# Transport relaxation rate of a two-dimensional electron gas: Surface acoustic-phonon contribution

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The transport relaxation rate $1/\tau$ of a two-dimensional electron gas due to scattering by thermally excited surface acoustic phonons is calculated. The temperature dependence of $1/\tau$ is found to be linear in $T$ for high temperatures, but decreases like $T^{\alpha}$ as $T$ drops below the Bloch-Grüneisen temperature for surface sound; $\alpha=7$ (5) for the deformation-potential (piezoelectric) interaction. The effect of a finite distance between the crystal surface and the two-dimensional electron gas is discussed. The results are compared with those that have been calculated for three- and two-dimensional phonons interacting with a two-dimensional electron gas. [S0163-1829(97)06611-3]

At low temperatures $T$, the mobility (or transport relaxation time $\tau$) of the two-dimensional electron gas (2DEG) in $GaAs/Al_xGa_{1-x}As$ heterostructures is mainly limited by scattering from impurities and imperfections. In a degenerate electron system, these scattering processes are temperature independent. They are thus described by a constant transport relaxation time $\tau_i$. Assuming Matthiesen's rule, the total transport relaxation rate can be written as $\tau^{-1}(T)=\tau_i^{-1}+\tau_{\rm ph}^{-1}(T)$, where $\tau_{\rm ph}(T)$ may be identified with the temperature-dependent effect of phonon scattering on $\tau$. Even though the rate $\tau_i^{-1}$ might not be well known, the temperature dependence of $\tau_{\rm ph}$ enables one to extract information on the electron-phonon scattering from mobility measurements. Indeed, an experiment of this kind has been performed by Störmer *et al.*[1] The results have been explained in terms of the scattering of three-dimensional (3D) phonons from 2D electrons using predictions of a theory developed by Price.[2] According to Price, the temperature dependence of $\tau_{\rm ph}^{-1}(T)$ undergoes a smooth transition from a linear $T$ dependence to a stronger $T^{\alpha}$ dependence as the temperature is reduced below $k_BT_{\rm BG}=2k_F\hbar c$, where $k_F$ is the Fermi wave vector of the 2DEG and $c$ is, depending on the phonon mode involved, the longitudinal or transversal sound velocity. The exponent $\alpha$ is equal to 5 for the piezoelectric electron-phonon interaction, whereas deformation-potential coupling leads to $\alpha=7$. $T_{\rm BG}$ determines the temperature at which short-wavelength phonons (phonon wave vector $q\approx 2k_F$) cease to contribute to the electron-phonon scattering processes. For $T<T_{\rm BG}$, only electron-phonon scattering through small angles remains ($q\ll 2k_F$). The transition from a high-temperature to a low-temperature regime can be viewed as a result of the phase-space restriction for electron-phonon interaction processes imposed at temperatures less than $T_{\rm BG}$. In this sense, the low-temperature range may be referred to as the Bloch-Grüneisen (BG) regime.[2,1] Note that $T_{\rm BG}$ is much smaller than the Debye temperature since the inverse lattice constant greatly exceeds $k_F$.

It is the purpose of this paper to discuss the contribution of thermally excited surface acoustic phonons (SAP's) to the transport relaxation rate of the 2DEG. Leal *et al.*[3] have already tried to address this problem. However, these authors replaced the surface phonons by a strictly 2D phonon system (i.e., a layer of vibrating atoms). Our calculations are based on the proper matrix elements for the interaction between SAP's and 2D electrons.[4] We shall show that the replacement of SAP's by 2D phonons leads to a different $T$ dependence and a different order of magnitude of $\tau_{\rm ph}^{-1}(T)$. In fact, the $T$ dependence of the relaxation rate due to scattering by surface phonons agrees with that arising from the interaction with 3D phonons and the contributions of SAP's and 3D phonons to $\tau_{\rm ph}^{-1}(T)$ can be of the same order of magnitude. This is in contrast to the results of Ref. 3 which predict that SAP's dominate $1/\tau_{\rm ph}$ once $T$ drops sufficiently below $T_{\rm BG}$. Moreover, our calculations show clearly that and how the effect of SAP's on $\tau_{\rm ph}^{-1}(T)$ depends on the distance of the 2DEG from the surface of the heterostructure.

