Insulator-to-quantum-Hall-liquid transition in an antidot lattice

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Measurements are reported of the Hall resistance ρ_{xy} and magnetoresistance ρ_{xx} of a two-dimensional electron gas containing a triangular lattice of approximately circular depleted regions (antidots). The transition from insulator to quantum Hall liquid shows a fixed point where the longitudinal resistance is independent of temperature; scaling exponents are determined. ρ_{xx} and ρ_{xy} oscillate with magnetic flux through the unit cell of the antidot lattice, although the dissipation results from hopping between localized states. [S0163-1829(97)11207-3]

There has been much recent interest in the insulator-to-quantum-Hall-liquid transition (IQHT) in disordered two-dimensional electron gas (2DEG) systems.[1–4] For a 2DEG with longitudinal sheet resistance $\rho_{xx} \gg h/e^2$ at zero magnetic field, ρ_{xx} decreases dramatically with increasing magnetic field B. At a critical magnetic field B_c where ρ_{xx} is of order h/e^2, the temperature derivative of the resistance changes from negative to positive, so at B_c, ρ_{xx} is independent of temperature. Furthermore, the transition is continuous, showing scaling similar to that seen in the insulator-to-superconductor transition in thin films.[5,6]

In previous experiments[1–4] the 2DEG has been made insulating because of scattering from random potential fluctuations. In this paper we report experiments on a 2DEG made insulating in a different way. We have eliminated the 2DEG in small, approximately circular regions (antidots). The disorder in this case presumably arises from the imperfections in the shapes and positions of the antidots. We determine localization lengths from variable-range-hopping behavior on both the insulating and quantum Hall sides of the transition. In addition to the exponent characterizing the divergent lengths we also measure the dynamical exponent that characterizes the IQHT. Although the IQHT is similar to that in disordered 2DEG systems, one feature is surprising. We find that at low B the localization length oscillates with magnetic flux through the unit cell of the antidot lattice with a period of about $h/2e$, and at high B the localization length oscillates with a period of about h/e.

Our samples are fabricated using a modulation-doped GaAs-$Al_{0.3}Ga_{0.7}As$ heterostructure. The density of the 2DEG formed at the GaAs interface, 95 nm beneath the surface of the $Al_{0.3}Ga_{0.7}As$, is 2.8×10^{11} cm^{-2}, and the mobility is 1.2×10^{6} cm^2/Vs at $T=1.5$ K in the dark. From these values we find that the mean-free path is 10 μm and the Fermi wavelength is 47 nm. A Hall bar is patterned on this heterostructure with a channel width of 16 μm and a distance between the potentiometric probes of 16 μm. Subsequently, a two-dimensional triangular lattice of holes with lattice constant $a=200$ nm is patterned in a resist using electron beam lithography. (See the inset in the lower left of Fig. 1.) Finally, wet chemical etching is used to remove a small amount of the surface of the $Al_{0.3}Ga_{0.7}As$, leading to a depletion of the 2DEG underneath. The diameter of the holes, measured by scanning electron microscopy, is about 110 nm, with fluctuations of $\sim$10 nm. The sample is mounted on the cold finger of a dilution refrigerator with a base temperature of about 25 mK. Both the longitudinal and Hall resistivities are measured using a lock-in amplifier at 8 Hz with a current of 0.1 nA.

Figure 1 shows ρ_{xx} versus B for different temperatures. Below the critical field $B_c=0.47\pm0.01$ T, ρ_{xx} decreases with T, whereas above B_c it increases with T. The fixed point occurs where the value of ρ_{xx} is 12.5 ± 0.9 kΩ, equal, within experimental error, to $h/2e^2=12.9$ kΩ. Whereas in conven-

