Phase-dependent thermal transport in Josephson junctions

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(Received 26 September 1996)

We calculated the energy current through a Josephson junction and found it to consist of three contributions: a quasiparticle current, an interference current, and a pair current, similar to the total electrical current in a Josephson junction. The quasiparticle part satisfies Onsager relations, and represents the normal dissipative heat current. The other two parts depend on the phase drop across the junction $\delta\theta$ and are related to pair tunneling. We show that the pair energy current, like the Josephson current, is nondissipative. It appears only when there is a voltage across the junction, and therefore oscillates in time. The interference energy current appears when either a voltage or a temperature drop is present. In the latter case, the interference current can flow in *either* direction, depending on the sign of cos($\delta\theta$). Thus, this part of the energy current can flow in the *opposite* direction to the temperature drop, causing a reduction in the dissipation as compared to what would occur without the interference current. Nevertheless, the second law of thermodynamics is not violated with respect to the *total* current. This effect is related to the coupling of quasiparticles and pairs in the superconducting electrodes comprising the junction. [S0163-1829(97)05006-6]

I. INTRODUCTION

The total electrical current in a superconductor-insulator-superconductor (SIS) Josephson junction consists of three parts: A quasiparticle current, an interference current, and the Josephson current.[1] The quasiparticle current represents a nonequilibrium response to a voltage or to a temperature drop across the junction. It is therefore associated with dissipation. The interference current[2,3] is proportional to the cosine of the phase drop across the junction, and is related to coupling of the quasiparticles and the condensate in the electrodes. The Josephson current flows in the absence of a voltage or a temperature drop, and is proportional to the sine of the phase drop across the junction. In a previous paper[4] we discussed the thermoelectric properties of a SIS Josephson junction. We have seen that only the quasiparticle current flows in response to a temperature drop. The fact that the interference current did not have a thermoelectric response was interpreted as an indication that this term was actually a nondissipative current. We also discussed the effect of the coupling of quasiparticles and condensate on the thermoelectric properties of the system.

These results lead us to the conclusion that we could expect interesting phenomena with respect to the *energy* transfer through the junction. Hence, we study here the thermal transport in a SIS Josephson junction and complete the description of thermoelectric transport in this system. We calculate the energy current through the junction, using perturbation theory. From the viewpoint of the theory of irreversible processes,[5] we expect the quasi-particle current to correspond to a heat current which satisfies the Onsager relations. The entropy production rate can be calculated from the heat current. This should give us the Joule heat generated by the normal current component. We also expect, in view of the quasi-particle-pairs coupling and the resulting interference current, that anomalous contributions will be found in the thermal transport.

In Sec. II we perform the perturbation theory calculation of the energy current through the junction. Three contributions emerge: one being the quasiparticle heat current and two additional pair related terms. A physical interpretation to the nature of the pair terms is presented in Sec. III. We conclude that these terms represent *reversible* energy transfer, which is facilitated by the coupling existing between quasiparticles and the condensate. We conclude in Sec. IV.

II. THE ENERGY CURRENT THROUGH A JOSEPHSON JUNCTION

A. The model

In order to determine the thermal current through a SIS Josephson junction, we calculate the total energy current flowing through the junction within the following model. The junction is comprised of two BCS bulk superconductors separated by an insulating barrier. We assume each superconductor is a particle reservoir in equilibrium and is characterized by a many-body BCS Hamiltonian H, a chemical potential μ, and a temperature T. The left-hand side (LHS) quantities are denoted by the subscript l and the right-hand side (RHS) quantities by r. The particle current is a tunneling current, therefore the total Hamiltonian includes a tunneling element H_T. The total Hamiltonian can be written as $H_{\text{tot}}=H_l+H_r+H_T$. The particle current is calculated using a microscopic perturbation theory, expanding in the small tunneling matrix element. The total particle current is equal to the rate of change of the averaged electron-number operator in the LHS reservoir P_l with respect to time.[1] The electric current is the particle current multiplied by the electric charge carried by the particle. We propose to calculate the energy current in a similar way: an electron, tunneling between the electrodes, carries a quantum of energy ϵ from one side and adds it to the other. In the case of a normal electron, the energy dissipated into the reservoir due to this process is the thermodynamic average of the difference $\epsilon-\mu$. Therefore, we will calculate the thermodynamic average of the time derivative of the operator $H-\mu P$.[5] We expect that with respect to the quasiparticles in the superconductor, this current will correspond to the heat carried by them. We assume that the oxidized layer separating the superconductors is a good heat insulator, thus heat is transported only by tunneling particles. According to the quantum-mechanical equation of motion, the energy change on the LHS is

$$\frac{dQ^l}{dt}=\left\langle \frac{d}{dt}(H_l-\mu_l P_l)\right\rangle=\frac{i}{\hbar}\langle[H_{\rm tot},H_l-\mu_l P_l]\rangle. \qquad (1)$$

