Effect of finite quantum-well width on the compressibility of a two-dimensional electron gas

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Double quantum-well structures have been used to measure the inverse compressibility (κ^{-1}) of a two-dimensional electron gas as a function of the confining quantum-well width. The lower layer is contacted independently and carrier density changes within this layer are used to measure κ^{-1} of the upper layer. As the well width increases we observe a decrease in κ^{-1} which may be attributed to the effect of the Hartree term. The inclusion of Hartree effects produces the correct trend of κ^{-1} with well width, although discrepancies still exist between the model and experiment. [S0163-1829(97)08411-7]

I. INTRODUCTION

The lateral transport properties of two, closely separated, two-dimensional electron gases (2DEGs) have been used to study a number of different interaction phenomena.[1–3] In particular, one 2DEG may be used as a detector to probe the compressibility of the adjacent 2DEG.[4] This may be achieved either by a capacitive technique,[4] or accurately determining the carrier density changes within the detector 2DEG.[5–7] The inverse compressibility κ^{-1} may be expressed as $\kappa^{-1}=N^2\delta\mu/\delta N$ where N is the carrier density and μ is the chemical potential. As has been shown in previous publications,[4] we can separate the chemical potential into three components due to many-body interactions μ_x, kinetic energy $\mu_{\rm KE}$, and Hartree effects μ_H. The size of $\delta\mu/\delta N$ is therefore determined by the relative changes of the interaction, Hartree, and kinetic components, and under certain conditions the inverse compressibility may be negative.[4] The first of these terms, the many-body interaction term, is negative and strongly dependent upon both the carrier density and quantum-well (QW) width whilst the kinetic term is positive and given by $\mu_{\rm KE}=N\pi\hbar^2/m_e$, where m_e is the effective electron mass. The third, Hartree component, is dependent upon the experimental structure and has been studied for both single[8] and double[4] quantum-well structures. In particular, it is important to note that the Hartree contribution to the measured κ^{-1} is negative. The reasoning behind the sign of the Hartree component is discussed in Ref. 4 and may be summarized in the following manner. As the top 2DEG's density is increased, the electron wave function moves towards the surface. The conduction-band edge at the upper interface of the top quantum well is therefore reduced in energy and the Hartree contribution at this point is positive. However, the conduction-band edge of the lower interface rises due to the changing position of the wave function. Since the bottom 2DEG, which is the detector in our system, is sensitive to changes in the conduction-band edge at the lower interface, this experiment measures a negative Hartree contribution.

We have used a range of samples with different well widths, enabling the effects of a finite width 2DEG upon the compressibility to be determined. We may quantify the terms contributing to κ^{-1} by considering the density $N=N_c$ at which the condition $\delta\mu_H/\delta N+\delta\mu_x/\delta N+\delta\mu_{\rm KE}/\delta N=0$ is satisfied and κ^{-1} changes sign. Our results show that as the well width increases from 95 to 200 Å, N_c increases. Self-consistent solutions of the one-dimensional Poisson and Schrödinger equations using the local-density approximation (LDA) to include many-body interactions show the same response for these well widths as the experimental data. The well-width dependence of the experimental N_c is clearly shown to be dominated by the Hartree component. There are, however, noticeable differences between the experimental data and the self-consistent calculations that may be attributable to the use of the LDA function in narrow wells.

II. EXPERIMENTAL METHODS

Measurements were performed on GaAs/$Al_{0.33}Ga_{0.67}As$ double quantum-well heterostructures grown by molecular-beam epitaxy. A range of devices was studied which had quantum-well widths, barriers widths, and barrier compositions as given in Table I. Hall bars were defined using optical lithography and wet etching, with AuGeNi Ohmic contacts that penetrated both conducting layers. Surface Schottky gates were made from NiCr/Au and consisted of a full-front gate and depleting gates over each mesa arm. The depleting gates were used to deplete locally the top 2DEG, enabling

TABLE I. Growth parameters of the samples, including their mobilities and carrier densities.

Sample	Barrier material	Barrier (Å)	Wells (Å) top	Wells (Å) bottom	N_{top} (10^{15} m^{-2})	N_{bottom} (10^{15} m^{-2})	μ_{top} (m^2 V^{-1} s^{-1})	μ_{bottom} (m^2 V^{-1} s^{-1})
A	$Al_{0.33}Ga_{0.67}As$	300	95	200	1.46	0.66	12	25
B	$Al_{0.33}Ga_{0.67}As$	300	100	100	2.79	2.40	20	20
C	AlAs	200	150	150	2.31	2.62	21	33
D	AlAs	75	150	150	2.32	2.52	32	40
E	$Al_{0.33}Ga_{0.67}As$	600	200	200	1.30	1.24	223	118
F	$Al_{0.33}Ga_{0.67}As$	200	200	200	3.45	2.27	35	22

independent contacts to the bottom layer to be achieved. For all measurements, the top and bottom layers were earthed together at the drain end of the Hall bar. Low-field Hall data were then used to determine the carrier density of the bottom 2DEG as a function of the full-gate bias. The magnetic field in this measurement was sufficiently low (0.2 T) that no quantization effects were seen. Longitudinal resistance measurements of both 2DEG's in parallel were also made as a function of gate bias at fixed magnetic fields. Analysis of the resistance oscillations was then used to calculate the relationship between upper 2DEG density and gate voltage. Four-terminal measurements were performed at a temperature of 1.5 K.

