Persistent currents from the competition between Zeeman coupling and spin-orbit interaction

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(Received 27 August 1996)

We investigate the role that the spin degree of freedom plays in persistent current and show that Zeeman coupling induces persistent current as conventional Aharonov-Bohm (AB) flux does by competing with spin-orbit (SO) interaction. We demonstrate that, in the presence of SO interaction, Zeeman coupling and AB flux break time-reversal symmetry through intrinsically different mechanisms. A corresponding interesting observable effect for persistent current is generally found and explicitly computed for a symmetric mesoscopic ring with noninteracting electrons. [S0163-1829(97)02707-0]

Persistent currents in multiply connected mesoscopic systems have attracted much attention in recent years.[1] Conventionally, persistent currents are produced by applying magnetic flux to the system and are regarded as one of the famous manifestations of the Aharonov-Bohm (AB) effect. Through a standard current-current coupling, the AB effect only involves the orbital degree of freedom. Since electrons have spin as well as charge, it is therefore of interest to study if the spin degree of freedom can play some effective and significant role in persistent current phenomena.

Recently, based on the discovery of the geometric phase,[2] many authors have investigated the persistent currents induced by the geometric phases, which originate from the interplay between an electron's orbital and spin degrees of freedom. Such interplay can be maintained by external electric and magnetic fields, which lead to spin-orbit (SO) interaction and Zeeman coupling, respectively. Loss *et al.* studied the textured ring embedded in inhomogeneous magnetic field.[3] On the other hand, Meir *et al.* showed that SO interaction in one-dimensional rings results in an effective magnetic flux.[4] Mathur and Stone[5] then pointed out that observable phenomena induced by SO interaction are the manifestations of the Aharonov-Casher (AC) effect[6] in electronic systems. Balatsky and Altshuler,[7] Choi,[8] and Oh and Ryu[9] studied the persistent currents produced by the AC effect.

So far the effects of SO interaction and Zeeman coupling on persistent currents have only been discussed separately. As is well known, SO interaction and Zeeman coupling are of quite different time-reversal transformation properties. SO interaction is time-reversal invariant and results in the AC flux, which induces the charge current in each single-particle electronic state. However, at finite temperature the thermal-equilibrium persistent currents always vanish in systems that possess time-reversal symmetry (TRS). On the other hand, Zeeman coupling breaks TRS. In particular, the finite-temperature persistent current is essentially a manifestation of the breaking of TRS due to certain external influence. We therefore expect that in a many-electron system Zeeman coupling can induce the persistent currents by competing with SO interaction, which cannot produce the finite-temperature persistent current by itself but provides a complete set of current-carrying single-particle states. This implies that Zeeman coupling can induce the persistent currents in a way that is totally independent of its inhomogeneity that results in the geometric phase.[3]

The purpose of this paper is to reveal the role of TRS in persistent current phenomena when the spin degree of freedom is fully taken into account. We show that by competing with spin-orbit interaction, Zeeman coupling can induce persistent currents in the mesoscopic rings with an odd number of electrons even in the presence of disorder and electron-electron interaction, though our illustrative example is a solvable model that is cylindrically symmetric and noninteracting. For noninteracting rings, we demonstrate that in the presence of SO interaction, as a result of the simultaneous spin and orbital quantum number dependence of the energy value and current magnitude for each single-particle Kramers doublet, the TRS breaking mechanism due to Zeeman coupling is intrinsically different from that due to AB flux, which was investigated by Kravtsov and Zirnbauer.[10] As the corresponding observable effect, we find that the direction of the currents induced by Zeeman coupling changes periodically with the particle number, while AB flux determines the direction of the induced currents by its sign alone. Throughout the discussion, we emphasize that the observability of the above-mentioned symmetry breaking effects can only be ensured in systems with a strong SO interaction that can produce an AC phase of order unity. We also point out that the zero-temperature and the low-temperature persistent currents derived for the perfect rings do not change qualitatively if disordered rings are considered. As the observed current magnitude in a strongly interacting GaAs-Al$_x$Ga$_{1-x}$As ring agrees well with the theoretical predictions for weak disorder and small number of transverse channels,[11] we finally have a numerical estimation for a simplified model of a ring formed by a two-dimensional electron gas (2DEG) on a semiconductor heterostructure, which provides a strong effective electric field and makes experimental observation possible.

