Critical conductivity of the quantum Hall system at higher Landau levels

Takamichi Terao
Department of Applied Physics, Hokkaido University, Sapporo 060, Japan
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The two-particle spectral function $S(q,\omega)$ is studied at the mobility edge on the quantum Hall system, in the lowest and higher Landau level. The profile and its scaling properties of $S(q,\omega)$ are numerically clarified. Our result shows that, in higher Landau level with the long-range random potential, the critical dissipative conductivity σ_{xx}^c is approximately equal to the universal value $0.5e^2/h$. This suggests that the critical conductivity σ_{xx}^c on the quantum Hall system with the long-range potential is independent of the Landau-level index N. [S0163-1829(97)12507-3]

In recent years, the localization-delocalization transition in the quantum Hall (QH) system has attracted much attention.[1,2] At the mobility edge, scaling behaviors have been studied in the localization length,[3] the spatial structure (multifractality) of the wave function,[4–6] the static and dynamic conductivity,[7,8] and the two-particle spectral function.[9–11] The localization length diverges as $\xi \propto |E-E_c|^{-\nu}$ at the mobility edge $E=E_c$ at the center of the Landau level, where ν is the critical exponent. The critical properties in the higher Landau level have not been fully clarified. It is shown that, for the correlation length of the random potential $\sigma \approx l$ (l is the magnetic length), the scaling behavior in the higher Landau level is indeed the same as that in the lowest Landau level, i. e., $\nu \approx 2.3$ is independent of the Landau level.[3]

A lot of work has been devoted to the critical conductivity σ_{xx}^c at $E=E_c$ in the QH systems.[9,11–14] The self-consistent Born approximation predicted the conductivity σ_{xx}^c depends on the Landau-level index N and the range of the potential.[12] Recently the critical dissipative conductivity σ_{xx}^c at $E=E_c$ is claimed to be universal in the lowest Landau level, $\sigma_{xx}^c = 0.5e^2/h$, irrespective of the range of the potential.[11] Chalker and Daniell have calculated numerically the two-particle spectral function $S(q,\omega)$ for a QH system in the lowest Landau level and with white-noise potential.[9] Using their numerical method, Huo, Hetzel, and Bhatt have performed the numerical calculations of the critical conductivity σ_{xx}^c in the lowest Landau level with different kinds of random potential.[11] They showed that the dissipative conductivity σ_{xx}^c at $E=E_c$ takes the universal value $0.5e^2/h$ in the lowest Landau level. Lee, Wang, and Kivelson have argued that the critical conductivity σ_{xx}^c is related to the fixed-point scaling amplitude Λ_c.[15] Their result and the reported value of the scaling amplitude Λ_c would support the universal critical conductivity $\sigma_{xx}^c = 0.5e^2/h$. While recent numerical results for σ_{xx}^c in the lowest Landau level seem to agree well with the universal value, the value of σ_{xx}^c in *higher* Landau level is still an open question.

In this paper, we study the two-particle spectral function $S(q,\omega)$ of noninteracting electrons in the center of the Landau level. We clarify the behavior of the two-particle spectral function $S(q,\omega)$ in the lowest and higher Landau level for two different random potentials, with δ scatterer (short-range) potential and the Gaussian-scatterer (long-range) potential. The crossover for $S(q,\omega)$ is shown between the small q^2/ω regime and large q^2/ω regime. In addition, we clarify the critical dissipative conductivity σ_{xx}^c in the higher Landau level with the short-range and the long-range random potential, using the method of Chalker and Daniell.[9] In the higher Landau level ($N=1$) with the long-range potential, the critical dissipative conductivity σ_{xx}^c becomes $\sigma_{xx}^c = 0.5e^2/h$ within the statistical uncertainty.

The Hamiltonian of the QH system is given by

$$\mathcal{H} = \sum_{NX} |NX\rangle \left(N + \frac{1}{2} \right) \hbar \omega_c \langle NX| + \sum_{NX} \sum_{N'X'} |NX\rangle \langle NX|V|N'X'\rangle \langle N'X'| \tag{1}$$

with the Landau wave function $|NX\rangle$, the cyclotron frequency ω_c, and a random potential V. We treat each Landau level separately, assuming that the magnetic field is strong enough with $|\langle NX|V|N'X'\rangle| \ll \hbar\omega_c (N \neq N')$. We consider the short-range, δ scatterer potential as

$$V(\mathbf{r}) = \sum_i V_i \delta(\mathbf{r} - \mathbf{r}_i). \tag{2}$$

Here the position of the scatterer $\mathbf{r}_i$ is randomly distributed with a concentration c_i, while its strength V_i is assumed to take either attractive or repulsive values, $V_i = \pm V_0$, with an equal probability to make the broadened Landau level symmetric. The concentration c_i is defined as $c_i \equiv N_i/N_L$, where N_i and N_L is the number of scatterers and the degeneracy of each Landau level, respectively. We also consider the scatterers finite ranged with Gaussian potentials,

$$V(\mathbf{r}) = \sum_i \frac{V_i}{2\pi\sigma^2} e^{-|\mathbf{r}-\mathbf{r}_i|^2/2\sigma^2}. \tag{3}$$

This makes the correlation of the random potential Gaussian as well.

