Defect modes of stratified dielectric media

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We present a transfer matrix treatment of the defect modes in a periodic stratified dielectric media containing an inserted array of defects. The eigenfrequency equation for the defect modes is given. Using the eigenfrequency equation we discuss the dependence of the defect modes on the structure parameters, and calculate the dispersion relations of the guided modes. [S0163-1829(97)08407-5]

Electromagnetic propagation in periodic media has been extensively studied for over several decades. As a result of the translational symmetry, the electromagnetic wave functions in periodic media are Bloch waves, and the frequency spectra have a band structure. The optical Bloch waves and the band structures of periodic stratified media were studied in the previous papers and works of Yeh *et al.*,[1] who treated the media as a one-dimensional periodic lattice and solved the eigenvalue problem. Recently, there has been great progress in the study of wave propagation in periodic media. The band theory has been used to deal with three-dimensional periodic dielectric structures called photonic crystals, and a complete forbidden gap, irrespective of propagation direction of the electromagnetic waves, has been discovered.[2]

In analogy to the impurity levels of semiconductors, defect modes will be created in photonic band gaps if an irregular region is introduced into the perfect periodic structures.[3] Much interest has been attracted to the defect modes for them having various applications in devices such as cavities, filters, and lasers.[3–7] A supercell method[3] and a Green's function approach[5–7] were employed to calculate the frequency of the defect modes.

In this paper, we report a transfer matrix treatment of the defect modes for 1D photonic band gap structures. The advantage of using the transfer matrix method to solve the eigenvalue problem is that it can avoid evaluating a high order determinant and makes the calculation of wave functions more easy.

We consider a multilayered system composed of dielectric slabs A and B stacked alternately along the z axis. The refractive indexes of the slabs are n_A and n_B, and their thicknesses are a and b, respectively. We concern ourselves with the TE waves. The treatment of the TM waves is similar. The electromagnetic field of the TE waves is described by a two-component wave function

$$\chi=\begin{pmatrix} E_y \\ icB_x \end{pmatrix}, \tag{1}$$

where E_y and B_x are the tangential components of electromagnetic field, which are continuous across the interfaces, and c is the velocity of light in vacuum. The transfer matrix related $\chi(z+\Delta z)$ to $\chi(z)$ can be written as

$$\mathbf{M}_\mu(\Delta z)=\begin{pmatrix} \cos k_\mu \Delta z & -\sigma_\mu^{-1}\sin k_\mu \Delta z \\ \sigma_\mu \sin k_\mu \Delta z & \cos k_\mu \Delta z \end{pmatrix}, \quad \mu=A,B, \tag{2}$$

where

$$\sigma_\mu=\left(n_\mu^2-\frac{\beta^2 c^2}{\omega^2}\right)^{1/2}, \tag{3}$$

$$k_\mu=\left(\frac{\omega^2}{c^2}n_\mu^2-\beta^2\right)^{1/2}, \tag{4}$$

β is the x component and k_μ is the z component of the wave vector. When $\beta>(\omega/c)n_\mu$, the electromagnetic field becomes evanescent, and k_μ should be replaced by

$$\kappa_\mu=\left(\beta^2-\frac{\omega^2}{c^2}n_\mu^2\right)^{1/2}=-ik_\mu. \tag{5}$$

In periodic systems, the wave function has the form of Bloch wave, i.e.,

$$\chi(d)=e^{iKd}\chi(0), \tag{6}$$

where $d=a+b$ is the period and K is the Bloch wave number. On the other hand, $\chi(d)$ is related to $\chi(0)$ by

$$\chi(d)=\mathbf{Q}\chi(0), \tag{7}$$

where

$$\mathbf{Q}=\mathbf{M}_B(b)\mathbf{M}_A(a). \tag{8}$$

Substituting (7) into (6) yields the following dispersion relation:

$$\cos(Kd)=\frac{1}{2}\,\mathrm{Tr}\mathbf{Q}. \tag{9}$$

According to Eq. (9), the allowed band is given by the condition $|1/2\mathrm{Tr}\mathbf{Q}|<1$. On this condition, K is a real number, and the Bloch wave is propagating. The forbidden gap occurs when $|1/2\mathrm{Tr}\mathbf{Q}|>1$. In the forbidden gap, the Bloch wave number takes the form[1] of $K=m\pi/d+iK_i$, and the Bloch wave is evanescent.

