

## Influence of electron-electron scattering on the electrical resistivity caused by oriented line imperfections

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By means of an exact solution of the Boltzmann transport equation, it is shown for a free-electron metal at low temperatures that electron-electron scattering has no effect (not even a  $T^2$  term) on the electrical resistivity caused by oriented line imperfections. [S0163-1829(97)04007-1]

### I. INTRODUCTION

It is of interest to consider the effect of electron-electron scattering on the electrical resistivity caused by oriented line imperfections. This question is not merely of academic interest, but is also related to a mechanism that has sometimes been invoked to explain anomalies in the low-temperature electrical resistivity of metals, as will be discussed in more detail later.

### II. THEORY

We consider a free-electron metal at low temperatures, supposing that the only scattering mechanisms are isotropic impurity scattering, scattering by oriented line imperfections, and electron-electron scattering. For a homogeneous system in a uniform applied electric field, the Boltzmann equation for the electron distribution  $f_{\mathbf{k}}$  is

$$\frac{\partial f_{\mathbf{k}}}{\partial t} - \frac{e\mathbf{E}}{\hbar} \cdot \nabla_{\mathbf{k}} f_{\mathbf{k}} = \left( \frac{\partial f_{\mathbf{k}}}{\partial t} \right)_{\text{coll}}, \quad (1)$$

where  $f_{\mathbf{k}}$  is the number of electrons having wave vector  $\mathbf{k}$  and  $(\partial f_{\mathbf{k}}/\partial t)_{\text{coll}}$  is the rate of change of the electron distribution due to scattering processes.<sup>1</sup> It is convenient to represent the rate of change of the electron distribution due to collisions by a collision operator acting on the electron distribution,  $C(f_{\mathbf{k}})$ . Making this substitution gives

$$\frac{\partial f_{\mathbf{k}}}{\partial t} - \frac{e\mathbf{E}}{\hbar} \cdot \nabla_{\mathbf{k}} f_{\mathbf{k}} = C(f_{\mathbf{k}}). \quad (2)$$

The total collision operator is the sum of the collision operators for scattering by isotropic impurities, scattering by oriented line imperfections, and electron-electron scattering. For scattering by static defects such as impurities and line imperfections, the collision operator in the Boltzmann equation has the form

$$C(f_{\mathbf{k}}) = \sum_{\mathbf{k}'} [W_{\mathbf{k}\mathbf{k}'} f_{\mathbf{k}'} (1 - f_{\mathbf{k}}) - W_{\mathbf{k}'\mathbf{k}} f_{\mathbf{k}} (1 - f_{\mathbf{k}'})], \quad (3)$$

where  $W_{\mathbf{k}'\mathbf{k}}$  is the transition rate for electron scattering from the plane-wave state  $\mathbf{k}$  to the plane-wave state  $\mathbf{k}'$ .

Scattering by oriented line imperfections is treated first. For randomly distributed line imperfections, we make the usual approximation that interference terms from scattering by different line imperfections can be neglected. The transition rate  $W_{\mathbf{k}'\mathbf{k}}$  for electron scattering by oriented line imperfections is then

$$W_{\mathbf{k}'\mathbf{k}} = N_d w_{\mathbf{k}'\mathbf{k}}, \quad (4)$$

where  $N_d$  is the number of line imperfections and  $w_{\mathbf{k}'\mathbf{k}}$  is the transition rate for scattering by a single line imperfection. Because of translation invariance in the  $z$  direction, the transition rate  $w_{\mathbf{k}'\mathbf{k}}$  for electron scattering by a line imperfection parallel to the  $z$  axis conserves the  $z$  component of wave vector  $k_z$ ; moreover, for  $k_z = k'_z$ , the transition rate is independent of  $k_z$ . Since the scattering is elastic, electron energy is also conserved, i.e.,  $\varepsilon = \varepsilon'$ . Because of cylindrical symmetry,  $w_{\mathbf{k}'\mathbf{k}}$  does not depend upon the direction of wave-vector transfer (in the  $xy$  plane)  $\mathbf{q} = \mathbf{k}' - \mathbf{k}$ . It is assumed that  $w_{\mathbf{k}'\mathbf{k}}$  is also independent of the magnitude of wave-vector transfer for  $q \leq 2k_F$ ; this assumption corresponds to the leading term for low-energy scattering by a short-range potential, i.e., in an expansion of the scattering rate in powers of the electron energy in the  $xy$  plane,  $E = \hbar^2(k_x^2 + k_y^2)/2m$ .<sup>2</sup> In order to illustrate a scattering rate having these properties, we let  $V(\rho)$  with  $\rho = \sqrt{x^2 + y^2}$  be the scattering potential of a line imperfection, treat the scattering by  $V(\rho)$  in Born approximation, and assume that the Fourier transform  $\Lambda(q)$  of  $V(\rho)$  is independent of  $q$  for  $q \leq 2k_F$ , giving