In the following we take the unit $\hbar=1$. The two-particle spectral function is defined by

$$S(\mathbf{r},\omega;E)\equiv\left\langle \sum_{\alpha,\beta}\delta\left(E-\frac{\omega}{2}-E_\alpha\right)\times\delta\left(E+\frac{\omega}{2}-E_\beta\right)\psi_\alpha(\mathbf{0})\psi_\alpha^*(\mathbf{r})\psi_\beta(\mathbf{r})\psi_\beta^*(\mathbf{0})\right\rangle, \tag{4}$$

where E_α and $\psi_\alpha(\mathbf{r})$ is the energy of the eigenstate α and its eigenvector, respectively, and the angular bracket denotes the ensemble average of the disordered system. The Fourier transform of the spectral function $S(q,\omega)$ can be written as

$$S(q,\omega)=\frac{\rho(E)}{\pi}\frac{q^2D(q,\omega)}{\omega^2+[q^2D(q,\omega)]^2}, \tag{5}$$

for small q and ω, where $D(q,\omega)$ is the wave-vector- and frequency-dependent diffusion function, and $\rho(E)$ is the density of states per unit area.[9,11] At the mobility edge, the diffusion function $D(q,\omega)$ depends only on q^2/ω and Eq. (5) can be rewritten as

$$\omega S(q,\omega)=\frac{\rho(E_c)}{\pi}\frac{(q^2/\omega)\tilde{D}(q^2/\omega)}{1+[(q^2/\omega)\tilde{D}(q^2/\omega)]^2}, \tag{6}$$

where $\tilde{D}(q^2/\omega)\equiv D(q,\omega)$.[9,11] Chalker and co-workers have shown that the wave function on the QH system shows anomalous diffusive behavior at the mobility edge.[9] For $q^2/\omega<a$ (a is a numerical constant), $\tilde{D}(q^2/\omega)$ becomes a diffusion constant D. For $q^2/\omega>a$, $\tilde{D}(q^2/\omega)$ reflects the property of anomalous diffusion and behaves as

$$\tilde{D}(q^2/\omega)\propto(q^2/\omega)^{-\eta/2}, \tag{7}$$

where η describes the anomalous diffusive behavior of the electron at the mobility edge,[2] and related to the generalized fractal dimension D_2 of the critical wave functions ($\eta=2-D_2$). From Eqs. (6) and (7), $\omega S(q,\omega)$ behaves as

$$\omega S(q,\omega)\approx\frac{\rho(E_c)D}{\pi}(q^2/\omega) \tag{8}$$

for small q^2/ω, and

$$\omega S(q,\omega)\propto(q^2/\omega)^{\eta/2-1} \tag{9}$$

for large q^2/ω. From Einstein's relation, the dissipative conductivity at $E=E_c$ can be obtained as[11]

$$\sigma_{xx}^c=e^2\rho(E_c)\lim_{q^2/\omega\to 0}\lim_{\omega\to 0}\tilde{D}(q^2/\omega). \tag{10}$$

From Eqs. (8) and (10), the critical conductivity σ_{xx}^c can be obtained by studying the behavior of $\omega S(q,\omega)$ for $q^2/\omega<a$. And the power-law dependence of $\omega S(q,\omega)$ for $q^2/\omega>a$ gives the value of the generalized fractal dimension D_2 of the critical wave function.

In this paper, we consider a QH system of size L under periodic boundary conditions in x and y directions. We take system size as $L=200l$, which is larger than those in previous studies of $S(q,\omega)$.[9,11] The Hamiltonian [Eq. (1)] is diagonalized for calculating $S(q,\omega)$ by the Lanczos algorithm.[6] It makes the numerical calculations of Eq. (4) quite effective with high accuracy. To check the finite-size effect, we also study $S(q,\omega)$ on QH system for several different system sizes. The calculated results show that the system size dependence is negligibly small for $L\geqslant 150l$, and the results for $L=150l$ and $L=200l$ make no difference within the statistical error. This shows that the system size we treat in these calculations ($L=200l$) is large enough. We have also performed an average over twelve samples.

