Self-consistent scattering description of transport in normal-superconductor structures

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We present a scattering description of transport in several normal-superconductor structures. We show that the related requirements of self-consistency and current conservation introduce qualitative changes in the transport behavior when the current in the superconductor is not negligible. The energy thresholds for quasiparticle propagation in the superconductor are sensitive to the existence of condensate flow ($v_s \neq 0$). This dependence is responsible for a rich variety of transport regimes, including a voltage range in which only Andreev transmission is possible at the interfaces, and a state of gapless superconductivity which may survive up to high voltages if temperature is low. The two main effects of current conservation are a shift towards lower voltages of the first peak in the differential conductance and an enhancement of current caused by the greater availability of charge transmitting scattering channels. [S0163-1829(97)05002-9]

I. INTRODUCTION

During the last few years, quantum transport in superconductor structures has been the object of renewed attention. Interest in this topic has grown as an extension of the research on electron transport in normal mesoscopic systems. A considerable amount of theoretical and experimental work has been devoted to coherent transport of quasiparticles in small structures containing normal and superconductor elements,[1] with the characteristic presence of phase-coherent Andreev reflection.[2,3] Theoretical studies of superconducting transport in inhomogenous structures based on a quasiparticle scattering picture are conventionally performed within the framework of the Bogoliubov–de Gennes (BdG) equations:[4]

$$\begin{bmatrix} H_0 & \Delta \\ \Delta^* & -H_0^* \end{bmatrix} \begin{bmatrix} u_n \\ v_n \end{bmatrix} = \varepsilon_n \begin{bmatrix} u_n \\ v_n \end{bmatrix}, \tag{1}$$

where $[u_n, v_n]$ and ε_n are the wave function and energy of quasiparticle n. In principle, the BdG equations must be solved in a self-consistent manner, so that the gap function $\Delta(\mathbf{r})$ is required to satisfy the condition

$$\Delta = g \sum_n u_n v_n^* (1 - 2 f_n), \tag{2}$$

g being the electron-phonon coupling constant. The implementation of the self-consistency condition (2) has often been neglected in the literature on superconducting transport, and this omission has not always been entirely or explicitly justified. The role of self-consistency in theoretical descriptions of superconducting transport has attracted some attention recently. Most importantly, it has been shown that current conservation is generally guaranteed only within a self-consistent scheme.[5–8] This result may be viewed as a particular case of a more general theorem in quantum statistical mechanics stating that conserving approximations must satisfy the mean-field equations.[9]

In a transport context, the requirement of current conservation can be satisfied only if the condensate carries a finite amount of current,[10] for which a nonzero phase gradient is needed: $\nabla \varphi \neq 0$, with $\Delta \equiv |\Delta| e^{i\varphi}$. In the asymptotic region, the superfluid velocity $v_s \equiv \hbar \nabla \varphi / 2m$ acquires a uniform value, so that the gap behaves as

$$\Delta(\mathbf{r}) = |\Delta| e^{2iqx}, \tag{3}$$

where $q \equiv m v_s / \hbar$ is half the Cooper pair momentum and x is the longitudinal coordinate in the lead. The occurrence of a nonzero q in the case of equilibrium superconducting flow is a basic result that has been known for a long time,[4,11,12] and a related study has been recently undertaken by Bagwell.[6] In Refs. 4,6,11,12, the quasiparticles, whose dispersion relation is modified by a finite q,[4] are assumed to be in equilibrium among themselves and with respect to the lattice at rest. With this assumption, the standard Ginzburg-Landau (GL) theory is derived from the microscopic BdG equations for systems with a nonzero current $\mathbf{j} = (e\hbar/m)|\psi|^2 \nabla \varphi$, where ψ is the (small) order parameter. The inclusion of a nonzero q is essential to describe the crossover from the Josephson effect between two weakly coupled superconductors to bulk flow in a single perfect superconductor.[7] The mismatch between a moving condensate and a population of quasiparticles in equilibrium with the solid at rest is responsible for the fast suppression of superconductivity as the superfluid velocity reaches the depairing value $v_d \equiv \Delta_0 / \hbar k_F$ (Δ_0 is the zero temperature, zero current gap and k_F is the Fermi wave vector). In a more general transport context, quasiparticles may not be in equilibrium with the condensate nor among themselves. Following the work by Blonder, Tinkham, and Klapwijk (BTK),[13,14] a number of papers have dealt with scattering descriptions of transport at finite bias in hybrid normal-superconductor (NS) structures. However, work on nonlinear transport through NS interfaces has generally not included the effect of moving Cooper pairs. The purpose of this article is to analyze *the combined effect of a nonzero superfluid velocity and a nonequilibrium population of quasiparticles*. Traditional work on nonequilibrium superconductivity[15] has indeed contemplated the combination of finite superfluid velocities and electric fields. However, such studies have been usually limited to the GL regime and to semiclassical descriptions of quasiparticle transport. Here we attempt to present a microscopic study based on the self-consistent resolution of the BdG equations. Because it relies on a quasiparticle scattering picture, this approach is similar in spirit to that of recent works on mesoscopic superconductivity,[16–18] where the concepts of normal mesoscopic transport[19,20] have been extended to include the presence of superconducting elements.[1]

A scattering description of transport in a realistic NS structure with an exact implementation of self-consistency is in general a demanding numerical task. In order both to lighten the required numerical work (and thus increase the scope of our study) and to isolate the essential physical facts, we have introduced several simplifying assumptions: (i) the gap amplitude $|\Delta|$ is assumed to be stepwise uniform; (ii) the self-consistency condition (2) is implemented asymptotically throughout the superconductor; and (iii) quasiparticle multiple scattering by more than one interface is assumed to be quasielastic and incoherent.

In the present work, we present a detailed analysis of the role of current conservation in nonlinear transport through hybrid NS structures, within the framework of the model outlined above. In particular, we discuss the nature of the assumptions involved, describe the numerical method, comment on some conclusions that can be inferred from general considerations, and present specific results for the current-voltage characteristics in several types of NS structures. Some preliminary results for the NS interface and a (symmetric) NSN structure were presented in Ref. 10. The combined effect of self-consistency and nonequilibrium has also been studied by Martin and Lambert[21] assuming coherent scattering and implementing condition (2) locally for NSN structures in which one interface is perfectly transmissive. Self-consistency at a NS interface in the case of zero current was already studied by McMillan,[22] who found that, at the scale of the coherence length ($\xi_0 \equiv \hbar v_F/\pi\Delta_0$), the gap shows a depression on the S side and a finite discontinuity at the interface. More recently, Bruder[23] has performed a similar calculation for the case of anisotropic superconductors. A systematic study of the self-consistent gap profile in a NSN structure for different values of the current and the barrier strength will be presented elsewhere.[24]

We wish to answer the questions: ''What are the effects of self-consistency?'' and ''when are they important?.'' While the bulk of this paper is devoted to the first question, an answer to the second question can be presented in rather simple terms. A clear result that emerges from the work presented here, as well as from related papers,[6,10,21] is that, at zero temperature, self-consistency is important when the superfluid velocity v_s is comparable to the depairing velocity v_d. Since v_s must be obtained self-consistently after an iterative resolution of the BdG equations, a general relation that yields v_s in terms of the applied voltage seems difficult to find. However, some conclusions can be reached from the following considerations: Let us consider a generic NSN structure, similar to any of those depicted in Figs. 1(c)–1(e) [the argument could be easily generalized to structures with one superconducting terminal, like those in Figs. 1(a) and 1(b)], with the possible inclusion of some additional quasiparticle scattering within the N and S regions. When the applied voltage V is sufficiently low, Andreev reflection is the only charge-transmitting channel. Incident electrons are reflected as holes, pumping Cooper pairs into the superconductor. In this low-voltage regime, and at zero temperature, all the current in S is carried by the condensate, with a value[4]

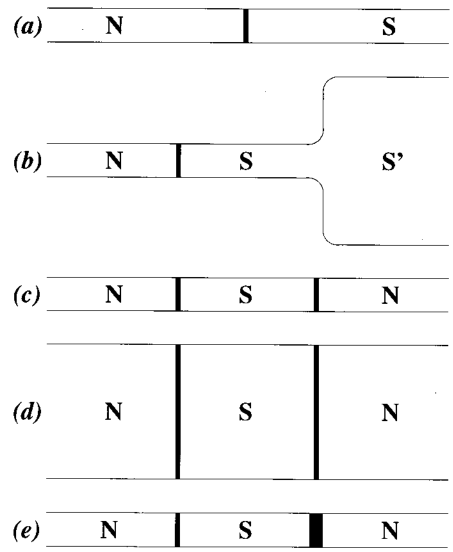

FIG. 1. Schematic representation of the structures to be studied: (a) Normal-superconductor junction (NS), (b) NSS′ structure with S' a wide superconductor for which $v_s \simeq 0$, (c) symmetric NSN structure, (d) symmetric NSN with many channels, and (e) asymmetric NSN (with a different barrier strength at each interface).