The transport relaxation rate associated with surface phonons can be written as

$$\frac{1}{\tau_{\rm sap}}=\sum_{\boldsymbol{q},\boldsymbol{k},\boldsymbol{k}'}[1-\cos\theta(\boldsymbol{k},\boldsymbol{k}')]W_{\boldsymbol{k},\boldsymbol{k}'}^{\pm\boldsymbol{q}}f_{\boldsymbol{k}}(1-f_{\boldsymbol{k}'}), \qquad (1)$$

where $\boldsymbol{q}$ is the 2D wave vector of a SAP, $\boldsymbol{k}$ and $\boldsymbol{k}'$ refer to the 2D wave vectors of an electron prior to and after an electron-phonon interaction, $\theta$ is the angle between $\boldsymbol{k}$ and $\boldsymbol{k}'$, and $f$ is the Fermi distribution function. The transition rate $W$ is given by

$$W_{\boldsymbol{k},\boldsymbol{k}'}^{\pm\boldsymbol{q}}=\frac{2\pi}{\hbar}\frac{|M_{\boldsymbol{k},\boldsymbol{k}'}^{\pm\boldsymbol{q}}|^2}{|\varepsilon(\boldsymbol{q})|^2}\left(N_{\boldsymbol{q}}+\frac{1}{2}\pm\frac{1}{2}\right)\delta(\epsilon_k-\epsilon_{k'}\mp\hbar\omega_q). \qquad (2)$$

The dispersion law for surface acoustic waves is $\omega_q=sq$, where $s$ denotes their sound velocity ($s=\xi c_t$, where $c_t$ is the transversal sound velocity and $\xi\approx 0.9$ for GaAs; cf. Ref. 4). The upper/lower sign in Eq. (2) refers to the emission/absorption of a SAP. $N_q=[\exp(\hbar\omega_q/k_BT)-1]^{-1}$ is the phonon distribution function. The matrix elements for the electron-SAP interaction are given by[4]

$$|M_{\boldsymbol{k},\boldsymbol{k}'}^{\pm\boldsymbol{q}}|^2=\frac{\hbar}{L^2\rho s}e^{-2\kappa qz_0}B_nq^n\delta_{\boldsymbol{k}\pm\boldsymbol{q},\boldsymbol{k}'}, \qquad (3)$$

 

$$B_n q^n = \begin{cases} B_2 q^2 = c_{DA} \Xi^2 q^2 & \text{for deformation-potential interaction} \\ B_0 = c_{PA} (e\beta)^2 & \text{for piezoelectric interaction.} \end{cases}$$

$B_0$ represents an angle average with respect to $\hat{\boldsymbol{q}}$. $L^2$, $\rho$, $\Xi$, $e$, and $\beta$ are the normalization area in the 2D plane of the electron gas, the mass density of GaAs, the deformation-potential constant, the electron charge, and the piezoelectric modulus, respectively. $c_{DA}$ and $c_{PA}$ are numerical coefficients of order unity. The exponential function accounts for a finite distance $z_0$ of the 2DEG from the surface. The decay of a surface acoustic wave towards the interior of the sample is determined by the decay length $(\kappa q)^{-1}$. For the deformation-potential interaction, only one coefficient $\kappa$ occurs that coincides with the decay constant of the longitudinal component of the surface wave $\kappa = \kappa_l \approx 0.8$. For the piezoelectric interaction, three different coefficients $\kappa_i \leqslant 1$, $i = 1,2,3$, appear. For the sake of simplicity, all of them are replaced by an effective decay constant $\kappa$.

The experimental results of Ref. 1 confirm that the screening of the electron-phonon coupling due to the 2DEG is not negligible. Screening is accounted for in Eq. (2) by the dielectric function[5]

$$\varepsilon(\boldsymbol{q}) = 1 + \frac{1}{a_B^* q} H(q), \tag{4}$$

where $a_B^*$ is the effective Bohr radius. Up to the function $H$, the latter expression agrees with the dielectric function of a strictly 2D electron system.[6] $H$ comprises the effects due to a finite thickness $d$ of the 2DEG. In the limiting cases $qd \ll 1$ and $qd \gg 1$ it approaches unity or behaves like $(qd)^{-1}$, respectively. This implies the following approximations for large (high-$T$ range) and small (low-$T$ range) wave vectors, $\varepsilon(2k_F) \approx 1$ and $\varepsilon(q \ll 2k_F) \approx (a_B^* q)^{-1}$, which will be used below.