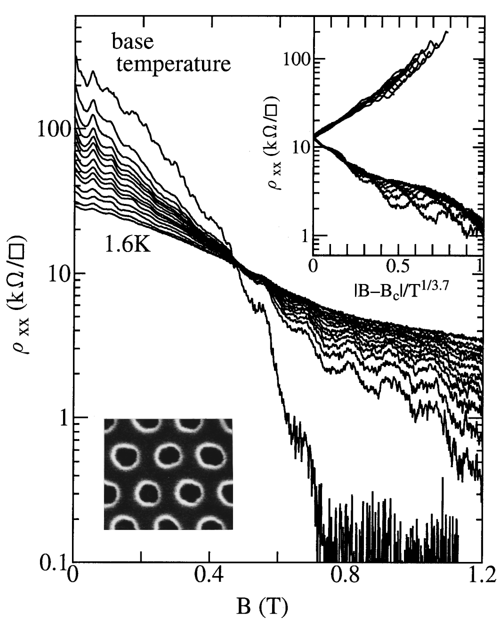

FIG. 1. Logarithm of ρ_{xx} vs magnetic field. At $B=0$ the curves, from top to bottom, correspond to temperatures $\sim$0.025, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.50, 0.60, 0.70, 0.80, 1.0, 1.2, 1.4, and 1.6 K. Data collapse is illustrated in the scaling plot in the top right inset. At the bottom left is shown an electron micrograph of the antidot lattice.

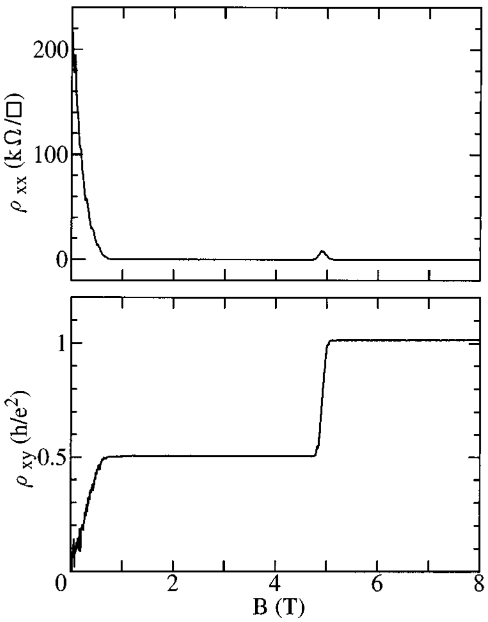

FIG. 2. Field dependence of ρ_{xx} and ρ_{xy} at the base temperature of the dilution refrigerator $\sim$25 mK. The electron temperature may be somewhat higher.

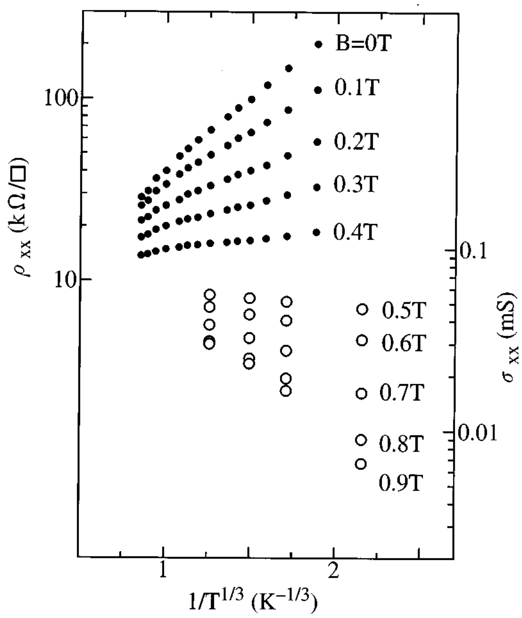

FIG. 3. Logarithm of ρ_{xx} (closed circles) in the insulator, and σ_{xx} (open circles) in the quantum Hall state, as a function of $T^{-1/3}$ for various magnetic fields. Below $\sim$1 K variable range hopping behavior is evident in both quantities.

tional 2DEG systems ρ_{xx} decreases monotonically with B near the IQHT, in the antidot lattice oscillations are seen at both the lowest and highest fields.