$$I_S = e v_s n A, \tag{4}$$

A being the transverse area and n the electron density. On the other hand, the current in the normal leads can be written as

$$I_N = \frac{2e^2}{h} N_{\rm ch} T_0 V, \tag{5}$$

where $N_{\rm ch}$ is the number of available channels at the Fermi level, and $T_0 \leqslant 1$ is the average transmission probability for electrons coming from the normal leads. Current conservation requires $I_S = I_N$. As the voltage increases, the condensate carries more current and self-consistency effects become important when $v_s \lesssim v_d$. A simple analysis reveals that this condition translates into

$$T_0 V \lesssim \alpha \Delta_0 / e, \tag{6}$$

where α is a number of order unity: $\alpha = 2$, $\pi/2$, and 4/3 in one, two, and three dimensions, respectively. Since the most interesting physics occurs for V comparable to Δ_0/e (at higher voltages, the superconducting features tend to become marginal), one infers from Eq. (6) that the effects of finite superfluid flow can only be important for T_0 close to unity. Thus, we may formulate the following criterion: *At voltages of order Δ_0/e, the effects of a finite condensate flow are important in NSN structures of uniform width containing transmissive NS interfaces.* The condition of equal width guarantees the possibility of achieving high currents in the superconductor at the voltages of interest. The argument above can be generalized to the case where the N and S wires have different widths. Let W_S and W_N be the widths of the superconductor and of the *narrowest* normal wire. One can show that, (i) when the superconductor is narrower than both normal wires ($W_S < W_N$), the condition (6) applies identically with T_0 defined differently but also bounded by a maximum value of one, reserved for structures with ideal contacts, and (ii), when S is wider than at least one normal wire ($W_S > W_N$), the right-hand side (rhs) of Eq. (6) must be multiplied by a factor $W_S/W_N > 1$, thereby raising the scale of voltages at which v_s becomes comparable to v_d (dilution effect) and hence making self-consistency unimportant.

It is interesting to note that the criteria formulated above exclude those structures with poor transmission from one of the normal sides as possible candidates to display the current-conservation effects discussed in this article. In particular, we cannot expect the physics of finite flow to be relevant in structures where electrons and holes experience weak localization in the N region, since that requires a small average tranmission: $T_0 \simeq \pi l/2L \ll 1$, with l the impurity mean free path and L the length of the disordered normal region.[25] This result holds even at the resistance minima characteristic of coherent Andreev reflection, because these are still comparable to the classical normal-state resistance.[26]

When studying nonlinear transport in superconducting structures within a current-conserving description, some ideas developed in conventional nonequilibrium superconductivity[15] are necessarily revisited, albeit from a new perspective based on a scattering picture. For instance, we reencounter the phenomenon of charge imbalance, whereby quasiparticles are not in equilibrium with the condensate.[27] In particular we find that lack of complete thermalization strongly reduces the efficiency with which the presence of quasiparticles tends to destroy superconductivity, and from Eq. (2) we identify a general reason for this behavior. The fact that a nonequilibrium population of quasiparticles can strengthen superconductivity has been shown for SNS structures both experimentally[28] and theoretically.[29] In Refs. 28 and 29 it is noticed that a nonzero superconducting order parameter may exist in the N segment at $T > T_c$ provided that equilibration of quasiparticles is incomplete. Here we predict a similar phenomenon by which, due to quasiparticle nonequilibrium, a superconductor (with $T < T_c$) may have $v_s > v_d$ and still display a nonzero order parameter.

The existence of a nonzero flow in the condensate gives rise to a rich variety of transport regimes. At low enough voltages, current is solely transmitted by Andreev reflection. As the voltage increases, so does v_s, and the quasiparticle dispersion relation becomes correspondingly distorted [compare the insets of Figs. 2(a) and 2(b)]. This causes a *lowering of the threshold voltage for quasiparticle transmission*, which occurs at $V = \Delta_-/e$, where $\Delta_\pm \equiv \Delta_0 \pm \hbar q v_F$. Above this voltage, Andreev transmission (AT) converting electrons into quasiholes becomes possible, while normal transmission (NT) as quasielectrons is still not available if $V < \Delta_+/e$. Some properties of this regime in which AT is the only quasiparticle transmission mechanism have been analyzed in Ref. 30, the most important being the insensitivity of transport to the presence of impurities, since these can only induce Andreev reflection and thus cannot degrade the current.

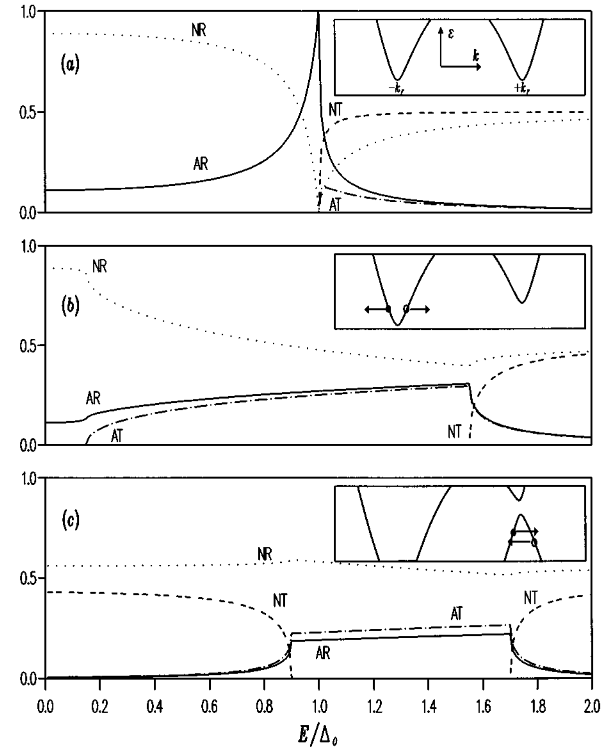

FIG. 2. Scattering probabilities for an electron incident on a NS junction from the normal lead. The superconducting condensate has a finite v_s and $Z=1$ is taken for the barrier at the interface. (a) $v_s/v_d=0$ (BTK); (b) $v_s/v_d=0.7$; and (c) $v_s/v_d=1.3$, GS regime, in which normal transmission at low energies is possible (here, $Z=1.1$ has been taken to show AR and AT probabilities separately). Insets: Schematic quasiparticle dispersion relation, $\varepsilon(k)$, for each case. Filled (empty) circles indicate electron- (hole-) like propagation.

As the voltage increases further, transport can evolve in different ways. For very transmissive structures, and at low temperatures, Δ_- may become negative and a state of gapless superconductivity (GS) is achieved [see the inset of Fig. 2(c)]. In less transmissive systems, v_s remains low and some regimes may be unstable.

This paper is arranged as follows: In Sec. II, we present the model employed in our calculations, and discuss its range of physical validity. In Sec. III, we describe the various scattering mechanisms which, with different energy thresholds, can operate in hybrid NS structures. Section IV deals with some transport properties that can be inferred from very general considerations. In Secs. V–IX, we present and discuss specific numerical results for several prototypical structures. Section V addresses self-consistent transport in the most basic structure: the NS interface. Several remarkable results (such as a negative peak in the differential conductance) are argued to be not observable in practice. However, their study paves the way for a deeper understanding of transport in more realistic structures such as NSS′ (Sec. VI), where S' is a wide superconductor with v_s always negligible, or a symmetric NSN (Sec. VII). The effect of several transverse channels (still within the regime of quasi-one-dimensional superconductivity) is studied in Sec. VIII, where the interesting conclusion is reached that the basic physics learned from one-dimensional models remains valid. Section IX is devoted to the asymmetric NSN structure, and a final summary is given in Sec. X. This paper is complemented by Appendix A, where we show that there is a well-defined asymptotic region in the S wire where the gap amplitude, as determined by the self-consistency condition, becomes independent of the longitudinal coordinate. This result underlines the physical relevance of results obtained within the approximation of asymptotic self-consistency.

II. THE MODEL

Our goal is to solve the BdG equations for several structures of interest. In Eq. (1), H_0 contains the kinetic energy plus a barrier potential of the form $V(x)=H\delta(x)$ at each NS interface. The parameter $Z=mH/\hbar^2 k_F$ is a dimensionless measure of the barrier strength.[13] As in other scattering studies of transport, we choose a basis of retarded scattering states. Each quasiparticle n is characterized by its energy E and by a discrete index α labeling the *incoming* scattering channel. Thus, quasiparticle $n\equiv(E,\alpha)$ is populated with probability $f_n=f_\alpha(E)=f_0(E-\tau\mu_\alpha)$, where $f_0(E)$ is the Fermi-Dirac distribution, μ_α is the chemical potential at the emitting reservoir, and $\tau=1(-1)$ for incoming electron (hole) channels (charge $e>0$ is assumed for electrons and a convention of $E>0$ for all n is adopted). For superconducting terminals, the noninteger value of the quasiparticle charge is not a problem because for them $\mu_\alpha\equiv 0$.

