Dynamic Ruderman-Kittel-Kasuya-Yosida indirect interaction

G. M. Genkin*
Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208
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We consider the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between two spins, when they are affected by a frequency-dependent magnetic field. It is shown that the RKKY indirect interaction, i.e., value of the coupling, its spatial dependence, and the periods of spatial oscillations, in the three-dimensional free-electron gas for transversal component of the spins depends on the frequency. For the high-frequency range $h\omega \leqslant \epsilon_F$ three main uncommensurable periods of oscillations k_F, $(1+\sqrt{2})k_F$, $(\sqrt{2}-1)k_F$ are found. The spatial dependence of the coupling for large $k_F r$ contains terms proportional to r^{-1} and r^{-2}. The value of the coupling is of the order $(k_F r)^2$ as much as in the static case. For the frequency range $h\omega \ll \epsilon_F$ the dynamic RKKY interaction contains the terms whose values and spatial periods are determined by the frequency of the magnetic field. It is shown that there is a frequency-dependent anisotropy of the dynamic RKKY interaction. [S0163-1829(97)01609-3]

The spin polarization of a degenerate electron gas has been used to describe the coupling [the Ruderman-Kittel-Kasuya-Yosida (RKKY) indirect interaction] between two spins in metals. Nowadays such RKKY interactions are investigated widely, particularly with the problems of coupling[1] in magnetic superlattices.

However, in the well-known RKKY interaction between two spins it is usually supposed that the spins are not affected by a frequency-dependent magnetic field (static case). We will consider the RKKY interaction, when our system is affected by a frequency-dependent magnetic field (dynamic case). We suppose that this external field is a circularly polarized uniform magnetic field $H^+(t)$ of the frequency ω (the dimension of our system is much less than the wavelength of a high-frequency electromagnetic field).

The response of a system of localized spins[2] to such a field has a rotational component that is determined by the field $H^+(t)$. Let us consider the RKKY interaction between the rotational components of two spins S_j^+ and S_i^+. The RKKY mechanism is produced by a contact potential V between the localized spin (position $\mathbf{R}_j$) and the conduction electron $s_{\rm el}$ (position $\mathbf{r}$). For our components V is given by

$$V=-AS_j^+ s_{\rm el}^- \delta(\mathbf{r}-\mathbf{R}_j), \tag{1}$$

where $S^{\pm}=S_x \pm iS_y$.

Each conduction spin therefore experiences[3] an effective field and the Fourier transform of this field is $H_{\rm eff}(\mathbf{q},\omega)=-A/(g\mu)S_j(\omega)$. The response of an electron gas to such field is determined by the transversal component of the dynamic susceptibility $\chi_{-+}(\mathbf{q},\omega)$. Then the induced rotational component of the conduction spin $s_{\rm el}^-$ arises; this component is caused by the alternating magnetic field $H_{\rm eff}$. Another localized spin in position $\mathbf{R}_i$ interacts with the induced rotational component. As a result, we have the effective interaction[4] to a second approximation in V

$$H_{\rm RKKY}=J(\omega,\mathbf{R}_{ij})S_i^+(\omega)S_j^+(\omega), \tag{2}$$

where

$$J(\omega,\mathbf{R}_{ij})=-\frac{A^2}{g^2\mu^2 V_0}\sum_{\mathbf{q}} \chi'_{-+}(\mathbf{q},\omega)\exp(i\mathbf{q}\mathbf{R}_{ij}),$$
$$\mathbf{R}_{ij}=\mathbf{R}_i-\mathbf{R}_j. \tag{3}$$

In Eq. (3), the real part of the transversal component of the nonuniform dynamic susceptibility of the electron gas is[2]

$$\chi'_{-+}(\mathbf{q},\omega)=\frac{2g^2\mu^2}{V_0}\sum_{\mathbf{k}} \frac{f_{\mathbf{k},\uparrow}(1-f_{\mathbf{k}+\mathbf{q},\downarrow})}{\epsilon_{\mathbf{k}+\mathbf{q},\downarrow}-\epsilon_{\mathbf{k},\uparrow}-h\omega},$$
$$\epsilon_{\mathbf{k},\sigma}=\epsilon_{\mathbf{k}}+\mu H_0\sigma, \epsilon_{\mathbf{k}}=\frac{h^2k^2}{2m}. \tag{4}$$

For Fermi distribution at $T=0$ we have

$$\chi'_{-+}(\mathbf{q},\omega)=\frac{12m\mu^2g^2N}{V_0}\frac{m}{h}\left(\frac{1}{k_{F\uparrow}^2}+\frac{1}{k_{F\downarrow}^2}-g(\omega_+,k_{F\uparrow}) + g(\omega_-,k_{F\downarrow})\right), \tag{5}$$

where

$$g(\omega,k_F)=\frac{m^2\omega^2-h^2q^2k_F^2}{hq^3k_F^3}\ln\left|\frac{m\omega+hqk_F}{m\omega-hqk_F}\right|, \tag{6}$$

for $H_0=0$

$$\omega_{\pm}=\omega\pm\frac{hq^2}{2m}, \tag{7}$$

$k_{F\uparrow}$ and $k_{F\downarrow}$ are the Fermi wave vector for spin-up and spin-down electrons, respectively.