We first derive the exact solution for a noninteracting ring in the presence of the cylindrically symmetric SO interaction, Zeeman coupling, and AB flux, based on the concepts of spin cyclic evolution and the corresponding Aharonov-Anandan (AA) phase.[12,13] For a 1D ring lying in the x-y plane with its center at the origin, in the presence of both electric and magnetic fields $\mathbf{E}=E(\cos\chi_1\mathbf{e_r}-\sin\chi_1\mathbf{e_z})$, $\mathbf{B}=B(\sin\chi_2\mathbf{e_r}+\cos\chi_2\mathbf{e_z})$ in the cylindrical coordinate system, the one-particle Hamiltonian for noninteracting electrons is given by

$$H=\frac{\hbar^2}{2m_e a^2}\left[-i\frac{\partial}{\partial\theta}+\phi+\alpha(\sin\chi_1\sigma_r+\cos\chi_1\sigma_z)\right]^2+\frac{\hbar\omega_B}{2}(\sin\chi_2\sigma_r+\cos\chi_2\sigma_z), \tag{1}$$

with $\sigma_r=\sigma_x\cos\theta+\sigma_y\sin\theta$, $\alpha=-eaE/4m_ec^2$, and $\omega_B=-geB/2m_ec$, where a is the ring radius, θ is the angular coordinate, and ϕ is the enclosed AB flux in unit of flux quantum. The exact eigenfunctions $\Psi_{n,\mu}$ and eigenvalues $E_{n,\mu}$ of Hamiltonian (1) are obtained as follows:

$$\Psi_{n,\mu}(\theta)=\exp(in\theta)\tilde{\psi}_{n,\mu}(\theta)/\sqrt{2\pi},\quad \mu=\pm, \tag{2}$$

with $\tilde{\psi}_{n,+}(\theta)=[^{\cos(\beta_n/2)}_{e^{i\theta}\sin(\beta_n/2)}]$, $\tilde{\psi}_{n,-}(\theta)=[^{\sin(\beta_n/2)}_{-e^{i\theta}\cos(\beta_n/2)}]$, and

$$E_{n,\mu}=\frac{\hbar\omega_0}{2}(n+\phi)^2+\frac{\hbar\omega_0}{2}(\alpha^2-\alpha\cos\chi_1)+\frac{\hbar\omega_n}{2}(1-\mu\cos\beta_n)+\mu\alpha\hbar\omega_n\cos(\beta_n-\chi_1)+\frac{\mu\hbar\omega_B}{2}\cos(\beta_n-\chi_2), \tag{3}$$

where $\omega_n=(n+\phi+\frac{1}{2})\omega_0$, $\omega_0=\hbar/M_ea^2$, and β_n is given by

$$\tan\beta_n=\frac{2\alpha\omega_n\sin\chi_1+\omega_B\sin\chi_2}{2\alpha\omega_n\cos\chi_1+\omega_B\cos\chi_2-\omega_n}. \tag{4}$$

From the eigenenergies (3), the persistent currents of the single-particle states can be expressed as

$$J_{n,\mu}=n+\phi-(\Phi^\mu_{AA}+\Phi^\mu_{\rm SO})/2\pi, \tag{5}$$

where $\Phi^\mu_{AA}=-\pi(1-\mu\cos\beta_n)$ is the geometric AA phase and $\Phi^\mu_{\rm SO}=-2\mu\pi\alpha\cos(\beta_n-\chi_1)$ the dynamical phase contributed by the SO interaction.[12,13]