The total current can be written in terms of a sum over quasiparticle states:

$$I(x)=\frac{2e\hbar}{m}\sum_n \operatorname{Im}[f_n u_n^*(x)\nabla u_n(x) - (1-f_n)v_n^*(x)\nabla v_n(x)], \tag{7}$$

where the factor of 2 accounts for spin degeneracy. The condensate current is identified with the term which is independent of the occupation probabilities $\{f_n\}$:

$$I_c(x)=-\frac{2e\hbar}{m}\sum_n \operatorname{Im}[v_n^*(x)\nabla v_n(x)], \tag{8}$$

On the other hand, the quasiparticle component of the electric current is given by the term linear in $\{f_n\}$:

$$I_{\rm qp}(x)=\frac{2e\hbar}{m}\sum_n f_n \operatorname{Im}[u_n^*(x)\nabla u_n(x)+v_n^*(x)\nabla v_n(x)]. \tag{9}$$

In Appendix A, we show that, except for unphysical oscillations at the scale of the Fermi wavelength, the amplitude of the rhs in Eq. (2) becomes independent of x at distances from the scattering region much greater than ξ_0. Hence, $\Delta(\mathbf{r})$ behaves asymptotically as in Eq. (3). This permits to describe transport in terms of a well defined scattering problem, since the scattering channels are known exactly. They are given by the solutions of the BdG equations for a perfect superconductor with a gap of the form (3).[4,12] The quasiparticle dispersion relation is[4]

$$\varepsilon(k)=\frac{\hbar^2 kq}{m}\pm\left[\left(\frac{\hbar^2}{2m}\right)^2(k^2+q^2-k_F^2)^2+|\Delta|^2\right]^{1/2}, \tag{10}$$

and the coherence factors are

$$u(k)=\frac{|\Delta|}{[(\varepsilon_k-\xi_{k+q})^2+|\Delta|^2]^{1/2}},$$

$$v(k)=\frac{\varepsilon_k-\xi_{k+q}}{[(\varepsilon_k-\xi_{k+q})^2+|\Delta|^2]^{1/2}}, \tag{11}$$

where $\xi_k=\hbar^2k^2/2m-E_F$.

In terms of the scattering channels, Eqs. (2), (8), and (9) are rewritten as

$$|\Delta|=\frac{g}{h}\int_0^{\hbar\omega_D}dE\sum_\lambda \frac{1}{\nu_\lambda}v_\lambda u_\lambda(1-2\tilde{f}_\lambda), \tag{12}$$

$$I_c=\frac{e}{\pi m}\int_0^\infty dE\sum_\lambda \frac{1}{\nu_\lambda}(q-k_\lambda)v_\lambda^2, \tag{13}$$

$$I_{\rm qp}=\frac{e}{\pi m}\int_0^\infty dE\sum_\lambda \frac{1}{\nu_\lambda}\tilde{f}_\lambda[k_\lambda+q(u_\lambda^2-v_\lambda^2)]. \tag{14}$$

In Eqs. (12)–(14), the subindex λ runs over all channels available at energy E. These are determined by the real solutions k_λ to the equation $\varepsilon(k)=E$. ν_λ is the absolute value of the group velocity for channel λ, i.e., $\nu_\lambda=(1/\hbar)|d\varepsilon(k_\lambda)/dk_\lambda|$. u_λ is an abbreviation for $u(k_\lambda)$, and similarly for v_λ. In the gap equation, a standard cutoff at the Debye energy has been introduced to ensure a finite $|\Delta|$. The occupation probabilities $\{\tilde{f}_\lambda\}$ are taken $\tilde{f}_\alpha=f_\alpha$ for the incoming states α, and

$$\tilde{f}_\beta=\sum_\alpha f_\alpha|S_{\beta\alpha}|^2, \tag{15}$$

for the outgoing channels β, where $|S_{\beta\alpha}|^2$ is the probability for a quasiparticle to be transmitted from α to β.

The term in the rhs of Eq. (14) which is explicitly linear in q may be written in the form $(2e\hbar/m)q\,\delta Q^*$, where

$$\delta Q^*\equiv\frac{1}{\hbar}\int_0^\infty dE\sum_\lambda \frac{1}{\nu_\lambda}\tilde{f}_\lambda(u_\lambda^2-v_\lambda^2) \tag{16}$$

is referred to as *quasiparticle charge imbalance*.[13,31] In all the cases that we have considered, its contribution to the quasiparticle current is negligible, namely, $(2e\hbar/m)q\,\delta Q^*/I_{\rm qp}<10^{-4}$.

The transmission coefficients $S_{\beta\alpha}$ are obtained from the solution of the scattering problem. The effect of the δ barrier is expressed through the matching conditions

$$\psi_L(0)=\psi_R(0),$$

$$\psi_R'(0)-\psi_L'(0)=\frac{2mH}{\hbar^2}\psi(0). \tag{17}$$

At this point we introduce the simplifying approximation that the gap amplitude is stepwise uniform, i.e., $|\Delta(x)|=|\Delta|\Theta(x)$. In this way, the calculation reduces to a generalization of the work by BTK (Ref. 13) to include a nonzero value of the superfluid velocity which will be determined by the requirement of current conservation.

To compute the transport properties of structures with more than one interface, such as NSN or NSS', we introduce the assumption of *incoherent* multiple scattering of quasiparticles by the interfaces. We wish to remark that this hypothesis is equivalent to that which in normal transport leads to a picture where the contribution $(h/2e^2)R/(1-R)$ of a given obstacle to the resistance[19] (R is the reflection probability) can be treated as an additive quantity. In the case of superconducting transport, the assumption of incoherent scattering does not lead to a simple additive rule for the resistance because of the presence of the condensate, which introduces qualitative differences in the required treatment. Within a picture of incoherent scattering, where probabilities—not amplitudes—are added, the description of transport in a finite S segment does not differ much from the case where S is a semi-infinite lead. Equations (12)–(14) still apply if, for $\tilde{f}_\lambda$, we use

$$\tilde{f}_\lambda = \sum_\alpha f_\alpha T_{\lambda\alpha}, \tag{18}$$

where $T_{\lambda\alpha}$ is a characteristic concept of Bayesian statistics: it is the probability that a quasiparticle found traveling in channel λ entered the structure initially from the incoming channel α.

As an example of how to compound scattering probabilities, let us consider the NSN structure. At the left interface, we may write

$$\hat{P}(q)\begin{bmatrix}|e_L^+|^2\\|h_L^+|^2\\|e_S^-|^2\\|h_S^-|^2\end{bmatrix} \equiv \begin{bmatrix}R_{ee}R_{eh}T'_{ee}T'_{eh}\\R_{he}R_{hh}T'_{he}T'_{hh}\\T_{ee}T_{eh}R'_{ee}R'_{eh}\\T_{he}T_{hh}R'_{he}R'_{hh}\end{bmatrix}\begin{bmatrix}|e_L^+|^2\\|h_L^+|^2\\|e_S^-|^2\\|h_S^-|^2\end{bmatrix} = \begin{bmatrix}|e_L^-|^2\\|h_L^-|^2\\|e_S^+|^2\\|h_S^+|^2\end{bmatrix}, \tag{19}$$

where the matrix elements are probabilities corresponding to a NS junction with a phase gradient $2q$. e and h refer to the type of quasiparticle and $\pm$ indicates the sign of the group velocity, positive if motion is to the right. The array on the lhs (rhs) of Eq. (19) contains the square moduli of the amplitudes for the incoming (outgoing) channels. The prime indicates scattering for particles coming from the S side. At the right interface, we have, analogously,

$$\hat{P}(-q)\begin{bmatrix}|e_R^-|^2\\|h_R^-|^2\\|e_S^+|^2\\|h_S^+|^2\end{bmatrix} = \begin{bmatrix}|e_R^+|^2\\|h_R^+|^2\\|e_S^-|^2\\|h_S^-|^2\end{bmatrix}. \tag{20}$$

Combining Eqs. (19) and (20), one may derive a linear relation giving the unknown probabilities $(|e_L^-|^2,|h_L^-|^2,|e_S^\pm|^2,|h_S^\pm|^2,|e_R^+|^2,|h_R^+|^2)$ in terms of the fluxes at the incoming channels, namely, $|e_L^+|^2,|h_L^+|^2,|e_R^-|^2,|h_R^-|^2$. The quantities $|S_{\beta\alpha}|^2$ or $T_{\beta\alpha}$ are obtained by assuming only one incoming channel is populated. Then, by introducing Eqs. (19) and (20) in Eqs. (12)–(14), the current and the gap can be computed.

Self-consistency at the NS interface is implemented as follows: At a given voltage, we solve for the scattering problem with guess values for q and $|\Delta|$ and compute the current in the N lead from the trivial normal counterpart of Eq. (14). The gap and the current in the superconductor are obtained from Eqs. (12)–(14). The new value of $|\Delta|$ is used as an input for the next iteration. The new q value is adjusted by requiring current conservation, $I_N = I_S = I_{\mathrm{qp}} + I_c$, and exploiting the fact I_c has an explicit linear dependence on q. With the new q and $|\Delta|$ the scattering calculation is performed again. The iterative process continues until self-consistency is achieved. Then, for a given value of the applied voltage, we have a pair of values $(q,|\Delta|)$ yielding scattering results that, when introduced in Eq. (2), give the same value of $|\Delta|$ and satisfy current conservation. By varying the voltage, we reproduce the self-consistent I-V characteristic of the NS structure. Generalization to structures with more than one interface is made following the indications given in the previous paragraph. The best guess values at a given step in the iteration are those of the last I-V point calculated. Self-consistency is especially difficult to attain at the beginning of every new transport regime. It is easily lost when the guess values lie very far from the true ones. For this reason, small voltage incremental steps (compared with the typical energy Δ_0) must be introduced to ensure success in the next I-V point.

