# Line shape of resonant tunneling between fractional quantum Hall edges

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We report experiments on resonant tunneling of charge $e/3$ *quasiparticles* through states bound on a quantum antidot. We find that at a given temperature $T$ the line shape of resonances fits predictions of Fermi and Luttinger liquid theories equally well, the difference between the *theoretical* line shapes being well within 1%. As $T$ is varied, the experimental resonances scale linearly with $T$, as expected for both Fermi and Luttinger tunneling for quasiparticles, and not as $T^{2/3}$ predicted by the Luttinger theory for electrons.
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The low-energy properties of an interacting one-dimensional (1D) fermion system are expected to be described by the Luttinger liquid theory,[1] with many properties qualitatively different from the more familiar Fermi liquid. Unambiguous experimental observation of a Luttinger liquid in 1D quantum wires is a difficult task, though. Problems arise because any impurities in the conducting channel can backscatter or localize the carriers and therefore destroy the Luttinger-like character of the system. Fortunately, there is another possible way of realizing a Luttinger liquid: at the edges of a two-dimensional electron system (2DES) in the fractional quantum Hall (FQH) regime.[2] In such chiral Luttinger liquid backscattering is not a problem since the left and right moving carriers are separated in the opposite edges of the sample by a macroscopic distance, so that localization does not occur, even with finite disorder.[3] Also in contrast to quantum wires, in FQH systems the only parameter of the Luttinger theory has a quantized value $g=\nu$, where $\nu$ is the filling fraction. Thus in FQH regime the Luttinger liquid resonant tunneling (RT) conductance has a truly universal line shape, independent of *any* sample parameters.[4] Generally, in FQH regime two kinds of charge carriers can tunnel:[2] electrons, but also fractionally charged quasiparticles. Initial indications consistent with Luttinger behavior have been reported in RT experiments on electron tunneling.[5]

The RT conductance of particles of charge $e/3$ between 1D Fermi liquids can be calculated easily.[6] In the low-temperature $kT \ll \Delta E$, low bias $V \rightarrow 0$ limit, where $\Delta E$ is the energy separation of the quantized resonant states at chemical potential $\mu$, the linear response conductance is a convolution of the intrinsic Lorentzian line shape with the derivative of the Fermi function:

$$G_T=\frac{e^2}{3h}\,\frac{\Gamma_L\Gamma_R}{4kT}\int d\epsilon\,\frac{1}{(\epsilon-\epsilon_0)^2+\Gamma^2}\cosh^{-2}\left(\frac{\epsilon-\mu}{2kT}\right). \quad (1)$$

Here $\Gamma_L$ and $\Gamma_R$ are the tunneling rates through the left and right barriers, $\Gamma=\frac{1}{2}(\Gamma_L+\Gamma_R)$ and $\epsilon_0$ is the energy of the resonant state. For $\Gamma \ll kT$, this simplifies to

$$G_T^{\mathrm{FD}}=\frac{e^2}{3h}\,\frac{\Gamma_L\Gamma_R}{\Gamma}\,\frac{\pi}{4kT}\cosh^{-2}\left(\frac{\epsilon_0-\mu}{2kT}\right), \quad (2)$$

whereas in the opposite limit $\Gamma \gg kT$ the resonance takes the usual Breit-Wigner (Lorentzian) form[7]

$$G_T=\frac{e^2}{3h}\,\frac{\Gamma_L\Gamma_R}{(\mu-\epsilon_0)^2+\Gamma^2}. \quad (3)$$

Experimental line shapes of resonances in the Coulomb blockade regime[8] agree well with the above theory in both thermally broadened Eq. (2) and intrinsic Eq. (3) regimes.

On the other hand theoretical work for RT between Luttinger liquids[4,9] predicts a universal line shape $\widetilde{G}$ determined only by the parameter $g$:

$$G_T^L=\widetilde{G}\left(c\,\frac{\epsilon_0-\mu}{T^{1-g}}\right), \quad (4)$$

where $c$ is a nonuniversal constant.