$$w_{\mathbf{k}'\mathbf{k}} = \frac{2\pi}{\hbar} \frac{\Lambda^2}{A^2} \delta_{k_z, k'_z} \delta(\varepsilon - \varepsilon'), \quad (5)$$

where  $A$  is the cross-sectional area of the sample in the  $xy$  plane. Since  $\Lambda(q)$  is constant for  $q \leq 2k_F$ , so is  $w_{\mathbf{k}'\mathbf{k}}$ . Conservation of  $k_z$  in Eq. (5) implies that a current in the  $z$  direction is not degraded by scattering from line imperfections oriented in the  $z$  direction; this is the ultimate reason why oriented line imperfections do not contribute to the electrical resistivity in the direction parallel to their orientation.

Substituting Eqs. (4) and (5) into the Boltzmann collision operator, Eq. (3), yields for the scattering by oriented line imperfections,

$$C_d(f_{\mathbf{k}}) = \frac{2\pi}{\hbar} \frac{n_d}{A} \Lambda^2 \sum_{\mathbf{k}'} \delta_{k_z, k'_z} \delta(\varepsilon - \varepsilon') (f_{\mathbf{k}'} - f_{\mathbf{k}}), \quad (6)$$

where  $C_d$  is the collision operator for oriented line imperfections and  $n_d$  is the number of line imperfections per unit area. If  $f_{\mathbf{k}} = g(\varepsilon)k_z$ , where  $g$  depends only on the electron energy  $\varepsilon = \hbar^2 k^2/2m$ , then it follows from conservation of energy and  $z$  component of the wave vector that

$$C_d[g(\varepsilon)k_z] = 0, \quad (7)$$

i.e.,  $g(\varepsilon)k_z$  is an exact eigenfunction of the collision operator with eigenvalue 0. On the other hand, if  $f_{\mathbf{k}} = g(\varepsilon)k_\alpha$  with  $\alpha = x$  or  $y$ , then because the scattering (in the  $xy$  plane) is isotropic, the net scattering into the state  $\mathbf{k}$  vanishes, leaving the net scattering out of the state  $\mathbf{k}$ . Equation (6) then gives

$$C_d[g(\varepsilon)k_\alpha] = -\frac{2\pi}{\hbar} n_d \Lambda^2 \rho(E) g(\varepsilon) k_\alpha, \quad \alpha = x \text{ or } y, \quad (8)$$

where  $E = \hbar^2(k_x^2 + k_y^2)/2m$  is the electron energy in the  $xy$  plane and  $\rho(E)$  is the two-dimensional density of states per unit area (for a single spin direction). But the two-dimensional density of states is a constant, independent of both  $E$  and  $k_z$ . If this were not the case, then different slices of the Fermi sphere parallel to the  $xy$  plane, having different values of  $E$  and  $k_z$  at the Fermi energy, would relax at different rates due to scattering by line imperfections, and the steady-state electron distribution in an applied electric field would be distorted from a uniformly shifted Fermi sphere. Substituting  $\rho(E) = m/2\pi\hbar^2$  in Eq. (8), we obtain

$$C_d[g(\varepsilon)k_\alpha] = -\frac{1}{\tau_d} [g(\varepsilon)k_\alpha], \quad \alpha = x \text{ or } y, \quad (9)$$

where  $1/\tau_d = n_d \Lambda^2 m/\hbar^3$ , i.e.,  $g(\varepsilon)k_\alpha$  with  $\alpha = x$  or  $y$  is an exact eigenfunction of the collision operator with eigenvalue  $-1/\tau_d$ .<sup>3</sup>

