Phase diagrams of flux lattices with disorder

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We review the prediction, made in a previous work [T. Giamarchi and P. Le Doussal, Phys. Rev. B **52**, 1242 (1995)], that the phase diagram of type-II superconductors consists of a topologically ordered Bragg glass phase at low fields undergoing a transition at higher fields into a vortex glass or a liquid. We estimate the position of the phase boundary using a Lindemann criterion. We find that the proposed theory is compatible with recent experiments on superconductors. Further experimental consequences are investigated.
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It is remarkable that after a decade of experimental and theoretical efforts, the phase diagram of type-II superconductors in a field is far from being completely elucidated.[1] Stimulated by the discovery of the high-T_c materials, a reexamination of the mean-field phase diagram unraveled two main new phenomena. First, it was realized,[2,3] and observed[4] that due to enhanced thermal fluctuations the Abrikosov lattice melts well below H_{c2} into a flux liquid. Second, it was argued that in the solid phase, pointlike disorder could produce a glassy state with barriers $U(j)$ diverging at small j, and thus characterized by the true vanishing of the linear resistivity even at finite temperature.[5,6] This was a significant departure from traditional models of thermally assisted flux flow, which assumed *finite* barriers between pinned states. A precursor sign of an instability towards a glass was also found in the flux liquid.[7] Both for technological applications of high-T_c materials and from a purely theoretical point of view, the understanding of the detailed properties of such a glassy phase is of paramount importance.

Two main phenomenological theories have been put forward to describe this glassy phase and to account for some of its properties observed in early experiments, mainly the observed continuous transition[8] from the glass to the liquid and giant thermal creep. The first approach is based on the gauge glass model,[5,9] and assumes a complete destruction of the Abrikosov lattice. The second approach retains the elastic lattice structure at small scale.[6] Although different in nature, both theories agreed that the disorder essential to produce the glassy low-temperature phase and the vanishing of the linear resistivity was also destroying at large scales the perfect flux lattice existing in mean-field theory. The low temperature phase was therefore generally expected to be a topologically disordered phase, lacking translational order. Several calculations supported this point of view. Elastic theory predicted at best a stretched exponential decay of translational order[6,10,11] (i.e., a power law growth of displacements) and general arguments tended to prove that disorder would always favor the presence of dislocations.[9] The vortex lattice seemed to be buried for good.

A few points did not naturally fit into the framework of these theories. Experimentally, a first-order transition between the glass phase and the liquid was observed at low fields[12,13] rather than the predicted continuous transition, observed at high fields. Also, decoration experiments of the flux lattice at very low fields (60 G) in several materials showed remarkably large regions free of dislocations.[14] On the side of theory, old calculations on the related disordered elastic random field XY model[15] as well as more recent scaling arguments for the vortex lattice[16] suggested, within a purely elastic description, a slower, logarithmic, growth of deformations. However, despite that fact, it remained unquestioned at that time that dislocations would always be generated by disorder, as argued in Ref. 15.

In a recent work we obtained a quantitative theory of the elastic vortex lattice[17,18] in the presence of point disorder.[19] Contrarily to previous approaches, it provides a description valid at all scales and demonstrates that while disorder produces algebraic growth of displacements at short length

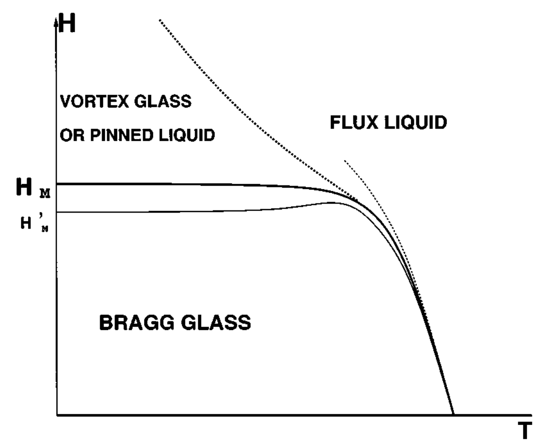

FIG. 1. The stability region of the Bragg glass phase in the magnetic field H, temperature T plane is shown schematically (thick solid line). Upon increasing disorder the region shrinks as indicated by the thin solid line (see text). The melting line of the pure system is shown as a dotted line, and the vortex glass transition line (or crossover to the pinned liquid) is shown as a thick dotted line. For clarity, the reentrant liquid at very low field, discussed in the text, is not shown.

