

Dissipative dynamics of vortex lines in superfluid ^4He

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We propose a Hamiltonian model that describes the interaction between a vortex line in superfluid ^4He and the gas of elementary excitations. An equation of irreversible motion for the density operator of the vortex, regarded as a macroscopic quantum particle with a finite mass, is derived in the frame of generalized master equations. This enables us to cast the effect of the coupling as a drag force with one reactive and one dissipative component, in agreement with the assumption of the phenomenological theories of vortex mutual friction in the two fluid model. [S0163-1829(96)07646-1]

I. INTRODUCTION

Since the discovery of quantized vortices in liquid helium II, it has been recognized that they might provide a mechanism for the coupling of the superfluid to the normal fluid. In the two fluid model, this coupling is represented by a mutual drag force with one dissipative and one conservative component, whose respective strengths can be measured investigating the attenuation of second sound at various temperatures.¹⁻³ Models for the friction coefficients which successfully fit the data up to 2.1 K have been presented in Refs. 4 and 5. The phenomenon of vortex mutual friction has been observed as well in rotating superfluid $^3\text{He-B}$;⁶ this fact brings support to the conjecture that such a mechanism is indeed a relevant source of dissipative processes in highly degenerate quantum fluids. It is also worthwhile to remind that in recent years, vortices have been seen to play a role in phase transitions taking place either in underpressurized ^4He (Ref. 7) or in supersaturated solutions of ^3He in ^4He .⁸

The interaction between the velocity field and the density fluctuations of the superfluid, which at nonvanishing temperatures are embodied in the normal fluid, can be accounted for within a Lagrangian description.^{9,10} However, microscopic descriptions of the interaction between the superfluid motion, especially when topological singularities are concerned, and collective excitations are not conclusive.^{2,3} The investigation of quantum tunneling of vortex lines in superconductors and superfluids¹¹⁻¹³ has now improved our comprehension of, for example, the role of the inertial mass of the vortex (see also, Refs. 14-16, and cited therein) and the influence of either pinning or dissipation on the tunneling rates. Indeed, the value of the vortex inertia is a fundamental parameter in any theoretical description of vortex dynamics and it remains a controversial issue.^{16,17} Different starting points assign to this inertia figures ranging from zero to infinity, the latter arising from a logarithmic divergence with the system size due to the renormalization effect induced by the condensate motion. On the other hand, the vortex mass is known to be finite in superconductors.¹⁸ Although the phenomenological approaches³ completely disregard inertial effects, the mass enters the description of the dynamics of a free vortex, known to be cyclotronlike,^{2,10,17} through a frequency parameter Ω , which would be a measurable quantity

if this time dependent regime were experimentally visualized. In particular, it has been recently shown¹⁰ that the cyclotron motion is a natural solution of the nonlinear Schrödinger equation applied to a vortex. Dynamical¹⁰ and thermodynamical¹⁹ methods have been proposed to measure the vortex inertial coefficient; our present purpose is neither to participate in the existing polemics, nor to propose a new model for the calculation of the vortex mass, but rather to assume that it is a numerical parameter and proceed along similar lines as those invoked in the well established cyclotron motion already discussed in textbooks.²

The aim of the present work is to propose a Hamiltonian model for the coupling between a rectilinear vortex immersed into the excitations of the superfluid, as will be discussed in Sec. II. Due to the translational symmetry of the problem along an axis parallel to the vortex, the problem is two dimensional, i.e., we consider a point vortex on a plane. We shall show in Sec. III that, if one considers the vortex as a quantum particle undergoing Brownian motion²⁰ in a heat reservoir, it is possible to establish the irreversible time evolution of its density operator within the generalized master equation (GME) approach.^{21,22} In this way, in Sec. IV we are able to derive dissipative equations of motion for the canonical position-momentum variables of the vortex and for its velocity. These variables can be seen to evolve under the combined effect of the usual hydrodynamical lift on a rotating cylinder, plus a drag force. If the coupling is linear in the excitation operators, the drag coefficients are governed by the dynamical susceptibility of the liquid. The consequences of the equations of motion thus obtained, the asymptotic velocity of the vortex and the relation of the current description to the phenomenological model are discussed in Sec. V, where the perspectives of the present approach are also outlined.