Suppose that we insert an array of N defects into the perfect periodic structure. As a result, the defect levels will be introduced into the forbidden gap. Since the frequency of the defect modes lie in the forbidden gap, the wave function is evanescent in the periodic region. The electromagnetic field, therefore, is confined in the irregular region. It is the localized property that makes the defect modes useful.

Let z_0 and z_N denote the coordinates of the beginning and the end of the inserted region, and a_2 and b_2 represent the thicknesses of the slabs of the detects, then $\chi(z_N)$ is related to $\chi(z_0)$ through a matrix

$$\chi(z_N)=\mathbf{W}\chi(z_0), \tag{10}$$

where

$$\mathbf{W}=[\mathbf{M}_B(b_2)\mathbf{M}_A(a_2)]^N. \tag{11}$$

We regarded the system as two semi-infinite periodic lattices coupled by an array of defects. Since we are interested in the localized modes, we set $K=m\pi/d+iK_i$ in the semi-infinite periodic regions, then the eigenfrequency equation for the defect modes can be obtained as the following:

$$\zeta(w_{11}+\xi w_{12})-(w_{21}+\xi w_{22})=0, \tag{12}$$

where w_{ij} are the elements of matrix $\mathbf{W}$, and

$$\xi=\frac{icB_x(z_0)}{E_y(z_0)}=(q_{22}-\alpha)/q_{12}, \tag{13}$$

$$\zeta=\frac{icB_x(z_N)}{E_y(z_N)}=-(q_{11}-\alpha)/q_{12}, \tag{14}$$

are two quantities proportional to the surface admittances, and

$$\alpha=\operatorname{sgn}(\eta)(\eta-\sqrt{\eta^2-1}), \tag{15}$$

$$\eta=\frac{1}{2}\operatorname{Tr}\mathbf{Q}, \tag{16}$$

with q_{ij} being the elements of matrix $\mathbf{Q}$.

First, we let $\beta=0$. The band gap structure and the defect modes for a typical disturbed periodic layered system are shown in Fig. 1(a), and the corresponding wave functions of the defect modes are plotted in Fig. 1(b). The refractive indexes are $n_A=1.5$ and $n_B=2.5$, and the thicknesses a_1 and b_1 for the host periodic part are such that $n_Aa_1=n_Bb_1=\lambda_0/4$, while those for the inserted part are such that $n_Aa_2=0.1\lambda_0$ and $n_Bb_2=0.4\lambda_0$, where λ_0 is a characteristic wavelength corresponding to the midgap frequency of a quarter-wavelength stuck. The inserted part contains $N=16$ defects, but we only see five defect modes appear in the gap.

Generally, when N slabs are coupled with each other, the eigenfrequency of the individual slabs will split into a band of N nondegenerate modes.[1] The separation of the modes is dependent on the couple strength. The stronger the couple, the wider the separation. When the separations are sufficiently large, part of the defect modes may be embedded into the continuum of the host periodic structure. On the other hand, the levels of the defect modes fall with an increasing of the size defects. Thus, the defect modes may be pulled out from the upper continuum as the defect size increases. This is illustrated in Fig. 2, where we plot the defect mode frequency as a function of n_Aa_2, keeping $n_Bb_2=0.4\lambda_0$, for the

