From Landau levels to universal fluctuations: Level statistics for lateral superlattices

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We study the energy-level statistics for electrons in a lateral two-dimensional superlattice in dependence on the strength of the modulation potential at a given perpendicular magnetic field. The *interminiband* statistics at some selected point in the magnetic Brillouin zone shows a transition from degenerate Landau levels to universal spectral correlations while the corresponding classical dynamics displays a crossover from regular cyclotron orbits to chaotic motion. In terms of the nearest-neighbor spacing distribution, we observe the occurrence of level repulsion for *all* nonzero modulation amplitudes, which indicates a discontinuous transition to universal fluctuations for the smallest level spacings when the rotational invariance of the system is broken by a finite superlattice potential. In order to test the universality of the transition on larger scales, we propose a random matrix model with one transition parameter for comparison. Confirmed by two different types of potential shapes, we detect an explicit influence of the potential shape on the level spacing distributions, which hence cannot be described by the one-parameter model. Using the spectral rigidity, however, universal long-range fluctuations in the regime of negligible Landau-level mixing are found, in agreement with our random matrix model. [S0163-1829(97)09907-4]

I. INTRODUCTION

Ever since the advent of quantum mechanics, important efforts have been undertaken to understand the connection between classical systems and their quantized versions. Up to now, the question of how the classical phase-space structure translates into the quantum system is still under lively discussion. One important approach to identify the type of classical dynamics is to analyze the statistical properties of the quantum-mechanical energy spectrum. It is a well-known empirical finding that energetically local correlations between the energy levels of classically chaotic systems are in many cases universal, and can be described by *random matrix theory*.[1] Traditionally, one has considered finite systems, e.g., billiards with hard walls, which provide discrete energy spectra and, in certain cases, allow analytical approaches.[2] As was demonstrated previously,[3,4] the universal behavior of spectral correlations is also present in extended systems like antidot superlattices, which resemble Sinai billiards[5] extended to an infinite plane.

Lateral superlattices, fabricated on the basis of GaAs-(Al_xGa_{1-x})As heterostructures, have become intensely investigated with respect to magnetotransport properties.[6] They consist of a two-dimensional electron system (2DES) in the x-y plane, subject to a perpendicular magnetic field $\mathbf{B}=B\hat{\mathbf{z}}$ (parallel to the epitaxial growth direction) and modulated by an electrostatic lateral periodic potential $V(x,y)$. A variety of experiments have been undertaken to study the dependence of magnetotransport properties of lateral superlattices on the modulation strength.[7]

Transport phenomena are decisively related to the energy spectrum near the Fermi energy. Therefore it is worthwhile to study spectral correlations with respect to a transition from weak to strong potential modulation compared to the Fermi energy. The latter case is often referred to as an antidot lattice, because the electrons at the Fermi surface are classically excluded from regions near the modulation maxima. The local spectral fluctuations should reflect the change of the classical dynamics from regular cyclotron orbits (in the limit of zero modulation) to chaotic motion. As already demonstrated elsewhere,[3] the chaotic limit translates into *universal* spectral correlations that can be reproduced by simple Gaussian random matrices.

Unlike generic systems with regular classical dynamics, however, the homogeneous 2DES exhibits a highly uniform energy spectrum of degenerate Landau levels. Whereas in generic regular systems of at least two degrees of freedom, semiclassical quantization results in the superposition of independent, uncorrelated subspectra — leading to Poisson statistics in the limit of infinitely many such subspectra — our system exhibits nongeneric correlations between the energy-level positions analogous to a harmonic-oscillator system. To the best of our knowledge, no previous research has been devoted to level statistics in the transition from degenerate Landau levels to universal correlations. This is the aim of our present work.

Continuous spectra of extended systems impose difficulties onto the task to define reasonable correlation functions for *intra band* statistics (one way to obtain discrete spectra is the restriction to a finite system size, cf. Ref. 8). In contrast to crystalline solids, however, the comparatively large superlattice constant (of the order of several hundred nanometers) leads to *many* relevant *minibands* near the Fermi energy, which makes the evaluation of *interminiband* statistics useful. Nevertheless, we emphasize that for practical purposes we have to average over a considerable energy range. The dependence of the classical phase space on energy due to the magnetic field and the smooth potential is not negligible in this range, in contrast to billiard systems with hard walls and in zero magnetic field, for instance. Therefore, for a given set of parameters there is no fixed structure of the classical phase space as a reference in our case (in contrast to billiards). This means that the level statistics obtained from a larger energy

interval corresponds to a mixture of different classical phase-space structures, which makes a direct mapping impossible. For the sake of dealing with realistic systems, we refrain from the possibility to enforce the semiclassical limit ($\hbar \to 0$) by unnaturally large lattice constants. The influence of this averaging over a broad energy range instead of performing genuinely local statistics around a certain energy will be discussed at a later point in this paper.

We restrict ourselves to a noninteracting 2DES described by the Hamiltonian

$$H = \frac{1}{2m^\star}[\mathbf{p} + e\mathbf{A}(x,y)]^2 + V(x,y), \tag{1}$$

where $m^\star = 0.067 m_e$ is the effective electron mass of GaAs, $\mathbf{A} = \frac{1}{2}(-By, Bx)$ the vector potential in the symmetric gauge, and $V(x,y)$ the superlattice potential. Spin degrees of freedom are neglected. It is the interplay of the quantizing magnetic field and the formation of a band structure caused by the periodic potential that determines the electronic energy spectrum.

