Scaling of the conductivity of Si:B: Anomalous crossover in a magnetic field

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The zero-temperature conductivity of Si:B with dopant concentrations near the metal-insulator transition exhibits scaling as a function of magnetic field with an anomalously large crossover exponent. The large value of δ is associated with unusual behavior of the magnetoconductance, which vanishes as a power law approaching the transition. This demonstrates that Si:B, which has an anomalous critical conductivity exponent in zero field, also exhibits unusual behavior in response to a magnetic field. [S0163-1829(97)07007-0]

Based on a number of experimental results, most notably the elegant measurements to very low temperatures by Paalanen *et al.*[1] in stress-tuned Si:P, the metal-insulator transition that occurs in doped semiconductors and amorphous metal-semiconductor mixtures is generally believed to be a continuous phase transition.[2] Various physical properties, such as the conductivity and the Hall coefficient, are expected to satisfy appropriate scaling relations as the transition is approached, and their critical behavior has been extensively studied in a variety of materials.[3] Despite considerable experimental and theoretical progress, there are a number of interesting unresolved issues in this area that continue to receive a great deal of attention. In particular, while most amorphous metal-insulator mixtures and most doped semiconductors, such as Ge:Sb,[4,5] have critical conductivity exponents near 1, the silicon-based materials [Si:P,[1] Si:B,[6] and Si:As (Ref. 7)] exhibit anomalously small conductivity exponents between $\frac{1}{2}$ and $\frac{2}{3}$.[8] A number of suggestions have been advanced to resolve this enigma, but none has gained general acceptance.[8–10]

The response to an external magnetic field of systems characterized by a critical conductivity exponent $\mu=1$ was investigated by Rosenbaum, Field, and Bhatt.[11] They showed that the conductivity of Ge:Sb, a system that exhibits the widely observed zero-field conductivity exponent $\mu=1$, obeys scaling with concentration and magnetic field with the crossover exponent $\delta=\frac{1}{2}$ predicted theoretically by Khmel'nitskii and Larkin.[12] In contrast, we show in this paper that the transport in a magnetic field of Si:B, a material that exhibits a zero-field conductivity exponent smaller than 1, is highly anomalous. The data for the zero-temperature conductivity in various magnetic fields of several samples of Si:B with different dopant concentrations lie on a single curve, but with a crossover exponent δ that is much larger than 1. This anomalously large value can be traced to the unusual concentration dependence of the magnetoconductance approaching the metal-insulator transition. Thus, two systems Ge:Sb and Si:B, which have different zero-field conductivity exponents, exhibit an even larger difference in their magnetic-field crossover exponent.

Seven metallic Si:B samples with dopant concentrations 4.11, 4.20, 4.30, 4.38, 4.56, 4.75, and 4.97×10^{18} cm^{-3} were measured at temperatures between 0.06 and 0.5 K in magnetic fields to 9 T. Sample characterization and measurement techniques are described in detail elsewhere.[13] The conductivity of a typical sample is shown as a function of $T^{1/2}$ in various fixed magnetic fields in Fig. 1.

The determination of the magnetic-field crossover exponent requires extrapolations of the measured values of the conductivity to zero temperature. We have recently shown[14] that the magnetoconductance of Si:B obeys a universal relation consistent with the general form expected for

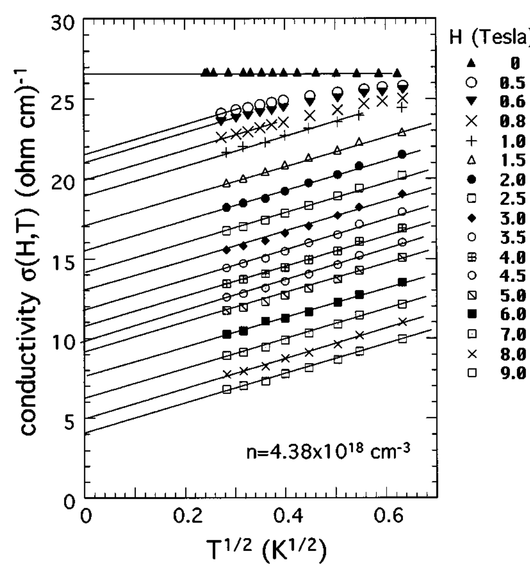

FIG. 1. For a typical metallic Si:B sample with boron concentration 4.38×10^{18} cm^{-3}, the conductivity $\sigma(n,H,T)$ is plotted as a function of $T^{1/2}$ at various fixed magnetic fields, as labeled. The lines (see text) yield zero-temperature conductivities, $\sigma(n,H,0)$ plotted in Fig. 2.

