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# Electromagnetic field generated by a charged particle moving slowly through a conducting media

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We calculate the spatial distribution of the electromagnetic (EM) field generated by the motion of a charged particle in a metal. Regimes of weak and strong spatial dispersion are considered analytically, and in the intermediate case we give numerical results. It is shown that the spatial distribution of the EM field is characterized by a certain length which plays the same role as the skin depth in the theory of the skin effect in metals. In the region behind the moving particle the distribution of the EM field has the form of a ''wake.'' Inside this wake the strength decays by a power law, and outside it decays exponentially. [S0163-1829(96)09345-9]

## I. INTRODUCTION

The electromagnetic (EM) fields generated by the motion of a charged point particle depend essentially upon the type of media in which the motion of the particle takes place. In vacuum, the generated fields are given by the Lienard-Wiechert potential and, as a result of the invariant principle of special relativity, particles moving with constant velocity produce no radiation.[1] In dielectric media, polarization gives rise to phenomena such as energy loss transition and Cherenkov radiation.[1–3] In plasmas, instead of polarization of the media the passage of the particle induces currents which in turn produce strong screening.[4] The screening radius depends on the speed of the particle $s$ and for $s=0$ it is the Debye radius which limits the range of the Coulomb interaction. The dielectric polarization of the medium gives a considerable contribution to the energy losses only for relativistic particles.[1–3] In contrast, in plasmas with a high concentration of carriers, even nonrelativistic particles could obviously induce correspondingly strong fields and currents. For example, in Ref. 5 the distribution of electrostatic potential and nonequilibrium electron density in the wake produced by a charge moving in electron gas has been calculated. The speed of the charge was assumed to be of the order of the Fermi velocity $v$. For such high speeds the principle contribution to the induced field comes from the plasmon excitations; therefore the field in the wake is a potential field and decays rather slowly. For the electrons in the wake the typical value of the transfer momentum is of the order of the Fermi momentum $p_F$. Therefore the spatial distribution of the wake potential is determined by Fourier components with large value of the wave vector $q \sim p_F/\hbar$. Such large values of $q$ require a quantum approach for the calculations of dielectric constant. In Ref. 5 the Lindhard dielectric function which accounts for electron transitions with $q \sim p_F/\hbar$ has been used.

In the present paper we consider the charge particle moving slowly, i.e., with speed $s$, much less than the Fermi velocity. In this case, the electron plasma behaves like a typical metal, where any electromagnetic excitation decays at a short distance. In the case of slowly moving particle the typical transfer momentum is $\Delta p \ll p_F$ and the classical kinetic equation can be used to calculate the response function. Now, unlike the case considered in Ref. 5, the relevant response function is the conductivity and not the dielectric function. The latter is meaningless in the electrodynamics of metals at frequencies less than plasma frequency.[6,7]

The distribution of the EM field produced by a slowly moving charge is important, for example, in radiative and acoustoelectric phenomena in metals and also in the theory of quasilinear Landau damping.[6] The charged particle moving slowly through the metal can be injected from an external source. All radiative losses are negligible at low speed. If this particle is a heavy ion, it can propagate with a constant velocity due to the channeling effect. There are also two important cases where light particles (electrons) move slowly with constant velocities through the metal. The first case is that of conduction electrons trapped by an acoustic wave of sufficiently large amplitude.[8] It is an important example of nonlinear Landau damping which gives rise to acoustic ''enlightenment.''[9] The second is the motion of charged dislocations in plastic deformation processes of metals or semiconductors.[10] The registration and analysis of EM fields generated by the mobile dislocations may be applicable for the control of the mechanical properties of materials. This would complement the sound control methods already used extensively.[11]

We shall show that the spatial distribution of the vortex EM field is characterized by a certain length which plays the same role as the skin depth in the theory of the skin effect in metals. Two different regimes of wave propagation can be realized in metals. The regime of weak spatial dispersion is relevant at room temperatures, whereas the regime of strong spatial dispersion occurs at low temperatures. Our calculations for the distribution of the generated EM field consider these two regimes, which correspond, respectively, to normal and anomalous skin effects.[7,12]

In the next section, we give the solution for the EM field in the form of Fourier integrals. In Secs. III and IV, respectively, we calculate these integrals in the limits of weak and strong spatial dispersion and analyze the spatial distribution of the EM field. In the weak dispersion case the EM field decays exponentially, just as in the case of normal skin effect. For the strong dispersion case the distribution is more complicated. It decays according to the power law in a certain region behind the particle and exponentially outside this region. For intermediate dispersion, the distribution is calculated numerically. This is done in Sec. V. Finally, in Sec. VI we summarize our conclusions.

