Superconductivity in a system with preformed pairs

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(Received 12 July 1996; revised manuscript received 20 September 1996)

We discuss the phenomenology of superconductivity resulting from the Bose condensation of preformed pairs coexisting with unpaired fermions. We show that this transition is more mean-field-like than the usual Bose condensation, i.e., it is characterized by a relatively small value of the Ginzburg parameter. We consider the Hall effect in the vortex-flow regime and in the fluctuational regime above T_c, and show that in this situation it is much less than in the transition driven entirely by Bose condensation but much larger than in usual superconductivity. We analyze the available Hall data and conclude that this phenomenology describes reasonably well the data in the underdoped materials of Y-Ba-Cu-O family but is not an appropriate description of optimally doped materials or underdoped La-Sr-Cu-O. [S0163-1829(97)02105-X]

I. INTRODUCTION

It is known for a long time that the excitation spectrum in underdoped high-T_c cuprates shows formation of a pseudogap at temperature T_s far above T_c; this phenomena was observed in the NMR responses[1] and in optics.[2–4] Recently, photoemission experiments showed that this phenomena can be attributed to the electrons in the corners of the Fermi surface which acquire a gap in these materials at about the same temperature at which a pseudogap is observed in optics and NMR.[5] Below T_c the value of the gap does not change significantly with temperature, instead the electron spectral function develops coherence peaks at the gap edges. These data invite the interpretation that this gap formation is due to the pairing of electrons in the corners of the Fermi surface into the bosons which later Bose condense at T_c. The description of the superconductivity in the cuprates as a Bose condensation of preformed pairs was proposed also in different physical contexts.[6–8] Unfortunately, all these scenarios would lead to the conclusion that superconducting transition is similar to the Bose condensation and has a wide fluctuation region near T_c. This conclusion does not agree with the data which show that the transition is more mean-field-like and that it is characterized by a small value of Ginzburg parameter. In this paper we show that the Bose condensation description and mean-field nature of the superconducting transition can be reconciled if Bose condensation happens against the background of the Fermi liquid and processes that convert bosons into the fermions on the Fermi surface are allowed. We formulate the model which describes this physics in Sec. II and derive its physical properties in Sec. III. Another problem of the descriptions based on Bose condensation is that it leads to a large value of the Hall effect in the superconducting state. Our analysis of the data shows that the usual Bose condensation is not consistent with the data whereas Bose condensation which happens against the background of the Fermi liquid might be consistent with the available Hall data in the underdoped Y-Ba-Cu-O materials but is not consistent with the data on optimally doped Y-Ba-Cu-O or underdoped La-Sr-Cu-O. We emphasize however that the data presently available are not sufficient to make a definite conclusion, especially for the underdoped materials; we discuss the data in more detail below in the Introduction and in Sec. IV. It is not important for the foregoing discussion what is the microscopic mechanism resulting in the formation of the preformed pairs but for the sake of concreteness we shall discuss the model where these pairs are formed from the electrons in the corners of the Fermi surface.

Qualitatively, the relative weakness of superconducting fluctuations in high T_c is clear from the following arguments. In these highly anisotropic materials the coherence length in c direction, $\xi_c(T=0)$, is much smaller than the interlayer distance, d, making them almost two-dimensional superconductors. In a purely two-dimensional superconductor Bose condensation would show up as Berezinskii-Kosterlitz-Thouless transition in which the superfluid density jumps from $\rho_S(T_c)\approx\rho_S(0)$ to $\rho_S=0$. Weak three-dimensional effects would only smear this transition a little. Such $\rho_S(T)$ dependence was not observed in any cuprates; instead the observed temperature dependence of $\rho_S(T)$ is mean-field-like in the broad temperature range even for underdoped cuprates, for instance in $YBa_2Cu_4O_8$ $\rho_S(T)\propto(T_c-T)$ for $T_c-T\gtrsim 0.05T_c$.[9] Note here that critical three-dimensional behavior of optimally doped $YBa_2Cu_3O_7$ reported in Ref. 10 does not contradict the conclusion that superconducting fluctuations are relatively weak. In this material the $\rho_S(T)$ dependence remains linear in T in a wide temperature range[11] and the mere fact that these critical fluctuations are three dimensional implies that they occur only in the vicinity of T_c where the correlation length in c direction becomes large, $\xi_c \gg d$.

Quantitatively, the strength of superconducting fluctuations in quasi-two-dimensional systems is determined by the superfluid density, ρ_S. The measured absolute values of ρ_S in cuprates turn out to be too large for the Bose condensation scenario; in $YBa_2Cu_4O_8$ the in-plane penetration lengths are $\lambda_a = 800$ Å and $\lambda_b = 2000$ Å.[12] Such penetration length would lead to the Kosterlitz-Thouless transition temperature $T_{KT} \approx 700$ K (here and below we assume that the individual planes constituting bilayers are strongly coupled so our estimates differ by a factor of 2 from the estimates in Ref. 7). This unrealistic value indicates that $\rho_S(T)$ must decrease by a factor of 10 before the thermal fluctuations become important in agreement with linear $\rho_S(T)$ dependence observed in Ref. 9. A related evidence of the weakness of superconducting fluctuations is provided by a small value of the Ginzburg parameter which is $G_i \sim 0.02$ in this material [see Eq. (9)].