scales, periodicity takes over at large scales and results in a decay of translational order *at most algebraic*.[20] One striking prediction is thus the existence of a glass phase with Bragg diffraction peaks. This result was derived within an elastic theory, assuming the absence of dislocations. However, the very result of our calculation, i.e., that quasi-long-range order survives, led us to advocate that dislocations would be much less relevant than commonly assumed.[17] The alleged importance of dislocations in a disordered system[9,15] made it mandatory to further investigate carefully whether allowing for dislocations would indeed modify the above result. The striking result[18] that we found based on energy arguments is that dislocations are *not favorable* for weak disorder in $d=3$. This implies self-consistently the existence of a thermodynamic glass phase, as far as energy and very-low-current transport properties are concerned, retaining a nearly perfect (i.e., algebraic) translational order and a perfect topological order. Since this phase exhibits Bragg peaks very much like a perfect lattice, it was called the ''Bragg glass.'' Because it retains a ''lattice'' structure and Bragg peaks, this glass phase is radically different from the vortex glass picture based on a random gauge model. In particular, since such a phase is nearly as good as a perfect lattice as far as translational order is concerned, it is natural to expect it to melt through a first-order phase transition. We proposed[18] that the phase seen experimentally at low fields was in fact the Bragg glass, solving the apparent impossibility of a pinned solid. This allowed us to account naturally for the first-order transition and the decoration experiments. Our prediction[18] that a new phase without topological defects should be stable at weak disorder, which also applies to the random field XY model,[22] received subsequent further support both from numerical simulations[23,24] and from analytical calculations in a layered geometry.[25,26]

Once the existence of a weak-disorder–low-field Bragg glass phase is established, the question arises of determining its limits of stability and phase boundaries. The Bragg glass phase should be stable as a self-consistent solution in the elastic limit, i.e., as long as $R_a \gtrsim a$.[18] This condition is violated when the field is increased and we proposed in Ref. 18 that upon raising the field the Bragg glass should undergo a transition into another phase, which could be a pinned liquid or another glass (vortex glass). A natural possibility then was that the critical point occurring on the melting line[13] was the end point of the transition line between the Bragg glass at low fields and a topologically disordered glassy phase (or a strongly pinned liquid) at higher field. We pointed out that the fact that this point can be lowered in field and raised in temperature by adding impurities was a hint that it was related to this transition. Such a field-driven transition corresponds to the destruction of the Bragg glass by proliferation of topological defects upon raising the field, which is equivalent to increasing the effective disorder, which favors dislocations. The other transition from the thermal liquid into the putative superconducting state at higher fields is presumably continuous. The topology of the phase diagram proposed in Ref. 18 is as depicted in Fig. 1. Its main features should be relevant for all type-II superconductors.

Several recent experiments can be interpreted to confirm the picture proposed in Ref. 18. Neutron experiments can be naturally interpreted in terms of the Bragg glass.[27,28] In BSCCO neutron peaks are observed at low fields and disappear upon raising the field.[29] The phase diagram of BSCCO has been recently explored in detail by overcoming spurious effects due to geometrical barriers.[30] It can increasingly be interpreted as a confirmation of our theory, the so-called second magnetization peak line being the candidate for the predicted field-driven transition. Since our proposal, this line has been investigated in more detail in BSCCO (Ref. 31) and found to be relatively temperature independent at lower temperatures and to be shifted downwards upon increase of point disorder through electron irradiation.[32] Also, similar types of phase diagrams are observed in a variety of materials, including YBCO (Refs. 12 and 33) and thallium compounds.[34] The fact that a controlled increase of point disorder through electron irradiation shifts the transition line to lower fields[35] is a strong indication that our picture is relevant in these materials as well.

In the present paper we follow up on the theory exposed in Ref. 18. We make more quantitative estimates of the phase diagram depicted in Fig. 1 using a generalized Lindemann criterion. We also explore in more detail some experimental consequences of our theory.