## II. SOLUTION OF MAXWELL EQUATIONS

Let us consider a point particle with charge $q$ moving in the metal along the $x$ axis with speed $s$. The distribution of electric $\mathbf{E}$ and magnetic $\mathbf{H}$ fields is given by the solution of the Maxwell equations

$$\nabla\times\mathbf{H}=\frac{4\pi}{c}[\mathbf{j}+q\mathbf{n}_x s\,\delta(x-st)\,\delta(\mathbf{r}_\perp)],$$

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}. \tag{1}$$

Here, $c$ is the speed of light and $\mathbf{r}$ is the radius vector in the plane perpendicular to the unit vector $\mathbf{n}_x$. The current density $\mathbf{j}$ is the linear response of the media to the motion of the charged particle. The second current density term (containing the $\delta$ function) is the charged particle in motion.

In general, because of the effects of spatial dispersion, the connection between the current $\mathbf{j}$ and the electric field $\mathbf{E}$ is nonlocal. Thus, it is convenient to solve the set of equations (1) in the Fourier representation,

$$\mathbf{E}(\mathbf{r},t)=(2\pi)^{-4}\int_{-\infty}^{\infty}d\mathbf{k}d\omega\mathcal{E}(\mathbf{k},\omega)\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)], \tag{2}$$

where the connection between field and current is local,

$$j_i(\mathbf{k},\omega)=\sigma_{ik}(\mathbf{k},\omega)\mathcal{E}_k(\mathbf{k},\omega)\quad (i,k=x,y,z). \tag{3}$$

In the isotropic medium the conductivity tensor $\sigma_{ik}(\mathbf{k},\boldsymbol{\omega})$ has two independent components.[4] The component $\sigma_\parallel$ ($\sigma_\perp$) characterizes the conductivity in the direction parallel (perpendicular) to the direction of the wave vector $\mathbf{k}$. These are given by

$$\sigma_\parallel(\mathbf{k},\omega)=\frac{3Ne^2}{2m}\frac{\nu-i\omega}{(kv)^2}\left[2+\frac{\omega+i\nu}{kv}\ln\frac{i\nu+\omega-kv}{i\nu+\omega+kv}\right] \tag{4}$$

and

$$\sigma_\perp(\mathbf{k},\omega)=\frac{3Ne^2}{4m}\frac{i}{kv}\left\{2\frac{\omega+i\nu}{kv}-\left[1-\frac{(\omega+i\nu)^2}{(kv)^2}\right]\times\ln\frac{i\nu+\omega-kv}{i\nu+\omega+kv}\right\}. \tag{5}$$

Here, $N$ is the concentration of the conduction electrons in the metal, $v$ is the Fermi velocity, $m$ is the effective mass of the electron, and $\nu$ is the electron relaxation frequency. We have defined the branch of the logarithm as $\ln(-1)=i\pi$.

It is convenient to separate all vectors into parallel and perpendicular (with respect to vector $\mathbf{k}$) components:

$$\mathbf{E}=\mathbf{E}_\parallel+\mathbf{E}_\perp,$$

$$\mathbf{n}_x=\frac{(\mathbf{n}_x\cdot\mathbf{k})}{k^2}\mathbf{k}+\frac{\mathbf{k}[\mathbf{n}_x\mathbf{k}]}{k^2}.$$

After Fourier transforming Eqs. (1) and eliminating $\mathbf{H}$ we obtain the following two equations for $\mathcal{E}_\parallel$ and $\mathcal{E}_\perp$:

$$\sigma_\parallel\mathcal{E}_\parallel+2\pi\delta(\omega-k_x s)qs(k_x\mathbf{k}/k^2)=0,$$

$$k^2\mathcal{E}_\perp=\frac{4\pi i\omega}{c^2}\{\sigma_\perp\mathbf{E}_\perp+2\pi\delta(\omega-k_x s)qsk^{-2}[\mathbf{k}[\mathbf{n}_x\mathbf{k}]]\}. \tag{6}$$