We treat next isotropic impurity scattering. Surprisingly, the eigenfunctions of the collision operator  $C_d$  for scattering by oriented line imperfections exhibited in Eqs. (7) and (9) are also exact eigenfunctions of the collision operator  $C_i$  for scattering by isotropic impurities. If  $f_{\mathbf{k}} = g(\varepsilon)k_\alpha$  with  $\alpha = x, y, \text{ or } z$ , then the effect of scattering by isotropic impurities is

$$C_i[g(\varepsilon)k_\alpha] = -\frac{1}{\tau_i(\varepsilon)} [g(\varepsilon)k_\alpha], \quad \alpha = x, y, \text{ or } z, \quad (10)$$

where  $1/\tau_i = \int n_i v \sigma(\theta) (1 - \cos\theta) d\Omega$ ,  $n_i$  is the density of impurities,  $v = \hbar k/m$  is the electron velocity,  $\sigma(\theta)$  is the differential cross section for scattering by an impurity, and the integration is over solid angle  $\Omega$ .<sup>4</sup> Although, in general, the relaxation time  $\tau_i$  depends upon electron energy, in a metal only the electron distribution near the Fermi surface is per-

turbed from equilibrium by an applied electric field. Consequently, for the electrical resistivity of a metal,  $\tau_i$  is evaluated at the Fermi energy  $\varepsilon_F$ .

The effects of scattering by isotropic impurities and oriented line imperfections are now combined to derive the electrical resistivity. Expanding the steady-state electron distribution to the first order in the applied electric field,

$$f = f_0 + f_1, \quad (11)$$

where  $f_0(\varepsilon)$  is the equilibrium Fermi distribution, substituting in the Boltzmann equation, Eq. (2), and noting that the equilibrium distribution  $f_0$  is unchanged by collisions, we obtain to the first order in the applied electric field,

$$-e\mathbf{E} \cdot \mathbf{v} \frac{df_0}{d\varepsilon} = C_i(f_1) + C_d(f_1). \quad (12)$$

Making use of the eigenfunctions of the collision operators in Eqs. (7), (9), and (10), we find that, if the applied electric field is in the  $z$  direction, the solution for  $f_1$  is

$$f_1 = \tau_i e \mathbf{E} \cdot \mathbf{v} \frac{df_0}{d\varepsilon} \quad (13)$$

and the electrical resistivity is

$$\rho_{zz} = \frac{m}{ne^2 \tau_i}; \quad (14)$$

whereas, if the applied electric field is in the  $xy$  plane,

$$f_1 = \frac{\tau_i \tau_d}{\tau_i + \tau_d} \mathbf{E} \cdot \mathbf{v} \frac{df_0}{d\varepsilon} \quad (15)$$

and the electrical resistivity is

$$\rho_{xx} = \rho_{yy} = \frac{m}{ne^2} \left( \frac{1}{\tau_i} + \frac{1}{\tau_d} \right). \quad (16)$$

Two theorems are immediate. First, a comparison of Eqs. (14) and (16) proves theorem I: oriented line imperfections do not contribute to the electrical resistivity in the direction parallel to their orientation. Second, the solutions for  $f_1$  in Eqs. (13) and (15) show that the steady-state electron distribution is a uniformly shifted Fermi sphere (to the first order in the applied electric field), even in the presence of anisotropic scattering by oriented line imperfections. Although the magnitude of the shift of the electron distribution depends upon the direction of the applied electric field, the shape of the shifted distribution remains spherical. But electron-electron collisions do not affect a uniformly shifted Fermi sphere, as is evident by making a Galilean transformation to the frame of reference in which the drift velocity of the electrons vanishes. This proves theorem II: electron-electron scattering has no effect (to the leading order) on the electrical resistivity caused by oriented line imperfections.

If one were to describe (incorrectly) the anisotropic scattering caused by oriented line imperfections by postulating an anisotropic wave-vector-dependent relaxation time  $\tau_d(\theta)$ , where  $\tau_d(\theta)$  is a (non-negative) function of the angle  $\theta$  between the wave vector  $\mathbf{k}$  and the orientation axis of the

line imperfections, then it is easily seen that both theorems proven above are violated. First, such a postulate implies that an electron excited at the Fermi surface with wave vector  $\mathbf{k}$  is relaxed to equilibrium uniformly over the entire Fermi surface by collisions with oriented line imperfections. Thus the component of electron velocity parallel to the orientation axis would not be conserved by these collisions, leading to a nonzero contribution by line imperfections to the electrical resistivity in the direction parallel to their orientation, which contradicts theorem I. To show this explicitly, we substitute

$$C_d(f_1) = -\frac{f_1}{\tau_d(\theta)} \quad (17)$$

in Eq. (12) and solve for  $f_1$ , obtaining

$$f_1 = \frac{\tau_i \tau_d(\theta)}{\tau_i + \tau_d(\theta)} \mathbf{E} \cdot \mathbf{v} \frac{df_0}{d\varepsilon}. \quad (18)$$