It is necessary to note that the frequency dependence of $J(\omega,r)$ is very essential. Moreover, the value of the coupling, its spatial dependence, and the periods of the spatial oscillations are frequency dependent. Note that the sign of the function $J(\omega,r=R_{ij})$ corresponds to a value of a difference of initial phases $\delta\phi$ between the alternating rotational

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components S_i^+ and S_j^+ (the plus sign if $\delta\phi=0$, the minus sign if $\delta\phi=\pi$). The largest value of the coupling is in the frequency range $h\omega\sim\epsilon_F$.

Let us present now the results for two frequency ranges[5] $h\omega\leqslant\epsilon_F$ and $h\omega\ll\epsilon_F$. We obtain the results following the mathematical scheme (Ref. 6): the contour of integration in Eq. (3) is deformed to go infinitesimally above a cut along the real q axis extending from q_i to $-q_i$ in the complex q plane, where at $\pm q_i$ there are the logarithmic singularities of the function $g(\omega,k_F)$. In the dynamic case these functions generate quadratic equations in q and the roots of these equations determine the logarithmic singularity.

We derive results for $J(\omega,r)$, however, the general formula is very cumbersome for all values k_Fr. Therefore we present results for $k_Fr\gg1$.

When $(\epsilon_F-h\omega)/\epsilon_F\ll1$, we obtain

$$\begin{aligned}J(\omega,r)=\frac{A^2Nm}{4\pi V_0h^2r}\Bigg(&2\{\cos k_Fr\ \cos er+(\sqrt{2}-1)\\&\times\cos[(1+\sqrt{2})k_Fr]-(\sqrt{2}+1)\cos[(\sqrt{2}-1)k_Fr]\}\\&+\frac{1}{k_Fr}\{2\sin k_Fr\ \cos er+(2\sqrt{2}-1)\\&\times\sin[(1+\sqrt{2})k_Fr]+(2\sqrt{2}+1)\\&\times\sin[(\sqrt{2}-1)k_Fr]\}\Bigg),\end{aligned}\tag{8}$$

where $e=[(2m)/h]^{1/2}(\epsilon_F-h\omega)^{1/2}$.

Note that the static case[7] is determined by the static nonuniform paramagnetic susceptibility $\chi_{zz}(\mathbf{q})$ of the electron gas. In contrast to the static case, where there is only one period $2k_F$ for the free-electron gas, in the high-frequency $h\omega\sim\epsilon_F$ dynamical case the spatial dependence of the RKKY interaction has a complicated form and there are three main uncommensurable periods of oscillation, k_F, $(1+\sqrt{2})k_F$, $(\sqrt{2}-1)k_F$. The reason is that in the dynamic case quadratic equations are different for $g(\omega_+)$ and $g(\omega_-)$. The roots q_i of these quadratic equations in the first order in e/k_F are $k_F\pm e$, $(\sqrt{2}\pm1)k_F$. In the static case there are only two roots $q_i=\pm2k_F$ for the free-electron gas. The spatial dependence is r^{-1} and r^{-2}, which are slower than in the static case, where there is the dependence r^{-3} for the three-dimensional free-electron system. If we compare the values of the coupling for the static case and the dynamic case in this frequency range, we have the same coupling for $k_Fr\sim1$. But for $k_Fr\gg1$, in the dynamic case the coupling is of the order $(k_Fr)^2$ as much as in the static case.

Let us consider the range $h\omega\ll\epsilon_F$. In this range we have

$$\begin{aligned}J(\omega,r)\sim\frac{1}{r}\Bigg[&\frac{\cos 2k_Fr}{r^2}(\cos ur+ur\ \sin ur)\\&+\frac{4u\ \sin ur}{r}(1-2\cos 2k_Fr)\\&+8u^2\cos ur\cos 2k_Fr\Bigg],\end{aligned}\tag{9}$$

where $u=(m\omega)/(hk_F)$.