After substituting the formal solution of $\mathcal{E}_\parallel$ and $\mathcal{E}_\perp$ into Eq. (2), one can then obtain the solution in the coordinate representation

$$E_x(x,\mathbf{r}_\perp,t)=i\frac{qs}{(2\pi)^3}\frac{\partial}{\partial x}\int_{-\infty}^{\infty}\frac{k_x dk_x d\mathbf{k}_\perp}{k^2\sigma_\parallel(\mathbf{k},k_x s)}\times\exp[ik_x(x-st)+i\mathbf{k}_\perp\cdot\mathbf{r}_\perp]+\frac{2q}{(2\pi)^2}\frac{s^2}{c^2}\frac{\partial}{\partial x}\times\int_{-\infty}^{\infty}\frac{k_\perp^2 dk_x d\mathbf{k}_\perp}{k^2}\frac{\exp[ik_x(x-st)+i\mathbf{k}_\perp\cdot\mathbf{r}_\perp]}{k^2-(4\pi i k_x s/c^2)\sigma_\perp(\mathbf{k},k_x s)} \tag{7}$$

and

$$\mathbf{E}_\perp(x,\mathbf{r}_\perp,t)=\frac{iqs}{(2\pi)^3}\frac{\partial}{\partial\mathbf{r}_\perp}\int_{-\infty}^{\infty}\frac{k_x dk_x d\mathbf{k}_\perp}{k^2}\times\left[\sigma_\parallel^{-1}(\mathbf{k},k_x s)+\frac{4\pi i k_x s/c^2}{k^2-(4\pi i k_x s/c^2)\sigma_\perp(\mathbf{k},k_x s)}\right]\times\exp[ik_x(x-st)+i\mathbf{k}_\perp\cdot\mathbf{r}_\perp]. \tag{8}$$

Here, $\mathbf{E}_\perp=\mathbf{E}-E_x\mathbf{n}_x$ and $\mathbf{k}_\perp=\mathbf{k}-k_x\mathbf{n}_x$ are the components of the electric field and wave vector in the plane perpendicular to the velocity of the test particle. These formulas are the formal solution for the spatial distribution of the electric field generated in the metal by the test particle.

In order to get the analytical expressions for the electric field it is necessary to perform the integration over the wave vector. This integration is not trivial. Obviously, it is not possible to integrate, in general, Eqs. (7) and (8) analytically. Therefore, below we shall consider two limiting cases, namely, weak and strong spatial dispersion. For the intermediate regime, we calculate the integrals numerically.

## III. WEAK SPATIAL DISPERSION

Weak spatial dispersion means that the following inequality is valid for typical values of the wave vector:

$$kv\ll\nu. \tag{9}$$

For slowly moving particles ($s\ll v$) weak spatial dispersion implies, of course, weak temporal dispersion also:

$$k_x s\ll\nu. \tag{10}$$

In the absence of spatial and temporal dispersions the conductivity of metals is given by the Drude formula

$$\sigma_{\parallel}=\sigma_{\perp}=\sigma=\frac{Ne^2}{m\nu}, \tag{11}$$

which can be obtained from Eqs. (4) and (5) in the limits expressed in Eqs. (9) and (10). Substitution of expression (11) into Eqs. (7) and (8) allows, after cumbersome calculations, for closed-form integration, and we obtain the following formulas for **E**:

$$E_x=-\frac{qs}{\sigma}\delta(x-st)\delta(\mathbf{r}_\perp)+\frac{qs}{8\pi\sigma}\frac{\exp\left[-\frac{1}{2}k_w(x-st)-\frac{1}{2}k_w\sqrt{(x-st)^2+r_\perp^2}\right]}{[(x-st)^2+r_\perp^2]^{3/2}}\times\left[(2+k_w\sqrt{(x-st)^2+r_\perp^2})\left(\frac{3r_\perp^2}{(x-st)^2+r_\perp^2}-1\right)-\frac{(k_wr_\perp)^2}{2}\right], \tag{12}$$

$$\mathbf{E}_\perp=\frac{qs}{4\pi\sigma}\frac{k_w\mathbf{r}_\perp\exp\left[-\frac{1}{2}k_w\sqrt{(x-st)^2+r_\perp^2}-\frac{1}{2}k_w(x-st)\right]}{[(x-st)^2+r_\perp^2]^{3/2}}\times\left\{1+\frac{1}{2}k_w\sqrt{(x-st)^2+r_\perp^2}+\frac{1}{2}k_w(x-st)\left[1+\frac{6\delta_w}{\sqrt{(x-st)^2+r_\perp^2}}+\frac{12\delta_w^2}{(x-st)^2+r_\perp^2}\right]\right\}. \tag{13}$$