III. RESULTS

Figure 1 shows the experimental and modeled carrier-density variations in the lower 2DEG as a function of the upper 2DEG density for samples *B*, *C*, and *F*. The theoretical results will be discussed in the following section. By considering the system as a parallel plate capacitor, we can relate these carrier-density changes within the lower 2DEG, δN_b, to the chemical potential changes $\delta\mu$ of the upper 2DEG. The carrier-density change δN_b, and inverse compressibility κ^{-1} of the upper 2DEG are given by

$$\delta N_b = \frac{\epsilon}{(d+d_b)e^2}\,\delta\mu = \frac{\epsilon}{(d+d_b)e^2}\left(\frac{\delta\mu_{\mathrm{KE}}}{\delta N_t}+\frac{\delta\mu_H}{\delta N_t}+\frac{\delta\mu_x}{\delta N_t}\right)\delta N_t, \qquad (1)$$

$$\kappa^{-1} = N_t^2\,\frac{(d+d_b)e^2}{\epsilon}\,\frac{\delta N_b}{\delta N_t}, \qquad (2)$$

where d is the distance between the electron gases, $d_b = (\epsilon/e^2)\,\delta\mu_b/\delta N_b$ the screening length of the bottom 2DEG, N_t and N_b refer to the top and bottom 2DEG densities, and ϵ is the effective dielectric constant. For the experimental devices considered, the screening length of the bottom 2DEG (Ref. 9) d_b is considerably smaller than the layer separation d, hence the bottom 2DEG has negligible affect on the measurements. In order to describe the observed density dependence we initially ignore the Hartree term[10] and consider the kinetic and interaction terms. The density dependence of the interaction term is approximately[11] that of $N_t^{-0.5}$. Therefore, at high 2DEG carrier densities, the interaction $\delta\mu_x/\delta N_t$ term is smaller than the positive kinetic term, leading to a positive compressibility and the lower 2DEG losing carriers as the upper layer is depleted. For low-carrier densities, the negative interaction term dominates and the inverse compressibility is negative. In this case, the lower 2DEG gains carrier as the upper is depleted. This process continues until the upper 2DEG mobility is sufficiently small, at low N_t, that it stops screening the lower 2DEG. At this point, determined by the degree of disorder in the upper 2DEG, the lower layer is depleted directly by the front gate and N_b decreases rapidly.

In order to compare the different devices, we determine the carrier density N_c of the top layer at which κ^{-1} becomes

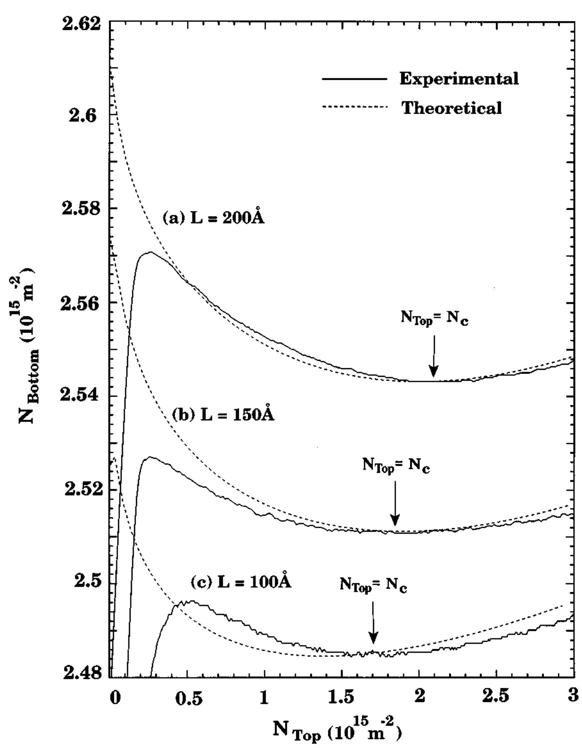

FIG. 1. Experimental (solid) and theoretical 3D LDA (dashed) results illustrating the carrier-density changes in the bottom 2DEG as the top 2DEG is depleted. Results from three of the samples studied are shown, which have QW widths of (a) 200 Å for sample *F*, (b) 150 Å for sample *C*, and (c) 100 Å for sample *B*. For clarity, each set of data is offset vertically. Arrows indicate the experimental densities N_c at which the compressibility of the top 2DEG changes sign.