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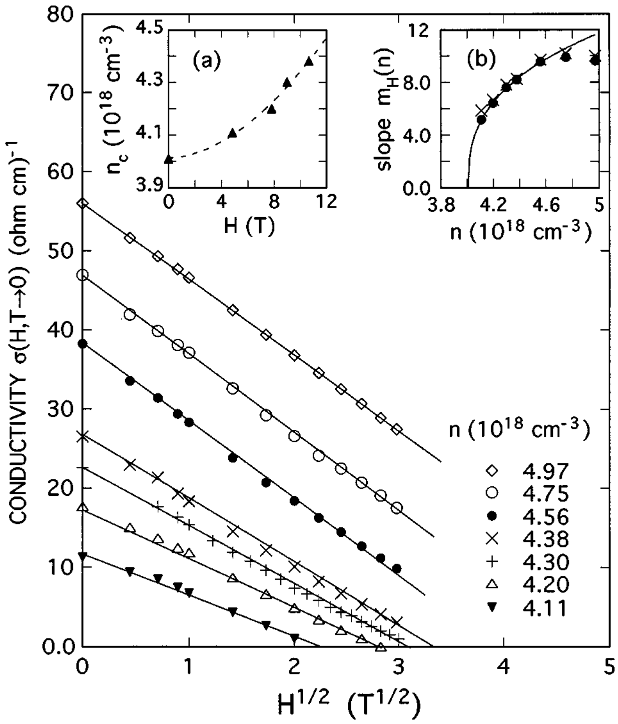

FIG. 2. Zero-temperature conductivities, $\sigma(n,H,0)$ versus $H^{1/2}$ for seven Si:B samples with different boron concentrations, as labeled. Inset (a) shows the critical concentration n_c versus magnetic field H; the dashed line is a fit to $\Delta n_c \approx H^{\delta}$, yielding $\delta=1.7$. Inset (b) shows the slope $m_H(n)$ (in units of $[\Omega^{-} \ \mathrm{cm}^{-1} T^{-1/2}]$), where closed symbols denote the slopes of the curves [see Eq. (2)] and crosses are values deduced from the data of Ref. 14. The solid curve is a fit to Eq. (3) of the four closed data points closest to the transition, yielding $n_c = 4\times10^{18}$ cm^{-3} and $\alpha=0.37$.

electron-electron interactions, $\Delta\sigma=[\sigma(H,T)-\sigma(0,T)] = KT^{1/2}F(H/T)$, with a function $F(H/T)$ associated with spin splitting that is approximately independent of dopant concentration n. For values $g\mu_B H > k_B T$ this simplifies to

$$\sigma(n,H,T)=\sigma(n,H,0)+m_T(n)T^{1/2}, \tag{1}$$

with a slope $m_T(n)$ that is independent of magnetic field for a given sample. Based on the form of the function $F(H/T)$ and the data above 2 T, the same limiting slope $m_T(n)$ is used in low fields to obtain the lines shown in Fig. 1 below the lowest temperature of 0.06 K reached in our experiments, where $g\mu_B H < k_B T$.

The zero-temperature conductivities $\sigma(n,H,0)$ are plotted in Fig. 2 as a function of the square root of the magnetic field for all seven samples of Si:B. The straight-line fits indicate that the zero-temperature conductivity obeys the relation

$$\sigma(n,H,0)=\sigma(n,0,0)-m_H(n)H^{1/2}. \tag{2}$$

We have determined the zero-temperature value of the conductivity by assuming the validity of the H/T scaling of Ref. 14 for the lowest relevant values of the magnetic field. The zero-temperature intercept for the four samples closest to the transition yield the n_c versus H plot shown in inset (a) of Fig. 2; a fit to $\Delta n_c \approx H^{\delta}$ yields a crossover exponent $\delta=1.7\pm0.4$. Different data analyses yield somewhat different values for the zero-temperature extrapolations and the crossover exponent. Nevertheless, we stress that all methods, including simple fits to the raw data, yield a crossover exponent larger than 1. The superlinear dependence of n_c on magnetic field is in contrast with Ge:Sb,[11] where a crossover exponent of $\frac{1}{2}$ indicates a sublinear, square-root dependence.