From these formulas it follows that the electric field decays exponentially with the distance from the test particle. The decay length is characterized by

$$\delta_w\equiv k_w^{-1}=\frac{c^2}{4\pi\sigma s}. \tag{14}$$

In the physics of metals, the weak spatial dispersion case is termed the normal skin effect. The skin depth for the normal skin effect is given by (see, e.g., Ref. 2)

$$\delta_n=\frac{c}{(2\pi\sigma\omega)^{1/2}}. \tag{15}$$

Here $\omega$ is the frequency of the monochromatic EM field impinging on the metal surface. Formulas (15) and (14) give, respectively, the decay length of the EM field generated by the external radiation impinging on the surface and by the particle moving in the bulk of the metal. These two lengths manifest the same physical property of the metal, namely, the screening of the EM field. Therefore, there is an intrinsic connection between the two lengths. In order to see this connection we take into account the fact that the test particle generates not a monochromatic wave but a wave packet with a frequency centered at $\omega_0=2\pi s/\delta_w$. Then, the decay length (14) can be obtained from the formula (15) by substituting $\omega_0\to\omega$ in Eq. (15) and solving for $\delta_n=\delta_w$.

It is easy to see that $E_x$ and $E_\perp$ decay more rapidly in front of the particle (i.e., $x-st>0$) than behind it ($x-st<0$). This difference is more drastic the closer the observation point is to the line of motion ($r_\perp\ll|x-st|$). Exactly on the line ($r_\perp=0$) the decay length of the EM field is $\delta_w$ for $x-st>0$ (ahead). For $x-st<0$ (behind) the decay length vanishes and in this region the field decays by a power law.

## IV. STRONG SPATIAL DISPERSION

The case of strong spatial dispersion,

$$kv\gg\nu,\quad kl\gg1,\quad l=v/\nu, \tag{16}$$

is realized in pure single-crystal metal samples at low temperatures. Under this condition we can calculate analytically only the asymptotics of the integrals, Eqs. (7) and (8). It is helpful to note that also in the case of the anomalous skin effect only the asymptotic solution can be calculated analytically.[13]

Each integral in Eqs. (7) and (8) contains two terms. The contribution of the longitudinal (transversal) component of the conductivity appears solely in the first (second) term. Let us first calculate the contribution of the longitudinal term which requires us to calculate the integral

$$I(x-st,\mathbf{r}_\perp)=\int_{-\infty}^{\infty}\frac{ik_xd\mathbf{k}}{\nu-ik_xs}\,\frac{\exp[ik_x(x-st)+i\mathbf{k}_\perp\cdot\mathbf{r}_\perp]}{2+[(i\nu+k_xs)/kv]\ln(i\nu+k_xs-kv)/(i\nu+k_xs+kv)]}, \tag{17}$$

which arises when Eq. (4) is substituted into Eqs. (7) and (8). The inequality (16) allows us to simplify the integrand in Eq. (17). Then the integration can be performed analytically:

$$I(x-st,\mathbf{r}_\perp)\approx\frac{i}{2}\int_{-\infty}^{\infty}e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp}d\mathbf{k}_\perp\int_{-\infty}^{\infty}\exp[ik_x(x-st)]\frac{k_xdk_x}{\nu-ik_xs}$$
$$=-\frac{8\pi^4}{s}\delta(\mathbf{r}_\perp)\left\{\delta(x-st)+\frac{\nu}{s}\Theta(st-x)\times\exp\left[-\frac{\nu}{s}(st-x)\right]\right\}. \quad (18)$$

Here, $\Theta(x)$ is the step function. Because of the $\delta$ function appearing in Eq. (18), the contribution of the longitudinal part is nonzero only at the site of the test particle. Thus in the case of strong spatial dispersion the distribution of the electric field is due mainly to the transversal component of conductivity. In Sec. V this conclusion will be confirmed also by numerical calculations.