In the following we consider mainly a potential of the form

$$V(x,y) = V_0 \left[\cos\left(\frac{\pi}{a}x\right) \cos\left(\frac{\pi}{a}y\right) \right]^{2\alpha}, \tag{2}$$

with period a and a steepness given by α. With regard to experimental activities,[7] we will focus our attention on the dependence of the energy spectrum on the modulation *strength*, i.e., the potential amplitude V_0. The role of the *shape* of the applied potential will also prove to be important. Yet before we proceed with our statistical approach, we will make a few general remarks about the properties of this spectrum.

The calculation of the magnetic band structure is complicated and highly sensitive to a change of the number n_Φ of magnetic flux quanta (h/e) threading the unit cell of the superlattice.[9] For weak modulation amplitudes one may ignore coupling between different Landau levels to obtain a spectrum similar to the well-known Hofstadter butterfly,[10] which is the solution of the Harper equation[11,12] for electrons in a magnetic field within the tight-binding approximation.[13] In general, if the number of flux quanta per unit cell is p/q, there are p energy bands per Landau level. For irrational n_Φ the spectrum becomes a fractal object, a so-called Cantor set.[14] Since those aspects have already been studied previously,[3,9,12,15] we will restrict the discussion in the present paper to an integer number of flux quanta per unit cell. In this case, the magnetic translations of the superlattice[16] are isomorphic to the conventional lattice translations, and one obtains a magnetic band structure $E_n(\mathbf{\Theta})$ characterized by a miniband index n and the magnetic wave vector $\mathbf{\Theta}$, which is restricted to a magnetic Brillouin zone (MBZ).[9]

II. SHORT-RANGE FLUCTUATIONS

In order to observe the universal fluctuations in the energy spectrum it is necessary to decompose the Hamiltonian into irreducible blocks,[1] which means that only one good quantum number varies in such a block. For generic Hamilton matrices one expects to observe *level repulsion* within each block, i.e., the distribution of nearest-neighbor level spacings S is

$$P(S) \propto S^\beta \quad \text{when} \quad S \to 0. \tag{3}$$

The exponent β in this expression depends on the symmetry of the block under consideration: $\beta = 1$ for time-reversal symmetric blocks (and those with some generalized time-reversal symmetry which involves additional geometric operations), $\beta = 2$ for blocks with broken time-reversal symmetry, and $\beta = 4$ in some systems with Kramers degeneracy (not relevant in our spinless system). For greater detail concerning this classification, we refer again to Ref. 1.

Since the submatrices $H(\mathbf{\Theta})$ for a fixed magnetic wave vector are already irreducible if $\mathbf{\Theta}$ is an unsymmetric point of the MBZ, we may detect the mentioned characteristics of $P(S)$ for the *interminiband* sequence $E_n(\mathbf{\Theta})$ with fixed $\mathbf{\Theta}$. We have to bear in mind, though, that for $V_0 = 0$ the system is still rotationally invariant. This leaves us with degenerate Landau levels, which is certainly no generic limit. How the spectral correlations change when this symmetry is broken by the modulation is by no means obvious.

In order to obtain significant statistics, we average the local spectral correlations over an energy range containing 1500–2000 energy levels. The classical dynamics in this energy range displays a crossover from regular orbits extended over several lattice cells to chaotic motion when the modulation strength is increased. This transition should be reflected in the observed level statistics.

Figure 1 shows the evolution of $P(S)$ for different modulation amplitudes at some unsymmetric $\mathbf{\Theta}$ point where time-reversal symmetry is already broken due to the presence of the magnetic field. For the unmodulated system, $V_0 = 0$, the density of states reads

$$D(E) = \frac{1}{2\pi\ell^2} \sum_{m=0}^{\infty} \delta[E - (m + \tfrac{1}{2})\hbar\omega_c], \tag{4}$$

where $\hbar\omega_c = \hbar eB/m^\star$ is the cyclotron energy, and $l = \sqrt{\hbar c/(eB)}$ is the magnetic length. According to the Landau counting of states there are $n_\Phi \equiv a^2/(2\pi\ell^2)$ energy levels per $\mathbf{\Theta}$ point in the MBZ (excluding spin degeneracy). The unfolded sequence of levels (with mean level spacing equal to unity) gives the spacing distribution

$$P(S) = \frac{1}{n_\Phi}[(n_\Phi - 1)\delta(S) + \delta(S - n_\Phi)]. \tag{5}$$

The unfolding procedure maps the cyclotron energy $\hbar\omega_c$, separating adjacent Landau levels, to n_Φ, which is equal to the Landau-level degeneracy. Note that this yields $\int_0^\infty SP(S)dS = 1$.

For weak modulation the δ peaks broaden and move closer according to the splitting of the Landau levels. Finally — in the regime of strong potential modulation — a single-peak structure emerges which turns out to match the universal distribution curve for the Gaussian unitary ensemble (GUE, $\beta = 2$) as predicted by random matrix theory for Hamilton matrices with broken time-reversal symmetry. This is typical for classically chaotic systems.