Figure 2 shows ρ_{xx} and ρ_{xy} over a wider range of B. Very wide Hall plateaus are observed, corresponding to filling fractions 2 and 1. From the transition between these, one finds electron density $n=1.8\times 10^{11}$ cm^{-2}, smaller than the value 2.8×10^{11} cm^{-2} found for the 2DEG without antidots. Presumably, depletion reduces the charge density not only in the etched region but also in the space between the holes. The onset of the filling fraction 2 plateau is very close to B_c, indicating that the transition is from the insulator directly to the quantum Hall liquid.

For a 2D magnetic-field-induced phase transition, the characteristic length scale ξ diverges as $|B-B_c|^{-\nu}$ and the characteristic energy or frequency vanishes as ξ^{-z}.[5] Because the states at the Fermi energy are localized on both sides of the IQHT, we can extract the exponent ν directly from the data.

At $B=0$ the resistance is typical of a two-dimensional insulator, as illustrated in Fig. 3. There ρ_{xx} is plotted for $B<B_c$ as a function of $T^{-1/3}$. Below about 1 K, the data follow the Mott variable-range-hopping prediction for two dimensions, $\rho=\rho_0\exp[(T_0/T)^{1/3}]$. Fitting the $B=0$ data for $T<1$ K to the form $\rho=\rho_0\exp[(T_0/T)^{\alpha}]$ we find $\alpha=0.35\pm 0.07$. From a fit with $\alpha=1/3$ we find $T_0=5.7\pm 0.3$ K. The localization length is related to T_0 by $\xi=(3/k_BT_0g)^{1/2}$, where g is the density of states of the two-dimensional disordered system. Because of the antidots g averaged over length scales larger than the size of the holes, is less than the density of states g_0 of the unpatterned 2DEG. We estimate that g is reduced by the fraction of area depleted by the antidots. We assume that the radius of the antidots is almost the same as the lithographic dimension. In this way we find $g=0.7g_0$ and $\xi=180$ nm, close to the antidot lattice constant, 200 nm. For each field $B<B_c$ we extract ξ in this way, using only data for $T<1$ K. Since hopping occurs at length scales larger than ξ and since the values of ξ we find are larger than the size of the holes, we conclude that our averaging of the density of states is consistent.

Above B_c the dissipation results from hopping between localized states in the quantum Hall liquid, and, as illustrated in Fig. 3, σ_{xx} follows the Mott form. Therefore, ξ may be extracted from σ_{xx} in the quantum Hall liquid, in the same way that we extract it from ρ_{xx} in the insulator. Fitting the data for $B<B_c$ to the power-law form, $\xi\propto|B_c-B|^{-\nu}$, we find $\nu=1.86\pm 0.08$. Using this value of ν we plot $\xi^{-1/\nu}$ versus B/B_c in Fig. 4. This shows that the localization length diverges with the same exponent on both sides of B_c. Note that the magnitude of ξ is also symmetric about B and that the localization length is always greater than the antidot lattice constant except at the highest fields.

Studies of disordered 2DEG systems have found good agreement with the proposed scaling relation[5]

$$\rho_{xx}=\rho_0 f_{\pm}(|B_c-B|/T^{1/z\nu}), \tag{1}$$

where f_+ is for $B>B_c$ and f_- is for $B<B_c$. To match the variable-range-hopping form we must have the asymptotic behavior $f_-(Y)\sim\exp(Y^{z\nu/3})$ for $Y=|B_c-B|/T^{1/z\nu}\rightarrow\infty$. When correlations are important in the insulator one observes hopping in the Coulomb gap which gives $\rho\sim\rho_0\exp[(T_0/T)^{1/2}]$ (Ref. 3) and one would expect the density of states to display critical phenomena. However, for our case we see no evidence of such a gap and conclude that g is weakly dependent on B near B_c for the temperatures studied. Therefore, as discussed above, $T_0\propto\xi^{-2}\propto|B_c-B|^{2\nu}$. However, the asymptotic behavior described above requires that $(T_0/T)^{1/3}\propto(|B_c-B|^{z\nu}/T)^{1/3}$, which then requires $z=2$. Using $\nu=1.86$ we expect ρ_{xx} to be a function of