In order to calculate this contribution it is convenient to change variables from Cartesian to cylindrical coordinates $(k_x,k_\perp,\varphi)$ in Eqs. (7) and (8). After performing the integration with respect to the angle $\varphi$ we obtain

$$E_x=\frac{q}{\pi}\frac{s^2}{c^2}\frac{\partial}{\partial x}\int_0^\infty k_\perp^3J_o(k_\perp r_\perp)Z_0(k_\perp)dk_\perp \quad (19)$$

and

$$\mathbf{E}_\perp=-\frac{q}{\pi}\frac{s^2}{c^2}\frac{\partial}{\partial\mathbf{r}_\perp}\int_0^\infty k_\perp J_0(k_\perp r_\perp)Z_2(k_\perp)dk_\perp, \quad (20)$$

where $J_0$ is the Bessel function of order 0 and

$$Z_n(k_\perp)\equiv\int_{-\infty}^{\infty}\frac{k_x^ndk_x}{k_\perp^2+k_x^2}\frac{\exp[i(k_xx-st)}{k_x^2+k_\perp^2-(4\pi ik_xs/c^2)\sigma_\perp},$$
$$\xi\equiv x-st,\quad n=0,1,2. \quad (21)$$

According to the Cauchy theorem the integral (21) is determined by the contribution of the poles and the branch point. The branch point is related to the presence of the logarithm appearing in Eq. (5) for the transversal conductivity. The position of this branch point is

$$k_x^{(b)}=i\frac{\nu s\pm v\sqrt{k_\perp^2(v^2-s^2)+\nu^2}}{v^2-s^2}. \quad (22)$$

The positions of the poles are the solutions of the ‘‘dispersion relation’’[14]

$$k_\perp^2+k_x^2=\frac{4\pi k_xs}{c^2}\sigma_\perp(\mathbf{k},k_xs). \quad (23)$$

In the limit of strong spatial dispersion, Eq. (16), relation (23) reduces to the algebraic equation

$$k_x=-i\delta_s^2(k_x^2+k_\perp^2)^{3/2}. \quad (24)$$

Here,

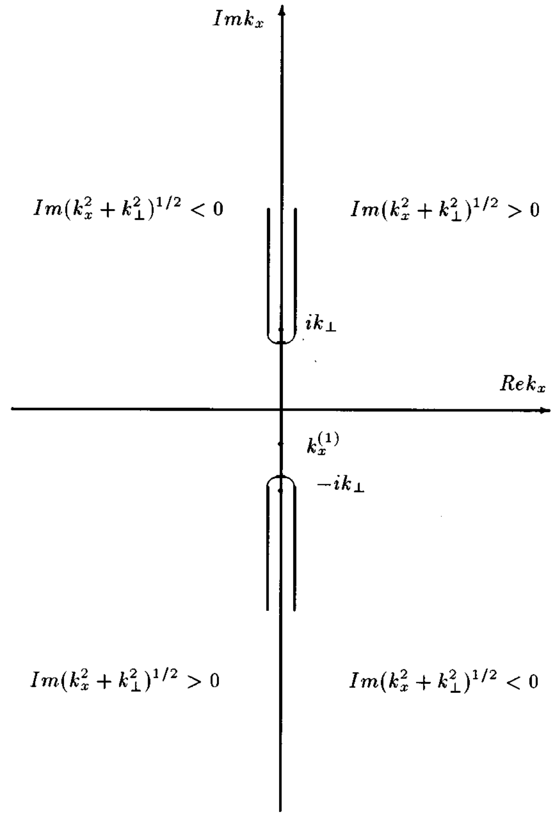


FIG. 1. The complex $k_x$ plane with the position of the pole (nearest to the real axis) and two branch points.

$$\delta_s\equiv k_s^{-1}=\left(\frac{c^2}{3\pi^2\sigma\nu}\frac{v}{s}\right)^{1/2} \quad (25)$$

is the typical decay length of the electric field in the strong spatial dispersion limit. Just as in the case of weak spatial dispersion treated above, this decay length can be obtained (to within a numerical factor) from the formula for the skin depth in the anomalous skin-effect conditions,[6,7,12,13]

$$\delta_a=\left(\frac{c^2l}{4\pi\omega\sigma}\right)^{1/3},$$

by replacing $\omega\to\omega_0=2\pi s/\delta_s$ and solving for $\delta_a=\delta_s$. We would like to note that $\delta_s$ (as well as $\delta_a$) does not depend on the relaxation frequency $\nu$ and, thus, corresponds to the case of collisionless Landau damping.