In this paper we report experiments on resonant tunneling of Laughlin quasiparticles through states bound on a lithographically defined potential hill (quantum antidot) between two edges of FQH liquid at $\nu=1/3$. We study the evolution of several resonances as a function of temperature, and find that at a given $T$ the line shape of resonances fits predictions of Fermi and Luttinger liquid theories equally well, the difference between the theoretical line shapes being well within experimental uncertainty. However, as $T$ is varied the RT peak width scales as $T^{1.05\pm0.07}$, consistent with quasiparticle tunneling for both Fermi and Luttinger liquids, and not as $T^{2/3}$ predicted by the Luttinger theory for electron tunneling.

Samples were fabricated from very low disorder GaAs/Al$_x$Ga$_{1-x}$As heterostructure material. The antidot-in-a-constriction geometry was defined by standard electron beam lithography on a pre-etched mesa with ohmic contacts [Fig. 1(a)]. A global back gate is separated from the 2DES by an insulating GaAs of thickness $\approx 430$ $\mu$m. The two front gates were contacted independently and were used to vary electron density $n$ (and thus $\nu$) in the constriction and to bring the two edges close enough to the antidot for tunneling to occur. In the experiment $\nu$ in the constriction is smaller than $\nu_B$, the filling factor in the bulk, as discussed in Ref. 10. We prepared 2DES with $n \approx 1\times10^{11}$ cm$^{-2}$ and mobility $2\times10^6$ cm$^2$/V s by exposing the sample to red light at 4.2 K. Experiments were performed in a dilution refrigerator with sample probe wires filtered at mK temperatures so that the total electromagnetic background at the sample's contacts $<2$ $\mu$V rms. The four-terminal magnetoresistance $R_{(2-3;\,1-4)}$ was measured with a lock-in amplifier at 12 Hz

 

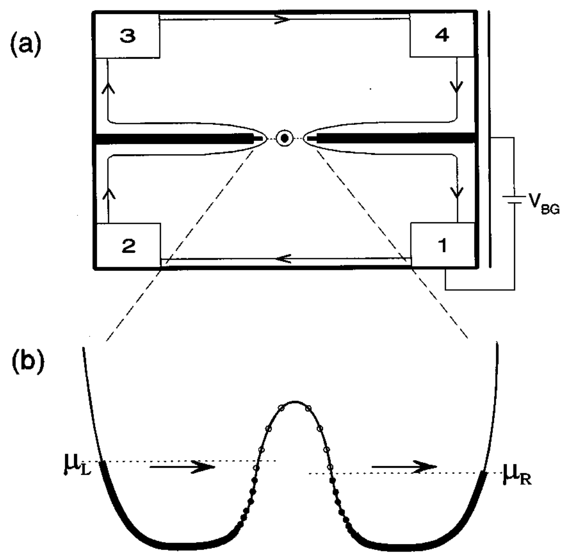


FIG. 1. (a) Illustration of an antidot sample. Numbered rectangles are Ohmic contacts, black areas are front gates in etched trenches and lines are edge channels. The back gate extends over the entire sample on the opposite side of the substrate. Dotted line represents tunneling. (b) Schematic energy landscape of the antidot. The excitation gap of the $\nu=1/3$ condensate forms the two barriers for tunneling, and the ladder of quantized states around the antidot has filled (●) and empty (○) states. Tunneling from left edge at chemical potential $\mu_L$ to the right edge at $\mu_R$ occurs through single level if $\mu_L-\mu_R, kT \ll \Delta E$.

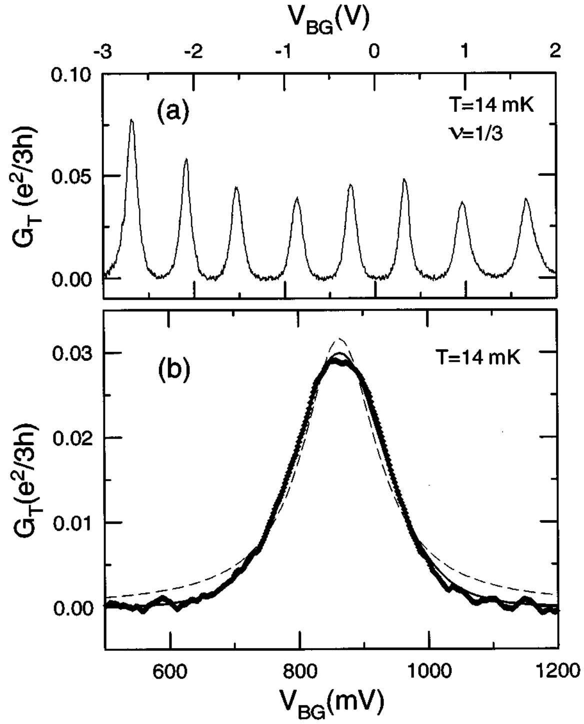


FIG. 2. (a) Tunneling conductance of quasielectrons vs back gate voltage at $\nu=1/3$, $B=8.18$ T. (b) An experimental resonant tunneling conductance peak (diamonds). Solid line is the best fit to Fermi liquid line shape, Eq. (2), dashed line to Lorentzian line shape Eq. (3).