The assumption of incoherent elastic scattering requires the existence of some quasi-elastic dephasing mechanism that randomizes the phase with negligible energy degradation. Thus, if l_ϕ is the dephasing length and L is the length of the superconductor, we require $L \gg l_\phi$. On the other hand, $L \gg \xi_0$ is needed for $|\Delta|$ to reach its asymptotic value. We also assume that quasiparticles do not relax within the superconductor among themselves nor with the superconductor, except for the indirect coupling required by the self-consistency condition. Our calculations show this lack of equilibration is responsible for notable transport properties. Relaxation is expected to be negligible when the charge imbalance[15,27] relaxation time, τ_ε, is much longer than the average residence time τ_r. In summary, we find that $L \gtrsim \max\{\xi_0, l_\phi\}$ and $\tau_\varepsilon \gg \tau_r$ must be satisfied for our model to be meaningful. For not very reflecting interfaces, $\tau_r \simeq L/v_F$, so that simultaneous fullfilment of the two above conditions requires $\tau_\varepsilon \gg \hbar/\Delta_0$. This strong inequality is realizable at sufficient low temperatures.[32] Another implicit assumption is that the condensate has a uniform chemical potential. This is necessarily the case when phase slips are effectively forbidden. For this reason we need operation far from the critical temperature ($T \ll T_c$).[33]

III. SCATTERING MECHANISMS

Transport properties of a hybrid normal-superconductor structure are strongly dependent on the microscopic scattering of quasiparticles. At an NS interface there are four characteristic scattering processes.[13] An electron coming from the N lead may undergo normal reflection (NR), normal transmission (NT) as a quasielectron, Andreev reflection (AR) as a hole, and Andreev transmission (AT) as a quasihole. Quasiparticle transmission into the superconductor can only occur above certain energy theresholds. The AT process takes place for $E > \Delta_-$, while NT requires $E > \Delta_+$. For sufficiently large q, we have seen that Δ_- may become negative before the Δ_+ threshold is reached. In such a case, NT into a nonconventional branch may occur for energies $0<E<|\Delta_-|$ [see inset of Fig. 2(c)]. If (Q_1,Q_2) are the charges transmitted to the quasiparticle (Q_1) and condensate (Q_2) current components of the superconductor in a scattering event for an electron of charge e coming from the N lead, we have (0,0) for NR, $(e,0)$ for NT, $(0,2e)$ for AR, and $(-e,2e)$ for AT. The charge going into the condensate is adjusted to preserve current conservation, taking into account the quasiparticle charge that is reflected into the N lead. The charge $2e$ absorbed by the condensate in the two Andreev processes can only be carried away from the NS interface if the Cooper pairs move, i.e., if $q\neq 0$. Since AR and AT contribute to loading the condensate, higher values of q are required as more of these two events occur, if current is to be conserved. Because Andreev events are the only possible processes at low energies, the superfluid velocity is expected to increase strongly with voltage in the low-voltage region.

The parameter Z is a dimensionless measure of the barrier scattering strength. If the S lead is made normal $(\Delta_0=0)$, then only NT and NR occur, with probabilities

$$|t|^2=1-|r|^2=\frac{1}{1+Z^2}, \tag{21}$$

where t (r) is the transmission (reflection) amplitude. The amplitudes for the scattering processes at an NS interface can be written exactly in terms of the one-electron coefficients r and t, provided that $q=0$ and the standard approximation is introduced that, for matching purposes, the quasiparticles wave vectors are replaced by $\pm k_F$. If a,b,c,d are the amplitudes for AR, NR, NT, and AT, respectively, one obtains

$$\begin{aligned} a&=u_0v_0|t|^2/D,\\ b&=(u_0^2-v_0^2)r/D,\\ c&=u_0t/D,\\ d&=v_0r^*t/D, \end{aligned} \tag{22}$$

where $D=u_0^2-v_0^2|r|^2$, and $u_0^2=1-v_0^2=[1+(E^2-|\Delta|^2)^{1/2}/E]/2$. For $E<|\Delta|$ the square root is replaced by $i(|\Delta|^2-E^2)^{1/2}$. Equation (22) applies to asymmetric barriers provided that r, t denote amplitudes for electrons coming from the left.[34] Explicit reference to amplitudes from the right has been removed by invoking unitarity of the scattering matrix. Equation (22) shows in a transparent manner that AR involves the transmission of two electrons, with the determinant D accounting for extra multiple reflections at the interface. AT is an involved process which requires tranmission of the electron with a simultaneous internal reflection of a hole within the superconductor. When current conservation is neglected $(q=0)$, AT is not important. Being available only for $E>\Delta_0$, it is always overshadowed by the more probable NT process (note that $|d|<|c|$), which has the same energy threshold.[13] The situation changes drastically when q becomes nonzero, because AT and NT open at different energies. In particular, we shall see that *there exists a range of voltages in which AT is the only quasiparticle transmission mechanism*. Note, however, that AT is forbidden in very transmissive interfaces with $|r|\simeq 0$.

In Fig. 2, we plot the four scattering probabilities for a barrier of intermediate strength $(Z=1)$ and several values of the superfluid velocity. The insets show the schematic dispersion relation in each case. For greater accuracy, the BdG equations have been solved exactly. For completeness, we show in Fig. 2(a) the scattering curves for $v_s=0$, retrieving results implicitly given in Ref. 13. Figure 2(b) deals with a moderate superfluid velocity $(v_s=0.7v_d)$ for which the initial threshold at $E=\Delta_0$ splits into $\Delta_\pm$. In the range $\Delta_-\leqslant E\leqslant\Delta_+$, we observe that AT occurs, while NT is not possible until $E\geqslant\Delta_+$. In this energy range AT is therefore the only mechanism of quasiparticle transmission. In Fig. 2(c), we present results for the case $v_s\geqslant v_d$. Formally, a negative Δ_- signals the onset of gapless superconductivity. With proper inclusion of self-consistency, it has been shown that *equilibrium* GS cannot exist within a perfect one-dimensional wire,[6] and only in a very small range of v_s values if three dimensionality is taken into account.[11] Lack of equilibration changes the picture qualitatively, permitting GS to be a well-defined, stable transport regime.[10] We will return to these scattering mechanisms, as well as to the transport regimes they define, when discussing numerical results for transport in specific structures.

IV. GENERAL REMARKS

In this section we pay attention to some general features of the model that will be helpful in the discussion of results later in this paper. If we focus on the condition of asymptotic self-consistency [see Eq. (12)], we note that the coherence factors u_λ, v_λ, defined in Eq. (11), are both real because k_λ is real. By factoring out the x dependence (which goes like e^{2iqx}) in Eq. (3), we obtain an equation for $|\Delta|$. Clearly, the properties of $u(k)$ and $v(k)$ will affect the magnitude of the gap.

Note the double sign appearing in Eq. (10) for the dispersion relation. When the superfluid velocity is moderate $(v_s<v_d)$, only the plus sign leads to a positive energy, and this is true for any k vector that may be considered.[35] However, when the superfluid velocity exceeds v_d, both signs lead to a positive quasiparticle energy in the neighborhood of $+k_F$, while none of them can yield a positive energy in the vicinity of $-k_F$. This is the GS regime shown in the inset of Fig. 2(c). A simple analysis reveals that $v(k)$ is positive for the plus solution of Eq. (10), and negative for the minus solution. Since $u(k)$ is always positive, we conclude that $\mathrm{sgn}[u(k)v(k)]<0$ for states in the new, unconventional branch. This implies that the presence of these new quasiparticles tends to cancel the contribution of the existing conventional quasiparticles to the rhs of the gap equation (12), i.e., with their presence they tend to *reinforce* the gap.[10,36] This behavior contrasts markedly with that found for thermal quasiparticles. In such a case, the states of $E\simeq 0$ becoming available with the onset of GS are populated with probability $f_\lambda\simeq\frac{1}{2}$, which contributes to depress the gap.[12] An important physical consequence is that, in the absence of quasiparticle equilibration, *a nonzero* $|\Delta|$ *can survive up to high voltages* (we have explicitly checked $eV\gtrsim 5k_BT_c$ in our calculations). This effect manifests itself as inefficient heating in the pres-

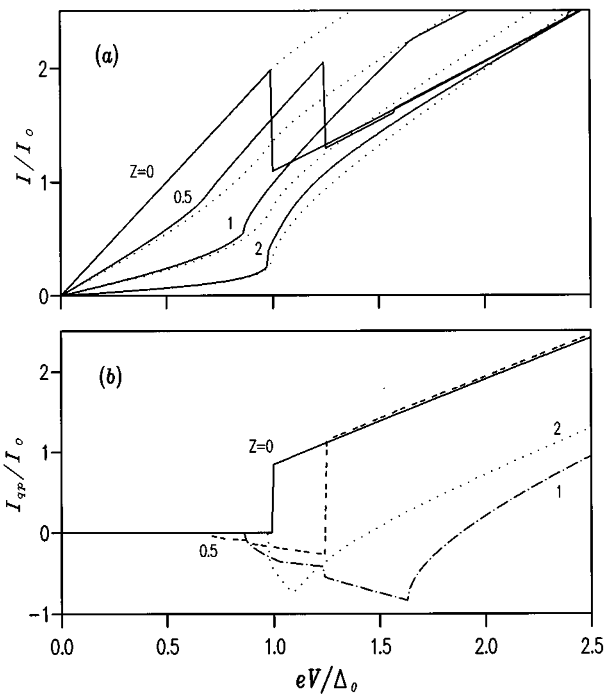

FIG. 3. I-V characteristics of a NS junction for different values of the effective barrier strength ($Z=0$, 0.5, 1, and 2). The current is given in units of $I_0=2e\Delta_0/[h(1+Z^2)]$. (a) Total electrical current; solid (dotted) lines are obtained from a self-consistent (non-self-consistent) calculation. (b) Quasiparticle component of the self-consistent current.