II. THE HAMILTONIAN MODEL

Let us first summarize the description of the free motion. The Hamiltonian for a cylindrical vortex parallel to the z axis in liquid helium at zero temperature is in charge of providing the Magnus force.² It reads

$$H_v = \frac{1}{2M} [\mathbf{p} - q\mathbf{A}(\mathbf{r})]^2 + M\Omega v_{s,y}, \quad (2.1)$$

where

$$\mathbf{A}(\mathbf{r}) = \frac{h\rho_s l}{2}(y, -x) \quad (2.2)$$

is the vector potential whose curl yields the vortex-velocity-dependent part of the Magnus force and the potential term $M\Omega v_{s,y}$ gives the superfluid-velocity-dependent part of this force. Here M is the dynamical mass of the vortex, ρ_s denotes the number density of the superfluid, v_s its velocity along the x axis, assumed to be uniform, h is Planck's constant, l the system length along the z axis and

$$\Omega = \frac{qh\rho_s l}{M}. \quad (2.3)$$

The quantity $q = \pm 1$ is the sign of the vorticity according to the right handed convention. Furthermore, at zero temperature, ρ_s coincides with the total density per unit mass ρ/m , being m the mass of a helium atom.

At this point it is convenient to remember the existing theoretical uncertainty regarding the vortex mass parameter M that should appear in dynamical calculations^{13,15,16} and keep in mind that in the phenomenological approaches³ the dynamical regime of the vortex is that in which the Magnus force balances the drag plus any applied force.²³ As stated in the Introduction, our viewpoint here is identical to that of former authors^{2,10,11,13} who assume a finite figure for the vortex inertia and consider the cyclotronlike motion of a free vortex as their starting point, with the frequency Ω as the leading parameter. Since it will be shown in Sec. IV that the dissipative motion is easily described in terms of the complex position variable $z = x + iy$ and the velocity dz/dt , we here write the complex Hamilton equation that stems from Eq. (2.1)

$$\frac{d^2 z}{dt^2} = i\Omega \left(\frac{dz}{dt} - v_s \right) \quad (2.4)$$

with the complex Magnus force at the right-hand side.

We now assume liquid helium to contain elementary excitations. For nonvanishing temperatures T , these excitations can be of thermal origin and thus give rise to the normal fluid, while at zero temperature they must be created by an external probe and yield a vanishing normal density ρ_n . If T is above 1 K, the normal component is mainly a gas of rotons, the phonons being the dominant excitations at lower temperatures. Therefore, at any temperature the interaction of the vortex line with these elementary excitations produces damped motion of the vortex. As stated in the Introduction, the main goal of this article is to construct a Hamiltonian model which enables us to obtain this dissipative behavior.

For this purpose, we shall consider a description of quantum dissipation similar to that recently presented in order to account for the irreversible evolution of solitons,²⁴ in which an effective Hamiltonian is constructed for the collective motion coupled to the residual excitations using the collective coordinate formalism.²⁵ This model exhibits unexpected

features²⁶ due to the fact that both the system and the reservoir have the same microscopic origin, which is just the case here discussed.