Substituting expressions (2)–(4) into Eq. (1), we obtain

$$\frac{1}{\tau_{\text{sap}}} = \frac{2^{3+n} m^* k_F^n B_n}{\pi \hbar^2 \rho s a^{3+n}} \int_0^a dx \frac{x^{3+n}}{\sqrt{1 - x^2/a^2}} \times \frac{e^{-4\kappa k_F z_0 x/a}}{|\varepsilon(2k_F x/a)|^2} \frac{e^x}{(e^x - 1)^2}, \tag{5}$$

where $m^*$ is the effective electron mass, $n = 2$ (0) for the deformation potential (piezoelectric) interaction, and $a \equiv 2\hbar k_F s / k_B T$. The decay of the surface waves with increasing distance from the surface is generally irrelevant if $4\kappa k_F z_0 \leqslant 1$. For GaAs/$Al_xGa_{1-x}$As heterostructures, $(4\kappa k_F)^{-1}$ is of the order of 3 nm. Let us first assume that $z_0$ is of this order and then consider how the result is changed due to a larger (more realistic) distance $z_0$ between the surface and the 2DEG. In the case under consideration, the BG temperature can be defined by $a = 1$, i.e., $k_B T_{\text{BG}} = 2\hbar k_F s$. $T_{\text{BG}} \simeq 6$ K is thus in the same range as for 3D phonons. In the high-temperature regime $a \ll 1$, formula (5) reduces to

$$\frac{1}{\tau_{\text{sap}}} = \frac{2^n c_n m^* k_F^{n-1}}{\hbar^3 \rho s^2} B_n k_B T, \tag{6}$$

where $c_2 = 5/16$ and $c_0 = 1$. That is, the transport relaxation rate exhibits a linear $T$ dependence independent of the electron-phonon interaction involved. This is a natural result because all SAP's that may contribute to scattering processes are thermally excited and a further rise of the temperature increases only the number of phonons. This number is proportional to $T$. For $a \gg 1$ (BG regime), Eq. (5) can be approximated by

$$\frac{1}{\tau_{\text{sap}}} = \frac{(5+n)!\,\zeta(5+n) m^* (a_B^*)^2}{\pi \hbar^2 k_F^3 \rho s} B_n \left( \frac{k_B T}{\hbar s} \right)^{5+n}, \tag{7}$$

i.e., the deformation-potential and piezoelectric interactions give rise to a $T^7$ and a $T^5$ dependence, respectively. Thus the power laws for the temperature dependence of $1/\tau_{\text{sap}}$ agree with those valid for 3D phonons in the high-temperature as well as in the BG regime. Neglecting the minor differences between $s$ and the longitudinal and transversal sound velocities, agreement is also found with respect to the other physical quantities determining the order of magnitude of the relaxation times. This implies that SAP's can contribute significantly to the temperature-dependent part of the transport relaxation rate if the distance between the surface and the 2DEG is small enough.

A value $z_0 > (4\kappa k_F)^{-1}$ results in the following modifications of the above expressions. The high-temperature result (6) is reduced by a factor of the order of $(4\kappa k_F z_0)^{-(2+n)}$ if the screening of the largest wave vectors that are of relevance in the integral (5) is small, $\varepsilon(1/2\kappa z_0) \approx 1$. For efficient screening or larger distances $z_0$, the reduction is given by $(2k_F a_B^*)^2/(4\kappa k_F z_0)^{4+n}$, i.e., the influence of the SAP's on the 2DEG decreases according to a power law in the quantity $(k_F z_0)^{-1}$. This modified high-temperature result remains valid until $T$ drops below a reduced BG temperature $T'_{\text{BG}} = T_{\text{BG}}/4\kappa k_F z_0$. For temperatures below $T'_{\text{BG}}$ the relaxation rate is again given by Eq. (7).