Another important argument against Bose condensation is provided by the Hall effect data near T_c. Bose condensation of charged particles would lead to a huge Hall effect in the superconducting state: $\sigma_b^{xy} = n_b ec/B$ where n_b is density of bosons and a large fluctuational contribution to the Hall effect above T_c. The existing data on the underdoped materials show that the Hall effect in the flux flow regime is large, but not as huge as follows from the Bose condensation model. Specifically, in 60 K material we extrapolate the data obtained in the flux flow regime at $T > 15$ K (Ref. 13) to zero temperature value $\sigma_{xy} = (4\times 10^5/B[T])(1/\Omega\ \mathrm{cm})$; this corresponds to the effective boson density $n_b \approx 0.02$ per in-plane copper atom which is too small.

A similar explanation of the pseudogap phenomena is based on the spin charge separation model.[8] In this model the gap formation is due to the pairing of spinons which carry no charge; such pairing does not lead to superconductivity; it happens only at lower temperature and is due to Bose condensation of holons. This model has the same difficulty as the condensation of the preformed pairs discussed above; there seems to be no reason to expect a narrow fluctuation region if the transition is driven by the Bose condensation of holons.

A somewhat different viewpoint on this problem is provided by the models which interpolate between BCS like transition in Fermi liquid and ordinary Bose condensation of preformed pairs as the interaction strength is varied.[14,15] In this framework the data discussed above would cause one to conclude that high-T_c cuprates are well inside the Fermi liquid regime and very far from the preformed pairs in contradiction to the observed gap formation above T_c in underdoped cuprates.

Another puzzling property of the superconducting transition is the change of the Hall effect sign occurring below T_c. This sign change is preempted by the negative fluctuational contribution to the positive Hall effect in the normal state;[16,17] qualitatively both the sign change in the superconducting phase and the fluctuational Hall effect can be explained if Cooper pairs which are responsible for superconductivity are in fact negatively charged. In this case these pairs give a large negative contribution to the Hall conductivity in the vortex state below T_c which is proportional to $1/B$, and produce negative fluctuational Hall conductivity observed in Ref. 16. Both Hall conductivity in the vortex state near T_c and the fluctuational conductivity above it can be described in the framework of the time dependent Ginzburg Landau (TDGL) equation; for this equation the negative sign of the Cooper pair implies that the imaginary part of the relaxation rate $\mathrm{Im}\gamma < 0$

$$\gamma \frac{\partial \Delta}{\partial t} = -\frac{\partial F}{\partial \Delta^*}. \tag{1}$$

Here F is the usual Ginzburg-Landau free energy.[18]

In the framework of the usual BCS theory the sign and the magnitude of $\mathrm{Im}\gamma$ (and therefore the effective charge of the Cooper pair) is determined by the derivative of the density of states at the Fermi surface, $\partial\nu/\partial\epsilon$, namely $\mathrm{Im}\gamma \sim -\partial\nu/\partial\epsilon$. This conclusion remains valid for any weak coupling BCS theory of superconductivity regardless of the nature of the interaction.[19] For the high-T_c cuprates $\partial\nu/\partial\epsilon$ is controlled by the proximity to a van Hove singularity and photoemission data show that the Fermi surface is always holelike, so that $\partial\nu/\partial\epsilon < 0$ and BCS theory would predict the hole sign of $\mathrm{Im}\gamma$ in contrast to the data. One can also relate $\partial\nu/\partial\epsilon$ to $\partial T_c/\partial\mu$ and avoid the use of photoemission data; this would lead to the prediction $\mathrm{Im}\gamma \sim -\partial T_c/\partial\mu$ which implies that the hydrodynamic contribution to the Hall effect is holelike for the underdoped cuprates and electronlike for the overdoped cuprates in a striking contrast to the study of $La_{2-x}Sr_xCuO_4$,[20] which reported the opposite correlation. We emphasize here that the sign change of the Hall effect in the superconducting state does not itself contradict the BCS theory, it is only the disagreement between the sign of $\partial\nu/\partial\epsilon$ (or $\partial T_c/\partial\mu$) and the sign of the hydrodynamic contribution to the Hall effect which indicates that the weak coupling BCS theory is not valid. In conventional, BCS-like superconductors, the Hall effect might change sign if $-\partial\nu/\partial\epsilon$ has the sign opposite to the sign of the charge carriers which is measured by the normal state Hall effect. The sign of the charge carriers in the normal state is determined by the topology of the Fermi surface. The sign change might occur if $\partial\nu/\partial\epsilon < 0$ on the electronlike Fermi surface or if $\partial\nu/\partial\epsilon > 0$ on the holelike Fermi surface.

Qualitatively, the notion of electronlike preformed pairs agrees with the non-BCS behavior of the Hall effect of the superconductive pairs, but it is difficult to reconcile both of them with the small value of the Ginzburg parameter and with a moderate Hall effect in the superconducting state. In this paper we resolve this dichotomy suggesting the model where preformed pairs coexist with usual fermions and show that in such systems the Hall effect might still be unusual but the Ginzburg parameter is small. One can justify this model using the following qualitative arguments.