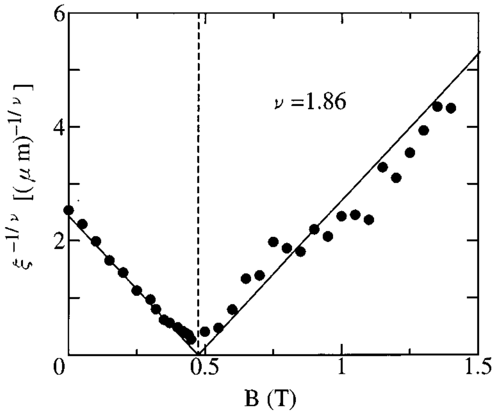

FIG. 4. Localization length ξ raised to the power $-1/\nu = -1/1.86$ as a function of B. Note that at $|B-B_c|=0.1$ T, the localization length is 3 μm, approaching the dimensions of the Hall bar.

$|B-B_c|/T^{1/3.7}$, and this describes the data quite well for small $|B-B_c|$, as seen in the inset of Fig. 1. It is clear that the oscillations do not scale in this way, because the positions of peaks and valleys are independent of T.

The value of ν (1.86) that we find is similar to the value found by Kravchenko *et al.*,[3] $\nu=1.6$, in Si field effect transistors, which show variable range hopping in a Coulomb gap. These authors assume $z=1$, the value of the dynamical exponent expected for pure Coulomb interactions. As discussed above, we do not need to make an assumption about the value of z. Wang *et al.*[2] find that the exponent $\kappa=1/z\nu=0.21\pm0.02$, in modulation-doped GaAs/Al_xGa_{1-x}As heterostructures, consistent with measurements for transitions between adjacent Hall plateaus in the integer quantum Hall effect. Our value of $\kappa=0.27\pm0.02$ is somewhat larger. The samples studied by Wang *et al.*[2] are much closer to the metallic limit at $B=0$ over the entire temperature range studied by these authors, showing weak localization (ln*T*) resistance rather than the strong localization indicated by variable range hopping resistance. Jiang *et al.*[1] have studied modulation-doped GaAs/Al_xGa_{1-x}As heterostructures with higher resistance than those of Wang *et al.* and find variable range hopping with no Coulomb gap as we do. Unfortunately, they have not extracted exponents from their data. Our results are in agreement with the observation of Shahar *et al.*[4] that the insulator to filling-fraction-2 quantum Hall state occurs when $\rho_{xx}=h/2e^2$.

As mentioned above, prominent oscillations are seen in $\rho_{xx}(B)$ (see Fig. 1). At low field, in the insulating regime, the period of these is (53 ± 11) mT, close to the value of the $h/2eA=59.7$ mT, where $A=\sqrt{3}a^2/2$ is the area of the unit cell of the antidot lattice, corresponding to a flux $h/2e$ through the unit cell. At high field, above the IQHT, the oscillations have approximately twice the period, (122 ± 27) mT, close to 119 mT, the value for one flux quantum per unit cell.

Figure 5 shows that on a plot of $T^{1/3}\ln\rho_{xx}$ versus B the size of the oscillations at low B is approximately independent of T from 0.15 K to 0.8 K. This indicates that T_0 oscillates with flux through the antidot with a period of about $h/2e$. This