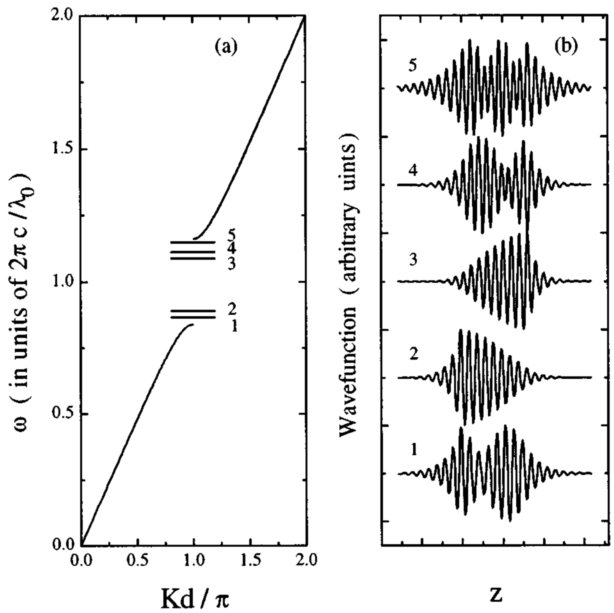

FIG. 1. (a) The dispersion relation of frequency vs the Bloch wave number and the levels of the defect modes for the disturbed periodic layered system with 16 defects. (b) The electric field distributions of the five-defect modes indicated in (a).

above mentioned structure of 16 defects. There are exactly 16 branches in a defect band, as shown in the figure. It should be noted that, compared with the finite multichannel waveguides,[1] the couple of the defects at present is stronger, since the couple is through a propagating field rather than an evanescent field. This stronger couple results in the wider separations of the defect modes.

Figure 3 shows the case that we keep $n_Aa_2+n_Bb_2$ as a constant $0.5\lambda_0$, and vary a_2 and b_2 simultaneously. In the figure the defect mode frequency is plotted as a function of n_Aa_2. When $n_Aa_2=0.25\lambda_0$, the system is reduced to a perfect periodic system, thus, no defect mode exists. The defect

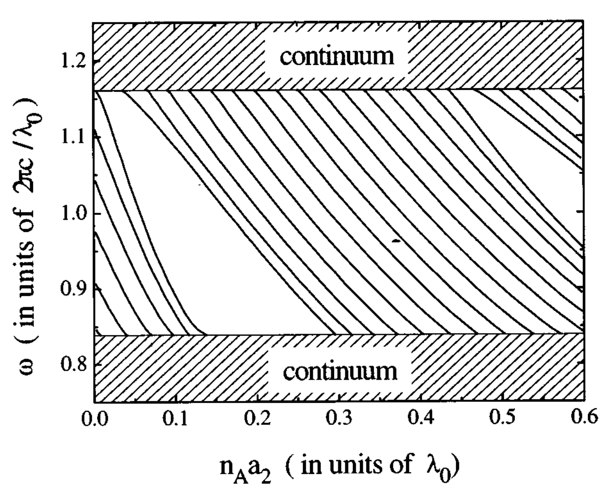

FIG. 2. The frequency of the defect modes as a function of optical thickness n_Aa_2 for the 16-defect system, the optical thickness n_Bb_2 is kept as a constant $0.4\lambda_0$. The shaded regimes are the continuum of the host periodic structure.

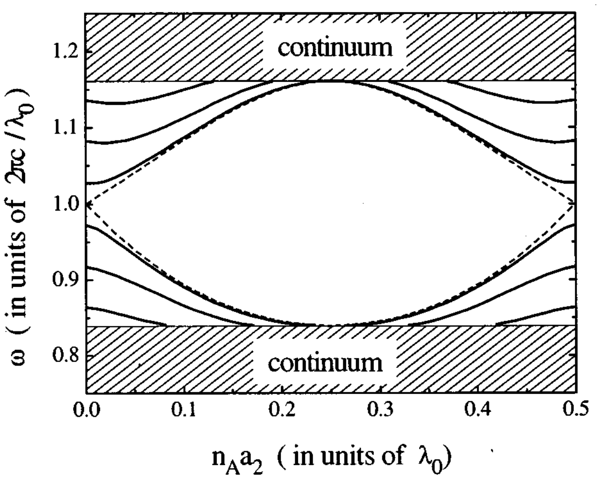

FIG. 3. The frequency of the defect modes as a function of optical thickness $n_A a_2$ for a 16-defect system, the total optical thickness of a bilayer ($n_A a_2 + n_B b_2$) is kept as a constant $0.5\lambda_0$. The dashed lines indicate the band edges of a periodic structure with the unit cell being $a_2 b_2$. The shaded regimes are the continuum of the host periodic structure with the unit cell being $a_1 b_1$.