Let us now compare our results with those following from a replacement of the SAP's by a 2D phonon system. In Ref. 3, the transport relaxation rate $1/\tau_{2\text{D}}$ due to deformation potential interaction between 2D phonons and the 2DEG has been calculated. Screening was not taken into account. As a result, $1/\tau_{2\text{D}}$ depends linearly on temperature for $T > T_{\text{BG}}$, but shows a $T^4$ dependence in the opposite case. Omitting the influence of screening in Eq. (5), $\varepsilon \equiv 1$, we find $1/\tau_{\text{sap}} \sim T^5$ [instead of $\sim T^7$ with screening; see Eq. (7)] in the BG regime. The magnitude of the rates can be compared replacing the number $N$ of lattice cells in the normalization area used in Ref. 3 by $L^2/b^2$, where $b$ is the lattice constant. Suppressing numerical prefactors, the ratio of the relaxation rates can be written as $(\tau_{2\text{D}})^{-1}/(\tau_{\text{sap}})^{-1} \simeq (bk_F)^{-1}$ in the high-temperature range, whereas $(\tau_{2\text{D}})^{-1}/(\tau_{\text{sap}})^{-1} \simeq a(bk_F)^{-1}$ is found for $T < T_{\text{BG}}$ (i.e., $a > 1$). Since

$bk_F \ll 1$, the replacement of surface waves by a 2D sound wave leads to a significant overestimation of the transport relaxation rate. The discrepancy becomes larger when we incorporate the effect of a finite distance $z_0 > (4\kappa k_F)^{-1}$.

The relations among the various transport relaxation times for 3D, 2D, and surface acoustic phonons can be understood by considering the interplay between the matrix element for the electron-phonon interaction and the density of states (DOS) of phonons. The matrix elements in Eq. (3) for SAP's have one power of $q$ more compared to those for 3D phonons.[4] This compensates exactly for the reduced DOS of SAP's, appearing in Eq. (1) when integrating over $\boldsymbol{q}$. The qualitative agreement of $\tau_{3D}$ and $\tau_{sap}$ is therefore natural. On the other hand, the matrix elements for 2D phonons[3] are constructed in analogy with those for 3D phonons and hence exhibit the same $q$ dependence. Consequently, the reduced DOS of 2D phonons is not compensated, resulting in an essentially different transport relaxation time.

We have seen that SAP's make a substantial contribution to the total phonon-induced transport relaxation rate in the small-$z_0$ and/or low-$T$ limits, i.e., when the largest wave vectors $q$ of the thermally excited SAP's become of the same order as $(z_0)^{-1}$. In this case, the influence of the surface cannot be considered as a small perturbation. This poses the question whether extended bulk phonons[2,1] represent an appropriate description of the corresponding elastic displacement modes in this regime. To answer this question, we have employed a formalism developed by Badalyan and Levinson,[7] which describes all branches of elastic modes on the same footing, i.e., both SAP's (decaying exponentially with increasing distance from the surface) and 3D phonons (nondecaying) are subject to the elastic boundary conditions imposed at the surface. It turns out that all elastic modes lead, up to numerical factors, to the same high-$T$ [Eq. (6)] and low-$T$ [Eq. (7)] limiting forms as far as $1/\tau_{ph}(T)$ is concerned. That is, bulk phonons provide a qualitatively correct description also in the case where the 2DEG is very close to the surface of the sample. Since the approach of Ref. 7 applies to the deformation-potential coupling, it remains to be seen whether this is true for the piezoelectric interaction as well.

In conclusion, we have shown that surface acoustic phonons lead to the same temperature dependence of the transport relaxation time as 3D phonons. They even contribute a term of the same order of magnitude if the distance $z_0$ between the surface and the 2DEG is smaller than $(4\kappa k_F)^{-1}$. For surface phonons, the onset temperature of the Bloch-Grüneisen regime depends on $z_0$ and is given by $k_B T_{BG} = 2\hbar k_F s/\max\{4\kappa k_F z_0, 1\}$.

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