Inset (b) shows the slopes $m_H(n)$ of the curves of Fig. 2, as well as values of the magnetoconductance $m_H(n)$ deduced directly from the H/T scaling of Ref. 14; these do not require zero-temperature extrapolations and confirm the overall validity of our procedure. The magnetoconductance decreases rapidly as the critical concentration is approached; a power-law fit to data for the four samples closest to the metal-insulator transition yields

$$m_H(n)=A(\Delta n/n_c)^{\alpha}, \tag{3}$$

with $\alpha=0.37$ and $n_c=4.01\times10^{18}$ cm^{-3}.[15]

Using the form for the critical behavior of the conductivity in zero magnetic field, $\sigma(n,0,0)=\sigma_0(\Delta n/n_c)^{\mu}$, we can rewrite Eq. (2) as

$$\sigma(n,H,0)/\sigma(n,0,0)=1-m_H(n)H^{1/2}/\sigma_0(\Delta n/n_c)^{\mu}. \tag{4}$$

Here $\Delta n=(n-n_c)$, n_c is the critical concentration and μ is the critical conductivity exponent in zero field. Scaling[12,16] with magnetic field requires

$$\sigma(n,H,0)/\sigma(n,0,0)=G(H^{-\delta}\Delta n)=F(H/H^*) \tag{5}$$

with a power law dependence of H^* on Δn in the critical region of the form

$$H^* \approx \Delta n^{1/\delta}. \tag{6}$$

Setting $B=A/\sigma_0$ and $x=H^{1/[2(\mu-\alpha)]}(\Delta n/n_c)$, comparison of Eqs. (4) and (5) yields a crossover function $G(x)=1-Bx^{(\alpha-\mu)}$ and crossover exponent $\delta=1/[2(\mu-\alpha)]$. Using $\mu=0.65$ for Si:B (Ref. 5) and $\alpha=0.37$, one obtains $\delta\approx1.8$, consistent with the value obtained from the fit to inset (a) of Fig. 2.

The ratio $\sigma(n,H,0)/\sigma(n,0,0)$ is shown as a function of magnetic field H on a log-log scale in Fig. 3(a); Fig. 3(b) demonstrates that all the curves can indeed be brought into coincidence by appropriate choices of H^*, corresponding to different rigid horizontal shifts for each sample. The inset to Fig. 3(b) shows H^* versus $\Delta n=(n-n_c)$ on a log-log scale. Deviations from a straight line are evident at the highest dopant concentration, indicating that the critical region is restricted: it extends no higher than $n\approx1.20n_c$, and probably includes only the four samples closest to the transition, or $n\leqslant1.10n_c$, in agreement with a recent suggestion of Stupp *et al.*[8] The exponent δ can be obtained from the inverse of the slope of the straight line region of inset (b). It depends on the breadth assumed for the critical range, as well as on the choice of the critical concentration n_c. For $n_c = 4.01\times10^{18}$ cm^{-3}, fits to four, five, or six data points yield $\delta=2.2$, 1.8, and 1.65, respectively. Equivalent fits for a much lower $n_c=3.90\times10^{18}$ cm^{-3} yield $\delta=1.4$, 1.2, and 1.15. All reasonable assumptions thus yield values for the crossover exponent that are well above 1, consistent with the superlinear behavior of Δn shown in Fig. 2, inset (a).

We now compare our findings with existing theories. The observation of scaling suggests that the dopant concentration