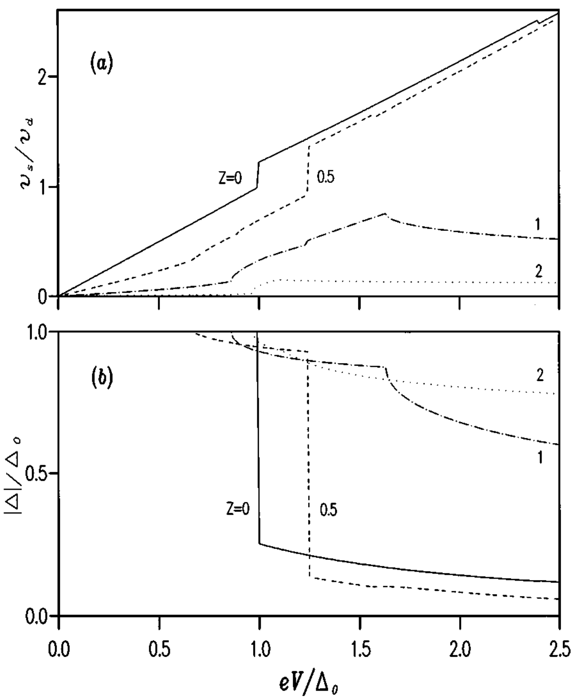

FIG. 4. Self-consistent parameters for the curves of Fig. 3: (a) superfluid velocity in units of the depairing velocity v_d; (b) magnitude of the order parameter in units of Δ_0.

ence of high-voltage transport. Lack of heating has been observed,[37,38] but it could also be due to a mere dilution effect. The GS state disappears quickly with increasing temperature. We shall see that, even at zero temperature, the directional randomization caused by the confining effect of two opaque barriers can also contribute to depress the gap. For not very long superconductors ($L\sim\xi_0$), the penetration of incoming quasiparticles below the gap can also be a cause of gap reduction.[21,24]

We end this section with some thermodynamic considerations. Although our system is not in equilibrium, we take advantage of the fact that the state of the system can be characterized by the properties of the emitting reservoirs, each of which is in internal equilibrium. The population of quasiparticle states and the properties of the condensate are determined by the chemical potentials at the reservoirs as well as by the current-conserving scattering processes within the structure. This picture of local equilibrium permits us to adopt a statistical definition for the free energy:

$$\mathcal{F}=\frac{1}{h}\int_0^{\hbar\omega_D}dE\sum_\lambda\frac{1}{\nu_\lambda}\{E_\lambda(\tilde{f}_\lambda-v_\lambda^2)+k_BT[\tilde{f}_\lambda\ln\tilde{f}_\lambda+(1-\tilde{f}_\lambda)\ln(1-\tilde{f}_\lambda)]\}. \tag{23}$$

This expression agrees with Eqs. (23) and (24) of Ref. 6 for a uniform superconductor with equilibrium flow, if the Fermi-Dirac distribution is replaced by $\tilde{f}_\lambda$ as defined in Eq. (15). In our calculations, we have checked that the free energy so defined is smaller than that which would be obtained by setting Δ to zero ($\mathcal{F}_S<\mathcal{F}_N$).

V. I-V AT NS

In Figs. 3 and 4, we present numerical results for several NS junctions of different barrier strengths. Here, as in the calculations shown in the following three sections, we have taken a bandwidth of $E_F=5$ eV, a cutoff energy of $\hbar\omega_D=0.1$ eV, and a zero temperature, zero current gap of $\Delta_0=1$ meV. These numbers yield a critical temperature $T_c=6.6$ K. Unless otherwise stated, the results shown throughout this paper correspond to zero temperature. In Figs. 3(a) and 3(b), we plot the total current and its quasiparticle component versus applied voltage. Non-self-consistent results are shown for comparison. Figure 4 shows the superfluid velocity and the gap amplitude determined self-consistently for each value of the voltage. In the fully transparent case ($Z=0$), the current I increases linearly as $(4e^2/h)V$ for $V<\Delta_0/e$. In this low-voltage regime, current is transmitted at the interface by AR and it is carried in S by the condensate. In this case, the AT channel is forbidden because $r=0$ for $Z=0$. Figure 4(a) shows that the superfluid velocity also rises linearly until it reaches the depairing value v_d, point at which the system enters the GS regime. This transition is marked by a sharp drop of the current to values close to $(2e^2/h)V$ corresponding to those of an ''ideal'' NN interface, i.e., a perfect N wire. In spite of the enhancement of v_s, the reduction in $|\Delta|$ is so strong that the condensate current becomes negligible. $|\Delta|$ decreases because, with GS, a new holelike branch appears that is however left empty because no quasiparticles come from the S side. On the other hand, the emerging electronlike branch is strongly filled (NT has a probability close to one) but that simply replaces the effect of the empty conventional holelike branch that has sunk below $E=0$. Because the vanished empty conventional electronlike branch has been replaced by an empty unconventional holelike branch, the net effect is a stronger cancellation on the rhs of Eq. (12) and a corresponding reduction of the gap. $|\Delta|$ is also diminished because, being overshadowed by NT, AR is no longer the dominant charge transmitting mechanism, and less current has to be carried by the condensate. We do not have a simple physical explanation to the small increase of v_s at the onset of GS. It is noteworthy that, in these conditions, the system still prefers energetically to have a small finite value of $|\Delta|$.

Except for the existence of AT, the situation with $Z=0.5$ is very similar to the transparent case. At $v\equiv eV/\Delta_0\simeq 0.6$, a slight increase in the differential conductance (dI/dV) is caused by the opening of the AT channel. A characteristic feature of AT is that the quasiparticle current is negative [see Fig. 3(b)], since it is transmitted by holes. The total current increases, however, because a new Cooper pair is added to the condensate in each scattering event. This translates into a faster increase (as a function of voltage) of the superfluid velocity [see Fig. 4(a)]. Figure 4(b) shows a slight decrease of the gap caused by the opening and (only) partial filling of a conventional channel. A much more drastic reduction of the gap takes place with the onset of GS at $v\simeq 1.2$, point at which the quasiparticle current begins to increase with voltage. The gap decay is even stronger than in the transparent case because the emerging electronlike branch is now less populated, since more electrons are normally reflected.

For $Z=1$ and $Z=2$, the barrier is less transmissive and the total current becomes smaller (note that in all the figures the current is plotted in units of $I_0\equiv 2e\Delta_0|t|^2/h$). Because of this, the superfluid velocity is also smaller, and the branch distortion is less important. The AT channel is therefore opened at higher voltages ($v\simeq 0.8$ for $Z=1$ and $v\simeq 0.95$ for $Z=2$). Still, there are some similarities with the case of $Z=0.5$, namely, an increase of v_s and of the condensate current, a negative quasiparticle contribution to the current, and a small $|\Delta|$ decay due to the low population of conventional quasiparticles caused by poor transmission.[10] As the voltage increases in the AT regime, GS is not eventually achieved because, being v_s low, the threshold Δ_+ for conventional NT is reached before Δ_- becomes negative. Unlike in the GS case, the onset of this new regime is not accompanied by a drop in the total current. Only a slight change in the slope of the current (visible at $Z=1$) reveals the existence of NT. A clearer signature appears in the plot of I_{qp}, where the slope changes from negative to positive (at $v\simeq 1.6$ for $Z=1$ and $v\simeq 1.1$ for $Z=2$). Conventional NT is accompanied by very moderate reductions of v_s, I_{c}, and $|\Delta|$, as well as by a rise in I_{qp}. The weak dependence of v_s and $|\Delta|$ on voltage in the high-voltage region is due to the poor transmittivity of the interfaces.

Figure 3(a) predicts a sharp decrease in the current through transmissive structures at a critical voltage value, and one wonders why it has never been observed. Typical measurements of the excess current predicted by the BTK model (dotted lines) have been performed in structures with a wide superconducting electrode (S'), with or without a narrower superconducting segment (S) connecting the N and S' leads. A wide superconducting lead S' may be described theoretically by a pair momentum that remains close to zero $(q\simeq 0)$ because of current dilution. The NS$'$ interface is well

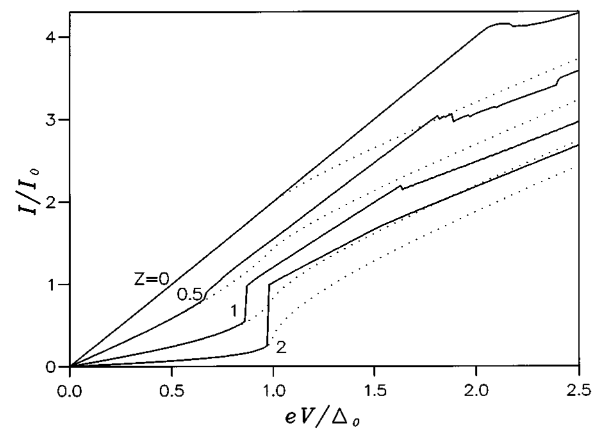

FIG. 5. I-V characteristics of a NSS$'$ structure for $Z=0$, 0.5, 1, and 2 at the NS interface (solid lines). Dotted lines correspond to the NS$'$ (non-self-consistent NS) case. Incoherent scattering has been assumed.

described by the BTK model without requiring self-consistency. In the next section we show that the NSS$'$ structure does also display an excess current that is even greater than in the NS$'$ case.