In this spirit, and considering that within a superfluid in its ground state the vortex exhibits a solitonlike behavior, we propose a vortex-plus-reservoir Hamiltonian, that modifies expression (2.1) as follows:

$$H = \frac{1}{2M} [\mathbf{p} - q\mathbf{A}(\mathbf{r}) - \lambda\mathbf{B}]^2 + M\Omega v_{s,y} + H_B \quad (2.5)$$

with \mathbf{B} a vector function of operators that represents the elementary excitations of the superfluid and H_B is the Hamiltonian of these excitations. In Eq. (2.5) the interaction term $H_{\text{int}} = -\lambda\mathbf{B} \cdot \mathbf{v}$ couples the reservoir and the vortex through the unperturbed velocity of the latter, being

$$\mathbf{v} = \left(\frac{p_x}{M} - \frac{\Omega}{2}y, \frac{p_y}{M} + \frac{\Omega}{2}x \right). \quad (2.6)$$

In the present approach, the Hermitian operator \mathbf{B} is associated to the creation of a density fluctuation in the liquid and could then be labelled by a transferred momentum \mathbf{q} . Up to lowest order, one may have for each component of the vector \mathbf{B}

$$B_{\mathbf{q}} = \frac{\hat{O}_{\mathbf{q}}^\dagger + \hat{O}_{\mathbf{q}}}{\sqrt{2}}, \quad (2.7)$$

where $\hat{O}_{\mathbf{q}}^\dagger (\hat{O}_{\mathbf{q}})$ is the Feynman-Cohen operator that creates (destroys) a density fluctuation quantum, i.e., a phonon or a roton

$$\hat{O}_{\mathbf{q}}^\dagger = \rho_{\mathbf{q}}^\dagger - \frac{1}{N} \sum_{\mathbf{k} \neq \mathbf{q}} \frac{\mathbf{k} \cdot \mathbf{q}}{k^2} \rho_{\mathbf{k}}^\dagger \rho_{\mathbf{q}-\mathbf{k}}, \quad (2.8)$$

being here N the number of atoms in the liquid. Furthermore, we realize that the term $\lambda^2 B^2 / 2M$ appearing in Eq. (2.5) can be absorbed into the Hamiltonian H_B , which is in charge of providing the equilibrium density vector of the reservoir.

III. THE GENERALIZED MASTER EQUATION

The Hamiltonian (2.5) is of the form system-plus-reservoir-plus-interaction.²¹ The standard reduction-projection procedure of nonequilibrium statistical mechanics,²⁰ enriched with the time convolutionless method developed by Chaturvedi and Shibata,²² has already proven to be useful to derive a generalized master equation (GME), with time-dependent coefficients, for the density operator σ of a particle interacting with a heat reservoir in the weak-coupling-non-Markovian limit.^{27,28} In this case, σ is the density operator of the vortex and the generalized master equation reads²⁶

$$\begin{aligned} \frac{d\sigma}{dt} + \frac{i}{\hbar} [H_v, \sigma] = & -\frac{\lambda^2}{\hbar^2} \int_0^t d\tau \{ [v_x, [v_x(-\tau), \sigma]] \\ & + [v_y, [v_y(-\tau), \sigma]] \} \phi(\tau) \\ & - i \frac{\lambda^2}{\hbar^2} \int_0^t d\tau \{ [v_x, [v_x(-\tau), \sigma]_+] \\ & + [v_y, [v_y(-\tau), \sigma]_+] \} \psi(\tau), \quad (3.1) \end{aligned}$$

where $[a, b]_+$ denotes an anticommutator. In this expression, the time dependent functions ϕ and ψ are the real and imaginary parts, respectively, of the correlation between heat bath operators,²⁹

$$\langle B_j(\tau)B_j \rangle = \phi(\tau) + i\psi(\tau) \quad (3.2)$$

for $j=x, y$, assuming an isotropic reservoir. If the Hermitian operator B_j is chosen according to Eqs. (2.7) and (2.8), the function

$$S_{\mathbf{q}}(\tau) = \phi_{\mathbf{q}}(\tau) + i\psi_{\mathbf{q}}(\tau) \quad (3.3)$$

is just the Fourier transform of the dynamical structure factor $S(\mathbf{q}, \omega)$ of helium II and is experimentally known for a wide range of transferred momenta.^{30,31}