II. MODEL

It is well established that the cuprate Fermi surface lies in the vicinity of the van Hove points. Moreover, it is remarkable how small is the dispersion of fermions near $(\pi,0)$

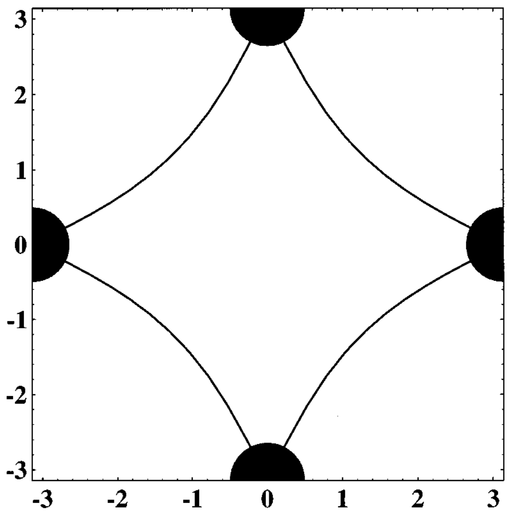

FIG. 1. Sketch of the Fermi line and region of the momentum space where pseudogap pairs is formed. The Fermi line shown here was obtained in the tight binding model with diagonal hopping $t' = -0.3t$; it is similar to the Fermi line observed in the underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Ref. 5). The shaded disks denote the part of the momentum space where a pseudogap was observed in the experiment. We shall assume that the fermions in these regions are paired into the bosons.

points in the underdoped cuprates according to the photoemission data.[5] It is natural to assume that interaction between these fermions can easily exceed their kinetic energy and that the interaction with momentum transfer $q \sim (\pi, \pi)$ is less repulsive than interaction with small momentum transfer. Such interaction gives fermions a gap which is due to the pairing in the antiferromagnetic or superconducting channels. In the weak coupling approximation the d-wave superconductive pairing dominates if the Fermi surface is not nested. In the cuprates both photoemission data[5] and band structure calculations[21] show that the Fermi surface is not nested, a simpler Fermi surface which agrees with the photoemission data shown in Fig. 1. Therefore, it is reasonable to assume that fermions near the corners of the Fermi surface (which lie inside the disks shown in Fig. 1) are paired into bosons, $b^\dagger$, with charge $2e$ and no dispersion; this is the key assumption of our model. So one-particle fermionic excitations acquire a gap; the soft modes appearing instead of these fermionic excitations are spinless bosons

$$H = \varepsilon b_q^\dagger b_q, \tag{2}$$

where ε is a phenomenological parameter of the model. Note that in this model the Bose condensation does not occur because bosons have no dispersion (i.e., are infinitely heavy). Another assumption of the model is that interaction, V, transferring electrons from the ''disks,'' where they are paired to the other parts of the Fermi surface (where Fermi velocity is large), is weak. This assumption can be justified in the spinon-holon model of charge separation[8] where this interaction is suppressed by gauge field fluctuations. If V is small we may neglect the effects of these transfer processes on the gap formation in the corners of the Fermi surface, clearly in this case the gap formation in the corners does not necessarily result in the superconductivity and it does not give a gap to the electrons away from the ''disks.'' At higher temperatures the effects of the remaining fermions can be neglected and bosons form a normal liquid without long range order. The boson-mediated Cooper pairing between remaining unpaired electrons results in the superconductivity only at sufficiently low temperatures.

The Hamiltonian describing this physics is

$$H = \sum_q \varepsilon b_q^\dagger b_q + \sum_{p,q}{}' V_{p,q}(b_q^\dagger c_{p\uparrow} c_{q-p\downarrow} + \text{H.c.}) + \sum_p \xi_p c_{p,\sigma}^\dagger c_{p,\sigma}; \tag{3}$$

here Σ' denotes the sum over the Brillouin zone excluding the ''disk'' area.

Because b describes fermions paired into the state with d-wave symmetry, $V_{p,q}$ also has this symmetry and we may approximate it by

$$V_{p,q} = Va^2(p_x^2 - p_y^2) \tag{4}$$

and neglect its q dependence at small q. Superconducting transition in this model occurs at T_c given by

$$\epsilon = g \ln \frac{\Lambda}{T_c}, \quad g = \frac{1}{(2\pi)^2} \int V_p^2 \frac{dp}{v_F(p)}, \tag{5}$$

where integral $\int dp$ is taken over the Fermi line and $\Lambda \sim \epsilon_F$ is the upper cutoff.

Depending on the parameter ϵ model (3) describes somewhat different physical situations. At $\epsilon \gg T_c$ even at low temperatures bosons exist only as virtual states; in this case the superconducting transition is almost conventional. At $\epsilon \sim T_c$ the density of bosons at $T \sim T_c$ is significant so the superconducting transition acquires some features of the Bose condensation. We anticipate that the former case is relevant for optimally doped cuprates whereas the latter is more appropriate for the underdoped ones. Model (3) is somewhat similar to the model of disordered quasilocalized pairs coexisting with the Fermi liquid introduced in Ref. 22; in the latter model the quasilocalized pairs are assumed to form resonances with energies E that are randomly distributed around the Fermi level. We do not know any experimental justification for this assumption and we believe that the phase transition in the presence of such large disorder in the energy levels will become quite broad.

