rm str}$ and the current densities $\mathbf{j}^{va}$ and $\mathbf{j}^{\rm str}$ we can calculate the energy of a fluctuating vortex as the sum of the well-known energy of a straight vortex plus the work performed by the Lorentz force $\mathbf{F} = s\mathbf{j}\times\vec{\Phi}_0$ to displace the pancakes by $\mathbf{u}(z)$. The current density $\mathbf{j}$ is the sum of $\mathbf{j}^{va}$ and $\mathbf{j}^{\rm str}$ and therefore the Lorentz force is also a sum of two terms $\mathbf{F}^{va}$ and $\mathbf{F}^{\rm str}$. As a result the contributions of the vortex-antivortex interaction and stray field to the energy of a fluctuating vortex can be calculated separately if one is interested in the terms up to second order in the displacements.

III. VORTEX-ANTIVORTEX ENERGY

A. Long distances

The contribution of the vortex-antivortex interaction E_{va} to the energy of a distorted vortex may be calculated from the interaction of a pair of pancakes separated by $\mathbf{r}_{mn} = (x_{mn}, y_{mn}, z_{mn}) = (x_m - x_n, y_m - y_n, z_m - z_n)$,[9,10]

$$\mathcal{E} \approx \epsilon_0 \begin{cases} 2\ln(\rho_{mm}/\xi), & n = m, \\ -\dfrac{s}{2\lambda}\exp\left(-\dfrac{|z_{mn}|}{\lambda}\right)\ln\left(\dfrac{\rho_{nm}}{\lambda}\right), & n \neq m. \end{cases} \tag{14}$$

Here $\epsilon_0 = s\Phi_0^2/(4\pi\mu_0\lambda^2)$, $\rho_{mn}^2 = x_{mn}^2 + y_{mn}^2$, $z_m = ms$, s is the layer spacing, and ξ the core radius of the pancake. For $z_m = z_n$, Eq. (14) applies to all distances $\rho_{mn} > \xi$, but for $z_m \neq z_n$, $\rho_{mn} \gg \lambda$ was assumed. The contribution E_{va} is composed of the self-energy of a vortex line and the interaction of this vortex with its image line of opposite orientation, namely,

$$E_{va} = \frac{1}{2}\sum_m \sum_n [\mathcal{E}(x_m - x_n, y_m - y_n, z_m - z_n) - \mathcal{E}(x_m + x_n, y_m - y_n, z_m - z_n)]. \tag{15}$$

The sums in Eq. (15) are over the pancakes of the *real* vortex in the superconducting half-space $x > 0$; thus $x_m, x_n > 0$. For a distorted vortex line parallel to the surface $x = 0$ we define pancake displacements $\mathbf{u}_m = \mathbf{u}_m(z_m) = (u_{xm}, u_{ym})$ by writing $x_m = x_0 + u_{xm}$ and $y_m = u_{ym}$. The linear elastic energy of the vortex is obtained by keeping in Eq. (15) only the terms quadratic in $\mathbf{u}_m$.

As a first step we consider random and isotropic displacements with ensemble averages $\langle u_{xm}\rangle = \langle u_{ym}\rangle = 0$, $\langle u_{xm}u_{xn}\rangle = \langle u_{ym}u_{yn}\rangle = f(|m-n|)$, $\langle u_{xm}u_{yn}\rangle = 0$. (It will turn out later that the assumption $\langle u_{xm}u_{xn}\rangle = \langle u_{ym}u_{yn}\rangle$ is not necessary, since only $u_x^2 + u_y^2$ enter when $2x_0 \gg \lambda$.) In this case the long-range interaction energy $E_{\rm int}$ of a vortex line of length L with its image then becomes from Eqs. (14) and (15) for $2x_0 \gg \lambda$

$$E_{\rm int} = -\frac{\Phi_0^2 Ls}{64\pi\mu_0\lambda^3 x_0^2}\sum_l \exp\left(-\frac{s}{\lambda}|l|\right)\langle(\mathbf{u}_l - \mathbf{u}_0)^2\rangle. \tag{16}$$