In this frequency range, besides the term that is proportional to $\cos 2k_Fr/r^3$, there are terms which have slow spatial dependence, r^{-2} and r^{-1}. These terms are determined by the frequency ω. Although for $ur\ll1$ the main term is $\cos 2k_Fr/r^3$, for $ur\sim1$ there are essential oscillations of the coupling with the period $2k_F$, which are amplitude modulated by an oscillation with the large spatial period u^{-1}. These oscillations have an amplitude that is also determined by the frequency ω.

Let us now note that for a static RKKY interaction a coupling is the same for all components (z and $\perp$), therefore a pairwise spin interaction between two spins i and j is proportional to $\cos\Theta_{ij}$. For a dynamic RKKY interaction there is a different situation. Indeed, by the action of a circularly polarized frequency-dependent magnetic field there is the rotational component of the spins and their pairwise RKKY interaction $J(\omega,r)$, which depends on the frequency of the magnetic field. However, the z component of the spins is static and their RKKY interaction is determined by the well-known function[7] $J_z(r)\sim\cos 2k_Fr/r^3$ for large k_Fr. We introduce the anisotropy of the dynamic RKKY interaction

$$A(\omega,r)=\frac{J(\omega,r)-J_z(r)}{J_z(r)}.\tag{10}$$

The anisotropy of the dynamic RKKY interaction between two spins depends on the frequency of the magnetic field and on the distance between the spins

$$A(\omega,r)\sim(k_Fr)^2\left(\frac{h\omega}{\epsilon_F}\right)^2.\tag{11}$$

The anisotropy $A(\omega,r)$ is not equal to zero if the frequency $\omega\neq0$ and only for $\omega=0$ the RKKY interaction is proportional to $\cos\Theta_{ij}$. The anisotropy increases with the growth of the frequency and it has a maximum at $h\omega\sim\epsilon_F$. For large distances $k_Fr\gg1$ the dynamic RKKY interaction is very anisotropic with $A(\epsilon_F,r)\sim(k_Fr)^2\gg1$.

The dynamic RKKY interaction might be observed in the neutron magnetic scattering experiment. Let us consider a composite system consisting of two thin ferromagnetic (FM) layers FM(1) and FM(2) separated by a metallic layer (trilayer). A linear-polarized laser beams on the frequency ω simultaneously imposed to both sides of the trilayer. By the action of a linear-polarized[8] light there are excited[9] linear-polarized dynamic components of spins $S_{ix}(\omega)$ and $S_{jx}(\omega)$ on the frequency ω of the FM(1) and FM(2). And the spin correlation function between two components of spins S_{ix} and S_{jx} for trilayers with different thicknesses of a metallic layer (under condition $k_Fr>1$) can be determined in the neutron-scattering experiment in the presence of pump lasers.

In conclusion, we have shown that the RKKY indirect interaction for dynamic cases, i.e., the value of the coupling, its spatial dependence, and the periods of spatial oscillations, depends on the frequency of the alternating magnetic field. This behavior is caused by the frequency-dependent dynamic magnetic susceptibility of an electron, which determines the indirect coupling.

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*Electronic address: gena@uranus.phys.nwu.edu

[1] See, for example, S. S. P. Parkin, N. More, and K. P. Roche, Phys. Rev. Lett. **64**, 2304 (1990); P. Bruno and C. Chappert, Phys. Rev. Lett. **67**, 1602 (1991); K. B. Hathaway, in *Ultrathin Magnetic Structures II*, edited by B. Heinrich and J. A. C. Bland (Springer-Verlag, New York, 1994).

[2] See, for example, P. W. White, *Quantum Theory of Magnetism* (Springer-Verlag, New York, 1983).

[3] We follow the scheme (Ref. 2) in considering the RKKY interaction.

[4] The components $S_i^-(\omega)$ and $S_j^-(\omega)$ have the same coupling.

[5] A range $h\omega > \epsilon_F$ requires separate consideration; our preliminary consideration shows that $J(\omega,r)$ decreases with the increase of the frequency.

[6] J. H. Van Vleck, Rev. Mod. Phys. **34**, 681 (1982).

[7] M. A. Ruderman and C. Kittel, Phys. Rev. **96**, 99 (1954); T. Kasuya, Prog. Theor. Phys. **16**, 45 (1956); K. Yosida, Phys. Rev. **106**, 893 (1957).

[8] The coupling between the components $S_{ix}(\omega)$ and $S_{jx}(\omega)$ is equal to one half of the coupling between $S_i^+(\omega)$ and $S_j^+(\omega)$.

[9] It is necessary to apply a powerful laser because the optical magnetic susceptibility is small. For example, the optical magnetic susceptibility of Fe is $(1.14\pm0.14)\times10^{-4}$ for $\lambda=6328$ Å according to G. S. Krinchik, *Physics of Magnetic Phenomenon* (Moscow State University Press, Moscow, 1976), p. 337.