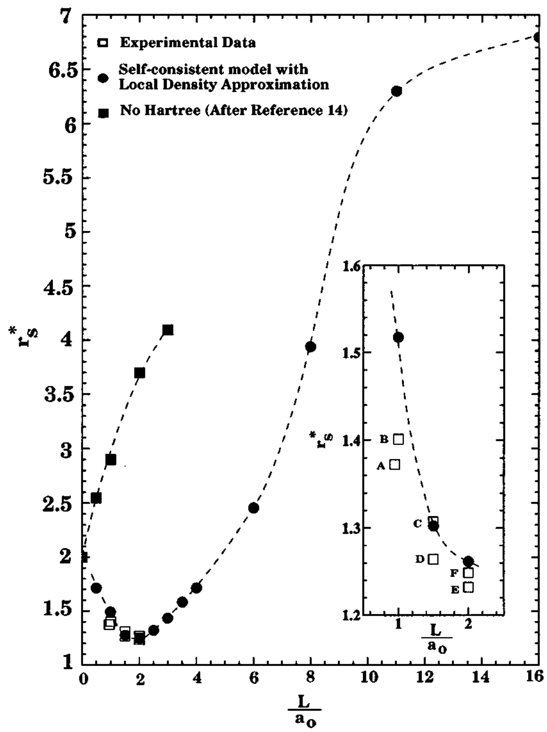

FIG. 2. The experimental and theoretical values of the 2D RPA parameter r_s^* are plotted as a function of the well-width parameter L/a_0, where a_0 is the Bohr radius (100 Å in GaAs) for (a) excluding Hartree effects, after Ref. 14 (solid squares), (b) self-consistent model incorporating the 3D LDA (solid circles) and, (c) the experimental points (open squares). The inset shows an expanded view. Dotted lines are drawn as guides for the eye only.

zero. This corresponds to when $\delta N_b/\delta N_t$ changes sign (see Fig. 1). The densities were then converted to the two-dimensional (2D) random-phase approximation (RPA) parameter r_s^* given by $r_s^{*2}=1/(\pi N_c a_0^2)$, where a_0 is the effective Bohr radius (≈ 100 Å in GaAs). These r_s^* values are plotted in Fig. 2 as a function of the parameter L/a_0, where L is the width of the upper quantum well.

IV. ANALYSIS AND DISCUSSION

We solved the 1D Poisson and Schrödinger equations self-consistently for the experimental structures, thereby including the Hartree effects. The densities of the upper and lower 2DEG's were then calculated as a function of front-gate bias.[4–6] As in Ref. 4, the approximation used for the many-body interactions was the 3D LDA proposed by Hedin and Lundqvist.[12] This 3D LDA function is evaluated at each point within the structure, so incorporating the calculated charge distribution. The model was found to be sensitive to changes in the dielectric constant, which was taken to be $11.9\epsilon_0$ and to be uniform throughout the heterostructure. Typically, a 1% change in ϵ results in a 4% change in r_s^{*2}. The results of the model are shown together with the experimental data in Fig. 1 for the three QW widths of 100, 150, and 200 Å. As may be seen, there is a qualitative fit between the model and experiment for all of the well widths considered. However, looking at the carrier density changes, there still remains a discrepancy between the experimental and theoretical data which increases for the narrower well widths. To account for differences between theory and experiment, it must be noted that the theoretical model uses the LDA which requires slow spatial variations in the density. This condition is clearly violated in the QW's and could account for the increasing mismatch between model and experiment with narrow wells.

Calculations for the compressibility of a 2DEG have been made using many different theories for the electron correlations as shown in Ref. 13 and references therein. These theories indicate that the inverse compressibility of a 2DEG of zero thickness changes sign ($\kappa^{-1}=0$) at $r_s^*\geqslant 2$. For a 3D electron gas, the corresponding RPA parameter is $r_s^*\geqslant 6$. It would, therefore, be anticipated that for quasi-two-dimensional electron gases, the critical r_s^* parameters would lie between the 2D and 3D limits, as shown in Ref. 14. The value of r_s^* will therefore increase with well thickness, contradicting the experimental data (see Fig. 2). In order to directly compare theory with experiment, the critical 2D RPA parameter was also determined from the model, using the carrier density N_c at which $\delta N_b/\delta N_t$ becomes zero. The qualitative behavior is similar to the experimental data, i.e., increasing the well width increases N_c (decreasing r_s^*), see inset to Fig. 2. The r_s^* parameters using the 3D LDA are also shown in Fig. 2 for a larger range of QW widths than used experimentally. For small well widths, r_s^* tend towards that of a zero-thickness 2D electron gas, whilst for the largest well widths r_s^* approaches the theoretical 3D value. There is, however, an initial decrease in r_s^* as the well width is increased. This variation in the critical RPA parameter may be attributed to the changing Hartree effects within the system as may be confirmed by comparing r_s^* determined with and without Hartree effects.

V. CONCLUSION

In summary, we have measured the inverse compressibility of a 2DEG as a function of the confining quantum-well width. To explain the observed trend with well width, we have used self-consistent calculations to determine the affect of the Hartree contributions which is specific to our structure and experimental technique. The results confirm the importance of the Hartree term and enable us to predict the variation of the experimental inverse compressibility with QW width. The observation of quantitative differences between the experiment and model indicates that the theory is not complete, and that it would be of interest to compare the experimental results with different interaction approximations.

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[10] Using the self-consistent calculations (discussed later), the Hartree term is found to be smaller than the interaction and kinetic components for a narrow quantum well but can become greater as the well width is increased.
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