VI. *I*-*V* AT NSS$'$

In Fig. 5, we show the I-V characteristic for a NSS$'$ structure. In S, the values of q and $|\Delta|$ are adjusted self-consistently, while in S', $q=0$ and $|\Delta|=\Delta_0$ are always taken. We assume a clean contact at the SS$'$ interface $(Z_S=0)$ with a zero-voltage drop. Non-self-consistent (NS$'$) results are also shown for comparison. We obtain the desired excess current, thus recovering qualitative agreement with experiments. Let us discuss first the $Z=0$ case. As in the NS interface, GS is reached for $v=1$ and a large current flux is carried by the unconventional quasielectron branch. However, these quasielectrons *can only be Andreev reflected at the SS$'$ interface*, and, after traveling through S, the quasiholes are normally transmitted at the NS interface with probability close to one. Thus, *Andreev reflection is enhanced* by the presence of S. The result is that, being AR still very probable, the slope of I continues to be $4e^2/h$ for $v>1$. The difference in behavior with the NS$'$ (dotted lines) and NS [Fig. 3(a)] cases is noticeable. As V continues to increase, the upper limit of the low-energy branch, $-\Delta_-$, eventually reaches the value Δ_0 (this occurs at $v=2$), and then NT from S to S' becomes possible. Since the increase in current is mostly channeled through the new NT channel, dI/dV shifts to a ''normal'' value of $2e^2/h$.

For high values of Z, the regime with only AT at the NS interface is bypassed, and the system jumps directly to GS. However, we have seen in the previous paragraph that the new flow of quasielectrons can only be Andreev reflected at SS$'$. The greater likelihood of AR causes a sharp increase in the current, as shown in Fig. 5.

We find it remarkable that, as compared with the NS$'$ (BTK) case, *the presence of a narrow S segment between the N lead and the wide S$'$ reservoir enhances the current for moderate-to-large voltage values*. In particular, Fig. 5 shows that the *excess current* due to Andreev reflection can be

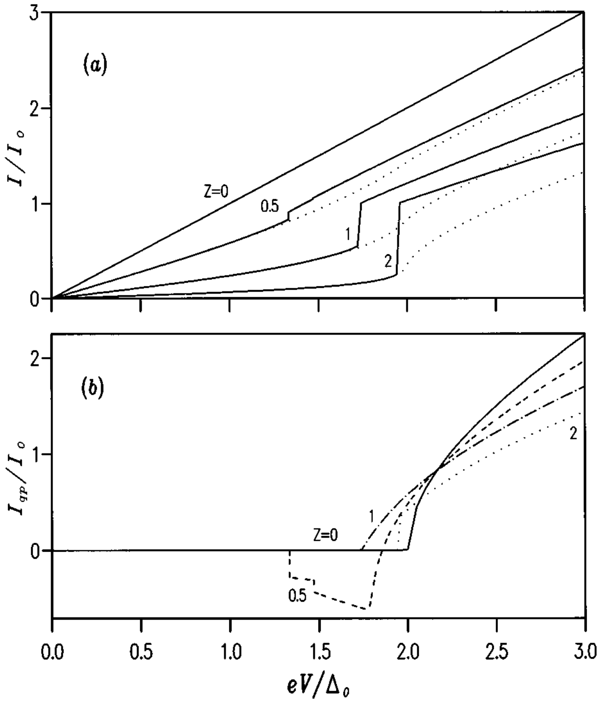

FIG. 6. Same as Fig. 3 for a symmetric NSN structure.

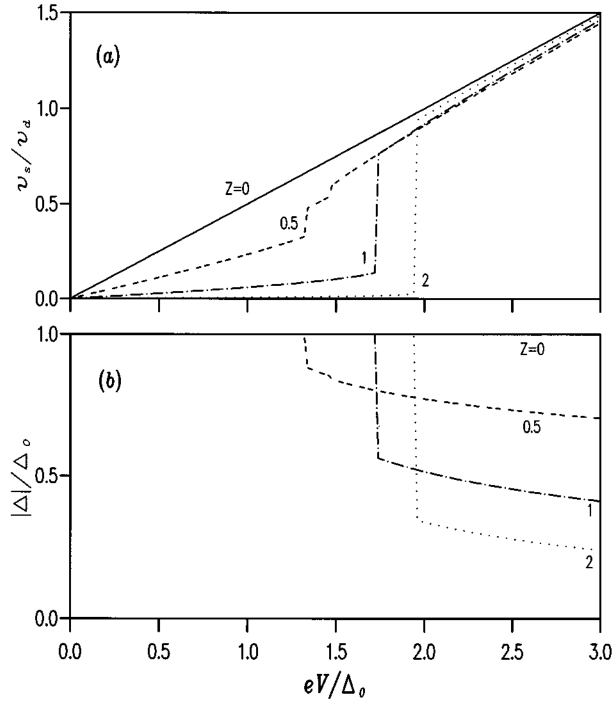

FIG. 7. Same as Fig. 4 for a symmetric NSN structure.

doubled in the case of transmissive contacts. The experimental observation of this effect would provide a proof of the existence of gapless superconductivity in the clean superconductor S.

VII. I-V FOR NSN

We have seen that the self-consistent I-V curves for a NS interface change qualitatively if the narrow S wire is made finite and a wide superconducting terminal is added on the other side. We will see in this section that the spectacular behavior found in Sec. V does not survive either in the other natural realization of the NS interface, which consists of attaching a second normal lead on the opposite side of the superconductor. For simplicity, we consider a symmetric NSN structure, with the same value of Z at the two NS interfaces. In such a case, the potential drops evenly at both interfaces and the existing electron-hole symmetry (the flux of incoming electron from the left mirrors that of holes from the right) simplifies the theoretical analysis. The more complicated case of an asymmetric NSN structure is studied in the next section.

The numerical results for a symmetric NSN are shown in Figs. 6 and 7. Non-self-consistent curves with analogous scattering assumptions are also shown. For $Z=0$, the I-V curve is identical to that of a perfect normal wire, namely, $I=(2e^2/h)V$. The reason is that a superconducting segment with fully transmissive NS interfaces cannot improve on the conductance of a perfect N wire,[39] because it cannot make the average transmission $T_0>1$ [see Eq. (5)]. Being $r=0$, AT is forbidden, and GS is achieved at $v=2$. This is clearly observed in the plot of I_{qp}, which becomes positive. Remarkably, the variation of the condensate current is such that the total current is unaffected by the onset of GS, showing the behavior of a perfect N wire at all voltages. Figure 7 shows that v_s and $|\Delta|$ are also unaffected by the presence of GS: v_s displays the same linear increase with voltage and $|\Delta|$ remains independent of it. The reason for the latter is that, as the new unconventional branch emerges [see the inset of Fig. 2(c)], both its quasielectron and quasihole states are occupied with very high probability (due to NT at the interfaces), and this contributes to reinforce the order parameter $|\Delta|$. However, this argument would merely explain an absence of strong reduction. The reason why $|\Delta|$ remains exactly constant has to do with the effective Galilean invariance of transport through a perfect wire. This is obviously a nonequilibrium effect (see Sec. IV) characteristic of structures with high transmission. Interestingly, the same prediction for the total I-V curve is obtained from a non-self-consistent calculation with $v_s=0$. For $Z=0.5$, AT opens at $v\simeq 1.3$, as signaled by the current jump in Fig. 6(a) and the negative values of Fig. 6(b). The enhancement of current is more abrupt and occurs at lower voltages than in the non-self-consistent calculation. As for the NS structure, the negative values of I_{qp} are more than compensated by the strong increase in I_c that is required to accommodate the extra Cooper pairs transferred in AT events. A small reduction of the gap (caused by the new presence of standard quasiparticles) is compensated by a jump in v_s. GS is reached at $v\simeq 1.8$, but, as in the transparent case, it is barely noticeable in the plot of the total current. In the regime between the onsets of AT and GS, possible impurities in the superconductor can only induce Andreev reflection of the quasiparticles. This results in an insensitivity (within this voltage range) of charge transport to the presence of impurities.[30]

For higher values of the barrier strength ($Z=1$ and $Z=2$), the jumps in the total current (at $v\simeq 1.7$ and 1.9, respectively) are much sharper than in the low Z cases. Comparison of Figs. 3(b) and 6(b) reveals that, as the voltage increases, the AT regime is bypassed and the system jumps directly to the GS state. A plausible reason for this behavior is the following: As the AT mechanism opens at both interfaces, the two conventional branches become strongly populated, which causes a reduction of the gap. On the other hand, frequent AT events force the condensate to carry an intense positive current, so that it can accommodate the extra flow of Cooper pairs. The two conditions (low $|\Delta|$, high I_c) can only be satisfied with a high value of v_s that in turn gives entrance to GS, which is what we observe. In other words, the AT regime is unstable for these structures, because it can only operate with values of v_s so high that the system shifts instead to GS. Figure 7 shows a jump in v_s and a decrease in $|\Delta|$ that corroborate the picture proposed above.