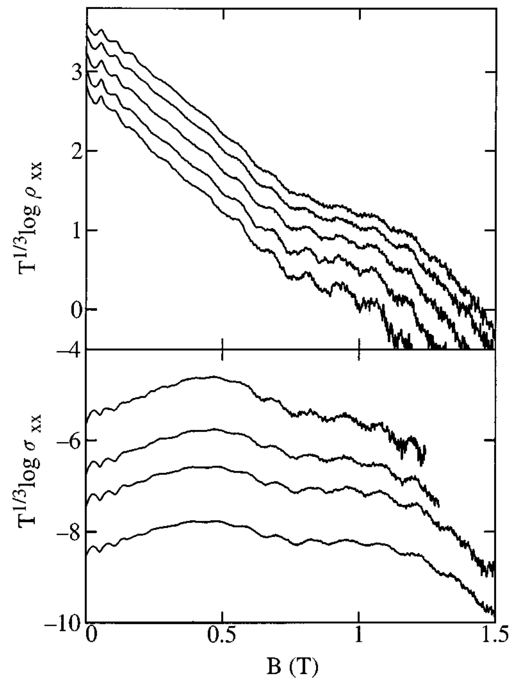

FIG. 5. Field dependence of $T^{1/3}\ln\rho_{xx}$ and $T^{1/3}\ln\sigma_{xx}$. The curves of ρ_{xx} from bottom to top correspond to temperatures 0.15, 0.25, 0.4, 0.6, and 0.8 K, and those of σ_{xx} from top to bottom correspond to 0.1, 0.2, 0.3, and 0.5 K. Because the oscillations in these quantities are approximately independent of temperature, we infer that the oscillations result from periodic variations of the localization lengths.

may result from oscillation of g or ξ or both. We suggest that the low-field oscillations result from variations of the localization length caused by coherent backscattering. Such an effect has been predicted for weak localization by Altshuler *et al.*[8] and observed by Sharvin and Sharvin.[9] Altshuler *et al.*[8] predict that the $h/2e$ oscillations die out when a flux of order h/e penetrates the conducting annulus surrounding an antidot. The lithographic area of the antidots is about 0.3 of the total, so it is not surprising that only a few periods of the $h/2e$ oscillations are observed.

Figure 5 also shows that when $T^{1/3}\ln\sigma_{xx}$ is plotted versus B the size of the oscillations at high B is approximately independent of T. This indicates that these oscillations result from periodic modulation of the localization length in the quantum Hall state. In the high-field limit, the localized states in the latter are expected to be those in which an electron is bound to an antidot, so it is not surprising that their radius oscillates with flux through the antidot. Oscillations with a period of about h/eA have also been observed in antidot lattices with higher carrier densities. However, we believe that the latter oscillations have a different origin because the samples are in the ballistic regime, where the mean-free path is larger than the lattice constant.[7]

Parenthetically, we note that the similarity of the slopes of $T^{1/3}\ln\rho_{xx}$ versus B in Fig. 5 is consistent with the value of κ determined above. The scaling function [Eq. (1)] predicts that the $d\ \ln\rho_{xx}/dB$ at B_c scales as $T^{-\kappa}$. The similarity of slopes in Fig. 5 thereby shows that $\kappa\sim1/3$, close to the value 0.27 determined above.

Although the general behavior of the IQHT is similar to that in conventional disordered 2DEG systems, it is quite different from that reported recently by Lutjering *et al.*[10] The latter authors have tuned a system similar to ours from a connected 2DEG containing an antidot lattice to an array of weakly coupled dots. However, the temperature dependence of their conductivity neither follows the Mott variable-range-hopping expression nor does it have a constant activation energy. We have no explanation for the differences between their data and ours.

The most surprising result of this work is the observation of magnetoresistance oscillations in the strongly localized regimes at both low and high field. The disorder, presumably arising from imperfections in the antidot lattice result in localization. However, the oscillations with a period of $h/2e$ result from modulation of the hopping conductivity in the low-field limit, and the antidots must be sufficiently uniform that the coherent backscattering effects do not average out. This is the case in the weak localization regime. Nguyen *et al.*[11] predicted $h/2e$ oscillations in an array of loops made of conductors in which the conductivity is by hopping. These have been observed by Poyarkov *et al.*[12] However, unlike our materials, in those studied previously theoretically and experimentally the localization length is smaller than the loop diameter. Above the IQHT, oscillations are also seen in the hopping contribution to the dissipation, but they have a period of about h/e. The observation of such oscillations clearly poses a challenge for theory.

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