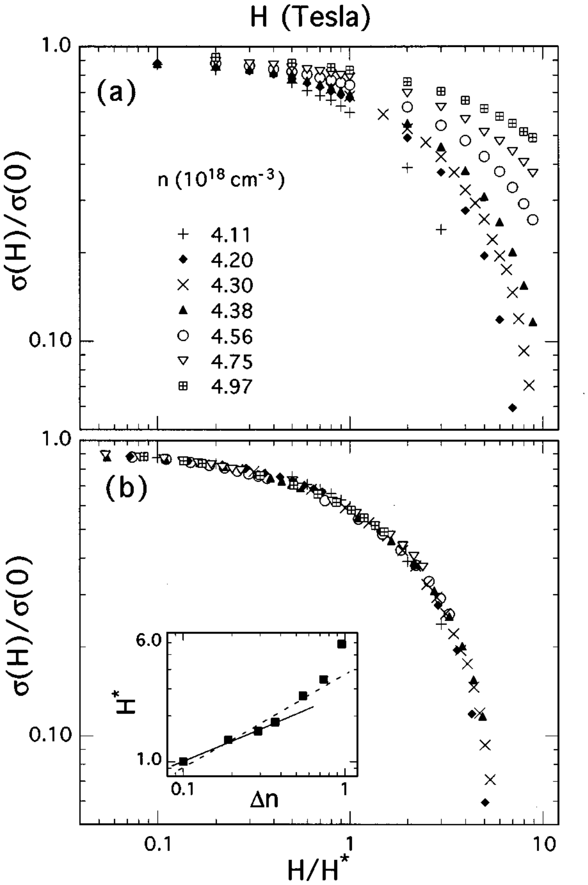

FIG. 3. (a) The ratio $\sigma(n,H,0)/\sigma(n,0,0)$ versus magnetic field H on a log-log scale for seven Si:B samples with dopant concentrations as labeled. (b) Scaled curves of $\sigma(n,H,0)/\sigma(n,0,0)$ versus H/H^* on a log-log plot. The inset shows H^* versus $\Delta n=(n-n_c)$ on a log-log scale for $n_c=4.01\times10^{18}$ cm^{-3}. The solid and dashed lines represent fits, respectively, to four and to six dopant concentrations nearest the metal-insulator transition.

and the magnetic field enter the conductivity through two lengths larger than the microscopic distances, namely, the localization length ξ and a magnetic length l_H. When orbital effects dominate, the conductivity is determined by the flux threading an area $\approx\xi^2$, the scaling function depends on $B\xi^2$ and the magnetic crossover exponent δ is given by $1/(2\nu)$. This was first argued by Khmel'nitskii and Larkin;[12] it is in fact quite general and follows from the gauge invariance of the coupling of the vector potential to the metallic order parameter.[17] In the absence of any complicating factors, rigorous bounds[18] on the exponent ν then imply that δ must be less than 0.75. Magnetic-field coupling to the spin degrees of freedom can produce another crossover exponent. Still, approximate calculations for noninteracting electrons[19] and for interacting electrons[20] yield a value of δ that is even smaller than for orbital effects. It is important to note, however, that the large crossover exponent in Si:B is associated with the unusual concentration dependence of the magnetoconductance and the value $\delta=1.7$ does not violate any theoretical bound.

The crossover behavior of the conductivity in a magnetic field has been investigated experimentally in Si:As by Shafarman *et al.*[21] and in Ge:Sb by Rosenbaum, Field, and Bhatt.[11] No definitive value of the crossover exponent was obtained for Si:As, where the observed behavior was attributed to changes in the critical exponent and to field-tuning of the critical concentration. On the other hand, it is instructive to compare the behavior of Si:B with that found for Ge:Sb,[11] which has a distinctly different crossover exponent $\delta=\frac{1}{2}$. The zero-field conductivity of Ge:Sb vanishes with the usual exponent $\mu\approx1$, and a magnetic field does not modify its value substantially.[5,11] The zero-temperature conductivity obeys Eq. (2) with a slope $m_H(n)$ that is constant and independent of concentration ($\alpha=0$). If plotted in Fig. 2, this would yield a set of parallel lines of equal slope for different dopant concentrations. Thus, the magnetoconductance of Ge:Sb is nonzero and negative (positive magnetoresistance) for any metallic sample, no matter how close its concentration to the critical value n_c; here a magnetic field drives any metallic sample toward the insulating phase.[22] The conductivity exhibits scaling as a function of magnetic field with a crossover exponent $\delta=1/[2(\mu-\alpha)]=\frac{1}{2}$, consistent with expectations for systems in which orbital effects dominate. The dependence of the critical concentration on H is sublinear (square root), varying strongly with H at small fields. In contrast, the critical conductivity exponent of Si:B has an anomalously low value[6] of $\mu\approx0.65$ in zero field (changing to the value 1 in a magnetic field[23]). The magnetoconductance $m_H(n)$ of Si:B (the slopes in Fig. 2) depends strongly on dopant concentration and decreases continuously as a power law to zero at n_c. Near the transition, a magnetic field drives the system neither toward insulating nor toward metallic behavior. The critical concentration exhibits a superlinear dependence on field with an anomalously large crossover exponent near 2. Thus, the large value of the crossover exponent of Si:B is associated with the very unusual behavior of its magnetoconductance.

To summarize, we show that Si:B, one of the prototypical silicon-based semiconductors whose anomalous behavior in zero field has long been an unresolved puzzle, also exhibits unusual behavior in response to a magnetic field. Its zero-temperature conductivity exhibits scaling as a function of magnetic field with an anomalously large crossover exponent δ. The large value of δ can be traced to the vanishing of the magnetoconductance amplitude approaching the metal-insulator transition. We note that measurements[24] of the susceptibility of Si:B have shown that the response of the spin degrees of freedom to a magnetic field becomes stronger as we approach the metal-insulator transition. Our paper demonstrates that the response of the charge degrees of freedom to an external magnetic field, namely, the magnetoresistance, shows the opposite behavior, becoming weaker as the transition is approached. To the best of our knowledge, there is currently no theoretical explanation for these experimental findings, which we hope will stimulate theoretical developments.