Let us consider a vortex lattice system in the presence of disorder. We can model the vortex lattice by stacks of coupled planes. The system is therefore described by layers of two-dimensional triangular lattices of vortices. We denote by R_i the equilibrium position of the vortices in the absence of disorder, labeled by an integer i, in the xy plane, and by $u(R_i,z)$ their in-plane displacements which are two-dimensional vectors (the vortex can only move within the plane). z is the coordinate perpendicular to the planes and along the magnetic field and $x=(r,z)$. The total energy is

$$H=\frac{1}{2}\int d^2r dz[(c_{11}-c_{66})(\partial_\alpha u_\alpha)^2+c_{66}(\partial_\alpha u_\beta)^2 + c_{44}(\partial_z u_\alpha)^2]+\int d^2r dz V(r,z)\rho(r,z), \tag{1}$$

where the density of vortex lines is simply defined by $\rho(x) =\sum_i \delta^{(2)}(r-R_i-u(R_i,z))$. The last term in Eq. (1) is the coupling to disorder. In the limit where many weak impurities act collectively on a vortex, point disorder can be modeled by a Gaussian random potential $V(x)$ with correlations $\overline{V(x)V(x')}=\Delta(r-r')\delta(z-z')$, where $\Delta(r)$ is a short-range function[36] of range ξ (the superconducting coherence length):

$$\Delta(r)=dU_p^2 e^{-r^2/2\xi^2}, \tag{2}$$

where d is the distance between layers and U_p a typical pinning energy per unit length along z.

In the high-T_c Abrikosov lattice, one has in principle to use nonlocal elasticity, and a calculation along the lines of Ref. 18 can be done simply by using the complete known wave-vector-dependent expressions[2,37] of the elastic constants in Eq. (1). Since we are only interested in nearest-neighbor correlations and want to obtain only an order of magnitude of the scales involved, we use simple constant elastic moduli. The physical properties of Eq. (1) were examined in detail in Refs. 17 and 18 and we just recall here the results needed for the phase diagram. The mean-squared relative displacements of two vortices separated by a distance r is

$$B(r)=\overline{\langle[u(0,0)-u(r,0)]^2\rangle}, \tag{3}$$

where $\langle\ \rangle$ denotes the thermal average, whereas $\overline{\cdots}$ is the disorder average. From $B(r)$ one defines two length scales R_c and R_a in the xy plane (and similarly L_c and L_a along z) such that $B_{\rm dis}(R_c)\sim\max(\xi^2,\langle u^2\rangle_T)$ (see below) and $B(R_a)\sim a^2$, respectively. R_c is the Larkin-Ovchinikov pinning length[38] directly related to the critical current, whereas R_a is the scale at which one enters the asymptotic regime with a logarithmic growth of the displacements. The model (1) leads to the Bragg glass phase with quasi-long-range translational order.

To determine the region of stability of the Bragg glass phase, we follow the arguments proposed in Ref. 18 that the elastic structure will become unstable when the displacement between two neighbors becomes of order of the lattice spacing a, i.e.,

$$B(r=a)\sim a^2. \tag{4}$$

To be more quantitative, one can introduce, as for the normal thermal melting, a Lindermann constant c_L and take for the criterion[39] of stability of the Bragg glass phase

$$B(r=a)=\overline{\langle[u(0,0)-u(a,0)]^2\rangle}=c_L^2a^2. \tag{5}$$

c_L, the Lindemann constant, is usually of the order of $c_L\sim0.1$–0.2 in the usual melting and we make here the assumption that c_L is indeed a constant independent of the field.[40]

From Eq. (5) one sees that both disorder and thermal fluctuations act together to increase the displacements. In fact formula (3.18) of Refs. 17 and 18 shows that $B(r=a)$ splits naturally into two parts $B(r=a)\approx2\langle u^2\rangle_T+B_{\rm dis}(r=a)$. One immediate consequence of Eq. (5) is therefore that the melting line should be pushed *downwards* in the presence of pointlike disorder. In fact the Bragg glass can disappear in two ways: (i) If the temperature is raised, it will melt to a liquid phase, and (ii) if the field is raised, which amounts to varying the effective disorder in the system, the system can become so disordered even at short length scales that dislocations will appear. Equation (5) gives thus the limit of stability of the BG phase in the H-T plane. Although the complete ''melting'' curve can be computed using the formulas for $B(r)$ obtained in Refs. 17 and 18, such a calculation is tedious and offers little insight. We therefore study mainly here the two limits of low temperature, where the transition is mainly field driven, and of temperature close to the melting curve in the absence of disorder.