Notice that the GME is a differential, rather than an integrodifferential, equation, since the unknown σ under the integral sign is taken at time t ; accordingly, it can be simplified if we define the following time-dependent parameters,

$$\frac{M}{2}\gamma(t) = -\frac{\lambda^2}{\hbar} \int_0^t d\tau \psi(\tau) \sin \Omega \tau, \quad (3.4)$$

$$\frac{M}{2}\mu(t) = \frac{\lambda^2}{\hbar} \int_0^t d\tau \psi(\tau) \cos \Omega \tau, \quad (3.5)$$

and

$$C(t) = \frac{\lambda^2}{\hbar^2} \int_0^t d\tau \phi(\tau) \cos \Omega \tau. \quad (3.6)$$

The velocities appearing in Eq. (3.1) are those of the free vortex displayed in Eq. (2.6) and their detailed time dependence is extracted from Hamilton's equations corresponding to the Hamiltonian (2.1), namely

$$\begin{aligned} v_x(t) &= [v_x(0) - v_s] \cos \Omega t - v_y(0) \sin \Omega t + v_s, \\ v_y(t) &= [v_x(0) - v_s] \sin \Omega t + v_y(0) \cos \Omega t. \end{aligned} \quad (3.7)$$

In terms of these quantities and using Eqs. (3.4) to (3.6), we can write

$$\begin{aligned} \frac{d\sigma}{dt} + \frac{i}{\hbar} [H_{\text{eff}}, \sigma] &= -C(t) \{ [v_x, [v_x, \sigma]] + [v_y, [v_y, \sigma]] \} \\ &+ \frac{i}{\hbar} \frac{M\gamma(t)}{2} \{ [v_x, [v_y, \sigma]_+] \\ &- [v_y, [v_x, \sigma]_+] \}. \end{aligned} \quad (3.8)$$

The effective Hamiltonian contains a renormalization to the vortex mass, induced by the coupling to the thermal reservoir, plus a drift contribution. Its expression is

$$H_{\text{eff}} = H_v + \frac{M\mu(t)}{2} (v_x^2 + v_y^2) + Mv_s \omega(t) v_x - Mv_s \gamma(t) v_y, \quad (3.9)$$

where $\omega(t) = \mu(t)|_{\Omega=0} - \mu(t)$. It is also worthwhile noticing that, being the system translationally invariant on the (x, y) plane, terms in H_{eff} proportional to v_x, v_y play no role in the dynamics.

It is important to observe that the validity of the GME (3.8) is more general than the weak-coupling approximation case. Indeed, if one expands the integral, time-dependent collisional kernel of the master equation in powers of the coupling parameter λ , as done, for instance, in Ref. 22, after a lengthy calculation one can realize that the form of the new GME is identical to Eq. (3.8), at least up to the fourth order in the expansion parameter, except for the fact that the coefficients (3.4) to (3.6) become polynomials in λ .

Finally, we should also mention that the most common assumptions considered in many applications are that the reservoir is purely harmonic and/or that the interaction term is linear in its coordinates. If this is not the current case, the nonvanishing mean values $\langle B_j \rangle$ must be considered²⁶ and modify the effective Hamiltonian (3.9). However, the equations of motion that we shall derive in the next section remain invariant, since these extra terms can be removed by a Galilean transformation. Moreover, it should be noticed as well that the correlation function $\langle B_k(\tau)B_j \rangle$ for $k \neq j$, which is in general a nonvanishing function, does not enter Eq. (3.1).