using excitation current 50 pA. Tunneling conductance $G_T$ between the two edges can then be calculated from $R_{(2-3;\,1-4)}$, if $\nu$ and $\nu_B$ both have a quantized value.[11] This condition was satisfied by tuning the sample to a plateau region $\nu=1/3$, $\nu_B=3/5$, and it was confirmed by measuring longitudinal and Hall resistance in the absence of tunneling, between the RT peaks.

Figure 1(b) shows schematically the energy landscape near the antidot. There is an edge channel around each of the front gates; at the edges the energy spectrum is continuous at $\mu$ and there is no gap for charged excitations. The size of the antidot is small enough that the quasiparticle states encircling the antidot are quantized. These quantized states are the resonant levels through which tunneling can occur, and for small enough Hall voltage ($\mu_L-\mu_R$) and low enough $T$ tunneling takes place through a single level.[12] These resonant levels can then be moved in energy relative to $\mu$ by changing either magnetic field $B$ or back gate voltage $V_{\rm BG}$ and thus line shapes of RT peaks can be measured.[13]

Figure 2(a) shows representative experimental $G_T$ as a function of $V_{\rm BG}$ at $\nu=1/3$. We clearly observe an interval of quasiperiodic resonant tunneling peaks on top of the $\nu=1/3$, $\nu_B=3/5$ plateau. The noise level in these data is $\sim 0.001e^2/3h$. Corresponding data can also be measured[10] by sweeping $B$. In Fig. 2(b) we plot an experimental RT peak and best fits to a Lorentzian [Eq. (3)] and thermally broadened Fermi liquid[15] (FD) [Eq. (2)] line shapes. The fits were done with two free parameters, the peak $G_p$ and the width $W$ of the resonance,

$$G_T^{\rm FD}=G_p\cosh^{-2}\left(\frac{V_{\rm BG}-V_{\rm BG}^r}{W}\right) \tag{5}$$

for Eq. (2), where $V_{\rm BG}^r$ is the position of the peak. We find that the FD line shape fits our data well, whereas the Lorentzian does not fit well. This holds true for all measured temperatures and for different resonant peaks.

This result shows that the experiment is not in the intrinsic width regime $\Gamma \gg kT$. The distinction between FD and Luttinger behavior cannot be made from the data of Fig. 2(b) because at a fixed $T$ the two line shapes differ less than 1% of $G_p$. This can be seen in Fig. 3, where we compare the two line shapes on linear and logarithmic scales. We note that there is no visible difference between the two on the linear scale. Only when the two functions are plotted on a log-log scale do we see that the tails of the functions differ. The Luttinger peak has a power law tail[9] $G_T^L(X)=3.3546(e^2/3h)X^{-6}$, $X\gg 1$, whereas the FD peak is an exponential in this regime: $G_T^{\rm FD}(X)=\frac{1}{6}(e^2/3h)\exp(-2X)$. In our experiment the rms noise level is $\sim 2\%$ of $G_p$, so the signal to noise ratio is 1 at $X\sim 3$.

In Fig. 4 we show the evolution of the width of a single resonance with temperature. The experimental temperature range from 12 to 70 mK is limited by the base temperature of the refrigerator, and, at high $T$ by the fact that the resonances start to overlap so much that meaningful analysis is impossible. $W$ was obtained from a two-parameter fit of Eq. (5). Note that use of FD line shape in fitting does not bias our analysis, since at each $T$ the Luttinger line shape is identical