To solve Eq. (24) it is necessary to select a single branch of the multivalued function $(k_x^2+k_\perp^2)^{1/2}$. We do this by cutting the complex plane $k_x$ as shown in Fig. 1. After squaring both sides of Eq. (24) we obtain a cubic equation for $(k_x\delta_s)^2$. Its solution is, ignoring superfluous roots,

$$k_x^{(1)}=-ik_s\sqrt{a-A-B},$$
$$k_x^{(2)}=-k_s\sqrt{-a-\frac{1}{2}(A+B)-\frac{i\sqrt{3}}{2}(A-B)},$$
$$k_x^{(3)}=-k_x^{*(2)}. \quad (26)$$

Here,

$$A=\left(\frac{a}{2}+\sqrt{\frac{1}{27}+\frac{a^2}{4}}\right)^{1/3},\quad B=-\left(-\frac{a}{2}+\sqrt{\frac{1}{27}+\frac{a^2}{4}}\right)^{1/3},$$

$$a=k_\perp/k_s. \tag{27}$$

In all formulas the roots are arithmetic. $A$, $B$, and $a$ obey the obvious relations:

$$0<A+B<a,\quad AB=-1/3,\quad a=A+B+(A+B)^3.$$

We use the Cauchy theorem to integrate Eq. (21). The contour of integration is a semicircle on the upper (lower) complex plane for $x-st>0$ $(x-st<0)$. Since the poles are situated only in the lower plane, it then follows that the spatial pattern of the EM field is very different for the regions in front of $(x-st>0)$ and behind $(x-st<0)$ the moving particle.

The EM field in front of the particle is due to the contribution of the branch point (22). The integral along the path surrounding the upper branch point is, after replacing $k_x$ by $iz$,

$$Z_n(k_\perp)=2i\,\mathrm{Im}\int_{k_x^{(b)}}^{\infty}dz(z^2-k_\perp^2)^{-1}z^n\exp[-(x-st)z]\times\left\{k_\perp^2-z^2+\frac{k_s^2 z}{\pi\sqrt{z^2-k_\perp^2}}\left[2\frac{sz+\nu}{v\sqrt{z^2-k_\perp^2}}-\left[1-\frac{(zs+\nu)^2}{v^2(z^2-k_\perp^2)}\right]\left(i\pi+\ln\frac{v\sqrt{z^2-k_\perp^2}-sz-\nu}{v\sqrt{z^2-k_\perp^2}+sz+\nu}\right)\right]\right\}^{-1}. \tag{28}$$

After substituting Eq. (28) into Eqs. (19) and (20) for the electric field, we obtain a double integral in which we first change the order of integration,

$$\int_0^\infty dk_\perp\int_{k_x^{(b)}}^\infty dz\cdots\equiv\int_{\nu/(v-s)}^\infty dz\int_0^{\sqrt{z^2-(\nu+sz)^2/v^2}}dk_\perp\cdots$$

and then change the variables,

$$k_\perp=z\sqrt{1-u^2},\quad z=\frac{\nu}{v-s}(1+z').$$

Further simplification is achieved using the following inequalities:

$$x-st\gg l,\quad l/\delta_s\gg 1. \tag{29}$$

The first inequality implies that our analytical results are valid for distances rather far from the moving particle. The second inequality is the condition of strong spatial dispersion, Eq. (16). Conditions (29) allow us to calculate the asymptote of the integral. The main contribution to the integral comes from the vicinity of the point $(z'=0,u=1)$. Expanding the integrand around this point gives the asympotic expression

$$E_x=-\frac{4\pi}{(ksl)^2}\frac{q}{l^2}\frac{s^2}{c^2}\exp[-(x-st)/l]\times\int_0^\infty z^3\exp\left(-\frac{x-st}{l}z\right)dz\times\int_0^1 dy\,y^3(1-y^2)J_0\left(\frac{r_\perp}{l}y\sqrt{2z}\right). \tag{30}$$

For the sake of brevity we do not give here the formula for $\mathbf{E}_\perp$. The integration variable $y$ is related to $u$ by $1-u=z'y^2$. Integrals in Eq. (30) can be calculated in closed form, yielding

$$E_x=\frac{2\pi q}{(k_sl)^2}\frac{s^2}{c^2}\frac{l^2}{(x-st)^4}\exp\left[-\frac{x-st}{l}-\frac{r_\perp^2}{2l(x-st)}\right]\times\left[\frac{r_\perp^2}{2l(x-st)}\left(1-\frac{r_\perp^2}{2(x-st)^2}\right)-1\right] \tag{31}$$

and

$$\mathbf{E}_\perp=-\frac{\pi q}{(k_sl)^2}\frac{s^2}{c^2}\frac{\mathbf{r}_\perp l}{(x-st)^4}\exp\left[-\frac{x-st}{l}-\frac{r_\perp^2}{2l(x-st)}\right]. \tag{32}$$

Formulas (31) and (32) describe the electric field in front of the test particle. Note that it decays exponentially with the distance $x-st$ and also with distance $r_\perp$.