This fluctuation-induced interaction is attractive and depends on the relative displacements $\langle(\mathbf{u}_l - \mathbf{u}_0)^2\rangle$ over a vortex length of order λ. Like a dipole-dipole interaction it decreases as $1/x_0^2$ and is isotropic in the components u_x, u_y. Since $s \ll \lambda$, one may approximate the sum in Eq. (16) by an integral,

$$E_{\rm int} \approx -\frac{\Phi_0^2 L}{32\pi\mu_0\lambda^3 x_0^2}\int_0^\infty dz\, \exp\left(-\frac{z}{\lambda}\right)g(z), \tag{17}$$

where $g(z) = \langle[\mathbf{u}(z) - \mathbf{u}(0)]^2\rangle$ is a correlation function. Formula (17) shows that the long-range interaction does not depend on the layer separation s. With the Fourier transform

$$\mathbf{u}(z) = \int_{-\infty}^{\infty}\frac{dp}{2\pi}\mathbf{u}(p)e^{ipz}, \tag{18}$$

the interaction energy (17) may be written as

$$E_{\rm int} = -\frac{\Phi_0^2}{32\pi^2\mu_0 x_0^2}\int_{-\infty}^{\infty}dp|\mathbf{u}(p)|^2\frac{p^2}{1+\lambda^2 p^2}. \tag{19}$$

B. General expressions

For calculations applying also to $\rho_{mn} < \lambda$, the general expression for the two-pancake interaction,[9,10]

$$\mathcal{E} = \frac{\Phi_0^2 s^2}{\mu_0}\int\frac{d^3\mathbf{k}}{8\pi^3}\frac{1}{1+k^2\lambda^2}\frac{k^2}{\kappa^2}e^{i\mathbf{k}\mathbf{r}_{mn}}, \tag{20}$$

has to be used, where as above $\kappa^2 = k_x^2 + k_y^2$, $k^2 = \kappa^2 + p^2$, and $\mathbf{r}_{mn} = (x_{mn}, y_{mn}, z_{mn})$. Since Eq. (20) is valid for all $\mathbf{r}_{mn}$, we may obtain from it the general vortex-antivortex contribution to the total elastic energy of a distorted vortex line, $E_{va}=E_{\rm self}+E_{\rm int}$, from Eq. (15). Expanding this to quadratic terms in $\mathbf{u}$ and using $s\Sigma\exp(ipz_l)=2\pi\delta(p)$ (for $|p|\leqslant\pi/s$) and $\int(dp/2\pi)|\mathbf{u}(p)|^2=\int dz\,\mathbf{u}(z)^2=\langle\mathbf{u}^2\rangle L$ we obtain

$$E_{va}=\frac{\Phi_0^2}{4\mu_0}\int\frac{d^3\mathbf{k}}{8\pi^3}|\mathbf{u}(p)|^2(f_{\rm self}+f_{\rm int}),\tag{21}$$

$$f_{\rm self}=\frac{k^2}{1+k^2\lambda^2}-\frac{\kappa^2}{1+\kappa^2\lambda^2},\tag{22}$$

$$f_{\rm int}=\left(\frac{k^2}{1+k^2\lambda^2}\frac{k_x^2}{\kappa^2}-\frac{1}{2}f_{\rm self}\right)e^{2ik_xx_0}.\tag{23}$$

Exactly the same result (21)–(23) is obtained from the anisotropic London theory in the limit $\lambda_c\to\infty$. Namely, the interaction energy of two London vortices at positions $\mathbf{r}_1(z)$ and $\mathbf{r}_2(z)$ is

$$\int d\mathbf{r}_{1\alpha}\int d\mathbf{r}_{2\beta}\,V_{\alpha\beta}(\mathbf{r}_1-\mathbf{r}_2),\tag{24}$$

with the anisotropic interaction $V_{\alpha\beta}(\mathbf{r}_1-\mathbf{r}_2)$ $(\alpha,\beta=x,y,z)$ given in Ref. 11 as a Fourier integral over $V_{\alpha\beta}(\mathbf{k})$. In the limit $\lambda_c\to\infty$, the general expression $V_{\alpha\beta}(\mathbf{k})=(\Phi_0^2/\mu_0)(1+k^2\lambda^2)^{-1}G_{\alpha\beta}(\mathbf{k})$ simplifies to a diagonal matrix with $G_{xx}=k_x^2/\kappa^2$, $G_{yy}=k_y^2/\kappa^2$, and $G_{zz}=1$ if z is along the c axis of the uniaxial superconductor. Inserting this $V_{\alpha\beta}(\mathbf{k})$ into Eq. (24) and integrating over the vortex and its image, we reproduce the result (21)–(23) of the pancake approach. From Eq. (23) one may derive the long-distance interaction (16) and (17).