The angular brackets in Eq. (1) represent a thermodynamic average over a grand-canonical ensemble. The operators are given by the following expressions. The electron-number operator is

$$P_l=\sum_{k,\sigma} C^\dagger_{k,\sigma}C_{k,\sigma}, \qquad (2)$$

where $C^\dagger_{k,\sigma}$ and $C_{k,\sigma}$ are single-electron creation and annihilation operators in the momentum (k) and spin (σ) representation. The momentum quantum number of the LHS (RHS) superconductor is denoted by $k(q)$. The tunneling Hamiltonian is

$$H_T=\sum_{k,q,\sigma} T_{kq}C^\dagger_{k,\sigma}C_{q,\sigma}+\text{H.c.}, \qquad (3)$$

where the tunneling matrix element is denoted by T_{kq}. In Josephson junctions the relevant physical parameter is a phase difference across the junction. Thus, we must keep track of the phases. Consequently, throughout the following calculations, the BCS order parameter Δ in the bulk superconducting electrodes will be a complex function.

Substituting the expressions for the operators into Eq. (1) we obtain for an isotropic superconductor

$$\frac{dQ^l}{dt}=\frac{2}{\hbar}\text{Im}\left[\sum_{k,q,\sigma}\xi_k T_{kq}\langle C^\dagger_{k,\sigma}C_{q,\sigma}\rangle+2T_{kq}\Delta^\dagger_k\langle C_{-k,-\sigma}C_{q,\sigma}\rangle\right], \qquad (4)$$

where $\xi_k\equiv\epsilon_k-\mu_l$ is the electron energy relative to the chemical potential.

B. Three contributions to the energy current

Following the recipe in Ref. 6, we calculate in the Appendix the energy change in the LHS electrode, using first-order perturbation theory. The energy current, flowing from left to right, is $-(dQ^l/dt)$ and will be denoted by $Q^l_{\rm tot}$. The result is

$$\begin{aligned}Q^l_{\rm tot}=Q^l_{\rm qp}+Q^l_{\rm qp\ pair}+Q^l_{\rm pair}=&\frac{4\pi}{\hbar}\int_{-\infty}^{\infty}dw\Theta(w^2-\Delta^2_{\max})N_lN_r|T_{lr}|^2[f_l(w)-f_r(w-\delta\mu)]\frac{w|w||w-\delta\mu|}{\sqrt{w^2-\Delta_l^2}\sqrt{(w-\delta\mu)^2-\Delta_r^2}}\\&+\frac{4\pi}{\hbar}\int_{-\infty}^{\infty}dw\Theta(w^2-\Delta^2_{\max})N_lN_r|T_{lr}|^2[f_l(w)-f_r(w-\delta\mu)]\cos[(\delta\theta+2\delta\mu t)]|\Delta_l||\Delta_r|\frac{w\,\text{sgn}(w)\text{sgn}(w-\delta\mu)}{\sqrt{w^2-\Delta_l^2}\sqrt{(w-\delta\mu)^2-\Delta_r^2}}\\&+\frac{4}{\hbar}\int_{-\infty}^{\infty}\int_{\infty}^{\infty}dwdw'\,\Theta(w^2-\Delta_l^2)\Theta(w'^2-\Delta_r^2)N_lN_r|T_{lr}|^2[f_l(w)-f_r(w')]\\&\times P\frac{\sin[(\delta\theta+2\delta\mu t)]}{w'-w-\delta\mu}|\Delta_l||\Delta_r|\frac{w\,\text{sgn}(w)\text{sgn}(w')}{\sqrt{w^2-\Delta_l^2}\sqrt{w'^2-\Delta_r^2}}. \end{aligned} \qquad (5)$$