The result of our analysis is that level repulsion can be detected even for the smallest modulation amplitudes. It is

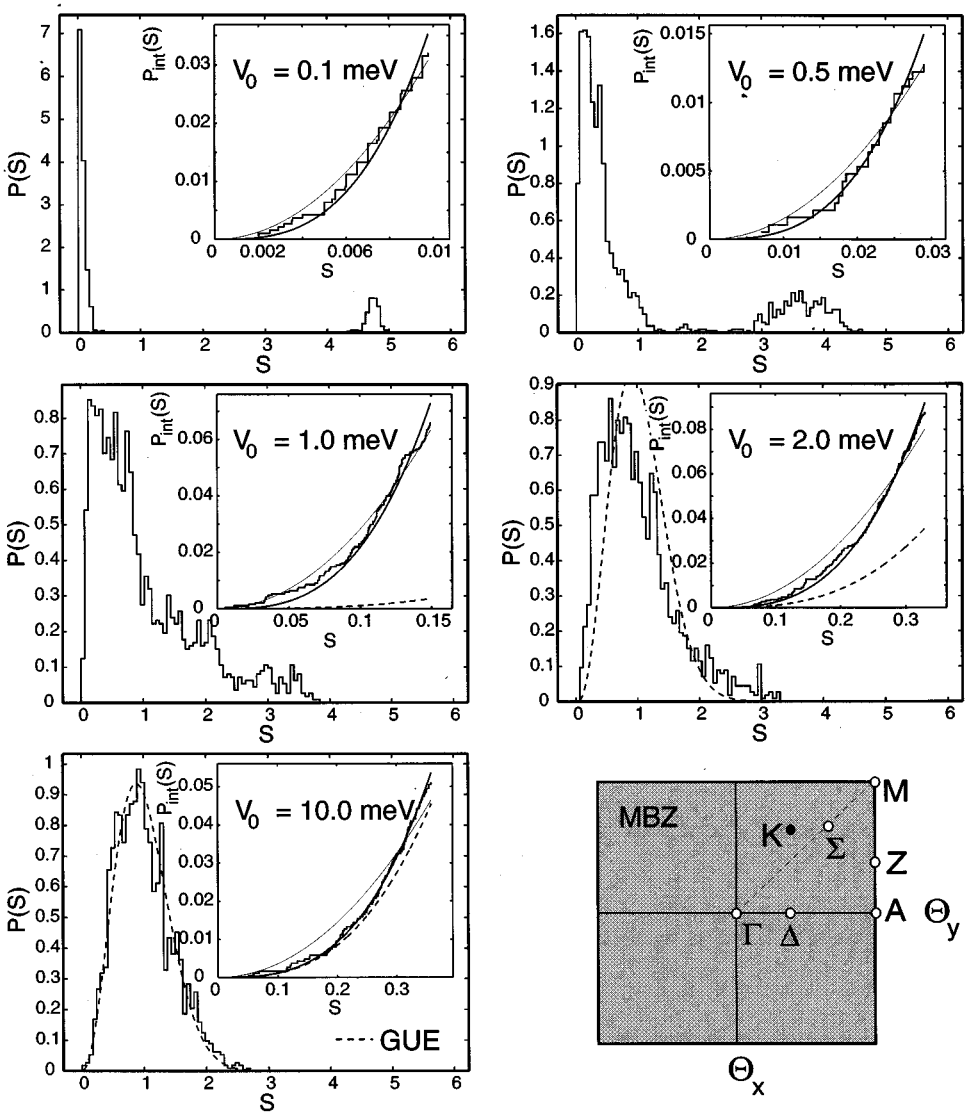

FIG. 1. Nearest-neighbor spacing distribution $P(S)$ of the unfolded sequence of roughly 1800 energy levels $E_n(\mathbf{\Theta})$ (from 1 to 38 meV) at the unsymmetric point $\mathbf{\Theta}=\pi(0.4,0.7)$ in the MBZ (marked K in the MBZ sketch at the bottom right). For different modulation amplitudes V_0 the remaining parameters have been chosen as $n_\Phi=5$, $a=600$ nm ($\hbar\omega_c=0.1$ meV), and $\alpha=4$ [cf. Eq. (2)]. The insets show the integrated spacing distribution $P_{\rm int}(S)$ in the region of level repulsion near $S=0$. The smooth curves in the insets are least-mean-square-deviation fits for linear (thin) and quadratic (thick) level repulsion as discussed in the text. The dashed curves correspond to GUE.

demonstrated by the insets of Fig. 1: Here we compare the integrated spacing distribution function

$$P_{\rm int}(S)=\int_0^S P(t)dt \tag{6}$$

with least-mean-square-deviation fits of the form C_1S^2 and C_2S^3 over a suitable range from 0 to $S_{\max}$. The latter is limited by the position of the maximum of $P(S)$, which is at $S_0\approx0.86$ for the GUE but goes to zero when V_0 vanishes. Since ever fewer spacings S are located in this relevant interval, our numerics is not significant enough to distinguish between linear and quadratic level repulsion for the smallest V_0 (cf. Fig. 1). Yet we are able to rule out *level clustering*, i.e., $P_{\rm int}(S)=C_0S+O(S^2)$ with $C_0>0$, which is typical of systems with regular classical dynamics. This is in contrast to results by Berry and Robnik,[17] who found persistent level clustering for a mixed classical phase space with coexisting regular and chaotic classical orbits. The Berry-Robnik-formulas hold rigorously only in the semiclassical limit; non-semiclassical deviations have been found in several works including ones by the authors themselves.[18]

Instantaneous level repulsion is consistent with theoretical investigations by Pandey,[19] who argued that the transition of spectral correlations in random matrix ensembles due to the loss of good quantum numbers, whose corresponding subspectra are independently superimposed, is discontinuous on small scales when the matrix dimension becomes infinitely large. Then the small-scale fluctuations should be those of the symmetry-breaking ensemble. In our case this transition is accomplished by breaking the rotational invariance with respect to the z axis, which results in the splitting of degenerate Landau levels. As already mentioned, the scale of level repulsion differs drastically for small and large modulation amplitudes, and shrinks to zero for $V_0\to0$. Ignoring inter-Landau-level coupling, one may argue that the splitting of