An interesting feature is that, at high voltages (GS state), $|\Delta|$ becomes smaller for higher Z. The reason for this is that, as Z increases, inner NR at the interfaces becomes very important and an effective directional randomization takes place in S, with a resulting equipartition of the four quasiparticle channels. However, because *only one half of the incoming scattering channels are occupied* (namely, electrons from the left and holes from the right), unitarity requires that *equipartition in S can only be achieved with* $\widetilde{f}_\lambda \simeq 1/2$ [see Eq. (18)], which necessarily leads to a strong reduction of the order parameter. In addition to this, in a realistic physical scenario, an increase of quasiparticle confinement would be accompanied by a stronger relaxation and an eventual thermalization. This effect would contribute to suppress $|\Delta|$ even further.

The dependence of the position of the first peak in the differential conductance (FPDC) with temperature and barrier strength has been studied in Ref. 10. There, it is shown that the FPDC shifts to lower voltages with increasing temperature and decreasing Z. More structured predictions about the behavior of the FPDC are made later in Sec. IX for the case of an asymmetric NSN structure.

VIII. NSN WITH MANY MODES

So far we have employed a one-dimensional model for our calculations. We wish to discuss the robustness of the main transport features against the existence of many propagating modes at the Fermi energy. Specific numerical results are presented in Fig. 8 for the case in which 20 transverse channels are available for propagation. We assume perfect interfaces so that the transverse quantum number is conserved. The different modes are not, however, totally independent, since propagation through them is sensitive to the values of q and $|\Delta|$, which depends in turn on the occupation of the whole set of modes.

We observe that the current jumps that appeared in the one-channel case are still present in a many-mode context and occur at similar voltage values. The physical reasons for the existence of these peaks are the same as in the one-dimensional model. However, the fact that the current enhancement takes place within a very narrow voltage range is not obvious if one notes that, nominally, the various thresholds should lie at different values of the voltage for each of the 20 available modes, since each has a different longitudinal k_F. Within a hard wall model for the transverse potential, the quantities $|\Delta| \pm \hbar v_F q$, which determine the thresholds in the one-channel case[10,21] must be replaced by $|\Delta| \pm f(m)\hbar v_F q$, where

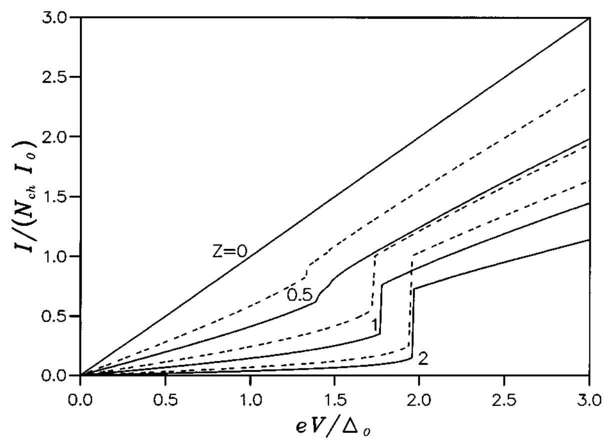

FIG. 8. I-V characteristics for a symmetric NSN structure with 20 transverse mode (solid), convenient rescaled for comparison with the single channel case (dashed).

$$f(m) = \left[1 - \left(\frac{m}{w}\right)^2\right]^{1/2}, \tag{24}$$

$w = 2W/\lambda_F$ (W is the device width), and m runs from 1 to the total number of channels. As V increases, quasiparticle transmission will occur first for the mode with the highest longitudinal energy (here, that with $m=1$), and one would expect naively that the other modes would open at correspondingly higher voltage values. Inspection of Fig. 8 shows, however, that well-defined jumps in the current take place within a very narrow voltage range, and in just one or a few steps. This points to the existence of a *cascade effect* whereby, as one mode enters the new regime (AT or GS), it induces a small sudden jump in v_s [see Fig. 6(a) for the single channel case], which helps anticipate the onset of the same regime for the following mode with smaller $f(m)$, which in turn will favor the entrance of the following mode, and so on. A plot of $I_{\rm qp}$, v_s and $|\Delta|$ (not shown) reveals that, for these quantities, the transition takes place in a slightly wider voltage range, and in a higher number of steps, although the naive expectation of 20 different steps is never observed. This points to a preference of the system for certain values of the total dI/dV from which it departs in a very small voltage range.

Another general feature is that current jumps occur at higher votages in the many-mode case. This is caused by the relative weight of channels with a smaller longitudinal k_F vector, which need higher q's to undergo the same transition. A complementary reason is that modes with smaller k_F are more strongly reflected at the interfaces and thus see a higher effective Z, which also tends to raise the voltage thresholds. For similar reasons, the presence of many modes tends to reduce the amplitude of the order parameter.

We conclude that the existence of a well-defined FPDC is preserved in a many-channel context because of its fundamental connection to the condensate flow. For this reason, the shift to lower voltages of the FPDC must be a clear signature of a nonzero superfluid velocity.

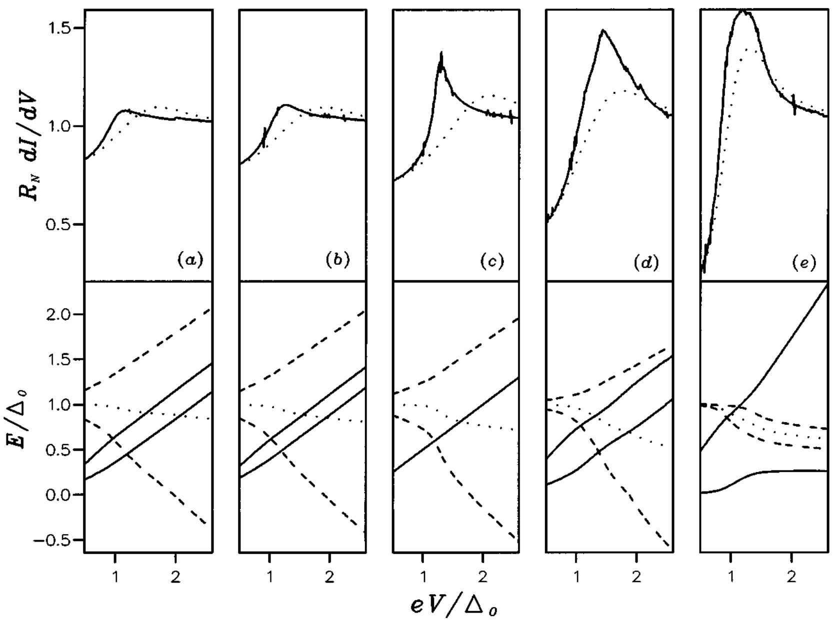

FIG. 9. Asymmetric NSN structure with fixed $Z_2=0.5$. For S, data of Pb have been taken. Temperature is $T=2$ K ($\ll T_c$). The upper part shows differential conductances for self-consistent (solid) and non-self-consistent (dotted) calculations. The normal resistance is $R_N=(h/2e^2)(1+Z_1^2+Z_2^2)$. The lower part shows (in units of Δ_0): Voltage drops at each interface (solid), magnitude of the order parameter $|\Delta|$ (dotted), and of Δ_+ and Δ_- thresholds (dashed). Values for Z_1 are: (a) 0.1, (b) 0.25, (c) 0.5, (d) 1, and (e) 2.

IX. ASYMMETRIC NSN

In Fig. 9, we present curves for an asymmetric NSN junction where one of the barriers is fixed at a low, nonzero value of Z (namely, $Z_2=0.5$), while the strength Z_1 of the other barrier varies from small to large values. The upper part of Fig. 9 represents the differential conductance for both the self-consistent and the non-self-consistent calculations, while the lower part shows the relevant energies of the problem, namely, the voltage differences at each interface, the magnitude $|\Delta|$ of the self-consistent order parameter, and the thresholds $\Delta_\pm$. The calculations have been performed for a finite temperature of $T=2$ K, with values of E_F, $\hbar\omega_D$, and Δ_0 which correspond to Pb, whose critical temperature is $T_c=7.2$ K. Because the voltage does not drop symmetrically at the two interfaces, the intermediate voltage at the superconducting segment is a third parameter (besides q and $|\Delta|$) that must be determined self-consistently. A practical method consists in requiring the current to be the same in the two N leads and adjust the voltage in S (for given q and $|\Delta|$) until this is achieved. With fixed voltage drops, the values of q and $|\Delta|$ are in turn determined as in the previous sections. We have found that, usually, I_N does not vary much in this second adjustment.