IV. THE EQUATIONS OF IRREVERSIBLE MOTION

We are now in a position to derive equations of motion for expectation values $\langle a \rangle$ of arbitrary observables a , which can be cast in the form

$$\begin{aligned} \frac{d\langle a \rangle}{dt} + \frac{i}{\hbar} \langle [a, H_{\text{eff}}] \rangle &= -C(t) (\langle [[a, v_x], v_x] \rangle + \langle [[a, v_y], v_y] \rangle \\ &+ \frac{i}{\hbar} \frac{M\gamma(t)}{2} (\langle [[a, v_x], v_y]_+ \rangle \\ &- \langle [[a, v_y], v_x]_+ \rangle). \end{aligned} \quad (4.1)$$

In order to derive equations of motion for the position and momentum components of the vortex we will restrict ourselves to the Markovian limit; in other words, we consider that the correlation indicated by ϕ, ψ is short lived, within the observational times. The parameters in Eqs. (3.4) and (3.5) become then time independent and after some algebra, elimination of the momentum permits us to write a unique complex differential equation for the expectation value of its velocity, that exhibits the effects of the coupling to the reservoir. This equation is

$$\frac{d^2\langle z \rangle}{dt^2} = i\Omega\beta \left(\frac{d\langle z \rangle}{dt} - v_s \right), \quad (4.2)$$

where the quantity that renormalizes the complex Magnus force is

$$\beta = 1 + \mu + i\gamma. \quad (4.3)$$

with μ and γ being the asymptotic values of Eqs. (3.5) and (3.4) respectively.

We here realize that the reservoir constituted by the excitations of the superfluid provides both a dissipative and a

conservative coupling, respectively measured by the parameters γ and μ . This is in agreement with the structure of the mutual friction force of the two fluid model.¹⁻³ Moreover, keeping in mind that if the density fluctuations of the superfluid carry a definite momentum \mathbf{q} , the heat bath correlation $\langle B_{\mathbf{q}} B_{\mathbf{q}}(\tau) \rangle$ is, to lowest order, just the Fourier transform of the dynamical structure factor, using standard relations of linear response theory³² one can readily show that

$$\mu + i\gamma = \frac{\lambda^2 l}{2\pi\hbar M} \chi(\mathbf{q}, \Omega), \quad (4.4)$$

where $\chi(\mathbf{q}, \Omega)$ is the dynamical susceptibility or response function of the liquid (per unit length) at momentum \mathbf{q} and energy $\hbar\Omega$. In the most general situation where thermal excitations cover the whole momentum range, a summation over \mathbf{q} should be applied on the right-hand side of Eq. (4.4). A consequence of this result is that the temperature dependence of the drag coefficients is provided by the variation of $\chi(\mathbf{q}, \Omega)$ with T ,³¹ it is then worthwhile to keep in mind that the harmonic oscillator heat bath employed in previous investigations of vortex coupling to excitations^{11,13} predicts a temperature-independent dissipation strength.²⁹ However, if the reservoir operators B_j are described by nonlinear functions of $\hat{O}_{\mathbf{q}}^\dagger, \hat{O}_{\mathbf{q}}$ rather than by the Feynman-Cohen operator (2.8), it is possible to show that the drag coefficients vanish at zero temperature;^{24,26} in such a case, the coupling is uneffective and the vortex moves freely governed by the Hamiltonian (2.1), as expected.

Equation (4.2) can be straightforwardly integrated, giving a mean value of the complex velocity operator

$$\frac{d\langle z \rangle}{dt} = \left[\frac{d\langle z \rangle}{dt} \right]_{t=0} - v_s e^{i\Omega(1+\mu)t} e^{-\Omega\gamma t} + v_s. \quad (4.5)$$

Since $\Omega\gamma$ is always a positive quantity, this expression contains exponential decay of the initial conditions, and the limiting value of the vortex velocity is then the superfluid one v_s .