For $\phi=0$ and $\omega_B=0$, Eqs. (3), (4), and (5) yield

$$E^0_{n,\mu}=\frac{\hbar\omega_0}{2}\left(n-\frac{\Phi^\mu_{\rm AC}}{2\pi}\right)^2,\quad J^0_{n,\mu}=n-\frac{\Phi^\mu_{\rm AC}}{2\pi}, \tag{6}$$

where the AC phase $\Phi^\mu_{\rm AC}=-\pi(1+2\mu p)$ is the sum of the AA phase $\Phi^\mu_{\rm AA}$ and dynamical phase $\Phi^\mu_{\rm SO}$, with $p=\sqrt{\alpha^2-\alpha\cos\chi_1+\frac{1}{4}}$ and $\Phi^+_{\rm AC}+\Phi^-_{\rm AC}=-2\pi$.[13] This gives the single-particle Kramers doublet $(\Psi^0_{n,\mu},\ \Psi^0_{-n-1,-\mu})$ with the degeneracy $E^0_{n,\mu}=E^0_{-n-1,-\mu}$ and the current relation $J^0_{n,\mu}=-J^0_{-n-1,-\mu}$ in which n is the orbital quantum number while μ is for spin. Due to this single-particle Kramers degeneracy, the entire set of energy eigenstates can be divided into two subsets $\{E_{n,\mu};\ \mu=\pm\}$, mutually degenerate to each other with one-to-one correspondence. For the purpose of later application, we divide p into its integer part m and fractional part f $(-0.5\leqslant f<0.5)$. We note that if $\Phi^\mu_{\rm AC}=0$ or $\pi(\bmod\ 2\pi)$, i.e., $f=-0.5$ or 0, then additional degeneracy appears and in each branch $\{E^0_{n,\mu}\}$ the two single-particle eigenstates with opposite currents become degenerate. Actually, such additional degeneracy is lifted by any weak disorder and consequently each of the single-particle states carries zero current, as illustrated in Fig. 1. As we will see later, so long as the SO interaction is strong enough to result in an AC phase of order unity, a complete set of current-carrying single-particle states resulting from the fractional f is essential to the observability of the low-temperature persistent currents that Zeeman coupling or AB flux induces by lifting of the Kramers degeneracy. Besides the generation of single-particle currents, the fractional part of the AC flux also plays an interesting role in the filling of the Fermi sea of the mesoscopic rings.

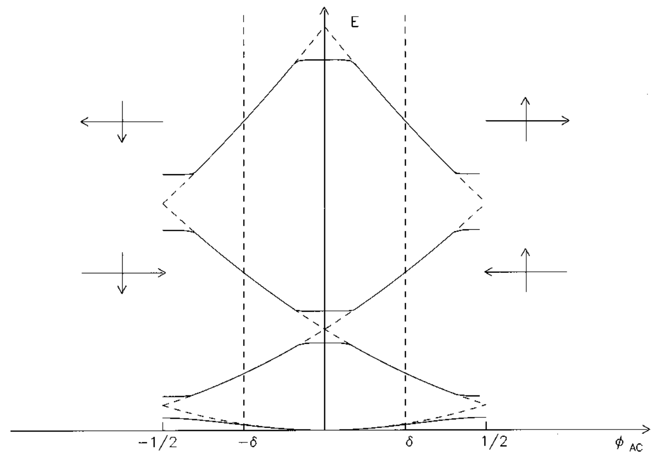

FIG. 1. The current-carrying single-particle states and their level spacing caused by the AC phase of order unity. $\pm2\pi\delta=\Phi^\pm_{\rm AC}(\bmod\ 2\pi)$. The vertical and the horizontal arrows indicate the spin orientation and the current direction, respectively.