C. Self-energy

From the self-interaction (22) the dispersive line tension P of an isolated flux line[12] or stack of pancakes is obtained,

$$P(p)=\frac{\Phi_0^2}{8\pi\mu_0\lambda^2}\frac{\ln(1+p^2\lambda^2)}{p^2\lambda^2},\tag{25}$$

which determines the linear elastic self-energy of a distorted vortex line,

$$E_{\rm self}=\frac{1}{2}\int_{-\infty}^{\infty}\frac{dp}{2\pi}p^2P(p)|\mathbf{u}(p)|^2.\tag{26}$$

This self-energy enters the fluctuation-caused interaction between vortex lines or between a vortex and the surface *indirectly* since it determines the amplitude of both the pinning-caused and thermal fluctuations. In real space this self-energy looks similar to Eq. (16),

$$E_{\rm self}=\frac{\Phi_0^2L}{16\pi\mu_0\lambda^4}\sum_{l\neq0}\exp\left(-\frac{s}{\lambda}|l|\right)\frac{\langle(\mathbf{u}_l-\mathbf{u}_0)^2\rangle}{|l|},\tag{27}$$

and (with $s\ll\lambda$) to Eq. (17),

$$E_{\rm self}\approx\frac{\Phi_0^2L}{8\pi\mu_0\lambda^4}\int_0^{\infty}dz\,\exp\left(-\frac{z}{\lambda}\right)\frac{g(z)}{z}.\tag{28}$$

Expressing the correlation function $g(z)$ via $\mathbf{u}(p)$,

$$g(z)=\frac{2}{L}\int_{-\infty}^{\infty}\frac{dp}{2\pi}\langle|\mathbf{u}(p)|^2\rangle(1-\cos pz),\tag{29}$$

we may write Eq. (28) as

$$E_{\rm self}=\frac{\Phi_0^2}{16\pi^2\mu_0\lambda^4}\int_{-\infty}^{\infty}dp|\mathbf{u}(p)|^2\ln(1+\lambda^2p^2),\tag{30}$$

which is just the above result (25) and (26).

Comparing Eqs. (17) and (28) one sees that if the correlation function $g(z)$ increases algebraically, $g(z)=\text{const}\times|z|^{\gamma}$, one has, for $2x_0\gg\lambda$,

$$E_{\rm int}=-(\gamma\lambda^2/4x_0^2)E_{\rm self}.\tag{31}$$

For example, a flux line diffusing thermally or in a random pinning potential may exhibit $\gamma\approx1$; thus one has $E_{\rm int}\approx-(\lambda^2/4x_0^2)E_{\rm self}$, which typically is not a small correction to $E_{\rm self}$.

We note here that the divergence for $|z|\to\infty$ of the displacement $\mathbf{u}(z)$ of a freely wandering thermally fluctuating vortex line does not affect our results since, as usual, it is tacitly assumed that the presence of other flux lines or of weak pinning, in combination with appropriate history and finite specimen size, limits $\mathbf{u}(z)$.

IV. STRAY-FIELD ENERGY

A distorted vortex line can be treated as a straight line plus a set of displaced pancakes located at the displaced positions (x_0+u_{xn},u_{yn},n) and a set of antipancakes located at the original positions $(x_0,0,n)$. An ideal vortex line exhibits no stray field; i.e., the stray field contribution to the interaction energy arises only from the work performed to create the pancake-antipancake pairs. Therefore, to find the contribution of the stray-field to the interaction energy $E_{\rm int}^{\rm str}$ we first consider two pancake-antipancake pairs located in the layers m and n at the points $(x_0,0,m)$, (x_0+u_{xm},u_{ym},m) and $(x_0,0,n)$, (x_0+u_{xn},u_{yn},n) as shown in Fig. 3 and calculate the work required to produce these two pairs. The total contribution of the stray field to the interaction energy is then the sum over the contributions of the pancake-antipancake pairs.