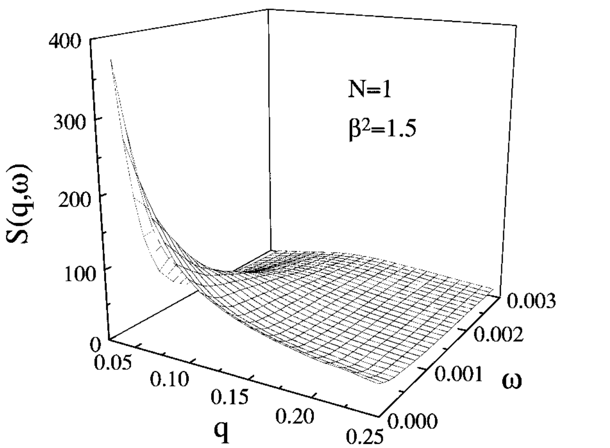

FIG. 1. The wave-number and frequency dependence of $S(q,\omega)$ for $N=1$ with Gaussian-scatterer potential ($\beta^2=1.5$).

Figure 1 shows the profile of the wave-number and frequency dependence of $S(q,\omega)$ for $N=1$ with Gaussian-scatterer potential $[\beta^2\equiv 1+(\sigma/l)^2=1.5]$ at $E=E_c$, for small q and ω. This is the first observation of $S(q,\omega)$ on a QH system in the higher Landau level. Figures 2 and 3 show the rescaled plot of the product of ω and $S(q,\omega)$ at $E=E_c$ for various q and ω. Figure 2 shows $\omega S(q,\omega)$ versus q^2/ω on QH system in the lowest Landau level ($N=0$). Filled squares show the result of $\omega S(q,\omega)$ for δ scatterer po-

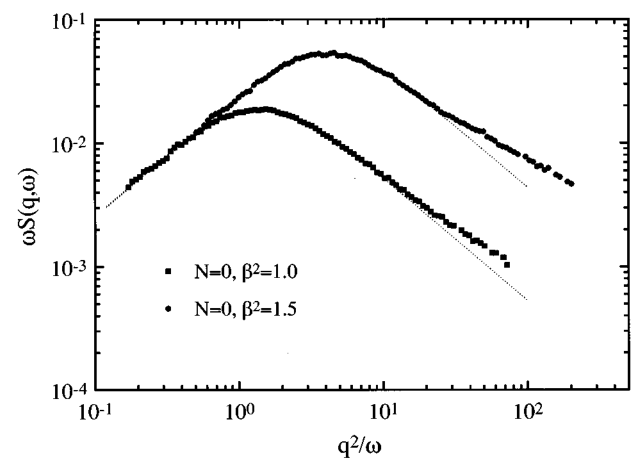

FIG. 2. The q^2/ω dependence of $\omega S(q,\omega)$ for $N=0$. Filled squares show the result for δ scatterer potential ($\beta^2=1.0$). The concentration of the scatterers is taken to be $c_i=10.0$. Filled circles show the result for the Gaussian-scatterer potential ($\beta^2=1.5$). The concentration of the scatterers is $c_i=3.0$. The dotted lines shows the relation written by Eq. (11).

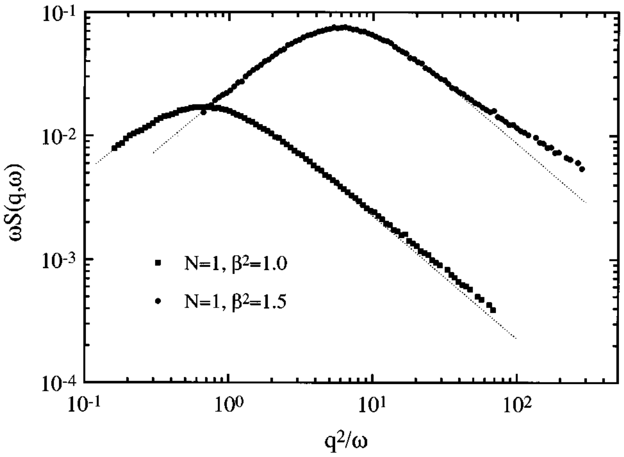

FIG. 3. The q^2/ω dependence of $\omega S(q,\omega)$ for $N=1$. Filled squares show the results for the δ scatterer potential ($\beta^2=1.0$). The concentration of the scatterers is taken to be $c_i=10.0$. Filled circles show the results for the Gaussian-scatterer potential ($\beta^2=1.5$). The concentration of the scatterers is $c_i=3.0$. The dotted lines show the relation written by Eq. (11).