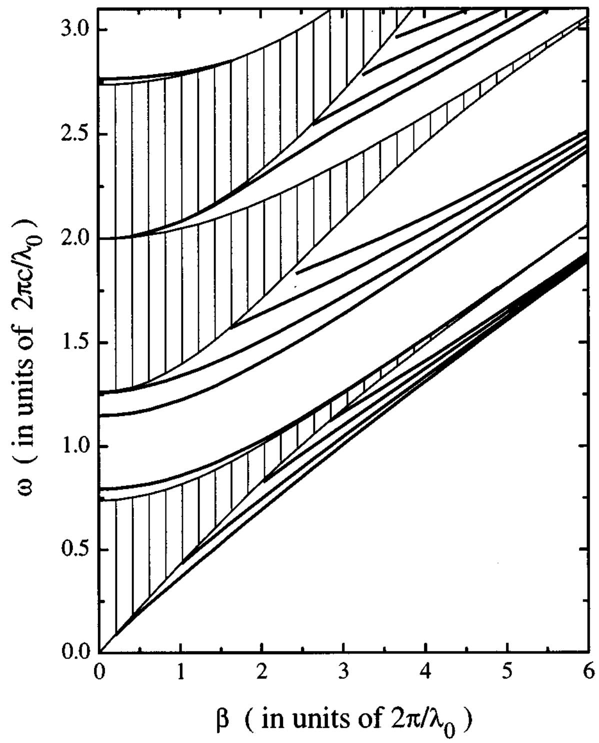

FIG. 4. The dispersion relation of guided modes for a disturbed periodic layered system with four defects. The shaded regimes are the continuum of the host periodic structure.

modes appear if the periodicity is broken by varying $n_A a_2$ (and $n_B b_2$) from $0.25\lambda_0$. The dashed lines in the figure indicate the band edges of a periodic structure with "$a_2 b_2$" being the unit cell. The regime closed by the dashed lines is the common forbidden gap regime. In this regime, no defect mode is found.

Now we consider the case of $\beta \neq 0$. The disturbed periodic stratified system, in fact, is a multichannel waveguide confined by two Bragg reflectors. To obtain a waveguide, one must confine the wave propagation. One of the purposes of searching photonic band gap structures is to find materials able to confine the electromagnetic propagation. Yeh and Yariv have suggested using a Bragg reflection to obtain a lossless confined electromagnetic propagation,[8] and the 1D photonic band gap materials have been used to perform photonic tunneling experiments recently.[9,10] The confinement effect of photonic band gap materials to the wave propagation, physically, is caused by the Bragg reflection. This confinement is different from that caused by the total internal reflection or by the negative dielectric function. In Fig. 4, we plot the dispersion relation of ω vs β obtained from (12) for a four-defect system with $n_A = 1.5$, $n_B = 3.5$ and $n_A a_1 = n_B b_1 = \lambda_0/4$, $n_A a_2 = 0.1\lambda_0$, $n_B b_2 = 0.4\lambda_0$. The shaded regimes in the figure are the continuous bands of the host periodic structure. When β is small, some defect modes are embedded in the continuum. With β increasing, all defect modes escape out from the continuum. In the regime of large β, each of the defect bands consists of four branches of guided modes.

The eigenfrequency equation (12) is also applicable to the finite multilayered system. If the semi-infinite periodic regions are replaced by bulk materials, Eq. (12) is still suitable, but ξ and ζ should be changed as

$$\xi = -(\beta^2 c^2/\omega^2 - n_0^2)^{1/2}, \tag{17}$$

$$\zeta = (\beta^2 c^2/\omega^2 - n_N^2)^{1/2}, \tag{18}$$

where n_0 and n_N are the refractive indexes of the bulk materials in $z < z_0$ and $z > z_N$. The dispersion curves of a four-channel waveguide obtained by replacing the semi-infinite periodic regions with bulk material of $n_0 = n_N = n_A = 1.5$ are shown in Fig. 5. The result is in agreement with those calculated in Ref. 1. Since the confinement effect of bulk materials is caused by the total internal reflection, thus, the wave number is limited by the condition $\beta > n_A \omega / c$. In the regime above the line $\omega = \beta c / n_A$, the field in the bulk material becomes propagating, thus, there is no confined mode. This is different from the case shown in Fig. 4, where the wave number is not subjected to such a limit. For the confinement of Bragg reflection, the evanescent field is the decaying Bloch wave, not the electromagnetic field itself. The field in the individual slabs may be propagating.