We start the calculation with the Lorentz forces $\mathbf{F}_n^{\rm str}$ and $\mathbf{F}_m^{\rm str}$ arising due to the stray field generated by both pairs and

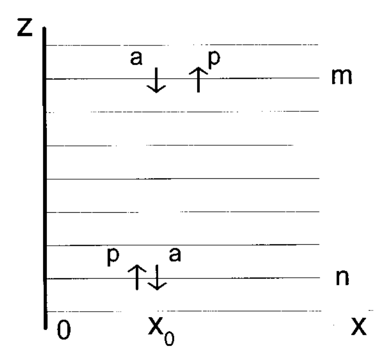

FIG. 3. Two pairs of pancake-antipancake located in the layers m and n. The antipancakes are located on the line $x=x_0$ and $y=0$, and the pancakes are shifted by $u_x(m)$ and $u_x(n)$ from this line.

acting on the pancakes residing in the mth and nth layers, $\mathbf{F}^{\rm str}_{m,n}=s\mathbf{j}^{\rm str}_{m,n}\times\vec{\Phi}_0$. Since an undisturbed vortex line exhibits no stray field, one has $\mathbf{F}^{\rm str}_n=\mathbf{F}^{\rm str}_m=0$ for $\mathbf{u}_n=\mathbf{u}_m=0$. If the vortex line is not perfectly aligned perpendicular to the layers, it generates a stray field and current densities $\mathbf{j}^{\rm str}_{m,n}$, resulting in forces $\mathbf{F}^{\rm str}_n$ and $\mathbf{F}^{\rm str}_m$ which initially are proportional to the displacements. Explicitly, the expressions for $\mathbf{F}^{\rm str}_n$ and $\mathbf{F}^{\rm str}_m$ read

$$\begin{aligned}F^{\rm str}_{xm}&=s\Phi_0[j^{\rm str}_y(2x_0+u_{xm}+u_{xn},0,m-n)\\&\quad-j^{\rm str}_y(2x_0+u_{xm},0,m-n)]\\&\approx s\Phi_0\frac{dj^{\rm str}_y}{dx}u_{xn},\end{aligned}\tag{32}$$

$$\begin{aligned}F^{\rm str}_{ym}&=-s\Phi_0[j^{\rm str}_x(2x_0,u_{ym}-u_{yn},m-n)\\&\quad-j^{\rm str}_y(2x_0,u_{ym},m-n)]\\&\approx s\Phi_0\frac{dj^{\rm str}_x}{dy}u_{yn},\end{aligned}\tag{33}$$

$$\begin{aligned}F^{\rm str}_{xn}&=s\Phi_0[j^{\rm str}_y(2x_0+u_{xm}+u_{xn},0,n-m)\\&\quad-j^{\rm str}_y(2x_0+u_{xn},0,n-m)]\\&\approx s\Phi_0\frac{dj^{\rm str}_y}{dx}u_{xm},\end{aligned}\tag{34}$$

$$\begin{aligned}F^{\rm str}_{yn}&=-s\Phi_0[j^{\rm str}_x(2x_0,u_{yn}-u_{ym},n-m)\\&\quad-j^{\rm str}_y(2x_0,u_{yn},n-m)]\\&\approx s\Phi_0\frac{dj^{\rm str}_x}{dy}u_{ym},\end{aligned}\tag{35}$$

where the derivatives $dj^{\rm str}_x/dy$ and $dj^{\rm str}_y/dx$ are taken at $(2x_0,0,|m-n|)$ and only the terms linear in the $\mathbf{u}_n$ and $\mathbf{u}_m$ are kept.