Evidently, for  $\mathbf{E} = E_z \hat{z}$ , the electrical conductivity  $\sigma_{zz} = j_z/E_z$  is affected by the scattering from line imperfections oriented in the  $z$  direction. Second, it is seen from the solution for  $f_1$  in Eq. (18) that the steady-state electron distribution is no longer a uniformly shifted sphere (to the first order in the applied electric field), but is distorted from its original spherical shape. Since the effect of electron-electron scattering is to restore the spherical shape of the electron distribution, electron-electron scattering would modify the steady-state electron distribution, thereby increasing the electrical resistivity caused by oriented line imperfections, which contradicts theorem II. (The increase of the resistivity when the distribution is modified follows from the variational theorem.<sup>5</sup>) Thus the assumption of a wave-vector-dependent relaxation time, which is often a physically appealing approximation, leads in this instance to qualitatively incorrect conclusions, and therefore the exact eigenfunctions of the collision operator must be used.

### III. DISCUSSION

At temperatures sufficiently low that electron-phonon scattering can be neglected, the electrical resistivity of a metal is expected to have the form,

$$\rho = \rho_0 + AT^2, \quad (19)$$

where  $\rho_0$  is the temperature-independent residual resistivity due to scattering by impurities, dislocations, and other static defects; and  $AT^2$  is the contribution to the resistivity from

electron-electron scattering. This behavior has been observed in potassium metal below about 1.5 K.<sup>6</sup> Contrary to the expectation for a free-electron metal, however, the coefficient of the  $T^2$  term was found to be sample dependent, varying by as much as a factor of 15. Two explanations of this sample dependence have been proposed. One attributes the temperature-dependent term to electron-electron scattering and electron-phonon scattering in the presence of a charge-density wave; and explains the sample dependence as a consequence of the uncontrolled orientational texture of the charge-density-wave  $\mathbf{Q}$ -domain distribution.<sup>7</sup> The other postulates a wave-vector-dependent relaxation time  $\tau_d(\mathbf{k})$  due to scattering by oriented dislocations (the scattering being caused primarily by the dislocation cores) and attributes the temperature-dependent term to the effect of electron-electron scattering on the resistivity from oriented dislocations in addition to the direct effect of electron-electron scattering; the sample dependence is then a consequence of variations in the dislocation density.<sup>8,9</sup>

Neutron-scattering experiments on potassium show that a well-annealed, carefully grown single crystal has a mosaic block structure, the block size being about 1 mm and the angular spread being about  $0.1^\circ$ .<sup>10,11</sup> Dislocations occur mainly in the small-angle boundaries between adjacent mosaic blocks, the average distance between dislocations being about 500 lattice parameters and the average dislocation density being less than  $10^6 \text{ cm}^{-2}$ . Since the distance between dislocations is much larger than the electron Fermi wavelength, the dislocations can be treated as independent scatterers. Since the mosaic block size is about 1 mm, it is likely that the dislocations are oriented over a length scale of an electron mean free path, which is about 0.1 mm in high-purity potassium at low temperatures. (In any case, this assumption is required by the mechanism proposed in Ref. 8.) Since large-angle scattering, which is important for the electrical resistivity, is caused mainly by the dislocation core, a dislocation can be approximated by a short-range line potential.<sup>12</sup> Consequently the foregoing theory is applicable to the mechanism proposed in Ref. 8. It has thus been shown here, using the exact eigenfunctions of the Boltzmann collision operator, that this mechanism vanishes in the leading order. (Even if it were present in the leading order, the dislocation density required is at least several orders of magnitude too large, as pointed out by Guban.<sup>13</sup>)

### ACKNOWLEDGMENT

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<sup>1</sup>For an introduction to the Boltzmann transport equation, see, for example, N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Saunders College, Philadelphia, 1976).

<sup>2</sup>This expansion is given by an effective range theory in two dimensions. For a derivation in three dimensions, see H. A. Bethe, *Phys. Rev.* **76**, 38 (1949).

<sup>3</sup>This agrees with the result given by E. A. Kaner and E. P. Feld-

man, *Fiz. Tverd. Tela* (Leningrad) **10**, 3046 (1968) [*Sov. Phys. Solid State* **10**, 2401 (1969)]. More generally, any distribution  $f_{\mathbf{k}}$  can be decomposed into an axially symmetric part, obtained by averaging  $f_{\mathbf{k}}$  over the azimuthal angle of the wave vector  $\mathbf{k}$ , and a nonsymmetric part, which is the remainder. Within the approximation that the transition rate is a constant, the symmetric and nonsymmetric parts are exact eigenfunctions of the col-

- lision operator with eigenvalues 0 and  $-1/\tau_d$ , respectively.
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