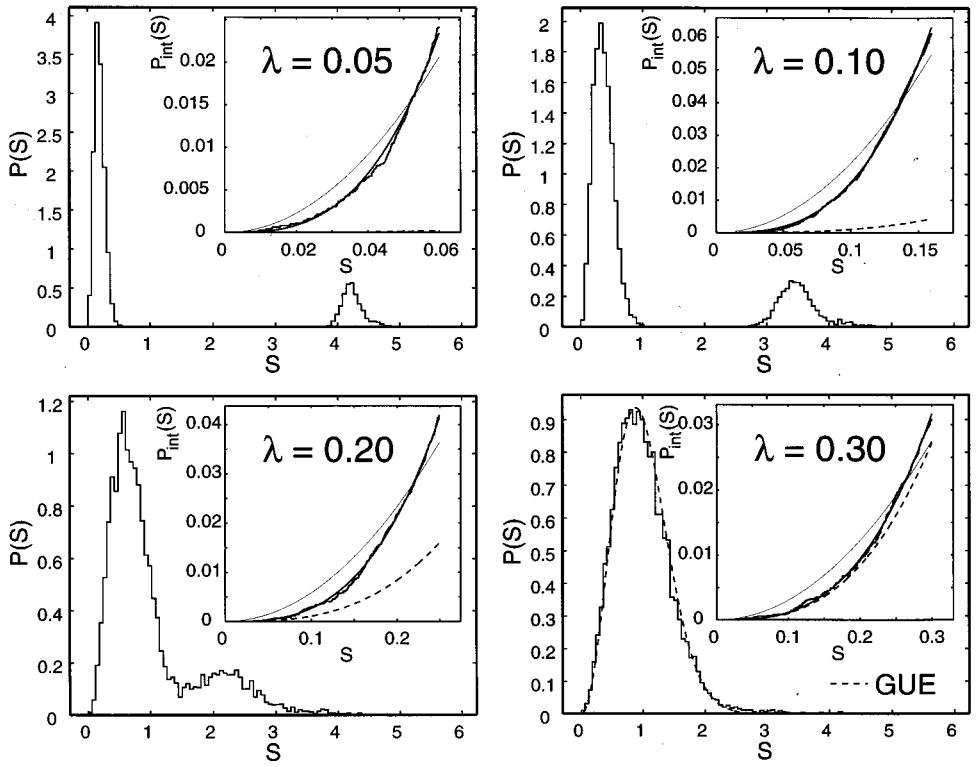

FIG. 2. Nearest-neighbor spacing statistics for the proposed random matrix model with $n_\Phi = 5$ and $N = 50$ averaged over 100 matrices of type (7). $P_{\text{int}}(S)$ (inset) clearly exhibits quadratic level repulsion for all $\lambda > 0$; like in Fig. 1, fits for linear (thin curve) and quadratic level repulsion (thick curve), as well as the GUE spacing distribution (dashed curve) are added.

the Landau levels is proportional to V_0. As long as this assumption holds one expects $S_{\max} \propto V_0$. This is approximately confirmed by our results in Fig. 1, where the appropriate scales $S_{\max}$ can be seen in the insets. Nevertheless the absolute scale of repulsion is dependent on the lattice parameters and the magnetic field (or n_Φ, respectively).

At this point we want to stress that n_Φ is a fundamental parameter which cannot be eliminated by any transformation with the aim to unite the quantities a, V_0, and $B = n_\Phi h/(ea^2)$. Classical dynamics does allow such a transformation.[20] At fixed energy $\tilde{E} = E/V_0$ it yields for a given *potential shape* a dependence on only one parameter, namely, $\gamma = eaB/(4\pi\sqrt{m^\star V_0})$. Quantum mechanics, however, introduces an explicit dependence on n_Φ. Therefore the Hamilton operator $\tilde{H}$ in units of V_0 is determined by $\gamma = n_\Phi h/(4\pi a\sqrt{m^\star V_0})$ *and* n_Φ independently. In the discussion about the crossover from Landau levels to quantum chaos, we thus have to specify the number of flux quanta per unit cell n_Φ, which is evident in the resulting n_Φ-specific nearest-neighbor statistics, cf. Eq. (5). Since any change in a can be absorbed in an appropriate modulation amplitude V_0 by keeping γ constant, a will remain unaltered in the studies presented in this paper.

Finally we remark that if the energy spectrum of some symmetric point of the MBZ is analyzed, one is able to see linear level repulsion (connected with some generalized time-reversal symmetry). For large V_0 the statistics of the Gaussian orthogonal ensemble (GOE) evolves. However, this needs a decomposition of the spectrum into independent subsets. Otherwise, as blocks for certain symmetric wave vectors $\boldsymbol{\Theta}$ are still reducible, the universal level repulsion may be masked by the superposition of two or more independent spectra.[3]

III. RANDOM MATRIX MODEL

For a given number of flux quanta per unit cell both limits of vanishing and very strong modulation strength are universal, that is, independent of a, α, and any other parameters specifying the potential shape. With this in mind, one may wonder whether the transition as a whole is — in any sense — universal.