For small ϕ and ω_B, the eigenvalues are of the form

$$E_{n,\mu}=E^0_{n,\mu}+\mu\hbar\omega_Bq/2p+\hbar\omega_0(n+1/2+\mu p)\phi, \tag{7}$$

with $q=\alpha\cos(\chi_1-\chi_2)-\frac{1}{2}\cos\chi_2$. The persistent current $J_{n,\mu}$ also changes from $J^0_{n,\mu}$. But this only leads to a high-order correction to the many-electron thermal-equilibrium persistent currents we will derive. Using Eqs. (5), (6), and (7), we can easily derive the low-temperature charge currents for small ϕ and ω_B in the noninteracting many-electron ring. For particle number M, there are three cases with $M=2N$, $M=4N+1$, and $M=4N+3$, respectively, where N is an arbitrary integer. We find the charge current for $M=2N$ is always zero when $\phi=0$ and $\omega_B=0$, even at zero temperature. This is actually a result of the Kramers theorem, which states that the Kramers degeneracy due to TRS only exists for odd M. Accordingly, the even M ground state at $\phi=0$ and $\omega_B=0$ is a singlet that carries no net current. It can be deduced that the dependence of the even M zero-temperature currents on the TRS breaking couplings are smooth and analytical. Kravtsov and Zirnbauer[10] have shown further that, for odd M electronic systems, the AB flux lifts the Kramers degeneracy and induces the persistent currents, which vary discontinuously with the flux at zero temperature.

For $M=4N+1$ and $4N+3$, if $\omega_B\ll N\omega_0$, $\phi\ll1$, and $T\ll N\hbar\omega_0/k_B$, we can consider only two single-particle states at and near the Fermi level to derive the currents because other states are much higher in energy. One is the highest occupied state $\Psi_{n_F,\rho}$ and the other is the lowest unoccupied state $\Psi_{-n_F-1,-\rho}$. These two states form the Kramers doublet as ω_B and ϕ equal zero. Denoting their energies as $\epsilon_\pm = \epsilon_0 \pm \epsilon$ and currents as $J_\pm = \pm J$, we have the thermal-equilibrium currents

$$\langle J \rangle = -J \tanh(\epsilon/k_B T). \tag{8}$$

To justify the above simplification, the energy-level spacing between $E_{n_F,\rho}$ and $E_{-n_F-1,-\rho}$ must be much smaller than the unperturbed spacing between $E^0_{n_F,\rho}$ (or $E^0_{-n_F-1,-\rho}$) and its nearest neighbor in energy in the branch $\{E^0_{n,\rho}\}$ (or $\{E_{n,-\rho}\}$). The level spacing of the latter kind is approximately $2|n_F|\hbar\omega_0 f$ for large $|n_F| \approx N$ with f the smaller of $|f|$ and $0.5-|f|$, and it almost vanishes at $\Phi^\mu_{\rm AC}=0$ and $\pi (\mathrm{mod}\ 2\pi)$ if the ring is weakly disordered. So, in general, besides the generation of single-particle currents, the strong SO interaction that results in the AC phase of order unity is also necessary to the validity of the above two-state approximation. We depict the current-carrying characteristic of the single-particle states and their level spacing in Fig. 1 to illustrate these two essential elementary facts, which result from the AC phase of order unity and constitute the basis of the following derivation.

Using Eqs. (6) and (7), for the system with Hamiltonian (1) and odd M noninteracting electrons, we obtain the thermal-equilibrium persistent currents

$$\langle J \rangle_M = -J_{F,+} \tanh\left\{ \frac{1}{k_B T} \left[\frac{q}{2p} \hbar\omega_B + \hbar\omega_0 J_{F,+} \phi \right] \right\}, \tag{9}$$