The superconducting transition at T_c can be described as a Bose condensation which occurs only because bosons become coherent due to the exchange of fermions. Alternatively, one might integrate out the bosons and get the fermion model with retarded short-range interaction. Both approaches lead to the same physical results. Here we shall adopt the Bose formalism because it is shorter and more physical in the regime when $\epsilon \sim T$ so the density of bosons is significant; we shall argue below that this regime is relevant for the underdoped cuprates. At T_c the gap begins to open on the remaining part of the Fermi surface,

$$\Delta(p) = V(p)\langle b \rangle. \tag{6}$$

Clearly $\phi=\langle b\rangle$ plays the role of the order parameter in this model; its thermal fluctuations are governed by the action $S(\phi)$ which is obtained after integrating out the fermion degrees of freedom:

$$S(\phi)=\sum_{\omega}\phi_{\omega}^{*}\left\{-i\gamma''\omega-g\left[\frac{T-T_c}{T_c}+\frac{\pi|\omega|}{8T_c}+\xi_0^2\left|\left(\nabla-\frac{2ie}{c}A\right)\right|^2\right]\right\}\phi_{\omega}-\frac{1}{2}\beta\int|\phi_t|^4dt. \quad (7)$$

Here we use imaginary time representation; we introduce coefficients

$$\xi_0^2=\frac{7\zeta(3)}{2(8\pi^2T)^2g}\int V_p^2v_F(p)dp,$$

$$\beta=\frac{7\zeta(3)}{2(4\pi^2T)^2}\int V_p^4\frac{dp}{v_F(p)}, \quad (8)$$

and $\gamma''=-1$; the latter we introduced to facilitate comparison with the usual time dependent Ginzburg-Landau equation where this coefficient is determined by a particle-hole asymmetry near the Fermi surface and is usually small. Generally, the coefficient γ'' has contributions from the bare action of the bosons, $S_b=-b^*\partial_t b-\epsilon b^*b$, described by the Hamiltonian (2) and from the fermions that we integrated out but the latter is always small in parameter T_c/ϵ_F leading to a simple result, $\gamma''=-1$.

Action of a generic form (7) but with different parameter values also describes the usual BCS type superconductivity in the Fermi liquid, Bose condensation, and interpolation between these two regimes.[14] The crucial difference between the interpolation scheme[14] and model considered here is that the latter leads to such parameters that the condensate amplitude, $|\phi|^2=g\tau/\beta$, always remains small even far from T_c and so the fluctuation region is narrow and the Hall effect never becomes too large.

III. RESULTS

The gradient term in the effective action (7) is determined by the fermion properties. As a result the superconducting transition is mean-field-like and thermal fluctuations become large only in the narrow vicinity, G_i, of the transition temperature; it is convenient to express it in terms of the screening length, λ_0. G_i is given by

$$G_i=\frac{(4\pi\lambda_0)^2T_c}{\sqrt{2}d\Phi_0^2}. \quad (9)$$

Here we define λ_0 as the value of the physical screening length interpolated from the vicinity of the transition temperature to low temperatures; it is expressed through the coefficients g,ξ_0^2 and β of the effective action (7) by

$$\lambda_0^2=\frac{c^2\beta d}{32\pi e^2g^2\xi_0^2}. \quad (10)$$

We computed the coefficient g,ξ_0^2 and β for the fermions with the spectrum $\xi_p=-2t(\cos p_x+\cos p_y)-4t'\cos p_x\cos p_y-\mu$. Because fermions in the disks of size p_0 around van Hove points are paired and do not contribute to the effective action we excluded these regions from the integrals over the Fermi surface (8). We get

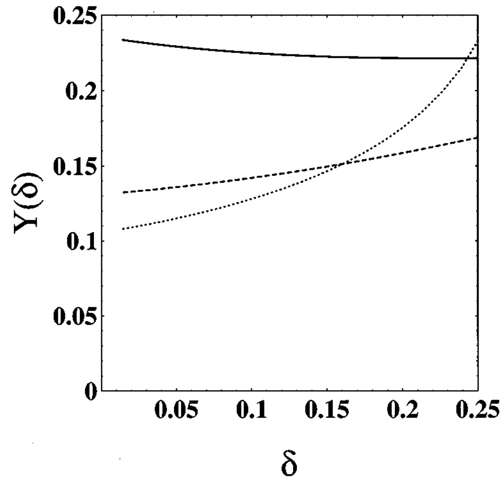

FIG. 2. Dimensionless function $\Upsilon(\delta)$ controlling the penetration depth λ_0^2 as a function of doping assuming that other parameters are constants for different ''disk'' sizes p_0 where a pseudogap is formed. Dotted line corresponds to $p_0=0$, dashed line was obtained for $p_0=0.2\pi$, and full line for $p_0=0.4\pi$.

$$\lambda_0^2=\frac{c^2d}{e^2t}\Upsilon(\delta), \quad G_i=\frac{16}{\sqrt{2}}\frac{T_c}{t}\Upsilon(\delta).$$

Here $\Upsilon(\delta)$ is a dimensionless function of the doping density δ which we plot in Fig. 2 for $t'=-0.3t$ and different sizes of the excluded regions, p_0. We observe that once the regions near the corners of the Fermi surface are excluded the doping dependence of the penetration length becomes relatively weak and the dependence on the size of the excluded regions become so far more important. Qualitatively we expect that the size of the excluded region becomes large in the underdoped bilayered cuprates where a large pseudogap was observed in spin responses and photoemission so that their penetration length is larger than the one in the optimally doped cuprates in agreement with the data. However, we cannot make a quantitative comparison because we do not know the value of p_0.