Many attempts have been presented to describe spectral correlation functions for broken symmetries theoretically by means of random matrices. Analytical expressions for $P(S)$ have been developed in the case of transitions from Poisson spectra (entirely uncorrelated energy eigenvalues) to all Gaussian ensemble spectra[21] as well as transitions between those ensembles. But none of these cases is appropriate for modeling the transition from Landau levels to Gaussian ensembles. A general approach for a weakly broken symmetry by Leyvraz and Seligman,[22] modeled as $H = H_0 + \lambda H_\kappa$, where H_0 belongs to some known ensemble, and λH_κ is a *small* perturbation, proves on a second-order perturbation level in λ that for $S \to 0$ the nearest-neighbor statistics exhibits the level repulsion of the perturbing ensemble H_κ. The latter is one of the Gaussian ensembles GOE, GUE, or GSE, the Gaussian symplectic ensemble. With H_0 being a Landau-level matrix, their method is not easily applicable because of the pathological level density (4). Furthermore, we are also interested in the behavior on larger scales (not only $S \to 0$) and for stronger symmetry breaking ($\lambda \to \infty$).

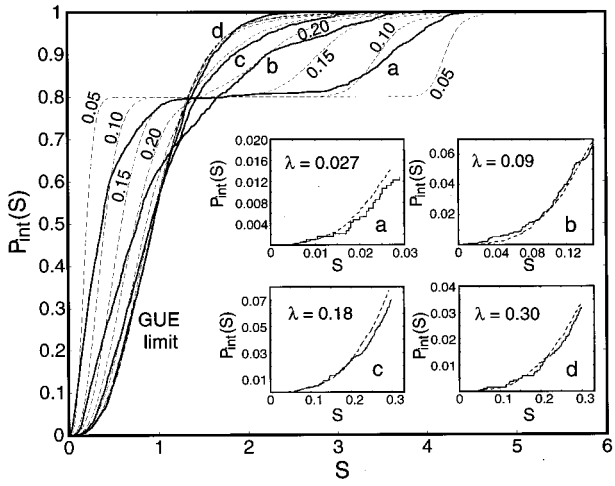

FIG. 3. Integrated spacing distribution $P_{\rm int}(S)$ with a close-up near $S=0$ (insets) for different modulation amplitudes. The solid curves a–d correspond to $V_0=0.5$, 1.0, 2.0, and 10.0 meV, respectively (with the remaining parameters as in Fig. 1). The dashed curves are determined by the random matrix model for different λ [in the main graph, $P_{\lambda\,\rm int}(S)$ has been drawn for values of λ from 0.05 to 0.5 in steps of 0.05].

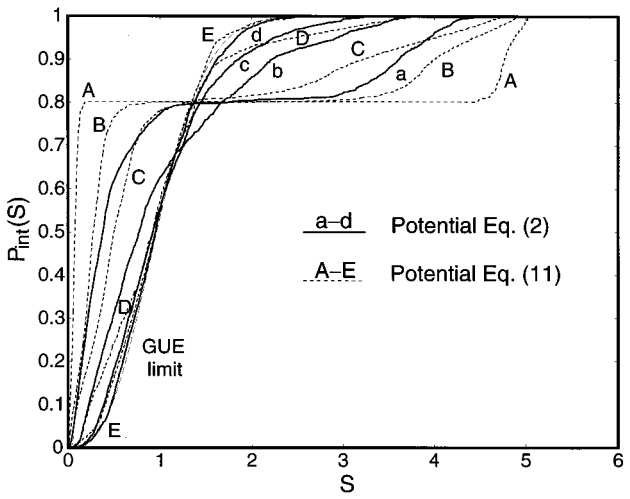

FIG. 4. Integrated spacing distribution $P_{\rm int}(S)$ of two different modulation potential shapes as indicated by line styles. The solid curves a–d correspond to $V_0=0.5$, 1.0, 2.0, and 10.0 meV, respectively, A–E (dashed) correspond to $V_0=0.01$, 0.05, 0.1, 0.2, and 0.5 meV. The remaining parameters are as in Fig. 1. The lowest 1800 energy levels of each spectrum were used for the analysis.

In order to address the question of universality via a numerical approach we therefore propose the following *random matrix model* and concentrate on the case of broken time-reversal invariance: Let $H_{\rm LL}$ be a $N\times N$ diagonal matrix with n_Φ-fold degenerate Landau levels (LL) with a (dimensionless) distance $E_{\rm c}$ and $H_{\rm GUE}$ a member of the Gaussian unitary matrix ensemble of dimension $N\times N$. The matrix H_λ consists of $H_{\rm LL}$ and an admixture of $H_{\rm GUE}$,

$$H_\lambda=\frac{1}{\sqrt{1+\lambda^2}}(H_{\rm LL}+\lambda H_{\rm GUE}). \tag{7}$$

The prefactor ensures that the energy range remains finite throughout the transition,[23] and we have $H_0\equiv H_{\rm LL}$, $H_\infty\equiv H_{\rm GUE}$. Since there are $M=N/n_\Phi$ degenerate blocks in $H_{\rm LL}$, the level density becomes (taking M to be even and shifting the energy by $-E_{\rm c}M/2$)

$$\rho_{\rm LL}(E)=\sum_{m=-M/2}^{M/2-1} n_\Phi\delta[E-(m+\tfrac{1}{2})E_{\rm c}]. \tag{8}$$

For the GUE with Gaussian distributed real and complex parts $\mathrm{Re/Im}H_{\rm GUE}^{(ij)}$ of the matrix elements as defined in Ref. 1 with variance $\mathrm{Var}\{\mathrm{Re/Im}H_{\rm GUE}^{(ij)}\}=1+\delta_{ij}$ (note that $\mathrm{Im}H_{\rm GUE}^{(ii)}=0$) Wigner's semicircle law[24] holds for the ensemble averaged spectral density

$$\rho_{\rm GUE}(E)=\frac{1}{2\pi}\sqrt{4N-E^2}\quad\text{for }|\mathrm{E}|\leqslant 2\sqrt{N}. \tag{9}$$

If we also want to place N levels within the interval $[-2\sqrt{N},2\sqrt{N}]$ in the LL limit, we must choose $E_{\rm c}=4n_\Phi/\sqrt{N}$ so that the mean density in this range becomes $\sqrt{N}/4$.