with $J_{F,+} = n_F^+ + \frac{1}{2} + p$, where for $M=4N+1$, $n_F^+ = N-m$ if $-0.5<f<0$ or $n_F^+ = -N-1-m$ if $0<f<0.5$; and for $M=4N+3$, $n_F^+ = -N-1-m$ if $-0.5<f<0$ or $n_F^+ = N-m$ if $0<f<0.5$. In deriving the two alternate cases of M, the filling with respect to the splitting of the Kramers doublet plays a crucial role and will be explained below. As T approaches 0^+, the dependences of $\langle J \rangle_{4N+1}$ and $\langle J \rangle_{4N+3}$ on ϕ and ω_B both tend to step functions. What we must point out here is that although Eq. (9) is explicitly derived from the model Hamiltonian (1), the existence of the persistent current induced by Zeeman coupling or AB flux alone in electronic systems with SO interaction is not subject to the absence of disorder or electron-electron interaction. Actually, as already indicated by the above calculation, so long as the time-reversal invariant systems with odd number of electrons have degenerate current-carrying ground states, either Zeeman coupling or AB flux can induce the persistent current by breaking TRS and lifting the Kramers degeneracy. Concerning AB flux only, this point has been emphasized in Ref. 10.

Back to the noninteracting rings, for large N, we see $J_{F,+}$ changes its sign from $M=4N+1$ to $4N+3$, but with its magnitude almost the same. This leads to $\langle J \rangle_{4N+1} \approx -\langle J \rangle_{4N+3}$ for $\phi=0$, and $\langle J \rangle_{4N+1} \approx \langle J \rangle_{4N+3}$ for $\omega_B=0$. So there exists an interesting and striking difference between the symmetry breaking effects of Zeeman coupling and of AB flux. The origin of such a difference is that when lifting the Kramers degeneracy, Zeeman coupling chooses the ground state with a specific spin orientation while AB flux chooses the ground state with a specific current direction, as illustrated in Fig. 2. This is obviously presented in Eq. (7) in which the energy shift comprises the two parts. One is of the form $\mu\hbar\omega_B$ in which only spin orientation

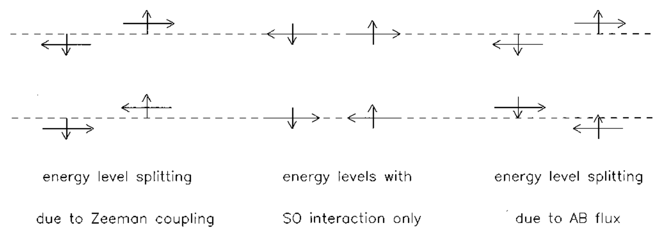

FIG. 2. The single-particle energy levels in the presence of SO interaction only and their shift caused by AB flux and Zeeman coupling.

matters while the other is of the form $J_n\phi$ in which only the sign of the current matters in finding the state with lower energy. Hence, under the influence of AB flux, the sign of the ground-state current never changes with the particle number. But for Zeeman coupling, instead of the sign of the current, the spin orientation of the ground state remains the same. We can understand why Zeeman coupling makes the current sign change periodically with the particle number as follows. Suppose the single-particle state at the Fermi level is $\Psi_{n_F,\rho}$, with $\rho=+$ if $q\omega_B/p<0$ (or $\rho=-$ if $q\omega_B/p>0$), when adding two electrons to the system, first the single-particle state $\Psi_{-n_F-1,-\rho}$ just above the Fermi level is occupied. The second state that becomes occupied is $\Psi_{m_F,\rho}$ and m_F is determined by the condition that $E^0_{m_F,\rho}$ is just above $E^0_{n_F,\rho}$ in the branch $\{E^0_{n,\rho}\}$. In short, adding the electrons two by two effectively lets the single-particle states in $\{\Psi_{n,\rho}\}$ be occupied one by one according to the single-particle energy levels $\{E_{n,\rho}\}$, which are only different from $\{E^0_{n,\rho}\}$ by a uniform shift $\rho\hbar\omega_B q/p$. This picture is true even if the ring is disordered. It is easy to verify from Eq. (6) that due to the noninteger part of $\Phi^\rho_{\rm AC}/2\pi$, the electron filling is not fully symmetric with respect to the quantum number $n+\rho m$. More explicitly, the fraction $1/2+\rho f$ plays an interesting role, which makes the orbital quantum number n_F of the top electron take the values $N-\rho m$ and $-N-1-\rho m$ for the two alternate cases of odd M, respectively. As a result, for large $|n_F|$, $m_F \cong -n_F$. This is to say that the ground-state current changes its sign periodically with the particle number and such sign dependence can survive at low temperature.