If the temperature is close to the pure melting line, $B(r=a)$ is dominated by thermal fluctuations. Since for weak disorder $R_a\gg a$, the disorder-induced displacements are negligible at the scale of nearest neighbors and one can compute Eq. (5) using thermal fluctuations only. One then easily recovers the pure melting line

$$T_m\approx4a^3\sqrt{c_{66}c_{44}}\,c_L^2. \tag{6}$$

Disorder effects will push the melting line slightly down, but effects should be negligible at low field for which the effective disorder is small enough. Upon increasing the field disorder-induced displacements will increase, forcing the transition line defined by Eq. (5) to go down to zero temperature at a finite field H_M. The scale at which disorder dominates can easily be obtained by looking at zero temperature. To obtain a reliable order of magnitude of the ''disorder-induced melting'' field H_M, it is necessary to know the precise $B(r)$ in the presence of disorder and not only its asymptotic forms. Fortunately such a calculation was performed in Refs. 17 and 18. Using formula (4.18) of Ref. 18 one gets

$$B(r)=\frac{a^2}{\pi^2}\tilde{b}(r/R_a). \tag{7}$$

For $r=R_a$ one has from Ref. 18 that $\tilde{b}\approx1$ while for $r<R_a$ one is in the random manifold regime and one can approximate $B(r)\simeq(a^2/\pi^2)(r/R_a)^{1/3}$. From the solution of Ref. 18 we know that the above formula is *quantitatively* correct, and not only asymptotics. Using Eq. (5) one finds that

$$a/R_a=(\pi c_L)^6. \tag{8}$$

Using $c_L=0.12$ gives $R_a\sim350a$. Thus the transition occurs well before the asymptotic regime. We will find that it does occur (e.g., in BSCCO) indeed deep into the random manifold regime. One also notes that in simplified models without an intermediate random manifold regime (where one directly goes from a Larkin regime to the asymptotic regime) the above formula would give $a/R_a=(\pi c_L)^{2/(4-d)}$. The transition then occurs for smaller values of R_a/a, in agreement with the results found in a special geometry.[25,26]

Using Eq. (8) and the expression (4.12) of Ref. 18 for R_a

$$R_a=\frac{2a^4c_{66}^{3/2}c_{44}^{1/2}}{\pi^3\rho_0^2U_p^2 2\pi d\xi^2}, \tag{9}$$

as well as $c_{66}=\epsilon_0/(4a^2)$ and $c_{44}\approx c\epsilon_0/(\gamma^2a^2)$ (single-vortex contribution) with $\epsilon_0=(\Phi_0/4\pi\lambda)^2$ and c a numerical constant.[37] One gets

$$a^3=\frac{4\pi^3}{(\pi c_L)^6}\frac{U_p^2}{\epsilon_0^2}2\pi d\xi^2\frac{\gamma}{\sqrt{c}}. \tag{10}$$

One thus obtains an expression for the transition field H_M naturally expressed in terms of some characteristic fields of the system:

$$H_M(T=0)=\frac{(\pi c_L)^4}{(16\pi)^{1/3}\pi^2}\left(\frac{\epsilon_0}{U_p}\right)^{4/3}H_{c2}^{2/3}H_{\rm cross}^{1/3}, \tag{11}$$

where we have introduced the crossover field $H_{\rm cross}=\pi c\Phi_0/(\gamma^2d^2)$ with $c\sim\ln(\gamma d/\xi)$ (Ref. 37) and $H_{c2}=\Phi_0/2\pi\xi^2$.