tential ($\beta^2=1.0$), and the concentration of scatterers is taken to be $c_i=10.0$. Filled circles show the result of $\omega S(q,\omega)$ for Gaussian-scatterer potential ($\beta^2=1.5$) and the concentration of scatterers is taken to be $c_i=3.0$. Both results show that two-particle spectral function at $E=E_c$ satisfies Eq. (6) and $\omega S(q,\omega)$ depends only on q^2/ω. Each dotted line shows the relation

$$\omega S(q,\omega)=\frac{\rho(E_c)}{\pi}\frac{(q^2/\omega)D}{1+[(q^2/\omega)D]^2}, \tag{11}$$

where the diffusion constant D is estimated from the numerical results for small q^2/ω in Fig. 2. We show the crossover for $\omega S(q,\omega)$ at $q^2/\omega\approx 10.0$ for $\beta^2=1.0$, and at $q^2/\omega\approx 25.0$ for $\beta^2=1.5$. For small q^2/ω, the calculated results are in good agreement with Eq. (11), where the diffusion function is equal to the diffusion constant D. For large q^2/ω, the property of anomalous diffusion appears and $\omega S(q,\omega)$ shows the power-law behavior written by Eq. (9).[9] Table I shows the values of the critical dissipative conductivity σ_{xx}^c and the generalized fractal dimension D_2 estimated from the anomalous diffusion exponent η in Eq. (9). In both cases σ_{xx}^c takes the universal value, $\sigma_{xx}^c=0.5e^2/h$, which is consistent with the previous studies.[11]

Figure 3 shows the calculated result of $\omega S(q,\omega)$ in the higher Landau level ($N=1$). Filled squares show the result for $\beta^2=1.0$ with the concentration of scatterers $c_i=10.0$. Filled circles show the result for $\beta^2=1.5$ with the concentration of scatterers $c_i=3.0$. Crossover for $\omega S(q,\omega)$ between small q^2/ω and large q^2/ω is shown at $q^2/\omega\approx 7.0$ for $\beta^2=1.0$, and at $q^2/\omega\approx 35.0$ for $\beta^2=1.5$. The values of the critical conductivity σ_{xx}^c and the generalized fractal dimension D_2 for $N=1$ are shown in Table I. The value of σ_{xx}^c for $\beta^2=1.5$ is approximately equal to $0.5e^2/h$, in accordance with that in the lowest Landau level. The value of σ_{xx}^c for $\beta^2=1.0$ is, however, apparently larger than $0.5e^2/h$. These results show that the critical conductivity σ_{xx}^c in higher Landau level depends on the correlation length of the potential, which is different from that in the lowest Landau level. This disagreement coincides with the scaling property of localization in the QH system. A recent finite-size scaling study suggested that, if one introduces an irrelevant scaling length $\xi_{\rm irr}$, the apparent lack of universality is reconciled on length scale $>\xi_{\rm irr}$.[2,3] And in the higher Landau level, the irrelevant length scale $\xi_{\rm irr}$ rapidly decreases and the single-parameter scaling is recovered for large σ when the correlation length of the potential σ is increased from $\ll l$ to $\sim l$. The values of the generalized fractal dimension D_2 are in agreement with our former results obtained from the multifractal analysis of the critical wave function within the statistical error.[6]

TABLE I. The values of critical conductivity σ_{xx}^c and the generalized fractal dimension D_2 of the wave function at the mobility edge, for different Landau level indices and the range of the random potential [$\beta^2\equiv 1+(\sigma/l)^2$]. Each value of D_2 is estimated from the anomalous diffusion exponent η in Eq. (9).

N	β^2	σ_{xx}^c	D_2
0	1.0	$(0.51\pm 0.03)e^2/h$	1.54 ± 0.06
0	1.5	$(0.49\pm 0.03)e^2/h$	1.48 ± 0.09
1	1.0	$(1.06\pm 0.08)e^2/h$	1.80 ± 0.09
1	1.5	$(0.47\pm 0.05)e^2/h$	1.45 ± 0.08

In conclusion, the scaling properties of the two-particle spectral function $S(q,\omega)$ at the mobility edge are studied on the QH system for the lowest and the higher Landau level numerically. The crossover for $S(q,\omega)$ between small q^2/ω regime and large q^2/ω regime is clearly observed. It is also shown that the critical conductivity σ_{xx}^c in the higher Landau level is approximately equal to $0.5e^2/h$ with long-range potential, in accordance with that in the lowest Landau level.

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