The results above do not depend on the dynamical part of the action (7), but it becomes important for the fluctuational conductivity:[23]

$$\delta\sigma_{xx}=\frac{1}{16d}\frac{e^2}{\hbar}\frac{T_c}{T-T_c}, \quad (11)$$

where d is the distance between planes. This result depends only weakly on the properties of the electrons if they form a Fermi liquid even with a large relaxation rate. Note that in the conventional Bose condensation scenario the real part of the relaxation time is absent[14] leading to a much larger fluctuational correction to the longitudinal conductivity. Thus, it would be important to understand whether this universal behavior (11) is indeed observed in high-T_c cuprates. Fluctuational Hall conductivity in low field is also controlled by the coefficients of the effective action (7):[19,24]

$$\delta\sigma_{xy}=\frac{e^2}{3\pi d}\gamma''\frac{T_c}{g}\frac{eH\xi_0^2}{c\hbar}\left(\frac{T_c}{T-T_c}\right)^2. \tag{12}$$

Here γ'' is the coefficient of the nondissipative term in the action (7); in this model $\gamma''=-1$. This contribution should be added to the normal state Hall conductivity. As a result a sign change of the Hall effect would occur above T_c at

$$\frac{T-T_c}{T_c}=\sqrt{2/3}\left|\frac{\gamma'' T_c}{g}\right|^{1/2}\frac{\xi_0}{l}, \tag{13}$$

where l is the mean free path; here we used the usual Drude formula, $\sigma_{xy}^n=(nec/B)(\omega_c\tau)^2$, for the conductivity in the normal state. If $|\gamma'' T_c/g|>(T_c/\mu)^2$ the correction to σ_{xx} is small and the Hall effect changes sign in the region where the longitudinal conductivity is still close to the normal state value.[17]

In the vortex state the hydrodynamic contribution to the Hall effect is[25–29]

$$\sigma_{xy}^V=\frac{2ec}{B}\gamma''|\phi|^2=\frac{H_c^2(T)}{2\pi(T_c-T)}\left(\frac{\gamma'' T_c}{g}\right)\frac{ec}{B}. \tag{14}$$

In the generic time-dependent Ginzburg-Landau theory[25,26] this contribution might be different by a numerical factor $\beta_V\simeq 1$; the physical effect taken into account by this numerical factor is an electric field generated by the moving vortex. This effect is small and $\beta_V\approx 1$ if the length, $\xi_E=4\xi_0\sqrt{2\sigma_n T_c\lambda^2}\approx\xi_0\sqrt{(8/\pi)\tau_{\rm tr}T_c}$, which sets the scale for the electric field variations, is long, $\xi_E\gg\xi_0$, which seems to be an appropriate limit for cuprates. In conventional notations[25,26] TDGL dimensionless parameter $u=(\xi_E/\xi_0)^2\ll 1$ for these materials.

In the Bose condensation scenario $\gamma''=1$, at low temperatures $|\phi|^2$ coincides with the boson density and the Hall conductivity $\sigma_{xy}=n_b ec/B$ is huge. In the present model $|\phi|^2\sim(T^2/gt)n_e$ is small leading to a smaller value of the Hall conductivity.

In the conventional BCS theory the coefficient γ'' is determined by the dependence of the density of states, $\nu(\mu)$, on the chemical potential $[\gamma''_{\rm BCS}=-(g/2)(\partial\ln T_c/\partial\mu)]$; here it is controlled by the bosons mediating the interaction between fermions. So, in the conventional BCS theory $|\gamma'' T_c/g|$ is small, $|\gamma'' T_c/g|\sim T_c/\mu$, whereas here it is large. Formally we get a large $|\gamma'' T_c/g|$ in the model (3) because we assumed that bosons are coupled to the pairs of electrons, not holes which introduced a large particle-hole asymmetry. This assumption can be justified if the fermion dispersion near van Hove points is small so that properties of the bosons are determined by the relative number of electrons and holes in the ''disk'' area. Further, if the number of electrons is small, the bosons are entirely electronlike and we get the phenomenological model (3); if the numbers of electrons and the holes in the ''disk'' area are close we would need to introduce two types of bosons (electronlike and holelike). This would lead to the effective action (7) with $\gamma''\ll 1$ and the resulting Hall effect would be much smaller.

These results show that γ'' is not necessarily related to $\partial\ln T_c/\partial\mu$ as was conjectured in Ref. 19. The arguments of Ref. 19 were based on the gauge invariance and on the assumption that T_c dependence on μ implies a dependence on the gauge invariant object $T_c[\mu+i(d/dt)]$. This is true in the BCS model with weak interaction where this dependence is due to the density of states dependence on the chemical potential. However, in a general case one should distinguish two sources of $T_c(\mu)$ dependence: the dependence via the energy of pairing electrons, ϵ_F, and the dependence via the total density of particles, n. The gauge invariance indeed requires that $T_c(\epsilon_F)$ is converted into the $T_c(\epsilon_F+i\omega)$ in the dynamical action but the dependence via the total density is not modified by the frequency so generally the quadratic term in the action is

$$S^{(2)}=-\sum_\omega g\ln\left(\frac{T}{T_c(\epsilon_F+i\omega,n)}\right)b_\omega^* b_\omega. \tag{15}$$

In other words, $n(\mu)$ dependence does not imply a non-gauge-invariant action; it can be reformulated in an explicitly gauge-invariant manner as a dependence on $\varphi=\nabla^{-2}(\nabla E)$.

In the phenomenological model (3) the T_c dependence on the doping, δ, is due to the interaction term, $V(\delta)$, so that T_c grows with doping. One possible microscopic mechanism of this dependence is suppression of the interaction $V(\delta)$ by the gauge field fluctuations discussed in Ref. 8 which becomes less in more doped systems.