Let us now examine the situation as described by the phenomenological theory¹⁻³ where the drag force is written as

$$f_D = -(\gamma_0 + i\gamma'_0) \left(\frac{dz}{dt} - v_n \right), \quad (4.6)$$

where γ_0, γ'_0 are the strengths of the dissipative and conservative components and v_n is the normal fluid velocity. It is also assumed that when equilibrium is reached, f_D can be expressed in the form $(\alpha - i\alpha')\rho_s qh(v_n - v_s)$. If one solves Newton's equation for a point particle with mass M moving under the Magnus and the drag force (4.6), one finds

$$\begin{aligned} \frac{dz}{dt} = & \left[\frac{dz}{dt} \right]_{t=0} - v_s - (\alpha' + i\alpha)(v_n - v_s) e^{i(\Omega - \gamma'_0 l/M)t} e^{-\gamma_0 l t/M} \\ & + (\alpha' + i\alpha)(v_n - v_s) + v_s, \end{aligned} \quad (4.7)$$

where α and α' can be written in terms of γ_0 and γ'_0 as

$$\alpha = \rho_s qh \frac{\gamma_0}{(\rho_s qh - \gamma'_0)^2 + \gamma_0^2}, \quad (4.8)$$

$$\alpha' = \frac{\gamma_0^2 + \gamma'_0(\gamma'_0 - \rho_s qh)}{(\rho_s qh - \gamma'_0)^2 + \gamma_0^2}.$$

The inverse relationships giving γ_0, γ'_0 in terms of α, α' can be found in Ref. 2. We see that in this case, the asymptotic velocity contains both the reactive and the resistive coefficients. However, measurements of second sound attenuation in helium II at temperatures below 1.5 K give values for α, α' around 10^{-2} , providing thus a negligible correction to the unperturbed velocity v_s .

On the other hand, it is important to notice that according to the general equation (4.1) derived in this work, the expectation value of the free velocity operator (2.6), whose complex counterpart reads

$$v = \frac{p}{M} + i \frac{\Omega}{2} z \quad (4.9)$$

satisfies the evolution law

$$\langle v(t) \rangle = \left[\langle v(0) \rangle - v_s \left(1 - \frac{\mu_0}{\beta} \right) \right] e^{i\Omega\beta t} + v_s \left(1 - \frac{\mu_0}{\beta} \right). \quad (4.10)$$

with $\mu_0 = \mu_{\Omega=0}$ [cf. Eq. (3.5)]. Equation (4.10) is remarkably close to the above expression (4.7) for vanishing normal fluid velocity. Indeed, for a normal fluid at rest, Eq. (4.10) is of the form (4.7), with coefficients $\tilde{\gamma}_0, \tilde{\gamma}'_0$ (or $\tilde{\alpha}, \tilde{\alpha}'$), given by

$$\begin{aligned} \Omega(\mu + i\gamma) &= -(\tilde{\gamma}'_0 - i\tilde{\gamma}_0) \frac{l}{M}, \\ \frac{\mu_0}{\beta} &= \tilde{\alpha}' + i\tilde{\alpha}. \end{aligned} \quad (4.11)$$

Elimination of μ and γ gives the relationship

$$\begin{aligned} \tilde{\alpha} &= \frac{\rho_s qh \tilde{\gamma}_0}{\tilde{\gamma}_0^2 + (\rho_s qh - \tilde{\gamma}'_0)^2} |\mu_0|, \\ \tilde{\alpha}' &= \frac{\rho_s qh (\tilde{\gamma}'_0 - \rho_s qh)}{\tilde{\gamma}_0^2 + (\rho_s qh - \tilde{\gamma}'_0)^2} |\mu_0|. \end{aligned} \quad (4.12)$$

The interesting similitude between these relations and those in Eqs. (4.8) gives support to the conjecture that the present model embodies substantial aspects of the mechanism responsible of damped vortex motion in superfluid helium. The differences between the relationships characterizing the phenomenological model in Eqs. (4.8) and the present ones in Eqs. (4.12), are due to the fact that the structure of the drag force is not identical in both approaches. In fact, a close look at Eqs. (4.2), (4.3), and (4.6) shows that the Hamiltonian description gives rise to an extra component of the force, proportional to the relative two fluid velocity $v_n - v_s$. This supplementary component is not removed by the assumption that the normal fluid lies at rest; however, it is also worthwhile to keep in mind that the assumption that the drag force is proportional to $\mathbf{v} - \mathbf{v}_n$ applies under the hypothesis of vanishing vortex mass.² If one is interested in getting rid of the extra force, a different model should be selected, so as to

bring the two fluid dynamics into the picture. Such an improvement does not consist of a simple modification of the Hamiltonian (2.5); instead, a totally different formulation is required stemming from a Hamiltonian description of the two fluids to which a suitable coupling is incorporated. This philosophy fits more specifically the spirit of macroscopic, fluid dynamical models and is thus beyond the scope of the present work.