Using Eqs. (32)–(35) we can present the work $\mathcal{W}_{mn}$ performed by the forces $\mathbf{F}^{\rm str}_n$ and $\mathbf{F}^{\rm str}_m$ during the creation of the two vortex-antivortex pairs as

$$\mathcal{W}_{mn}=s\Phi_0\left(\frac{dj^{\rm str}_y}{dx}u_{xm}u_{xn}+\frac{dj^{\rm str}_x}{dy}u_{ym}u_{yn}\right).\tag{36}$$

The contribution of the stray field to the interaction energy is then given by the sum

$$E_{\rm str}=\frac{1}{2}\sum_{n,m}\mathcal{W}_{mn}.\tag{37}$$

Since $s\ll\lambda$, this sum over m and n can be approximated by an integral over z and z'. Using the Fourier transform (18) for $u_x(z)$, $u_y(z)$, $j^{\rm str}_x(\mathbf{r})$, and $j^{\rm str}_y(\mathbf{r})$ and the formulas (12) and (13) for the Fourier transforms of the current density $\mathbf{j}^{\rm str}$ we finally obtain

$$\begin{aligned}E_{\rm str}&=\frac{\Phi_0^2}{4\pi^2\mu_0}\int_{-\infty}^{\infty}dp\frac{p^2|\mathbf{u}(p)|^2}{(1+p^2\lambda^2)}\\&\quad\times\int_0^\infty\frac{q^2e^{-2qx_0}dq}{q+(1+p^2\lambda^2)\sqrt{q^2+p^2}+p^2\lambda^2\sqrt{\lambda^{-2}+q^2+p^2}}.\end{aligned}\tag{38}$$

In the stray-field energy (38) one can distinguish the contributions from three characteristic intervals of the distortion wave number p. Using our assumption $x_0\gg\lambda$ and noting that in the second integral in Eq. (38) the main contribution comes from $q\approx x_0^{-1}$, one finds that (a) large $p\gg\lambda^{-1}$ yield

$$E_{\rm str}=\frac{\Phi_0^2}{32\pi^2\mu_0\lambda^4x_0^3}\int dp\,\frac{|\mathbf{u}(p)|^2}{|p|^3},\tag{39}$$

(b) medium p with $x_0^{-1}\ll p\ll\lambda^{-1}$ yield

$$E_{\rm str}=\frac{\Phi_0^2}{16\pi^2\mu_0x_0^3}\int dp|\mathbf{u}(p)|^2|p|,\tag{40}$$

and (c) small $p\ll x_0^{-1}$ yield

$$E_{\rm str}=\frac{\Phi_0^2}{32\pi^2\mu_0x_0^2}\int dp|\mathbf{u}(p)|^2p^2.\tag{41}$$

These three expressions allow to estimate the stray-field energy when the characteristic wavelength $2\pi/p$ of the vortex fluctuation is known.

V. ATTRACTION TO THE SURFACE

A. Long-wavelength distortions

Combining the self-energy of the fluctuating vortex line $E_{\rm self}$, Eq. (30), with the interaction energy with its image $E_{\rm int}$, Eq. (19), and with the stray-field energy $E_{\rm str}$, Eq. (38), we get the total fluctuation-caused energy $E_{\rm tot}=E_{\rm self}+E_{\rm int}+E_{\rm str}$ of a fluctuating vortex at a distance $x_0\gg\lambda$ from a planar surface.

Comparing these contributions we first note that the (bulk) self-energy is the dominating contribution, i.e., the interaction between the displaced pancakes of the same line minus this interaction for the undisplaced pancakes. While $E_{\rm self}$ does not depend on the vortex position, the two perturbation terms $E_{\rm int}$ and $E_{\rm str}$ decrease with increasing distance x_0 to the surface.

Next we note that the stray-field term may be disregarded except for very long wavelengths of the fluctuations, $E_{\rm str}\ll|E_{\rm int}|$ for $px_0\gg1$ and $E_{\rm str}\approx-E_{\rm int}$ for $px_0\ll1$. This means that for long wavelengths of the displacements $\mathbf{u}(z)$ the contributions of the interaction with the image line and of the stray-field energy *nearly compensate*. For long wavelengths with $p\lambda\ll1$ one has

$$E_{\rm str}+E_{\rm int}=\frac{\Phi_0^2}{4\pi^2\mu_0}\int_{-\infty}^{\infty}dp|\mathbf{u}(p)|^2p^2\beta(p),\tag{42}$$

where

$$\beta(p)=\int_0^\infty dq\,\frac{q^2e^{-2qx_0}}{\sqrt{q^2+p^2}+q}-\frac{1}{8x_0^2}. \tag{43}$$