As a numerical estimate of the melting field H_M for BSCCO with $H_{\rm cross}\sim 1$ T, $H_{c2}\sim 100$ T, $U_p/\epsilon_0=0.4$, and $c_L=0.12$ gives $H_M\sim 400$ G in good agreement with the observed experimental values.[32] The fact that this field is well below the decoupling field validates *a posteriori* the calculation [note also that $B(r=0,z=d)$ is still small at the transition]. The general shape of the phase diagram is in agreement with the one of Fig. 1. Note that some nonlinear effects, such as screening of disorder by thermal fluctuations or by interactions at short scales, may not be captured directly by the Gaussian theory of Ref. 18. They can be incorporated by a renormalization of the effective disorder $U_p(T)$. Such effects were computed in the flux *liquid* using the renormalization group (RG) in Ref. 7 and it was shown that the pinning length was renormalized upward (and thus the effective pinning strength downward) by a factor of $\exp[(T/T_{\rm dp})^3]$ where $T_{\rm dp}\sim(U_p^2 d\xi^2 c\epsilon_0/\gamma^2)^{1/3}$ is the single-vortex depinning temperature.[1] It would be interesting to compute these effects in the solid as well. On general grounds that thermal fluctuations can only weaken the disorder, one expects an additional curvature *upward* of the Bragg glass instability line $H_M(T)$ when T increases beyond $O(T_{\rm dp})$.

In Fig. 1 two main regions can be distinguished: If the temperature is high, the stability line is nearly indistinguishable from the melting line of the pure system. This regime corresponds to the case where $R_a(T=0)\gg a$. In that case the translational order is only affected at distances huge compared to the lattice spacing, and the modification compared to a pure lattice is negligible as far as the melting is concerned. This part of the stability line is therefore nearly identical to the melting of a pure lattice and one can expect the transition to be first order. The Bragg glass melts to a liquid phase, nearly insensitive to disorder.

If the field is increased, one will shorten $R_a(T=0)$. The disorder itself is now able to make dislocations proliferate. In particular even at $T=0$ disorder destroys the Bragg glass. In this range of field and at low T the transition line flattens as a function of temperature, since it is controlled mainly by the disorder. The phase into which the Bragg glass ''melts'' at low T is relatively poorly understood. It is characterized by the absence of translational order and of Bragg peaks. Since it is dominated by point disorder, dislocations will decorrelate along z and thus can lead to an entangled state. There should still be some amount of pinning at low T, but whether such a phase is a true glass with diverging barriers, similar to the proposed vortex glass of Ref. 9, or simply a very viscous form of the liquid phase remains controversial. This phase could also retain hexatic order (hexatic glass) at least in a portion of it since, at least at a naive level, similar arguments for the survival of hexatic topological order (no unbound disclinations) as for translational topological order in the Bragg glass can be given. If the phase is a true glass phase, then it should melt thermally to the liquid, on the thick-dotted line of Fig. 1. Whether a true vortex glass phase exists in *untwinned* samples is an important, still open, and controversial question[41] which may need to be settled by high-sensitivity[42] measurements. Since the low-temperature phase is in any case much more continuously related to the liquid phase, one can expect now the transition to become second order. The Bragg glass therefore provides one natural explanation for a change of the order of the (thermal) melting transition, as well as for the existence of a field-induced transition.

One should also point out that at *very low* fields $(B\sim H_{c1})$ where screening is important, $a\gtrsim\lambda$, a similar (inverted) field driven transition should also occur when the field is lowered, from the Bragg glass to a pinned liquid (or another glass) as suggested by the decoration images of Ref. 14. As shown in Ref. 7 the liquid becomes unstable for $B<B_0\exp[-(T/T_{\rm dp})^3]$, presumably to a glass or a pinned liquid (see Ref. 43 for an experimental evidence of this line).

The proposed field-induced transition between the Bragg glass (BG) and the putative vortex glass (VG) being just characterized by an injection of dislocations, it is not necessarily linked to a decoupling between the layers. As a consequence one does not expect the critical current along z to become zero at the field-induced transition, at least for a low-anisotropy system like YBCO. Of course it is always possible that in materials with high anisotropy like BSSCO dislocations prefer to appear first between the planes and the BG-VG and decoupling transition coincide. Let us, however, emphasize that it does not need to be so and that we also expect our transition to occur in purely isotropic systems. Another argument against the field-induced transition being a simple thermal decoupling transition[37] is the fact that such a transition could not extend down to zero temperature. In any case measurements of the critical current perpendicular to the plane, in particular in YBCO, should help to separate between the two effects.