IV. ANALYSIS OF THE DATA

Equations (12),(14) can be directly compared with the data. Note here that fluctuational Hall conductivity and Hall conductivity in the flux flow regime are controlled by the same dimensionless parameter $(\gamma'' T_c/g)$; this is a general feature of any hydrodynamic description based on time-dependent Ginzburg-Landau equation. Experimental verification that one gets the same parameter if it is extracted from the Hall data in the fluctuational regime above T_c or if it is extracted from the data in the vortex flow regime would be a very important proof of the validity of the hydrodynamic approach. The comparison of these parameters becomes more complicated in weakly anisotropic materials such as $YBa_2Cu_3O_7$ where the fluctuational data are further complicated by the crossover between two- and three-dimensional behaviors; to avoid these problems it is better to compare the data obtained on more anisotropic materials.

First we compare the values of $(\gamma'' T_c/g)$ obtained on similar optimally doped materials. The extensive study[16] of the fluctuation regime in $Bi_2Sr_2Ca_2Cu_3O_x$ shows that in the regime of 2D fluctuations $\delta\sigma_{xy}\approx 0.08$ $1/\Omega$ cm at $B=0.7$ T. Using the value $dH_{c2}/dT\approx 2$ T/K and $d=18.5$ Å we obtain $(\gamma'' T_c/g)\approx -0.003$. Unfortunately we are not aware of the Hall effect data in the vortex-flow regime on this material, so we compare this value with the other optimally doped cuprates. It is convenient to characterize Hall conductivity data in the vortex-flow regime by the value of $\sigma_{xy}(0)$ obtained by a linear extrapolation to low temperatures. For $YBa_2Cu_3O_7$ we use extrapolated value $\sigma_{xy}(0)=(2\times 10^5/B[T])(1/\Omega$ cm) (Ref. 30) and $dH_c/dT=0.02$ T/K; we get $(\gamma'' T_c/g)\approx -0.03$. For $Tl_2Ba_2CaCu_2O_8$ we use $\sigma_{xy}(0)=(3\times 10^3/B[T])(1/\Omega$ cm),[31] and $dH_c/dT=0.01$ K/T;[32,33] we get $(\gamma'' T_c/g)\approx -0.0016$. The fit of the fluctuational Hall conductivity data obtained on the same sample agrees with theoretical predictions if one chooses

$dH_{c2}/dT=1$ T/K and $(\gamma'' T_c/g)\approx -0.002$.[33] These data indicate that the hydrodynamic approach is likely to be valid but do not allow one to make a definite conclusion. They also show that in optimally doped materials $\epsilon\sim g \gg T_c$, so the bosons may exist only as virtual states of electron pair.

The situation is different for underdoped bilayered cuprates. We take extrapolated value $\sigma_{xy}=(4\times 10^5/B)(1/\Omega\ \mathrm{cm})$ (Ref. 13) and $dH_c/dT=0.006$ T/K (Ref. 34) appropriate for 60 K $YBa_2Cu_3O_{7-x}$; we get $(\gamma'' T_c/g)\approx -0.9$ in agreement with our initial expectations that bosons exist as real electron pairs in these materials. However, the data on the underdoped $La_{2-x}Sr_xCuO_4$ lead to a different conclusion. Here we take $\sigma_{xy}=(300/B)(1/\Omega\ \mathrm{cm})$ (Ref. 20) and $dH_c/dT=0.006$ T/K (Ref. 35) for the material with $x=0.1$; we get $(\gamma'' T_c/g)\approx -0.001$. This estimate implies that bosons are unlikely to exist as real pairs in underdoped $La_{2-x}Sr_xCuO_4$. We emphasize that we do not know of any data which would allow us to check that the hydrodynamic approach remains valid for underdoped materials.

The independent check of the validity of the hydrodynamic (time dependent Ginzburg-Landau) description in the flux flow regime is provided by the Hall angle data *in the weak field region* $B\ll H_{c2}$. In the framework of the effective action (7) it is directly related with the same dimensionless parameter $(\gamma'' T_c/g)$ which we extracted from the Hall conductivity[25,26]

$$\tan\theta_H=\frac{\gamma''}{\gamma'\ln(\xi_E/\xi_0)}=\left(\frac{\gamma'' T_c}{g}\right)\frac{8}{\pi\ln(\xi_E/\xi_0)}. \tag{16}$$

The data[13] for the Hall angle tangent in 60 K $YBa_2Cu_3O_{7-x}$ show that its value extrapolated to $T=0$ is $\tan(\theta_H)\approx 1$, for 90 K $YBa_2Cu_3O_7$ it is much smaller, $\tan(\theta_H)\approx 10^{-2}$, finally for $x=0.1$ $La_{2-x}Sr_xCuO_4$ $\tan(\theta_H)\approx 10^{-3}$.[20] All these values are in reasonable agreement with the above estimates for the parameter $(\gamma'' T_c/g)$ and usual expectation that $\ln(\xi_E/\xi_0)\sim 1$.