The branch point in the lower complex plane gives, by symmetry, the same contribution for $x-st<0$. However, in this region, the contribution of the poles dominates that of the branch point. The principal contribution to the integral (21) comes from the pole nearest to the real axis, $k_x^{(1)}$. Considering the contribution of only this pole, we obtain

$$E_x=-\frac{2q}{k_s^3}\frac{s^2}{c^2}\frac{\partial}{\partial x}\int_0^\infty\frac{k_\perp^3 J_0(k_\perp r_\perp)(a-A-B)^{1/2}\exp[-k_s|x-st|\sqrt{a-A-B}]dk_\perp}{(A+B)(2A+2B-3a)}. \tag{33}$$

The dependence of $a$, $A$, and $B$ on $k_\perp$ is given by Eq. (27).

After changing to the new variables,

$$y=A+B,\quad k_\perp dk_\perp=\tfrac{1}{2}k_s^2(1+3y^2)dy,$$

Eq. (33) becomes

$$E_x=qk_s\frac{s^2}{c^2}\frac{\partial}{\partial_x}\int_0^\infty\sqrt{y}(1+y^2)J_0(k_s r_\perp\sqrt{y+y^3})\times\exp(-k_s|x-st|y^{3/2})dy. \tag{34}$$

Now we distinguish two different cases corresponding to two different asymptotics of the integral (34).

Case I:

$$1\ll k_s r_\perp\ll(k_s|x-st|)^{1/3}\ll k_s l. \tag{35}$$

Case II:

$$1\ll(k_s|x-st|)^{1/3}\ll k_s r_\perp\ll k_s|x-st|\ll(k_s l)^2 k_s r_\perp. \tag{36}$$

In case I the main contribution to integral (34) comes from small values of $y$ $(y\sim[k_s(x-st)]^{-2/3})$. Hence the argument of the Bessel function can be taken as zero. After this simplification, the integral can be easily calculated and we obtain the following result for the electric field $E_x$:

$$E_x=\frac{2}{3}\frac{s^2}{c^2}\frac{q}{(x-st)^2}. \tag{37}$$

Returning to the variables $(k_x,k_\perp)$ we find that

$$|k_x|\sim|x-st|^{-1}\quad\text{and}\quad k_\perp\sim k_s|k_s x-st|^{-1/3} \tag{38}$$

give the main contribution to the asymptotic formula (37). Note that because of inequalities (35) the condition of strong spatial dispersion is satisfied for the values (38).

For case II, the principal contribution is due again to small values of $y$ but now the argument of the Bessel function is much greater than one [cf., Eq. (36)]. Hence, substituting the asymptotic expression for the Bessel function,

$$J_0(k_s r_\perp\sqrt{y})\approx\sqrt{\frac{2}{\pi k_s r_\perp}}y^{-1/4}\cos\left(k_s r_\perp\sqrt{y}-\frac{\pi}{4}\right), \tag{39}$$

into integral (34), and changing the variable $(y=z^2)$, we obtain

$$E_x=2qk_s^2\frac{s^2}{c^2}\left(\frac{2}{\pi k_s r_\perp}\right)^{1/2}\mathrm{Re}\int_0^\infty z^{9/2}\times\exp\left(-k_s\left|x-st\right|z^3+ik_s z-\frac{\pi}{4}\right)dz. \tag{40}$$

The asymptote of Eq. (40) is calculated by the method of steepest descent. The result is

$$E_x=-\frac{2\sqrt{2}}{9\sqrt{3}}q\frac{s^2}{c^2}(x-st)^{-2}\left(\frac{k_a^3 r_\perp^3}{k_s|x-st|}\right)^{1/2}\times\exp\left(-\frac{k_s r_\perp}{3}\sqrt{\frac{2r_\perp}{3|x-st|}}\right)\times\cos\left(\frac{k_s r_\perp}{3}\sqrt{\frac{2r_\perp}{3|x-st|}}-\frac{\pi}{4}\right), \tag{41}$$

which shows that the electric field decays exponentially and with fast oscillations. In **k** space the main contribution to the asymptote (41) is due to

$$k_x\approx k_s(r_\perp/|x-st|)^{3/2},\quad k_\perp\approx k_s(r_\perp/|x-st|)^{1/2}. \tag{42}$$