We denote the phase difference $\theta_l-\theta_r$ across the junction by $\delta\theta$. $E_k=\sqrt{\xi_k^2+\Delta_k^2}$ is the BCS quasiparticle energy. The difference between quasiparticle chemical potentials of the two superconductors is $\mu_r-\mu_l\equiv\delta\mu$. Note that in all the integrals in Eq. (5) a θ function like $\Theta(w^2-\Delta^2_{\max})$ restricts the quasiparticle energy w to be above the BCS gap. In the first two integrals this is $\Delta_{\max}\equiv\max[\Delta_l,\Delta_r]$, where Δ_l and Δ_r are assumed constant. Also note that for convenience we omitted the $\hbar$ in the second part of the argument of the sin and cos functions. The quasiparticle distribution function is denoted by $f(w)=1/[\exp(w-\mu)/k_BT]$. The normal-metal densities of states (DOS) are denoted by $N_l(\xi)$ and $N_r(\xi)$ whereas the tunneling matrix element is T_{lr}. The third integral on the RHS is the principal value of the integral over the pole in the integrand, and is denoted by P.

As evident in Eq. (5), the total current breaks up into three parts. The first integral is the normal heat current due to tunneling of quasiparticles, and is understood within the semiconductor model.[7] It represents the energy carried by the tunneling quasiparticles, which is dissipated in the electrodes. Like its electric counterpart,[1] it ensues from the normal spectral-density functions $A_kA_q(w)$ [see Eq. (A6) in the Appendix]. It vanishes when $\delta\mu=\delta T=0$. Similar to the behavior in bulk superconductors, $Q^l_{\rm qp}$ decreases exponentially in (Δ/k_BT) as the temperature is reduced below T_c. As expected, this term also satisfies Onsager's relations. This can be shown by expanding the energy dependent functions $N_l(\xi)$, $N_r(\xi)$, and $T_{lr}(\xi)$ to first order in the electronic energy ξ. As explained in Ref. 4, the expansion must be carried out in Eq. (A6) and not in Eq. (5). We find that $L^s_{21}\equiv Q^l_{\rm qp}/\delta T$ is equal to TL^s_{12}, which was obtained in Ref. 4. The notation L^s_{ij} depicts the elements of the 2×2 quasiparticle thermoelectric transport coefficient matrix of the system in the superconducting state.

In addition to the normal heat current, the energy current Eq. (5) includes two anomalous terms. These contributions-represent the energy transported by the *pairs*. Unlike the quasiparticle current, this energy cannot be dissipated because there is no heat reservoir which corresponds to pair states: the condensate forms a single-equilibrium, many-body quantum state. Therefore, we will argue that these terms represent the reversible exchange of the energy into the surrounding magnetic field. In Sec. III we present the arguments which lead us to this understanding. In the Appendix we find that these terms originate from the *mixed* term in Eq. (A6) that is proportional to $A_k(w)B_q(w)$. This stands in contrast to the electrical current, where both the Josephson current and the interference current are given by the anomalous term proportional to $B_k(w)B_q(w)$. In the case of energy transport, this latter term vanishes in the calculation. The new mixed term is dependent on $\delta\theta$ and implies a novel mechanism of energy transport, which is related to the coupling between the quasiparticles and the condensate in the bulk superconductor electrodes. As we show below, this coupling leads to an *effective* transfer of energy between the electrodes. Note that there is another mixed term in Eq. (A6) which is proportional to $B_k(w)A_q(w)$. This term does not depend on the phase drop across the junction, and contributes to the quasiparticle current. The two pair terms in Eq. (5) are proportional to cos($\delta\theta$) and sin($\delta\theta$), respectively. The term $Q_l^{\text{qp pair}}$ is analogous to the electrical interference current, whereas Q_l^{pair} resembles the Josephson current. However, we emphasize that mathematically they originate from a different source. Both $Q_l^{\text{qp pair}}$ and Q_l^{pair} vanish as the temperature of the system approaches the superconducting transition temperature T_c.

III. INTERPRETATION

In order to gain some physical insight of the nature of the anomalous pair terms, we study Q_{tot}^l in special cases and calculate the entropy production rate σ related to them. Since we interpreted Q_{qp}^l as the heat current of quasiparticles, we can assume that the entropy current is given by Q_{qp}^l/T_l. The entropy production rate is given by the thermodynamic relation[5]

$$\sigma=\frac{Q_l}{T_l}+\frac{Q_r}{T_r}. \tag{6}$$

Equation (6) represents the fact that the heat transferred by the carriers from the LHS to the RHS is conserved. Hence, when summing the reverse heat currents one is left with the heating in the system (e.g., Joule heating). This can be expressed as the entropy production rate in the system. For the sake of clarity, we study two special cases: a junction biased only by a voltage and a junction biased by a temperature drop.