Another physical property of the phenomenological model (3) is anomalous thermopower in the normal state. The magnitude of this effect is very sensitive to the value of ϵ_R/T where $\epsilon_R=\epsilon-g\ln(\lambda/T)$ is the effective chemical potential of the pairs. We have only a rough estimate of this parameter based on the following arguments. The boson density in the phase space is $n_0=n_B(\epsilon_R/T)\lesssim 1$ (here n_B is Bose factor), so $\epsilon_R/T=\ln 1/n_0\gtrsim 1$; such ϵ_R makes possible the scattering of electrons with energies larger than ϵ_R resulting in a large relaxation rate for these fermions. Because this relaxation mechanism is effective only for fermions above the Fermi energy it results in a large particle-hole asymmetry and leads to a large thermopower. Assuming that this contribution to the relaxation rate $1/\tau_B$ is much larger than the typical relation rate for the fermions with energies less than ϵ_R we get Seebeck coefficient

$$S_0=\ln\left(\frac{1}{n_0(T)}\right)n_0(T)=\frac{g\ln\left(\frac{T}{T_c}\right)}{T}\exp\left(-\frac{g\ln\left(\frac{T}{T_c}\right)}{T}\right), \tag{17}$$

which is much larger than the usual value, T/ϵ_F, for the normal metal. The sign of the thermopower is positive. Its temperature dependence is nonmonotonic, at $G_i T_c\ll T-T_c\ll T_c$ the thermopower decreases with temperature due to the temperature dependence of $\epsilon_R=g\ln(T/T_c)$, at higher temperatures, $T-T_c\gg T_c$, the temperature dependence of ϵ_R becomes negligible and thermopower becomes small and it increases with temperature. The sign and the value of the thermopower are in agreement with the experiment,[36] but its temperature dependence at high temperatures is not. This is not very surprising because this model does not describe the transport properties at high temperatures which are due to a new physics associated with the appearance of low energy modes. The disagreement between the predictions of the model (3) and data implies that these low energy modes are responsible for the temperature dependence of the thermopower at high temperatures.

V. CONCLUSION

The model (3) applies to the superconductivity in the underdoped cuprates where the gap opens above T_c; we expect a more usual transition in the overdoped cuprates. The crossover from underdoped to overdoped occurs in the framework of the model (3) if ϵ and V is increased with doping; at large ϵ the transition can be described in terms of the virtual pair formation and becomes very similar to a usual BCS picture. However, even in this regime the contribution of these virtual pairs to the γ'' coefficient in time dependent Ginzburg Landau equations can be much larger than the contribution coming from the density of states dependence near the Fermi surface and may result in a sign change of the Hall effect. In the optimally doped cuprates n_0 is still nonzero and we expect a large hydrodynamic contribution to the Hall effect and large positive thermopower.

In the optimally doped cuprates and in the underdoped ones above the temperature of the pseudogap formation one expects new physical effects due to the appearance of new low energy modes. These soft modes are responsible for the anomalous transport relaxation rates. Another probe of the effect of these modes in the optimally doped cuprates (where they are expected to exist down to the transition temperature) is the fluctuational conductivity which should no longer be given by universal form (11). It is important to determine experimentally whether fluctuational conductivity agrees with a phenomenological Fermi liquid picture with large relaxation rate which gives universal form (11), if the data do not fit the universal form (11) it means that this phenomenological Fermi liquid picture is not applicable at all even for the in-plane properties.

A model similar to (3) but in real space also describes a phase transition of the system of superconducting grains embedded in the normal matrix. In this case the mixed b^*cc term corresponds to the Andreev reflection at the NS boundary. In this system the Hall effect in the superconducting state is governed by the particle hole asymmetry of the grains and may change sign close to T_c.

In conclusion we have shown that the phenomenological description of the superconductivity which follows from the concept of preformed pairs coexisting with electrons on some patches of the Fermi surface agrees semiquantitatively with available data on Hall conductivity in the fluctuation and flux flow regime and with the small value of the Ginzburg parameter for underdoped bilayeres cuprates. However, in order to describe the data on optimally doped bilayered cuprates or underdoped La-Sr-Cu-O one needs to assume that the value of the chemical potential for these pairs is large so that preformed pairs exist only as virtual states. The important necessary ingredients of this model are (1) the assumption that the pairs have very little dispersion of their own and (2) their coupling to the electrons on the Fermi surface is weak. The hydrodynamic contribution to the Hall effect in this model is controlled by the pairs and has electronlike sign; it explains the Hall sign change observed experimentally.

It is not possible to test thoroughly the predictions of the model because experiments which give data on the Hall and longitudinal conductivity in the fluctuational and vortex flow regime obtained on the same sample are scarce. Such data on underdoped (spin gapped) materials do not exist at all. It would be very important to verify experimentally that hydrodynamic approach is still valid for the underdoped cuprates (i.e., that the parameters extracted from the fluctuational regime are the same as those extracted from vortex-flow regime) and that the parameters needed for the hydrodynamic description are indeed in agreement with the picture of preformed pairs coexisting with fermions as we conclude here using a limited number of data.

Note added in proof. For the sake of completeness we would like to note the works[37–40] where similar models comprised of fermions and Bose-like particles were considered. Although similar, the present model is different in a few crucial aspects which follow from the assumption that these bosons represent pairs of fermions in the corners of the Fermi surface. In particular, here bosons have almost no dispersion of their own, their density is small and temperature independent, and their decay rate into electrons is low.

ACKNOWLEDGMENTS

We wish to thank G. Blatter, P. Coleman, A. Millis, and T. M. Rice for helpful discussions and A. Samoilov for showing us his unpublished data. The major part of this work was performed at ITP, Santa Barbara, whose hospitality and support via Grant No. PHY94-07194 are gratefully acknowledged; L. Ioffe is also grateful to ETH for its hospitality.

[1] M. Takigawa, Phys. Rev. B **42**, 243 (1991); H. Zimmerman, M. Mali, M. Bankay, and D. Brinkman, Physica C **185-189**, 1145 (1991).

[2] L. D. Rotter, Z. Schlesinger, R. T. Collins, F. Holtzberg, C. Field, U. W. Welp, G. W. Crabtree, J. Z. Lin, Y. Fang, K. G. Vandervoort, and S. Fleshler, Phys. Rev. Lett. **67**, 2741 (1991).