Our numerical studies of this model yield that the level spacing distributions (in units of the mean level spacing) for a given λ converge for $N\gg n_\Phi$. In the case $n_\Phi=5$, for example, $N\geqslant 50$ is sufficient to achieve practical independence of N.

Figure 2 shows nearest-neighbor spacing distributions $P_\lambda(S)$ for some values of λ. Clearly one can make out the crossover from LL statistics to GUE statistics. In the integrated distribution $P_{\rm int}(S)$ level repulsion again shows up — here the fit for quadratic level repulsion is significant in contrast to the one for linear level repulsion. Concerning level spacings in this range far below the cyclotron energy, it turns out that the prefactor in $P_{\rm int}(S)=C_2S^3$ fitting the distribution $P_{\lambda\,\rm int}(S)$ near $S=0$ when $\lambda\ll 1$ is proportional to λ^{-2}; in other words, we find a scaling law

$$P_\lambda(S)\propto\left(\frac{S}{\lambda}\right)^2+O(S^3)\quad\text{for }\lambda\ll 1; \tag{10}$$

a similar law for V_0 when C_2 is fitted to the physical system could not be established.

Comparison of the intermediate distributions $P_\lambda(S)$ with the ones of the physical system (Fig. 3) reveals that the transition does not occur in the same way. This suggests that there might be an explicit dependence of the distribution curves on the *shape* of the modulation potential. In order to confirm this conjecture, we compare the integrated spacing distributions of the potential (2) with some of the additive potential

$$V(x,y)=V_0\left[\cos\left(\frac{2\pi}{a}x\right)+\cos\left(\frac{2\pi}{a}y\right)\right]. \tag{11}$$

As a result (cf. Fig. 4), we find a pronounced difference between the transitions of these systems: The distribution curves intersect each other several times and do not seem to follow any universal pattern valid for both transitions equally. Note that no assumption whatsoever has been made about the relation between the modulation amplitudes of the systems.

We believe the reason for this behavior to be the highly energy-dependent phase space, which is determined by the potential modulation shape and amplitude. As we have already pointed out, the classical phase space can be uniquely characterized by two independent parameters: the energy in units of the modulation potential amplitude, and $\gamma = eaB/(4\pi\sqrt{m^\star V_0})$; this requires, however, a fixed potential shape.[20] Changing the shape of the potential also changes the classical phase-space structure. Therefore an energetically local statistical analysis performed for a semiclassical spectrum with a high density of energy levels is expected to yield an equally nonuniversal crossover of the spacing distributions as for our distribution curves obtained from averaging over a finite-energy range. A practically local analysis in the semiclassical regime lies beyond our numerical capacities, though, since an enlargement of the lattice constant requires also a high-energy regime, hence enormous Hamilton matrices, in order to stay in the domain of (eventually chaotic) classical motion extended over several lattice cells.

To summarize, in spite of the fact that we observe a *universal level repulsion exponent* (here $\beta = 2$) the transition on scales up to the order of the cyclotron energy (or n_Φ, respectively) is not independent of the chosen system and thus cannot be generally matched by a random matrix model with only one transition parameter.

It should be mentioned that even the level statistics on the smallest scales is influenced by the explicit shape of the modulation potential. Consider again the additive potential (11), which yields an integrable Hamilton function for infinitely small V_0. This is connected with a Poisson-like level spacing distribution, i.e., $P_{\text{int}}(S) \propto S$ for $S \to 0$. Features of level clustering are indeed found for sufficiently small modulation amplitudes (of the order of one tenth of the cyclotron energy with parameters like in Fig. 1). Here level repulsion cannot be resolved in our analysis, since the relevant range near $S = 0$ is too small and hence does not contain enough level spacings in order to yield a useful statistics.

IV. LONG-RANGE FLUCTUATIONS

While the nearest-neighbor spacing distribution measures short-range correlations of the energy levels, the spectral rigidity $\Delta_3(L)$ (first introduced by Dyson and Mehta[25]) characterizes fluctuations over a longer spectral range. It is defined as the mean-square deviation of the integrated level density or ''spectral staircase'' $N(e)$ (e denoting the unfolded sequence of levels in units of the mean level spacing) from its best linear approximation $Ae + B$ over a spectral distance L,

$$\Delta_3(L) = \left\langle \frac{1}{L} \int_{e_0}^{e_0+L} [N(e) - Ae - B]^2 de \right\rangle_{e_0}, \tag{12}$$

where $\langle \rangle_{e_0}$ symbolizes averaging over the whole spectrum. In Fig. 5 our numerical results for different modulation amplitudes are displayed. For comparison, curves for the Gaussian unitary ensemble spectra with the asymptotics[1]

$$\Delta_{3\,\text{GUE}}(L) = \frac{1}{2\pi^2}[\ln(2\pi L) - 0.672\,784\ldots] + O(L^{-1}), \tag{13}$$