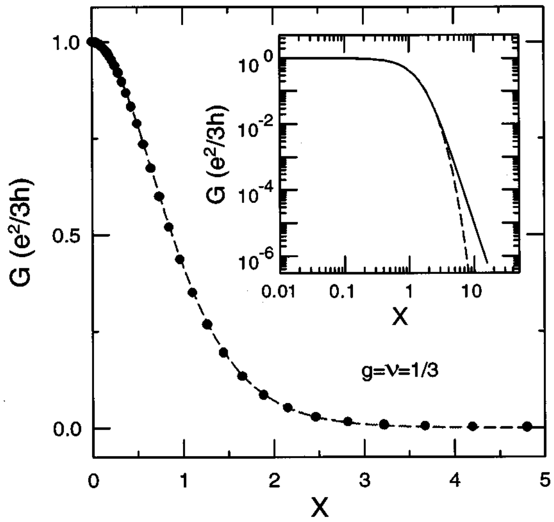


FIG. 3. Comparison of Fermi-liquid RT line shape Eq. (2) (dashed line) with the Luttinger liquid result (dots). $X \sim (V_{\rm BG}-V^r_{\rm BG})/T^{2/3}$ for Luttinger peak, $X \sim (V_{\rm BG}-V^r_{\rm BG})/T$ for Fermi peak. $G(X\rightarrow 0)$ is normalized to $(e^2/3h)(1-X^2)$. Inset shows the same functions in a log-log scale: solid line is the Luttinger result, dashed is the Fermi peak.

within our experimental accuracy. The tail contributions from the nearest and the second nearest neighboring peaks were also included in the analysis, which is important at higher $T$. We note that no simple power law of the form $W = aT^p$, as expected from Eqs. (2) or (4), fits the data in the whole experimental temperature range, although in the high-$T$ limit the data approach the linear dependence $W \sim T$, in agreement with Eq. (2). Below $\sim$25 mK the width saturates, which can be caused by several mechanisms: (i) approach of

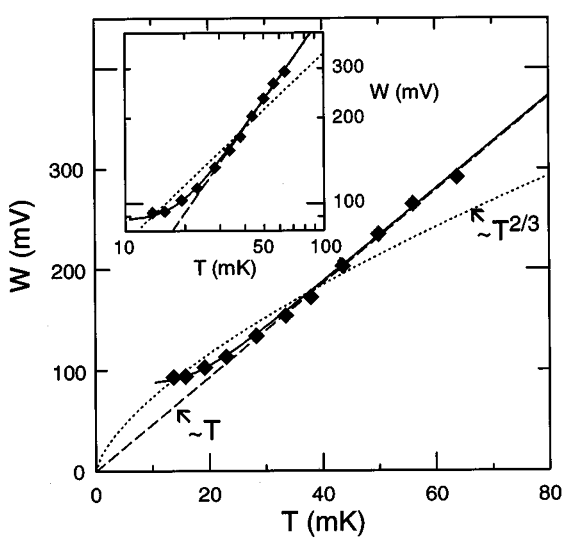


FIG. 4. Width of a quasiparticle resonance as a function of temperature (size of diamonds gives error bars). Dotted line gives $W \sim T^{2/3}$ (expected for electron Luttinger tunneling), dashed line is $W \sim T$ (expected for quasiparticle tunneling in both Fermi and Luttinger liquids). Solid curve is a two-parameter best fit including effects of Joule heating and ac excitation voltage. Inset shows the same in a log-log scale.

the intrinsic $W$ regime, (ii) electron Joule heating by excitation bias and electromagnetic noise, and (iii) smearing by the ac excitation voltage.

(i) If we approach the intrinsic regime, a contribution from $\Gamma$ in the full convoluted line shape Eq. (1) would make $W$ less temperature dependent at low $T$, but would also add a constant to it at high $T$, i.e., $W = aT + W_0$. It is clear from Fig. 2(b) that $\Gamma$ cannot be large, since the RT line shape is sufficiently distinct from Lorentzian even at 14 mK. Furthemore, by determining $W_0$ from the data of Fig. 4 we get an estimate $\Gamma < 0.5$ mK. This definitely rules out significant broadening from finite $\Gamma$.