This may be written as

$$\beta(p)=\int_0^\infty dq\,\frac{q^2e^{-2qx_0}}{\sqrt{q^2+p^2}+q}-\frac{1}{2}\int_0^\infty dq\,qe^{-2qx_0}$$
$$=p^2\int_0^\infty dt\,te^{-2|p|x_0t}\left(\frac{t}{\sqrt{t^2+1}+t}-\frac{1}{2}\right)$$
$$=-\frac{p^2}{2}\int_0^\infty dt\,te^{-2|p|x_0t}(\sqrt{t^2+1}-t)^2. \tag{44}$$

For not too small wave numbers $x_0^{-1}\ll p\ll\lambda^{-1}$ the contribution of small values $t\ll 1$ dominates in Eq. (44), yielding

$$\beta(p)=-\frac{1}{8x_0^2};$$

i.e., the total interaction with the surface is attractive and decreases as x_0^{-2}. For small wave numbers $|p|\ll x_0^{-1}$ the contribution of large $t\gg 1$ dominates in Eq. (44), and thus

$$\beta(p)=-\frac{p^2}{8}\int_0^\infty dt\frac{e^{-2|p|x_0t}}{t}\approx-\frac{p^2}{8}\ln\frac{1}{|p|x_0},$$

$$E_{\rm str}+E_{\rm int}=-\frac{\Phi_0^2}{32\pi^2\mu_0}\int_{-\infty}^{\infty}dp|\mathbf{u}(p)|^2p^4\ln\frac{1}{|p|x_0}. \tag{45}$$

This means that a sinusoidally deformed vortex with long wavelength $2\pi/p\gg 2\pi x_0$ is attracted to a planar surface *weakly*, since p is small in Eq. (45), with a potential decreasing logarithmically with the distance x_0.

B. Thermal fluctuations

From the above results we see that the energy of a vortex line which fluctuates with constant amplitude $\langle|\mathbf{u}|^2\rangle$ *decreases* when it approaches the surface. This effect causes an attraction of a thermally fluctuating vortex line to the surface. The attractive force in this case is obtained as the derivative of the free energy $F(x_0)$ with respect to the distance x_0. The free energy of the fluctuating vortex line is given by

$$F(x_0)=-k_BT\ln Z(x_0). \tag{46}$$

The statistical sum $Z(x_0)$ may be written as a functional integral over the displacement field $\mathbf{u(z)}=(u_x,u_y)$,

$$Z(x_0)=\int\mathcal{D}[\mathbf{u}(z)]\exp\left(-\frac{E_{\rm tot}\{\mathbf{u}(z)\}}{k_BT}\right). \tag{47}$$

This integral may be evaluated by using the discreteness of the displacement field $\mathbf{u}(z)$, which is defined only at the layer positions $z=z_n$. The integral then becomes a product of double integrals over the displacement components u_{xn} and u_{yn} from $-\infty$ to ∞. Alternatively, the functional integral may be evaluated in Fourier space as a product of double integrals over the (complex) Fourier coefficients $u_x(p)$ and $u_y(p)$. Both methods yield the same result. Expressing $E_{\rm tot}=E_{\rm self}+E_{\rm int}+E_{\rm str}$ in the form (which defines G)

$$E_{\rm tot}(x_0)=\int_{-\infty}^{\infty}\frac{dp}{2\pi}|\tilde{\mathbf{u}}(p)|^2G(p,x_0)=\frac{1}{L}\sum_p|\tilde{\mathbf{u}}(p)|^2G(p,x_0), \tag{48}$$

the Gaussian integrals are easily taken and one obtains

$$F(x_0)=\tilde{F}_0+k_BT\sum_p\ln G(p,x_0). \tag{49}$$

Writing $G(p,x_0)=G_0(p)+G_1(p,x_0)$ with the main term G_0 originating from $E_{\rm self}$ and the perturbation G_1 from $E_{\rm int}+E_{\rm self}$ one gets the free energy

$$F(x_0)=F_0+k_BT\sum_p\frac{G_1(p,x_0)}{G_0(p)}. \tag{50}$$

Inserting here G_0 from $E_{\rm self}$, Eq. (30), and G_1 from $E_{\rm int}$, Eq. (19), and disregarding the stray-field contribution $E_{\rm str}$, Eq. (38), which for the here relevant large p values is much smaller than $|E_{\rm int}|$, one obtains

$$\frac{G_1(p,x_0)}{G_0(p)}=-\frac{\lambda^4p^2}{2x_0^2(1+\lambda^2p^2)\ln(1+\lambda^2p^2)}. \tag{51}$$