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[1] M. A. Paalanen, T. F. Rosenbaum, G. A. Thomas, and R. N. Bhatt, Phys. Rev. Lett. **48**, 1248 (1982); **51**, 1896 (1983).

[2] There continue to be claims that this is a discontinuous transition. See, for example, A. Mobius, Phys. Rev. B **40**, 4194 (1989); J. Phys. C **18**, 4639 (1985); A. Mobius, D. Elefant, A. Heinrich, R. Muller, J. Schumann, H. Vinzelberg, and G. Zies, *ibid.* **16**, 6491 (1983); A. Mobius, H. Vinzelberg, C. Gladun, A. Heinrich, D. Elefant, J. Schumann, and G. Zies, *ibid.* **18**, 3337 (1985).

[3] For a recent review, see D. Belitz and T. R. Kirkpatrick, Rev. Mod. Phys. **66**, 261 (1994).

[4] S. B. Field and T. F. Rosenbaum, Phys. Rev. Lett. **55**, 522 (1985).

[5] Y. Ootuka, H. Matsuoka, and S. Kobayashi, in *Anderson Localization*, edited by T. Ando and H. Fukuyama (Springer-Verlag, Berlin, 1988), p. 40.

[6] P. Dai, Y. Zhang, and M. P. Sarachik, Phys. Rev. Lett. **66**, 1914 (1991).

[7] P. F. Newman and D. F. Holcomb, Phys. Rev. B **28**, 638 (1983); W. N. Shafarman, D. W. Koon, and T. G. Castner, *ibid.* **40**, 1216 (1989).

[8] We note, however, that H. Stupp, M. Hornung, M. Lakner, O. Madel, and H. v. Lohneysen, Phys. Rev. Lett. **71**, 2634 (1993), have recently suggested that analysis of data for samples very near the transition yields $\mu \approx 1.3$.

[9] T. R. Kirkpatrick and D. Belitz, Phys. Rev. Lett. **70**, 974 (1993).

[10] V. Dobrosavljevic and G. Kotliar, Phys. Rev. Lett. **71**, 3218 (1993); Phys. Rev. B **50**, 1430 (1994).

[11] T. F. Rosenbaum, S. B. Field, and R. N. Bhatt, Europhys. Lett. **10**, 269 (1989).

[12] D. E. Khmel'nitskii and A. I. Larkin, Solid State Commun. **39**, 1069 (1981).

[13] P. Dai, Y. Zhang, and M. P. Sarachik, Phys. Rev. B **45**, 3984 (1992).

[14] S. Bogdanovich, P. Dai, M. P. Sarachik, and V. Dobrosavljevic, Phys. Rev. Lett. **74**, 2543 (1995).

[15] This value of n_c is roughly 1% lower than the earlier determination of Refs. 6 and 13.

[16] P. Pfeuty, D. Jasnow, and M. E. Fischer, Phys. Rev. B **10**, 2088 (1974); S. Singh and D. Jasnow, *ibid.* **11**, 3445 (1975).

[17] M. Biafore, C. Castellani, and G. Kotliar, Nucl. Phys. B **340**, 617 (1990).

[18] J. Chayes, L. Chayes, D. S. Fisher, and T. Spencer, Phys. Rev. Lett. **57**, 2999 (1986).

[19] C. Castellani and G. Kotliar, Physica A **167**, 294 (1990).

[20] R. Raimondi, C. Castellani, and C. Di Castro, Phys. Rev. B **42**, 4724 (1990).

[21] W. N. Shafarman, T. G. Castner, J. S. Brooks, K. P. Martin, and M. J. Naughton, Phys. Rev. Lett. **56**, 980 (1986).

[22] As Rosenbaum *et al.* (Ref. 11) point out, however, weak-disorder perturbation theory for noninteracting electrons predicts a negative magnetoresistance, in contrast to the positive magnetoresistance they observed experimentally.

[23] P. Dai, Y. Zhang, and M. P. Sarachik, Phys. Rev. Lett. **67**, 136 (1991).

[24] M. P. Sarachik, D. R. He, W. Li, M. Levy, and J. S. Brooks, Phys. Rev. B **31**, 1469 (1985).