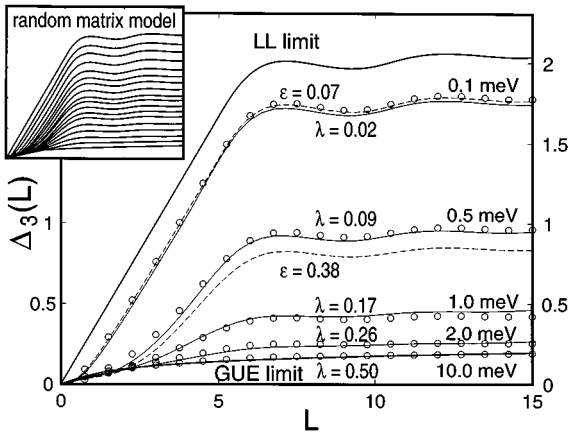

FIG. 5. Spectral rigidity $\Delta_3(L)$ for different modulation amplitudes (the remaining parameters have been chosen as in Fig. 1): Curves for the physical system (circles) are compared with theoretical curves for fivefold-degenerate Landau levels $\Delta_{3\,\text{LL}}(L)$, equidistantly split levels $\Delta_{3\,\varepsilon}(L)$ (dashed lines with the values of ε), and the GUE (labeled GUE). The inset at the left displays schematically the spectral rigidity $\Delta_{3\,\lambda}(L)$ for the random matrix model with different values λ from 0 (LL limit) to 0.5 (approaching the GUE limit). Suitable curves (solid lines with values of λ) have been drawn in the main graph for comparison.

and theoretical curves for n_Φ-fold degenerate Landau levels are also shown.

The theoretical curves were obtained by using the relation between $\Delta_3(L)$ and the number variance $\Sigma^2(L)$, which describes the fluctuations of the number of levels e within a spectral range of length L:[1]

$$\Delta_3(L) = \frac{2}{L^4} \int_0^L (L^3 - 2L^2 r + r^3) \Sigma^2(r)\, dr. \tag{14}$$

It is easily shown that, for n_Φ degenerate Landau levels,

$$\Sigma^2_{\text{LL}}(L) = (n_\Phi - L)L \quad \text{for } 0 \leqslant L < n_\Phi, \tag{15}$$

and $\Sigma^2_{\text{LL}}(L + n_\Phi) = \Sigma^2_{\text{LL}}(L)$. This periodicity eventually leads to a convergence of $\Delta_{3\,\text{LL}}(L)$ for $L \to \infty$ since the kernel in Eq. (14) becomes smoother and the integral tends to average over more and more periods of $\Sigma^2_{\text{LL}}(L)$. Replacing $\Sigma^2_{\text{LL}}(L)$ by its average over one period, namely, $n_\Phi^2/6$, yields

$$\lim_{L \to \infty} \Delta_{3\,\text{LL}}(L) = \frac{n_\Phi^2}{12}. \tag{16}$$

Furthermore, $\Delta_{3\,\text{LL}}(L) = n_\Phi L/15$ if $L \leqslant n_\Phi$. The saturation of the spectral rigidity on large scales reflects the fact that there are no more fluctuations on scales beyond the cyclotron energy.

For weak potential modulation $\Delta_3(L)$ is well described by a simple model of equidistantly split levels (thus incorporating the observation of level repulsion) whose separation ε is still much smaller than 1 [so that $(n_\Phi - 1)\varepsilon \ll n_\Phi$, the Landau-level distance]. In this situation one can as well derive an expression for $\Sigma^2_\varepsilon(L)$ and obtain $\Delta_{3\,\varepsilon}(L)$ by integration. Taking the parameter ε to be the mean intra-Landau-level spacing from the numerical data of the physical system (in units of the overall mean level spacing), this model is

able to describe the spectral rigidity for surprisingly large ε (cf. Fig. 5). The reason for this model being appropriate is that $\Delta_3(L)$ measures long-range fluctuations rather than short-ranged details of the level distribution. Therefore more sophisticated models are not required as long as the intra-Landau-level separation does not affect scales comparable to the cyclotron energy.

The results for the random matrix model are also included in Fig. 5. The inset shows the development for selected values of λ. Comparison of suitable curves $\Delta_{3\lambda}(L)$ with those of our physical system demonstrates a very good agreement for modulation amplitudes up to 0.5 meV. Deviations for higher amplitudes will be discussed toward the end of this section.

If we deal with a small perturbation of the Landau levels, we may assume for the mean intra-Landau-level spacing $\varepsilon \propto V_0$. This is well reflected in Fig. 5, which includes the numerically determined values of ε for the two smallest modulation amplitudes. For the random matrix model, we may argue that in the case of the inter-Landau-level coupling being negligible, we basically superpose a $n_\Phi \times n_\Phi$ matrix $H_{\mathrm{GUE}}^{(n_\Phi)}$ over each Landau level where the energy corrections are distributed according to Wigner's semicircle law (9) with $N \to n_\Phi$. Note that in this regime the unfolding procedure is still barely affected by the Landau-level splitting, so that the eigenvalues of $\lambda H_{\mathrm{GUE}}^{(n_\Phi)}$ are rescaled by a constant factor given by the mean level spacing of the unperturbed system $S_0 = E_c/n_\Phi$. This leads to a mean intra-Landau-level spacing $\varepsilon \propto \lambda$. For the parameters we have chosen in this paper ($n_\Phi = 5$, $N = 50$, $S_0 = 0.57$) the mean spacing between the eigenvalues of $H_{\mathrm{GUE}}^{(5)}$ is $S = 2.34 \pm 0.01$, which gives $\varepsilon = \lambda S/S_0 = 4.11\lambda$, thus $V_0 \propto \lambda$. This is in excellent agreement with the values in Fig. 5. It shows that the assumption of no Landau-level mixing is still reasonable for the physical system when $V_0 \lesssim 0.5$ meV. Nevertheless, the deviations of our simplified form for $\Delta_{3\varepsilon}(L)$ from the numerical values are obvious. Remember that a proportionality between λ and V_0 could not be established on small scales. This discrepancy, which is evident when Fig. 3 is compared with Fig. 5, can be attributed to our averaging over a large energy range. This certainly influences the observed scales of quadratic level repulsion and is affected by the specific lattice shape. Concerning long-range fluctuations ($L \gtrsim n_\Phi$), $V_0 \propto \lambda$ is, however, a good approximation because only the *average* level splitting ε is relevant, and it does not matter how *precisely* it is achieved. This indicates only a weak dependence on the specific modulation potential as long as the Landau-level mixing may be ignored.