A. A voltage biased junction

First we consider a system in which a voltage $V=-\delta\mu/e$ $(e>0)$ appears across the junction (e.g., an external current that is larger than the critical current is applied to the junction), but the temperature drop across it vanishes. Inserting Eq. (5), and the equivalent expression for Q_{tot}^r [which can be easily derived from Eq. (5)] into Eq. (6), we obtain the net entropy production rate in the Josephson junction. Note that we also included the pair terms in the derivation of σ, even though the contribution of these terms is not dissipative, as we discussed above. In the limit of small bias we find

$$\begin{aligned}\sigma=&\frac{V}{T}\frac{4\pi e}{\hbar}\int_{-\infty}^{\infty}dw\Theta(w^2-\Delta_{\max}^2)N_lN_r|T_{lr}|^2[f_l(w)-f_r(w-\delta\mu)]\frac{|w||w-\delta\mu|}{\sqrt{w^2-\Delta_l^2}\sqrt{(w-\delta\mu)^2-\Delta_r^2}}\\&+\frac{V}{T}\frac{4\pi e}{\hbar}\int_{-\infty}^{\infty}dw\Theta(w^2-\Delta_{\max}^2)N_lN_r|T_{lr}|^2[f_l(w)-f_r(w-\delta\mu)]\cos[(\delta\theta+2\delta\mu t)]|\Delta_l||\Delta_r|\frac{\text{sgn}(w)\text{sgn}(w-\delta\mu)}{\sqrt{w^2-\Delta_l^2}\sqrt{(w-\delta\mu)^2-\Delta_r^2}}\\&+\frac{V}{T}\frac{4e}{\hbar}\int_{-\infty}^{\infty}\int_{\infty}^{\infty}dwdw'\Theta(w^2-\Delta_l^2)\Theta(w'^2-\Delta_r^2)N_lN_r|T_{lr}|^2[f_l(w)-f_r(w')]\\&\times P\frac{\sin[(\delta\theta+2\delta\mu t)]}{w'-w-\delta\mu}|\Delta_l||\Delta_r|\frac{\text{sgn}(w)\text{sgn}(w')}{\sqrt{w^2-\Delta_l^2}\sqrt{w'^2-\Delta_r^2}}.\end{aligned} \tag{7}$$

It is not surprising to recognize in Eq. (7) the expressions for the total electric current through the junction, which have been worked out in this notation in Ref. 4. Using these results, we can rewrite Eq. (7) as

$$\sigma T=I_{\text{tot}}V=(I_{\text{qp}}+I_{\text{qp pair}}+I_{\text{pair}})V, \tag{8}$$

where $I_{\text{qp}}V$ is the normal Joule heat due to the quasiparticle current. The terms $I_{\text{qp pair}}V$ and $I_{\text{pair}}V$ are *time dependent* and represent the energy transfered by pairs of electrons which traverse the electric field. This does not contribute to the heating of the system. Indeed, the time average of these terms is zero. The product $I_{\text{qp pair}}V\sim\cos\delta\theta(t)$ was introduced and discussed in Ref. 4, in view of the fact that the interference current was found to have no thermoelectric properties. The fact that the net entropy production rate derived from the energy currents gives us the terms $I_{\text{qp pair}}V$ and $I_{\text{pair}}V$ is interesting. First, it implies that they can indeed be interpreted as electric power. Moreover, even though the pair energy currents originate in the mixed $A_k(w)B_q(w)$ term rather than the $B_k(w)B_q(w)$ term (which is the origin of the electric currents), we conclude that the physical processes underlying the phase-dependent electrical currents also drive the phase-dependent energy currents. In particular, the "$\cos\delta\theta$" term results from a coupling between the quasiparticles and the condensate in the bulk superconductors. We shall therefore refer to this term as the interference energy current. Our explanation of these anomalous power terms is the following. We argue that the energy ($\sim eV$) carried by the coupled quasiparticles and the pairs is stored in the surrounding magnetic field via the inductancelike behavior of the Josephson coupling. Such a process is reversible, i.e., it is not associated with dissipation in the system. This is consistent with the change of sign of the power with time. The change in the magnetic field, in turn, is manifested by a change in the phase drop across the junction. The same mechanism also corresponds to the product $I_{\rm pair}V\sim\sin\delta\theta(t)$. In this case only the pairs carry energy across the junction. We believe that this is the physical process that gives rise to the ac Josephson current that is measured when there is a nonzero voltage across the junction. Note that part of the energy carried by the pairs is dissipated as radiation.