It is worthwhile to further investigate the influence of electron-electron interaction on the above results. The experimental observation of the persistent currents in a GaAs-Al$_x$Ga$_{1-x}$As ring shows that electron-electron interactions do not significantly change the value of persistent current, and the observed magnitude is in good agreement with the theoretical prediction for weak disorder and small number of channels.[11] We therefore expect our results are at least qualitatively and probably quantitatively related to some real systems, which are quasi-one-dimensional and weakly disordered. We make a numerical estimation for an InAs ring.[14] The Hamiltonian is of the form

$$H_{\rm InAs} = \frac{1}{2m}\left(\mathbf{p} - \frac{e\mathbf{A}}{c}\right)^2 + \hbar\kappa[\boldsymbol{\sigma}\times\mathbf{p}]_z - \frac{ge\hbar}{4mc}\boldsymbol{\sigma}\cdot\mathbf{B}, \tag{10}$$

where $m=0.023m_e$ is the effective mass, $\hbar^2\kappa = 6.0\times 10^{-10}$ eV cm is the SO coefficient, and $g=15$. Here the effective

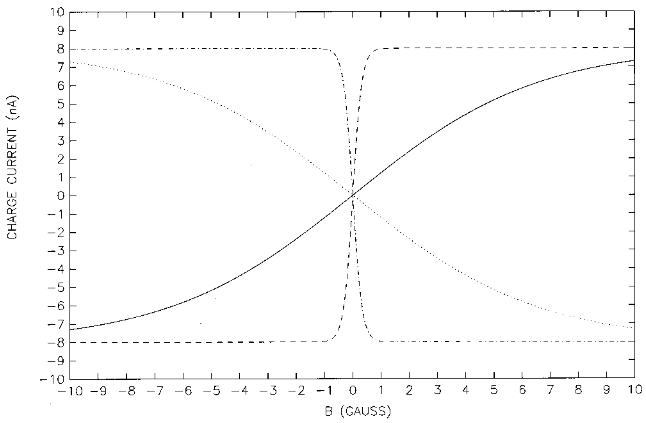

FIG. 3. The charge currents in the InAs ring. The dashed and the dashed-dotted lines are for $M=4N+1$ and $M=4N+3$ at $T=10$ mK. The solid and the dotted lines are for $M=4N+1$ and $M=4N+3$ at $T=100$ mK.

electric field is in the z direction, hence $\chi_1=\pi/2$. For the loop of radius $a=1$ μm, the dimensionless coefficient α is found to be $ma\kappa=1.8$, which is large enough to result in an AC phase of order unity.[13] The Fermi velocity v_F is approximately 3×10^7 cm s^{-1}, corresponding to $|n_F|\approx60$ and $I_F=ev_F/2\pi a\approx8$ nA. Using the exact solution in Eqs. (2), (3), and (4), we numerically compute the charge currents due to Zeeman coupling in the noninteracting system with fixed particle number. The dependence of the currents on the magnetic field is depicted in Fig. 3 for various temperatures, with the zero-temperature limit as a step function. The striking dependence of the sign or the direction of the charge currents on the particle number is also depicted in Fig. 3. We expect these phenomena can be observed in experiments within the reach of existing technology.

In conclusion, we have shown that in the presence of strong SO interaction, Zeeman coupling can induce persistent current by breaking TRS and lifting the Kramers degeneracy. We have demonstrated that the TRS breaking mechanism due to Zeeman coupling is intrinsically different from that due to AB flux. We have found that the direction of the persistent currents induced by Zeeman coupling changes periodically with the particle number, while AB flux determines the direction of the induced currents by its sign alone.

T. Z. Q. and Y. S. Y. would like to thank Professor C. L. Wang for kind help.

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