(ii) It is well known that at low $T$ the electron system can have temperature $T_e$ higher than the lattice temperature $T_L$ (assumed to equal the bath $T$) due to weak electron-phonon coupling. Steady state is reached when the electron-phonon relaxation rate $\tau^{-1}$ is such that the heat input power $P_{\rm in}$ to the electron system is equal to power dissipated to the lattice. Assuming that the dominant relaxation mechanism is piezoelectric coupling to acoustic phonons ($\tau^{-1} \sim T^3$), we have[16] $P_{\rm in} = \beta(T_e^5 - T_L^5)$. Using $W \sim T_e$ we get $W = A(\beta P_{\rm in} + T_L^5)^{1/5}$, introducing only one extra fitting parameter, $\beta P_{\rm in}$. $\beta$ is a function of sample parameters that are impossible to estimate reliably, therefore we do not attempt to extract numerical value for $P_{\rm in}$. Below we will show that (ii) is the dominant broadening mechanism in our sample.

(iii) The effect of finite excitation voltage $V_{\rm ac}$ can be evaluated by calculating the response of the sample for $V_{\rm ac} = V^p_{\rm ac}\sin(\omega t)$. Since the experimental value of $\frac{1}{2}e^* V^p_{\rm ac} \approx 0.9$ $\mu$eV is known, we do not introduce any additional uncertainty in the analysis, and get an experimental value for the ac broadening at any given $T$. Specifically, at $T = 14$ mK the extra width due to finite $V_{\rm ac}$ is $\sim$1 mK, which is potentially measurable, but cannot explain the full experimental broadening, and is therefore a secondary effect compared to electron heating.

Figure 4 shows a two-parameter fit of FD resonance with heating and $V_{\rm ac}$ included (solid curve). We see that the low-$T$ saturation of the width can be explained by the two discussed mechanisms. The fit gives us an estimate for $T_e$ as a function of $T_L$, e.g., $T_e = 19$ mK at $T_L = 14$ mK. In addition, instead of assuming the power law $\tau^{-1} \sim T^3$, we can let the exponent $p$ be an additional free parameter. The best fit then gives $p = 3.3$, a further confirmation of the validity of the analysis. The line shape at $T_L = 14$ mK with effects of heating and $V_{\rm ac}$ included also fits the experimental RT peak well, being practically indistinguishable from the FD fit given in Fig. 2(b).

Finally, we note that it is not absolutely clear whether the theory[4,9] can be applied to quasiparticle tunneling in a situation where $G_p \ll e^2/3h$. Also, the confining potential in experiments is smooth, and so far chiral Luttinger liquid theory has only been considered in the case of sharp confining potentials.[17] In Ref. 14 a RT line shape for quasiparticles was calculated based on chiral Luttinger edge picture in the regime $G_T \ll e^2/3h$ using perturbative techinques. We compared our results also with these predictions in the same way as with Eq. (2) but did not find satisfactory agreement.

In conclusion, we have measured the line shape of quasiparticle tunneling resonances at $\nu = 1/3$ between two edges of the FQH condensate through states circulating an antidot. Line-shape analysis at a fixed temperature is consistent with both Luttinger and Fermi liquid pictures of edge excitations, which cannot be distinguished experimentally. The temperature evolution of the resonances in our sample geometry behave, at least phenomenologically, as Fermi liquids. Recently theoretical treatment of quasiparticle resonant tunneling has been introduced;[18] the predictions of Ref. 18 are significantly similar to those of Eq. (2) in the regime of our experiments.

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[11] The calculation gives $G_T=(R_{(2-3;\,1-4)}-R_L)/[R_H^2-R_H(R_{(2-3;\,1-4)}-R_L)]$, where $R_L=(h/e^2)(1/\nu-1/\nu_B)$ and $R_H=h/\nu e^2$. In the limit $G_T\ll e^2/3h$ this reduces to $G_T\approx(R_{(2-3;1-4)}-R_L)/R_H^2$ directly proportional to the measured $R_{(2-3;1-4)}-R_L$.

[12] The antidot levels (radius is 300 nm) are spaced by $\sim 10$ $\mu$eV at $\nu=1/3$ (Ref. 10).

[13] Tunneling of quasiparticles of charge $e/3$ is possible because the two edges are separated by the $\nu=1/3$ QH condensate. (Ref. 10). Earlier experimental work (Ref. 5) employed the point contact geometry, where resonances are created by random impurities and where both electron and quasiparticle tunneling are possible. The antidot geometry circumvents the need for uncontrollable impurities by bringing in an ''artificial impurity,'' the quantum antidot, between the two point contact gates. Effectively, this geometry was considered theoretically in Ref. 14.

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