An important aspect, which we have not yet discussed at length, is the *saturation* of $\Delta_3(L)$ for $L > L_{\max} \approx n_\Phi$. In the case of the LL limit we have already mentioned that this is a consequence of the uniform Landau-level spectrum, leading to the saturation value given by Eq. (16). But even in the strongly modulated system one observes a saturation beyond the cyclotron energy. By means of periodic orbit theory for finite systems, Berry has shown that $L_{\max}$ is connected with the shortest classical periodic orbits:[26]

$$L_{\max} = \frac{h\overline{d}}{T_{\min}}, \tag{17}$$

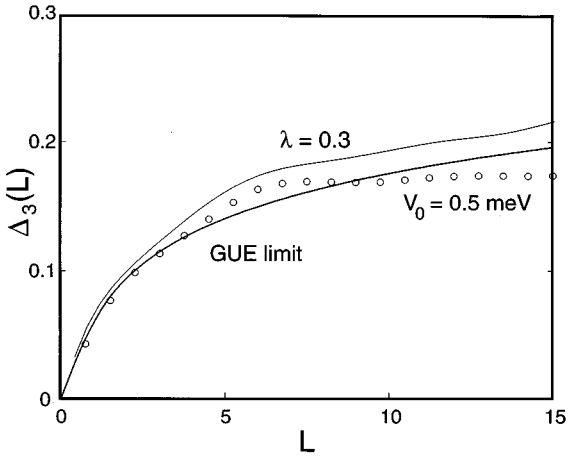

FIG. 6. Spectral rigidity $\Delta_3(L)$ for the additive potential $V(x,y) = V_0[\cos(2\pi x/a) + \cos(2\pi y/a)]$ for $V_0 = 0.5$ meV (with $a = 600$ nm, $n_\Phi = 5$, energy range 1–38 meV). The physical system (circles) is compared with the random matrix model (solid curve) for $\lambda = 0.3$. One can see the saturation of $\Delta_3(L)$ as a consequence of the classical phase-space structure.

where $\overline{d} = n_\Phi/(\hbar\omega_c)$ is the mean (physical) level density in the system, and $T_{\min}$ the shortest period. The latter can be estimated by regular cyclotron orbits, which yields $L_{\max} \approx n_\Phi$. The actual orbits are, however, distorted cyclotron orbits[27] and strongly dependent on the applied modulation potential.

Obviously, the specific phase space structure plays an important role on large scale fluctuations. As can be seen in Fig. 6, the saturation of the spectral rigidity is especially pronounced for the additive potential $V(x,y) = V_0[\cos(2\pi x/a) + \cos(2\pi y/a)]$, which has saddle points between its maxima. Our random matrix model can certainly not account for such a saturation because it leads *by definition* to the pure GUE limit.

V. CONCLUSION

In summary, our investigations have led to the conclusion that the transition from degenerate Landau levels to a spectrum with universal fluctuations of the Gaussian ensemble type displays features of universality, which are, however, restricted to the smallest scales. On those scales, which depend on the modulation strengths, we observe quadratic level repulsion that indicates the broken time-reversal invariance of the system.

On energy scales of the order of the cyclotron energy, the transition appears to be well described by a random matrix model with one parameter as long as Landau-level mixing is negligible. Fluctuations on intermediate scales smaller than the cyclotron energy have been found to be nonuniversal, as demonstrated in terms of the nearest-neighbor spacing statistics. The transition rather depends explicitly on the modulation potential shape, which is in accordance to its influence on the classical phase-space structure. It is very unlikely that a change of the potential shape could be canceled by some appropriate scaling of the independent parameters $\gamma = n_\Phi h/(4\pi a\sqrt{m^\star V_0})$ and $\tilde{E}/V_0$ (while n_Φ remains fixed)

given the various ways one could construct a lattice periodic function.

The long-range fluctuations, too, show a dependence on the physical realization of the transition, which can be verified by studying different types of potentials, e.g., alternatively to Eq. (2), the potential $V(x,y)=V_0[\cos(2\pi x/a)+\cos(2\pi y/a)]$. This should be related to the shortest closed classical orbits as was pointed out by Berry for finite-size systems.[26]

We want to emphasize that in order to keep close to experimental parameters, we did not attempt to approach the semiclassical limit where truly energetically *local* level statistics can be performed. The classical dynamics undergoes a transition from regular cyclotron orbits to chaotic trajectories that strongly depends on the shape of the modulation potential and on the energy. Hence it is most doubtful that even energetically local level statistics in the semiclassical limit could be described by one-parameter random matrix models. This, however, is a necessary condition to make contact to the theory of a *universal transition parameter* $\Lambda \propto[\alpha/\overline{S(E)}]^2$ as introduced by French *et al.*[28] for the breaking of a good symmetry, for instance, rotational invariance, in random matrix models. Here α is the mean symmetry-breaking matrix element (corresponding to V_0 or λ, respectively), whereas $S(E)$ is the *local* mean level spacing at energy E. In any case, such an analysis would require extremely large Hamilton matrices beyond our present possibilities.

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