According to Eq. (5) the energy currents corresponding to pair transport oscillate in time because of the relation $2eV/\hbar=d(\delta\theta)/dt$. The phenomenon corresponds to the mechanism given above. Note that, like the quasiparticle current, when $T_l=T_r=0$ the quasiparticle-pair term is nonzero only for $eV>2\Delta$. This is the electrical energy needed to break a pair. The response of the interference energy current to a voltage can be calculated by analogy to the normal thermoelectric coefficient, i.e., "$L_{21}^{\rm qp\ pair}$" $\equiv Q_l^{\rm qp\ pair}/V$. We use quotation marks in order to indicate that "$L_{21}^{\rm qp\ pair}$" is not a phenomenological thermodynamic coefficient. The result is that the coefficient oscillates in time like $\cos[\delta\theta(t)]$ due to the time dependence of the phase drop in the presence of a voltage across the junction. However, when averaging over time the coefficient vanishes. This behavior supports our identification of the interference current as a nondissipative current. In Ref. 4 a similar calculation showed that "$L_{12}^{\rm qp\ pair}$" $\equiv I_l^{\rm qp\ pair}/\delta T=0$ (at every instant of time). One might be tempted to state that this is consistent with the Onsager relations. However, we note that the pair currents cannot be incorporated into the theory of irreversible processes. We only state that the interference current, and the heat-current analog, possess no thermoelectric properties.

B. A temperature biased junction

Next we consider the case where the junction is biased by a temperature drop across the junction, but the voltage across it vanishes. It turns out that there is only a contribution from $Q_l^{\rm qp}$ and $Q_l^{\rm qp\ pair}$. Using Eq. (5) we find

$$Q_l^{\rm tot}=Q_l^{\rm qp}+Q_l^{\rm qp\ pair}=\frac{8\pi N_lN_r|T_{lr}|^2\delta T}{\hbar T}\int_{\Delta_{\rm max}}^{\infty}dw\left(-\frac{df}{dw}\right)\frac{w^2[w^2+|\Delta_l||\Delta_r|\cos(\delta\theta)]}{\sqrt{w^2-\Delta_l^2}\sqrt{w^2-\Delta_r^2}},\tag{9}$$

where we made the usual linear-response approximation $f_l^{T+\delta T}(w)-f_r^T(w)\approx-(df/dw)w\,\delta T/T$. The functions N_l, N_r, and $|T_{lr}|^2$ were taken at the Fermi energy. The first term in the square brackets on the RHS of Eq. (9) corresponds to the quasiparticle heat conductance in the superconducting state. The second term in those brackets is a novel effect in which the energy current depends on the phase drop across the junction. This latter part has the very interesting property that it can conduct energy in the direction *opposite* to the temperature drop across the junction (depending on the phase drop across the junction). This enables us to control the quasiparticle heat conductance by manipulating the interference energy current via the phase drop. However, note that the net heat conductance will always be in the direction of the temperature drop, in agreement with the second law of thermodynamics.

The anomalous properties of the interference energy current are exhibited when examining the entropy production rate for a temperature drop across the junction. Inserting Eq. (9) into Eq. (6) we find

$$\sigma=\frac{8\pi N_lN_r|T_{lr}|^2(\delta T)^2}{\hbar T^3}\int_{\Delta_{\rm max}}^{\infty}dw\left(-\frac{df}{dw}\right)\frac{w^2[w^2+|\Delta_l||\Delta_r|\cos(\delta\theta)]}{\sqrt{w^2-\Delta_l^2}\sqrt{w^2-\Delta_r^2}}.\tag{10}$$

Defining $L_{22}\equiv Q_l/\delta T$, we see that Eq. (10) breaks up into two parts. The first part of the sum is the quasiparticle contribution, which can be written as $L_{22}^{qp}(\delta T)^2/T^2$. Indeed, this is the thermodynamic expression for the normal entropy production rate. The second part can be written as "$L_{22}^{\rm qp\ pair}$" $(\delta T)^2/T^2$. "$L_{22}^{\rm qp\ pair}$" is not a thermodynamic coefficient. It depends on the cosine of the phase drop across the junction, which implies that the "entropy production rate" associated with $Q_l^{\rm qp\ pair}$ can be negative. Equation (10) implies that it is possible to control the quasiparticle Joule heating by tuning the phase difference across the junction. Note that the total entropy production is always non-negative, as required by the second law of thermodynamics.