The suggestion that there may be two different glass phases could seem farfetched. There is a case, however, mostly of theoretical interest at present, where it *should* happen as a direct consequence of our Bragg glass considerations. This is for d-dimensional vortex line systems with correlated disorder or equivalently in $(d-1)$-dimensional quantum bosons with disorder. It is reasonably well established theoretically, numerically, and experimentally that a Bose glass phase exists for these systems in $d=3$ [i.e., $d=2+1$ (2 space, 1 time dimension) for bosons]. It is also believed that this phase lacks translational long-range order in the plane perpendicular to the columns. Indeed, since the vortices are localized along the columns, one can roughly view the properties in the perpendicular plane, as those of a $(d-1)$-dimensional system with pointlike disorder.[44] For the $d=3$ vortex problem, dislocations are therefore expected to appear (as they presumably appear for $d=2$ systems with pointlike disorder). In higher dimensions, however, this need not be the case. For instance, in $d=4$ for vortex systems $(d=3+1$ for quantum particles) one is led, by similar arguments as in Ref. 18 to *two distinct localized phases*. For weak disorder no dislocation will appear, giving a Bose glass with topological order. This ''Bragg-Bose glass'' phase is the equivalent for columnar defects of the Bragg glass one occurring for pointlike disorder. For stronger disorder, dislocations will destroy the topological order perpendicular to the columns, giving back the ''conventional Bose glass,'' i.e., the continuation of its $d=2+1$ version. At the transition between these two different Bose glass phases, unbinding of dislocations loops (cylinders) should occur. An interesting point is that in the ''conventional Bose glass'' these dislocations loops will remain pinned to the columnar defects, and it will thus still be a true glass. This Bose glass phase would be in some sense analogous, in the case of point disorder, to the putative vortex glass. However, for pointlike disorder it is much less obvious that such a phase exists as a genuine thermodynamic phase in $d=3$.[45] Let us note that the correlated disorder problem was studied analytically within an elastic theory in Ref. 44 using a variational method and in Ref. 46 using RG methods: The phase described there is thus the ''Bragg-Bose glass.'' The difference between the two phases should be apparent only at scales larger than the distance between unpaired dislocations.

Several consequences of our theory could be further checked in experiments. First, since the Bragg glass phase has translational order and the vortex glass has not, neutron experiments should observe a destruction of the Bragg peaks at the same location[47] as the transition observed by magnetic measurements. Such a feature seems to be consistent with the existing experimental data in BSSCO,[29] but a more detailed experimental investigation would be needed to check this point in other materials as well. Another clear distinction between the two phases should be observed when a cycling in current similar to the one of Ref. 27 is performed. Such cycles, taking the system above the critical J_c and then back to zero, are expected to heal the lattice and to expel out of equilibrium dislocations. One can therefore expect good healing in the Bragg glass phase, as is indeed the case, since dislocations can only exist as out of equilibrium object. On the other hand, the same cycle performed in the VG phase should make little difference on the neutron diffraction pattern since the *equilibrium* state already contains unpaired dislocations.

Finally one expects the barriers to vary very differently in the two glass phases. In the Bragg glass phase, elasticity is strong. Pinning can only be collective and one expects therefore weak barriers at short length scales. This implies a small critical current. On the other hand, since creep can only occur collectively, the barrier should grow very rapidly with decreasing current. Standard creep arguments[1] in the presence of the Lorentz force $f\sim j$ show that $V\sim \exp[-U(j)/T]$ with the optimal barrier $U(j)=\max_L(L^\theta - juL^d)\sim(1/j)^\mu$ with $\mu=\theta/(d+\zeta-\theta)$, and lead to μ=0.7–0.8 at intermediate currents (random manifold exponents) and μ=0.5 at very small currents ($\theta=d-2$, $\zeta=0$).[18] Taking the dispersion of elastic moduli into account leads to higher values of μ in the intermediate regime.[1] On the other hand, in the VG phase barriers should be significantly larger at short length scales since the nearly destroyed lattice has additional effective degrees of freedom, such as free dislocations, and can thus adapt more easily to the pinning potential. The critical current should therefore increase when approaching the field-induced transition. The onset of entanglement at the transition could also increase the critical current because of flux cutting barriers.[48] On the other hand, in the VG phase the barriers should grow much more slowly with decreasing current since there is no need for collective motion, or even remain finite if the phase is simply a crossover from the liquid phase. Some estimates of the exponents for the gauge glass model gave very small exponents of the order of $\mu\sim$0.1–0.2. One can therefore expect I-V characteristics evolving with fields like the ones shown in Fig. 2. Such a behavior is in good qualitative agreement with the observations of a second peak in relaxation measurements when the transition is passed. More refined transport or relaxation measurements should help in deciding on the behavior of the barriers. Note that peak effects upon raising the field were observed[49] in two-dimensional (2D) materials. There the increase of the critical current, as compared to the predictions of a 2D Larkin-Ovchinikov theory, was interpreted in terms of an elastic instability towards dislocations. It was not discussed at that time whether the vortex state at low field in these experiments was or not a true glass with topological order and diverging barriers at low current.