Finally, we consider an example that the defects in the inserted region are not arranged periodically. Suppose that the symbol "0" stands for the defect bilayer "$a_2 b_2$," and the symbol "1" stands for the normal bilayer "$a_1 b_1$," and they are arranged following the Fibonacci sequence, which is generated following the recursion rule $S_n = S_{n-1} S_{n-2}$ with $S_0 = 0$ and $S_1 = 1$. The successive Fibonacci chain is 0, 1, 01, 010, 01001, 01001010, 0100101001001, ... The total number of the bilayers for the nth generation is F_n, and the number of the defects is F_{n-1}, where F_n is the Fibonacci number defined as $F_n = F_{n-1} + F_{n-2}$ with $F_0 = F_1 = 1$. By matching the product order in Eq. (11) with the Fibonacci sequence we can calculate the defect modes. The defect modes in a forbidden gap are shown in Fig. 6 for different

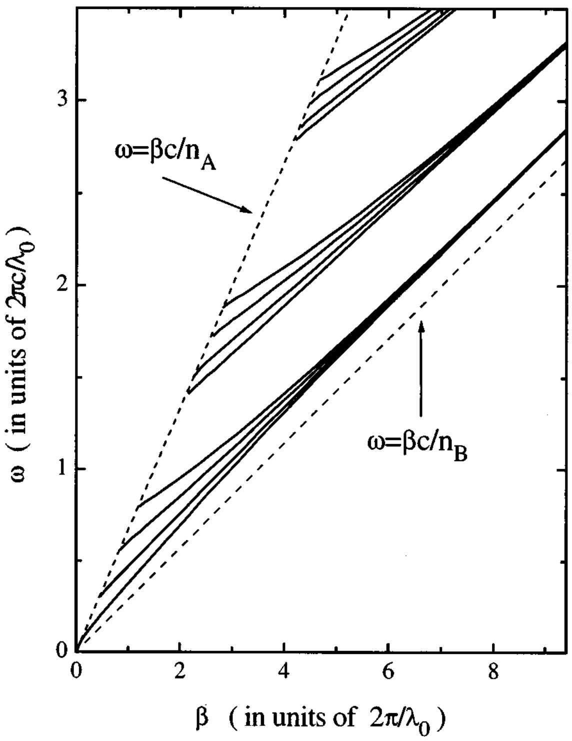

FIG. 5. The dispersion relation of guided modes for a finite four-channel waveguide.

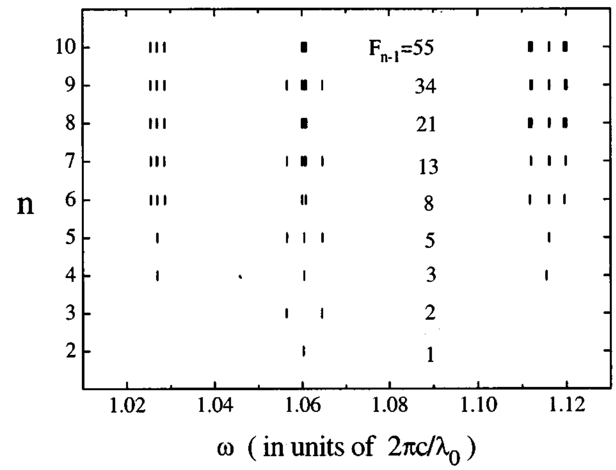

FIG. 6. The frequencies of the defect modes for the case that the defects in the inserted region are arranged following the Fibonacci sequence, n is the generation order of the sequence.

generations of the Fibonacci sequence, where we keep $\beta = 6\pi/\lambda_0$ as a constant. The number of the modes is exactly equal to F_{n-1}, i.e., the number of the defects, although some modes are too close to be resolved in the figure.

To summarize, we have given the eigenfrequency equation for the defect modes of disturbed periodic layered media. The dependence of defect modes on the structure parameters is discussed. The eigenfrequency equation can be used to determine the resonant frequency and to calculate the dispersion relations in device designs. It can also be used to deal with a variety of eigenvalue problems of multilayered systems as long as we modify properly the form of the transfer matrix.

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