In order to understand the physical mechanism underlying this peculiar interference current, it is useful to rewrite Eq. (5) in a more explicit form. Starting from Eq. (A6) and substituting Eq. (A5), we write down the four terms given by the product $A_k(w)B_q(w)$. Then we perform the integration over the quasiparticle energies dw and dw'. Since we set $\delta\mu=0$, the expression for the interference energy current is reduced to a sum of two terms:

$$Q_l^{\rm qp\ pair}=\frac{4}{\hbar}\sum_{k,q}\frac{|\Delta_k||\Delta_q|}{E_q}|T_{lr}|^2\cos(\delta\theta)[(f_l^{T_l}-f_r^{T_r})\delta(E_k-E_q)+(1-f_l^{T_l}-f_r^{T_r})\delta(E_k+E_q)],\tag{11}$$

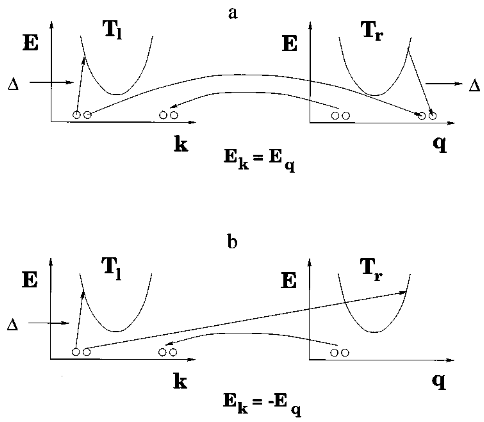

FIG. 1. A schematic description of the processes that correspond to the interference energy current. On the LHS and RHS we have the quasiparticle energy spectrum of the electrodes. This illustrates the mechanisms that give rise to an effective transport of energy from one side to the other (see text).

where we substituted $u_q v_q = |\Delta_q|/2E_q$. Following Langenberg,[2] we associate the terms on the RHS of Eq. (11) to processes illustrated in Fig. 1. Those processes involve the tunneling of a quasiparticle, which is represented by $|T_{lr}|^2$ in Eq. (11). However, in order to account for the dependence on the phase drop $\delta\theta$, one must assume that another process also occurs and the two processes must be superimposed. The other process is that a pair tunnels. Then it breaks up into two quasiparticles, one of which tunnels back. The mechanism of energy transfer is the following: consider process *a* in Fig. 1 for the case $\Delta_l = \Delta_r = \Delta$. Initially we have an excitation E_q on the RHS. Then a pair tunnels to the LHS and breaks up. The final state is an excitation E_k on the IHS. We see that breaking a pair on the LHS removes an energy 2Δ from that side. On the other side, the recombination of the pair releases an energy 2Δ. Hence, *effectively* the energy 2Δ is transferred from the LHS to the RHS. Process *b* in Fig. 1 involves the removal of an energy 2Δ from the LHS electrode. Note that the time reversed versions of processes *a* and *b* also occur. The sum of all these processes determines the net energy transfer by the interference energy current. We emphasize that this is *not* energy carried by the pairs (i.e., the charge carriers), as is the case for quasiparticle transport. The entropy generated in the electrodes is due to the combined effect of tunneling pairs and the process of breaking and recombining pairs in the electrodes. The dependence on the phase drop ensues from the tunneling of particles as coherent pairs across the junction. Note that the contribution of the interference energy current, Eq. (9), vanishes when $T_l = T_r$. This is explained by the fact that the time reversed processes cancel each other out when $T_l = T_r$.