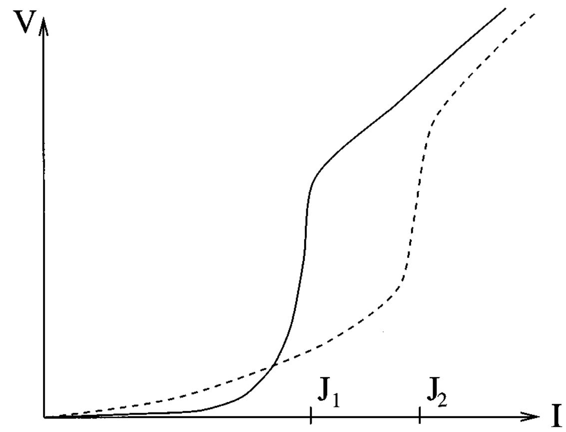

FIG. 2. I-V characteristics in the Bragg glass phase and in the vortex glass (or pinned liquid) are shown schematically, respectively, as solid and dashed lines. One goes from small (J_1) to larger (J_2) critical currents, but rapidly divergent to weakly divergent (or finite) barriers, when increasing the field.

It is important to note that the Lindemann criterion used here is *not* a detailed theory of the transition when dislocations proliferate, which is not yet available. It represents one possible mechanism of instability dominated by short length scales. It thus provides a reasonable upper bound for the instability field H_M since the Bragg glass certainly cannot self-consistently survive if $R_a<a$. However, it cannot be excluded that the Bragg glass is unstable before this limit, as could be the case, for instance, if, because of the weakening of translational order at large distances in the Bragg glass compared to a real solid, unbound dislocations start to appear first at large length scales compared to a. In that case this additional phase (which may or may not be a true glass) would also melt through a first-order transition with good short-distance translational order properties. This could happen, for instance, if unbound dislocations appear first at scales between a and $R_c(T)$, thus affecting the critical current but not the first-order melting. This is one possible scenario for YBCO where the second peak line is observed well below the tricritical point (though a clear interpretation there is more delicate due to additional twin boundaries and the fact that the second peak region appears quite broad). Finally, it would prove very interesting to investigate in more detail the phase diagrams of a variety of compounds including organic superconductors[50] and heavy fermion compounds[51] as well as the thallium family. Indeed these phase diagrams show remarkable similarities, and we expect that these can also be interpreted using the ideas of Ref. 18 and the present paper.

In conclusion, we have examined in some detail the implications, for the phase diagram of type-II superconductors, of the existence of a glass phase with translational order: the Bragg glass. The existence of this phase immediately implies the existence of a field-driven transition in the phase diagram, and thus provides a natural interpretation of several of the experimentally observed features of the phase diagram of BSCCO, YBCO, and TlBCCO, namely, a change from a first-order melting transition to a continuous transition when the field is increased and the existence of a field-induced transition. We interpret this last transition as the destruction of the Bragg glass phase by spontaneous injection of unbounded dislocations into a topologically disordered glass phase or liquid. This transition, being disorder driven, should extend down to $T=0$. We have estimated the position of this transition using a Lindemann criterion and discuss further experimental consequences.

Note added in proof. We received recently an interesting paper by D. S. Fisher (unpublished) where refined energy and scaling arguments are presented confirming the existence of a dislocation-free phase at weak disorder, thus providing additional theoretical support in favor of the Bragg glass theory of Refs. 17 and 18.

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