The case $Z_1\gg Z_2$ [Figs. 9(d) and 9(e)] is easy to understand. Because of its poor transmittivity, interface 1 acts as a bottle neck for current flow, and its properties, which are those of a NS tunnel junction, determine the global transport behavior. Because transmission is low, the inclusion of self-consistency does not introduce changes in the position of the first peak, although it affects its height. As in BTK, the first peak lies, characteristically, at $V\simeq\Delta_0/e$ in both cases. Figure 9(c) shows that, when the two barriers are identical, one obtains the largest differences between the two descriptions, with the peaks located in quite different positions. By symmetry, the non-self-consistent peak must be at $V\simeq 2\Delta_0/e$ (i.e., when the voltage drops by Δ_0/e at each interface). The inclusion of a finite condensate flow gives rise to the lowering of the voltage threshold and, for this reason, the FPDC tends to stay in the neighborhood of $V\simeq\Delta_0/e$. Finally, when $Z_1\ll Z_2$ [Figs. 9(a) and 9(b)], the bottleneck shifts to the second interface. However, since this is already quite transmissive ($Z_2=0.5$), we do not reproduce the transport behavior of a NS tunnel junction. Again, the presence of v_s is responsible for important differences. While the BTK-type calculation tends to send the peak back to the neighborhood of $V\simeq\Delta_0/e$ (as Z_1 decreases), the self-consistent calculation predicts a peak position below this threshold (see also the curve for $Z=0.5$ in Fig. 3). The evolution of the FPDC as a function of Z_1 is summarized in Fig. 10.

The lower curves of Fig. 9 show a correlation between the apparition of the first peak in the self-consistent dI/dV and the reaching of the Δ_- threshold by the voltage drop at the interfaces. As AT opens, there is a corresponding enhancement of the current.

The continuous evolution from small to high Z_1 could be realized experimentally by means of a scanning tunneling

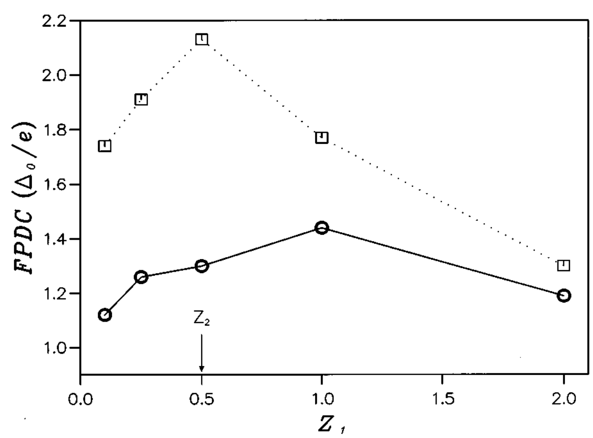

FIG. 10. Evolution of the position of the first peak in the differential conductance as a function of the barrier Z_1 for a fixed $Z_2=0.5$. Self-consistent (non-self-consistent) results are represented with circles (squares).

microscope measurement. A normal tip could be applied to a superconducting particle located on top of a normal substrate. Z_1 could be varied by changing the tip-particle distance, and the Z_2 value characterizing the island-substrate contact would be a fixed parameter. The same experiment could be performed on S islands of different sizes with different levels of current dilution. One expects transport through broad islands to be well described by a non-self-consistent calculation, while the effect of a condensate flow should be essential to understand the transport behavior of a narrow island.

X. CONCLUSIONS

We have computed the self-consistent current-voltage characteristics of several types of hybrid normal-superconductor structures. We have focused on the new physical features arising from the implementation of self-consistency and from the related condition of current conservation ($I_N = I_S$). We have shown on rather general grounds that the self-consistency effects are important in transmissive structures where current is not diluted in the superconductor.

The existence of current flow in the superconducting condensate ($v_s \neq 0$) introduces qualitative changes in the scattering of quasiparticles, affecting even the energy thresholds for propagation. The new structure of scattering channels gives rise to a rich set of transport regimes, including voltage ranges in which only Andreev transmitted quasiparticles can enter the superconductor or in which a peculiar form of gapless superconductivity exists. We have seen that, when applied to the isolated NS interface, the implementation of self-consistency predicts remarkable transport properties which, however, are not observed in practice if proper boundary conditions are assigned to the superconductor side. We have explicitly shown that, if another normal lead, or a wide superconductor (for which $v_s \simeq 0$), is attached on the opposite side of S, then the predicted I-V curves vary qualitatively. The two main features are (i) a lowering of the voltage at which the first peak in the differential conductance occurs, and (ii) a global enhancement of current in a wide range of voltages due to the increased availability (favored by $v_s \neq 0$) of charge-transmitting channels. In the case of NSN, we have shown these effects persist in the presence of many propagating channels at the Fermi energy. Predictions (i) and (ii) are quite robust and should be observable. The effect of current increase should be particularly clear in NSS′ structures, where S is narrow and S' is wide, in the form of an enhancement of the excess current. Such an effect would be a strong indication of the existence of gapless superconductivity in the narrow superconductor. For asymmetric NSN structures in which at least one of the barrier strengths can be tuned at will, we suggest experiments which would permit the identification of finite condensate flow effects.

The implementation of self-consistency in transport calculations puts a strain on the numerical resources and necessarily reduces the range of situations one can explore. By introducing some approximations (see the Introduction), we have reduced the extra numerical requirements to a minimum. The simplicity of the resulting model has allowed us to identify the essential physical features associated to current conservation. A shortcoming is that the predictions obtained, although rich in structure, are of a semiquantitative character. The development of numerical methods to compute transport properties at finite current densities with quantitative predictive power for interesting structures is an important challenge. Together with the work of Refs. 10,21, we have attempted here to give some steps in this direction. We hope that the present work will stimulate more research on the rich physics of current-conserving transport in normal-superconductor structures.

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APPENDIX A: ASYMPTOTIC SELF-CONSISTENCY

A quasiparticle of energy E approaching the structure through channel α can be scattered into any of the outgoing channels β with probability amplitude $S_{\beta\alpha}$. Far enough from the scattering region ($|x| \to \infty$), the wave function is of the form

$$\psi_\alpha(x) = \begin{bmatrix} u_\alpha(x) \\ v_\alpha(x) \end{bmatrix} = \phi_\alpha(x) + \sum_\beta \left(\frac{\nu_\alpha}{\nu_\beta}\right)^{1/2} S_{\beta\alpha}\phi_\beta(x), \tag{A1}$$

where ν_λ is the group velocity of channel λ. The scattering channels are the free propagating solutions for a perfect superconductor with Cooper pair momentum $2q$. The wave function for channel λ is[4]

$$\phi_\lambda(x) = \begin{bmatrix} u_\lambda e^{iqx} \\ v_\lambda e^{-iqx} \end{bmatrix} e^{ik_\lambda x}\eta_\lambda(x), \tag{A2}$$

where $\eta_\lambda(x)$ is an indicator function taking value 1 if point x lies in the lead of channel λ and 0 otherwise. u_λ, v_λ are the coherence factors for vector k_λ [see Eq. (11)]. For greater clarity, we refer to vectors of incoming channels α with letter k and those of outcoming channels β with p. Inserting Eqs. (A1) and (A2) into Eq. (2) with $\Delta(x) = |\Delta| e^{2iqx}$, we obtain

$$\begin{aligned} |\Delta| = \frac{g}{h}\int_0^{\hbar\omega_D} dE \sum_\alpha \frac{(1-2f_\alpha)}{\nu_\alpha} \Bigg[v_\alpha e^{-ikx}\eta_\alpha(x) \\ + \sum_\beta \left(\frac{\nu_\alpha}{\nu_\beta}\right)^{1/2} S^*_{\beta\alpha} v_\beta e^{-ipx}\eta_\beta(x)\Bigg]\Bigg[u_\alpha e^{ikx}\eta_\alpha(x) \\ + \sum_{\beta'} \left(\frac{\nu_\alpha}{\nu_{\beta'}}\right)^{1/2} S_{\beta'\alpha} u_{\beta'} e^{ip'x}\eta_{\beta'}(x)\Bigg]. \end{aligned} \tag{A3}$$

To achieve asymptotic self-consistency, we take the limit $|x|\to\infty$ and neglect two types of terms: (i) $e^{i(p'-p)x}$ with $\beta \neq \beta'$, which goes roughly as $e^{2ik_F x}$ and yields unphysical oscillations in $|\Delta|$ at the scale of the Fermi wavelength; (ii) $e^{i(k-p)x}$ appears because of the electron-hole coherence between α and β. Since k and p take values within an interval of width ξ_0^{-1}, the integrated term becomes negligible for $|x|\gg\xi_0$ (energy is integrated within and effective interval of order Δ_0). Thus, we may write

$$|\Delta|=\frac{g}{h}\int_0^{\hbar\omega_D}dE\sum_{\alpha}(1-2f_{\alpha})\left[\frac{v_{\alpha}u_{\alpha}}{\nu_{\alpha}}\eta_{\alpha}(x)+\sum_{\beta}\frac{v_{\beta}u_{\beta}}{\nu_{\beta}}|S_{\beta\alpha}|^2\eta_{\beta}(x)\right]. \tag{A4}$$

Noting that $\tilde{f}_{\beta}=\Sigma_{\alpha}f_{\alpha}|S_{\beta\alpha}|^2$ and $\tilde{f}_{\alpha}=f_{\alpha}$, as well as the unitarity condition $\Sigma_{\alpha}|S_{\beta\alpha}|^2=1$, we are able to reproduce Eq. (12) of the main text. Equations (13) and (14) can be easily obtained with the same set of assumptions.

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