As a final remark, we wish to recall that every time-dependent quantity here presented owes this dependence to the special model feature that makes room to a finite, although unknown, inertial coefficient of the vortex. This parameter rules the evolution since it appears in both the conservative and the decay time scale [cf. Eqs. (4.5), (4.7), and (4.10)]; we then realize that as pointed out in Ref. 10, experimental detection of the time-dependent regime would thus provide a means of measuring the vortex inertia. It should be kept in mind that the present results concerning the dynamics cannot be extrapolated down to $M=0$; in fact, the free Hamiltonian (2.1), the frequency (2.3), the vortex-reservoir Hamiltonian (2.5) and the velocity (2.6) become meaningless in such a case. However, the asymptotic velocity does not depend upon the mass, since its value causes the Magnus and the drift force to cancel each other in the absence of inertial effects.

V. DISCUSSION AND SUMMARY

Let us now examine further the characteristics of the model here presented and its relationship to the phenomenological description of dissipation. On the one hand, it is important to keep in mind that the vortex mass is assumed to vanish in the phenomenological two-fluid model of mutual friction, where the velocity arises from the balance between the Magnus and the drag forces; it should be noticed that this regime is also the time-asymptotic form of a Newton-like equation of motion if the vortex mass is finite.³ The precise value of the vortex inertial coefficient is thus not important in the limiting regime, although it influences the dynamics at finite times through the frequency Ω [cf. Eqs. (2.3) and (4.7)], which is the relevant parameter of the model. In this context, it is important to keep in mind that the coupling to the thermal excitations further renormalizes the vortex mass; in fact, inspection of the effective Hamiltonian (3.9) shows us that the kinetic energy has been changed into $M(1 + \mu)\mathbf{v}^2/2$.

On the other hand, the phenomenological theories introduce a mutual friction whose drift and dissipative components are proportional to the relative two-fluid velocity $\mathbf{v}_n - \mathbf{v}_s$. The vortex velocity, either with respect to the superfluid or to the normal one, is determined by the force balance when inertial effects vanish; consequently, it depends upon the parameters of the drag. Instead, the present model should be regarded as a description in the reference frame of the normal fluid, i.e., both v_s and $d\langle z \rangle/dt$ refer to the local velocity \mathbf{v}_n of the heat reservoir in the neighborhood of the vortex. The coefficients that measure the drag effects thus depend upon the strength of the coupling to the thermal excitations and upon their dynamical response; however, the fluid dynamics of the elementary excitations is not explicitly contemplated.

We believe that the model here presented covers most aspects of the description of dissipative dynamics of a vortex line in helium II and opens possibilities towards further improvements, among which, a definite one is the introduction of the motion of elementary excitations, to properly account for mutual friction in the sense of phenomenological theories. With respect to previous calculations of the drag coefficients carried, for example, in Refs. 4 and 5, our model, being quantal in nature, is not subjected to either the low-temperature limitations of a hydrodynamical description as pointed out in (Ref. 4), or to uncertainties associated to a classical approach to the roton-vortex collisions.⁵ It may be also mentioned that the model holds as well for vortex motion in liquid ³He; quantum statistics only enters the characterization of the excitations making the heat reservoir, which would consist of the zero sound phonons of the fermion liquid. No special differences with the present results would be expected in that case, except from the fact that the larger core size of vortices in ³He could probably enlarge the inertia parameter M with a subsequent decrease in the oscillation frequency Ω .

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