[3] D. N. Basov, R. Liang, B. Dabrowski, D. A. Bonn, W. N. Hardy, and T. Timusk (unpublished); C. C. Homes *et al.*, Physica C **254**, 265 (1995).

[4] T. M. Rice, S. Gopalan, and M. Sigrist, Europhys. Lett. **23**, 445 (1993); A. J. Millis, L. B. Ioffe, and H. Monien, J. Phys. Chem Solids **56**, 1641 (1995).

[5] D. M. King, D. S. Dessau, A. G. Loeser, Z.-X. Shen, and B. O. Wells, J. Phys. Chem. Solids **56**, 1865 (1996).

[6] A. S. Aleksandrov, Phys. Rev. B **48**, 10 571 (1993).

[7] V. Emery and S. Kivelson, Phys. Rev. Lett. **74**, 3253 (1995); Nature **374**, 434 (1995).

[8] L. B. Ioffe and A. J. Millis, Phys. Rev. B **54**, 3645 (1996).

[9] A. Schilling, F. Hullinger, and H. R. Ott, Physica C **168**, 272 (1990).

[10] S. Kamal, D. A. Bonn, N. Goldenfeld, P. J. Hirschfeld, R. Liang, and W. N. Hardy, Phys. Rev. Lett. **73**, 1845 (1994).

[11] D. A. Bonn, R. Liang, T. M. Riseman, D. J. Baar, D. C. Morgan, K. Zhang, P. Dosanjh, T. L. Duty, A. MacFarlane, G. D. Morris, J. H. Brewer, and W. N. Hardy, Phys. Rev. B **47**, 11 314 (1993).

[12] D. N. Basov, R. Liang, D. A. Bonn, W. N. Hardy, B. Dabrowski, M. Quijiada, D. B. Toner, J. P. Rice, D. M. Ginsberg, and T. Timusk, Phys. Rev. Lett. **74**, 598 (1995).

[13] J. M. Harris, Y. F. Yan, O. K. C. Tsui, Y. Matsuda, and N. P. Ong, Phys. Rev. Lett. **73**, 1711 (1994).

[14] C. A. R. Sa de Melo, M. Randeria, and J. R. Engelbrecht, Phys. Rev. Lett. **71**, 3202 (1993).

[15] J. Maly, K. Levin, and D. Z. Liu (unpublished).

[16] W. Lang, G. Heine, W. Kula, and R. Sobolewski, Phys. Rev. B **51**, 9180 (1995).

[17] R. Jin and H. R. Ott, Phys. Rev. B **53**, 9406 (1996).

[18] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon Press, Oxford, 1990), Pt. 2, p. 45.

[19] A. G. Aronov, S. Hikami, and A. I. Larkin, Phys. Rev. B **51**, 3880 (1995).

[20] Y. Matsuda, T. Nagaoka, G. Suzuki, K. Kumagai, M. Suzuki, M. Machida, M. Sera, M. Hiroi, and N. Kobayashi, Phys. Rev. B **52**, R15749 (1995).

[21] A. I. Lichtenstein, O. Gunnarson, O. K. Andersen, and R. M. Martin (unpublished).

[22] G. M. Eliashberg, JETP Lett. **46** Suppl., S81 (1987).

[23] L. G. Aslamasov and A. I. Larkin, Sov. Phys. Solid State **10**, 875 (1968).

[24] S. Ullah and A. T. Dorsey, Phys. Rev. B **44**, 262 (1991).

[25] A. T. Dorsey, Phys. Rev. B **46**, 8376 (1992).

[26] N. B. Kopnin, B. I. Ivlev, and V. A. Kalatsky, J. Low Temp. Phys. **90**, 1 (1993).

[27] M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, JETP Lett. **62**, 811 (1995).

[28] A. van Otterlo, M. Feigel'man, V. Geshkenbein, and G. Blatter, Phys. Rev. Lett. **75**, 3736 (1995).

[29] A. I. Larkin and Yu. N. Ovchinnikov, Phys. Rev. B **51**, 5965 (1995).

[30] J. M. Harris, N. P. Ong, P. Matl, R. Gagnon, L. Taillefer, T. Kimura, and K. Kitazawa, Phys. Rev. B **51**, 12 053 (1995).

[31] A. V. Samoilov, Z. G. Ivanov, and L.-G. Johanson, Phys. Rev. B **49**, 3667 (1994).

[32] A. Wahl, A. Maignan, C. Martin, V. Hardy, J. Provost, and Ch. Simond, Phys. Rev. B **51**, 9123 (1995).

[33] A. V. Samoilov (unpublished).

[34] J. W. Loram, K. A. Mirza, J. R. Cooper, and W. Y. Liang, Phys. Rev. Lett. **71**, 1740 (1993).

[35] J. W. Loram, K. A. Mirza, J. R. Cooper, N. Athanassopoulou, and W. Y. Liang (unpublished).
[36] J.-S. Zhou and J. B. Goodenough, Phys. Rev. Lett. **77**, 151 (1996).
[37] I. O. Kulik, Int. J. Mod. Phys. B **2**, 851 (1988).
[38] R. Friedberg and T. D. Lee, Phys. Rev. B **40**, 6745 (1989).
[39] L. Ioffe *et al.*, Int. J. Mod. Phys. B **3**, 2065 (1989).
[40] J. Ranninger and J. M. Robin, Physica C **253**, 279 (1995).