IV. CONCLUSION

We showed that the tunneling energy current in a Josephson junction is similar to its electrical counterpart. The quasiparticle current represents energy transport by quasiparticles. This energy is dissipated in the electrodes which act as heat reservoirs. As expected, the quasiparticle heat current satisfies the relations imposed by nonequilibrium thermodynamics. The interference and pair energy currents are nondissipative processes, since they correspond to a mechanism related to the tunneling of pairs. If $T_l = T_r = 0$ and $V \neq 0$, these currents are nonzero and oscillate in time. We understand this as a reversible exchange of energy with the magnetic field surrounding the junction. The pair energy current, unlike the Josephson current, vanishes if the voltage and the temperature drop across the junction are zero. The interference term, like its electrical counterpart, is a result of energy exchange due to the combined processes of breaking and creating pairs and the tunneling of pairs. (In dirty materials, impurities in the electrodes could enhance pair breaking; this process is not accounted for here.) For $\delta T \neq 0$ and $V = 0$ one can reduce the heat transport and the dissipation of the quasiparticle current to a minimum value, determined by Eqs. (9) and (10). The process responsible for this effect originates in the interplay between quasiparticles and the condensate. The interference term affects the dissipation and heat transport in the system in a reversible fashion, i.e., it can flow in either direction, depending on the sign of $\cos(\delta\theta)$. This effect can be measured as a modulation of the heating of the electrodes of a Josephson junction as function of an applied magnetic flux (e.g., in a SQUID setup).

We believe the microscopic description of the interference energy current given above is relevant to the phenomenological ''convective heat conductance'' in a superconducting bar, which was suggested by Ginzburg.[8] According to Ginzburg, the thermal transport of a superconducting bar, to which a temperature gradient is applied, will include a new contribution, in addition to the normal contributions. This term is due to the breaking and recombination of pairs at the edges of the bar, as a result of the discontinuity imposed by the edges. In our case the edges form the junction.

ACKNOWLEDGMENTS

Partial support for this work was provided by the Israel Science Foundation, the U.S.-Israel Binational Science Foundation, and a grant from the office of the vice president for research at the Tel-Aviv University. We would like to thank V. L. Ginzburg for stimulating discussions. We also benefited from fruitful discussions with A. Shnirman.

APPENDIX A: CALCULATION OF THE ENERGY TRANSFER THROUGH A JOSEPHSON JUNCTION

In order to calculate the energy change in the LHS electrode, given in Eq. (4), we need to evaluate the thermodynamic average of the product of two electron operators. In general, to first order in perturbation theory, the average of an operator $O(t)$ (where t denotes time) is

$$\langle O(t)\rangle = -i\int_{-\infty}^{t} dt_1 \langle [O(t),V(t_1)]\rangle \exp(\eta t_1). \quad \text{(A1)}$$

The square brackets denote the commutators of the two operators. The operator V is the perturbation in the Hamiltonian, and η is a small parameter which is eventually taken to zero. It represents the assumption of an adiabatic perturbation. In our calculation $V = H_T$ and O is the product of electron operators on the RHS of Eq. (4). Performing the first-order calculation, we obtain averages of commutators of products of two C operators (e.g., $\langle [CC,CC]\rangle$). Applying the commutation rules, we end up with averages over products of four electron operators. These break up into pairs, corresponding to the LHS and RHS electrodes. At this stage it is convenient to define two types of functions:

$$G^{>}_{k\sigma}(t,t')=-i\langle C_{k\sigma}(t)C^{\dagger}_{k\sigma}(t')\rangle,$$

$$F^{>}_{k\sigma}(t,t')=-i\langle C_{k\sigma}(t)C_{-k-\sigma}(t')\rangle. \tag{A2}$$

In a similar way we define the functions $G^{<}$ and $F^{<}$ by reversing the order of the operators and the sign. The functions $G^{>}$ and $G^{<}$ can be used to construct the normal single-particle Green's function. Similarly, the functions $F^{>}$ and $F^{<}$ can be used to construct the anomalous Green's function, which describe pair tunneling. Recognizing these functions in the first-order expansion of Eq. (4), we can write the energy change in the LHS in the following way:

$$\frac{dQ_l}{dt}=-8\mathrm{Re}\sum_{kq}\int_{-\infty}^{t}dt_1\exp(\eta t_1)\Big\{\xi_k|T_{kq}|^2[G^{<}_k(t_1,t)G^{>}_q(t,t_1)-G^{>}_k(t_1,t)G^{<}_q(t,t_1)]+\frac{1}{2}\xi_kT_{kq}T_{-k-q}[F^{\dagger>}_k(t_1,t)F^{<}_q(t,t_1)$$
$$-F^{\dagger<}_k(t_1,t)F^{>}_q(t,t_1)]+\Delta^{\dagger}_kT_{kq}T_{-k-q}[G^{<}_k(t,t_1)F^{>}_q(t_1,t)-G^{>}_k(t,t_1)F^{<}_q(t_1,t)]+\Delta^{\dagger}_k|T_{kq}|^2[F^{<}_k(t_1,t)G^{>}_q(t,t_1)$$
$$-F^{>}_k(t_1,t)G^{<}_q(t,t_1)]\Big\}. \tag{A3}$$

In Eq. (A3) we denote the LHS (RHS) momentum by $k(q)$. Next, we Fourier transform the Green's functions into frequency space and express them by their spectral densities

$$G^{><}_k(t,t_1)=e^{-i\mu_l(t-t_1)}\int_{\infty}^{\infty}\frac{dw}{2\pi}e^{-iw(t-t_1)}(\pm i)f(\pm w)A_k(w),$$

$$F^{><}_k(t,t_1)=e^{-i\mu_l(t+t_1)}e^{i\theta_l}\int_{\infty}^{\infty}\frac{dw}{2\pi}e^{-iw(t-t_1)}(\pm i)f(\pm w)B_k(w). \tag{A4}$$

The superconducting phase in the F function is introduced by the creation-annihilation operators of a pair, as postulated in the modified Bogliubov transformation.[1] The spectral densities are

$$A_k(w)=2\pi[|u_k|^2\delta(w-E_k)+|v_k|^2\delta(w+E_k)],$$

$$B_k(w)=2\pi u_kv_k[\delta(w-E_k)-\delta(w+E_k)], \tag{A5}$$

where $|u_k|^2=1/2(1+\xi_k/E_k)$ and $|v_k|^2=1/2(1-\xi_k/E_k)$ are the absolute value squared of the coherence factors. The product u_kv_k can be written as $|\Delta_k|/2E_k$. In these relations $E_k=\sqrt{\xi_k^2+\Delta_k^2}$ is the BCS quasiparticle energy spectrum and ξ_k is the electron energy spectrum relative to the chemical potential. Substituting Eqs. (A4) and (A5) into Eq. (A3) we obtain

$$\frac{dQ_l}{dt}=\frac{8}{\hbar}\mathrm{Im}\sum_{k,q,\sigma}\int_{-\infty}^{\infty}\int_{\infty}^{\infty}\frac{dwdw'}{(2\pi)^2}[f_l(w)-f_r(w')]\Big[\xi_k|T_{kq}|^2\frac{A_k(w)A_q(w')}{w-w'-\delta\mu+i\eta}$$
$$+\frac{1}{2}\exp[-i(\delta\theta+2\delta\mu t)]\xi_kT_{kq}T_{-k-q}\frac{B^{\dagger}_k(w)B_q(w')}{w-w'-\delta\mu+i\eta}+\exp[-i(\delta\theta+2\delta\mu t)]\Delta^{\dagger}_kT_{kq}T_{-k-q}\frac{A_k(w)B_q(w)}{w-w'-\delta\mu+i\eta}$$
$$+\Delta^{\dagger}_k|T_{kq}|^2\frac{B_k(w)A_q(w')}{w-w'-\delta\mu+i\eta}\Big], \tag{A6}$$

where $\delta\theta$ is the phase difference across the junction. The difference between the quasi-particle chemical potentials of the 2 superconductors is denoted by $\mu_r-\mu_l\equiv\delta\mu$. The next stage is to perform the sum over the momentum and spin. The latter gives a factor of two. The sum over momentum is transformed into an integral over the electronic spectra ξ_k and ξ_q. The integration nullifies terms of the integrand which are odd in these variables. Substituting the explicit expressions of the spectral densities into Eq. (A6) and integrating the expressions we obtain Eq. (5). Since the tunneling matrix element is invariant under time reversal, we used the relation $T_{kq}T_{-k-q}=|T_{kq}|^2$. Note that the second term in Eq. (A6) is odd in ξ_k and therefore it will not contribute to the energy current. Also note that in the above derivation we assumed that the normal DOS functions and the tunneling matrix element were independent of ξ_k and ξ_q.

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