This yields $F(x_0)=F_0+F_1(x_0)$ with

$$F_1(x_0)=-k_BTL\frac{\lambda^2}{x_0^2}\int_0^{\pi/s}\frac{dp}{2\pi}\frac{\lambda^2p^2}{(1+\lambda^2p^2)\ln(1+\lambda^2p^2)}. \tag{52}$$

Our final result for the position-dependent part of the free energy of a thermally fluctuating vortex line is thus, in the limit of a very large ratio λ/s,

$$F_1(x_0)\approx-k_BTN\frac{\lambda^2}{4x_0^2}\frac{1}{\ln(\pi\lambda/s)}, \tag{53}$$

where $N=L/s$ is the number of layers. In the interval $10\leq\lambda/s\leq 100$ a good approximation with relative deviation less than 2% is

$$F_1(x_0)\approx-k_BTN\frac{\lambda^2}{4x_0^2}\frac{1.1}{\ln(1.1\lambda/s)}. \tag{54}$$

Interestingly, the fluctuation-caused perturbation $F_1(x_0)$ of the *free energy* exactly coincides with the perturbation of the vortex *energy* $E_{\rm tot}$ which would be obtained by inserting the unperturbed and spatially constant fluctuation amplitude $\langle|\mathbf{u}(p)|^2\rangle=k_BT/[p^2P(p)]$ resulting from the line tension $P(p)$, Eq. (25). However, the thus obtained spatial dependence of $E_{\rm tot}$ is exactly compensated by the spatial variation of the fluctuation amplitude $\langle u^2\rangle$ calculated from the total energy, not only from the self-energy, of the vortex line.[4] Therefore, the correct energy of the thermal fluctuations of a pancake stack is independent of its distance from the surface. Namely, this energy is just k_BT per pancake, irrespective of the total quadratic potential which the pancake feels. As follows from our above calculations, this effective potential becomes softer when the vortex approaches the surface. This softening is due to the interaction with the image pancakes, and it is slightly reduced by the stray-field energy. As a consequence, the thermal fluctuations, and typically also the pinning-caused fluctuations, of the pancakes *increase* with decreasing distance of the pancake stack from the surface. The two consequences of this long-range surface interaction are thus the spatially varying fluctuation amplitude and the surface attraction, which is caused only by the entropy term in the free energy but not by a spatially varying vortex energy.

VI. SUMMARY

We have calculated the interaction of a fluctuating vortex line with the planar surface of a layered superconductor with no Josephson coupling between the layers. The boundary conditions are satisfied by adding the magnetic field of an image vortex and the stray field. The stray-field energy is smaller than the interaction with the image except when the vortex exhibits long-wavelength distortions, which may be caused, e.g., by pinning or by the presence of other vortices. The relevant wavelengths $2\pi/p$ of the thermal fluctuations of a vortex line are short, $\lambda^{-1}<p\leqslant\pi/s$ [cf. Eq. (52)], where $\lambda=\lambda_{ab}$ is the magnetic penetration depth and $s\ll\lambda$ the layer spacing.

As our main result we find that the free energy (53) of a thermally fluctuating vortex line decreases with increasing distance $x_0\gg\lambda$ to the surface proportional to $T\lambda^2/x_0^2$. This means that the vortex is attracted to the surface by a long-range force which increases with increasing temperature T as $T/(1-T^2/T_c^2)$, at least as long as the penetration depth $\lambda(T)\approx\lambda(0)/(1-T^2/T_c^2)^{1/2}$ stays smaller than the distance x_0. For smaller distances $x_0<\lambda$ our theory (the stray-field contribution) is not yet complete; further extensions to finite Josephson coupling (noninfinite anisotropy) and to nonplanar surfaces are in preparation.

The long-range attraction (53) of a fluctuating vortex is analogous to the Casimir effect[5] in metals, which considers the interaction of the electromagnetic fluctuations with their images and predicts an attractive force between two closely spaced metal plates. This analogy was recently extended to predict a long-range van der Waals attraction between thermally fluctuating vortex lines in anisotropic superconductors.[13]

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