Again, these values satisfy the condition of strong spatial dispersion for case II. In summary, results (37) and (41) show that there are two different regions in the space behind the particle. These two regions are separated by a cubic parabola

$$k_s|x-st|=(2/27)(k_s r_\perp)^3. \tag{43}$$

Inside this parabola the field decays by a power law [cf. Eq. (37)], with $|x-st|$, whereas outside it decays exponentially, Eq. (41), with $r_\perp$. This pattern defines the wake produced by the charged particle moving in the conducting media.

The method for calculating the perpendicular component of the electric field, $\mathbf{E}_\perp$, and is exactly the same as the one detailed above for the field $E_x$, so we simply give the final result.

Case I:

$$\mathbf{E}_\perp=-\frac{2}{3}q\frac{s^2}{c^2}\frac{\mathbf{r}_\perp}{|x-st|^3},$$

$$1\ll k_s r_\perp\ll(k_s|x-st|)^{1/3}\ll k_s l. \tag{44}$$

Case II:

$$\mathbf{E}_\perp=\frac{qk_s}{12|x-st|}\left(\frac{2r_\perp}{3|x-st|}\right)^{5/2}\frac{s^2}{c^2}\exp\left(-\frac{k_s r_\perp}{3}\sqrt{\frac{2r_\perp}{3|x-st|}}\right)\times\cos\left(\frac{k_s r_\perp}{3}\sqrt{\frac{2r_\perp}{3|x-st|}}-\frac{\pi}{4}\right)\frac{\mathbf{r}_\perp}{r_\perp}, \tag{45}$$

$$1\ll(k_s|x-st|)^{1/3}\ll k_s r_\perp\ll k_s|x-st|\ll(k_s l)^2 k_s r_\perp.$$

The magnetic field generated by the moving particle can now be calculated immediately using the second equation in Eq. (1). Because of the cylindrical symmetry of the problem, the magnetic field consists only of the $H_\varphi$ component. In front of the particle it is

$$|H_\varphi| = \frac{\pi q}{3(k_s l)^2}\frac{s}{c}\frac{r_\perp}{(x-st)^3}\exp\left(-\frac{x-st}{l}-\frac{r_\perp^2}{2l(x-st)}\right), \tag{46}$$

$$x-st \gg l.$$

Behind the moving particle, the magnetic field is given by the following.

Case I:

$$|H_\varphi| = \tfrac{2}{9}\Gamma(\tfrac{2}{3}) q k_s^3 \frac{s}{c}\frac{r_\perp}{[k_s(x-st)]^{5/3}}, \tag{47}$$

$$1 \ll k_s r_\perp \ll (k_s|x-st|)^{1/3} \ll k_s l.$$

Case II:

$$|H_\varphi| = \left(\frac{2}{3}\right)^{3/2} q\frac{s}{c}\frac{k_s}{|x-st|}\sqrt{\frac{r_\perp}{|x-st|}}\exp\left(-\frac{k_s r_\perp}{3}\sqrt{\frac{2r_\perp}{3|x-st|}}\right) \times \cos\left(\frac{k_s r_\perp}{3}\sqrt{\frac{2r_\perp}{3|x-st|}}-\frac{\pi}{4}\right), \tag{48}$$

$$1 \ll (k_s|x-st|)^{1/3} \ll k_s r_\perp \ll k_s|x-st| \ll (k_s l)^2 k_s r_\perp .$$

These two cases are the same as the ones introduced before [cf. Eqs. (35) and (36)].

## V. NUMERICAL RESULTS

In the intermediate region of parameters,

$$\delta_s \sim \delta_w \sim l. \tag{49}$$

It is necessary to recur to numerical calculations for the electric field $E_x$, Eq. (7). Two terms contribute to the electric field: One is related to the longitudinal conductivity $\sigma_\parallel$ and another to the transversal conductivity $\sigma_\perp$. The transversal part can be calculated directly without any transformations due to the good convergence of the integrals. On the other hand, the longitudinal part is singular [see Eq. (18)]. The singularity comes from the region of large values of $k$. To calculate the contribution of $k \gg 1/l$ we expand the function $1/\sigma_\parallel$, Eq. (4), with respect to the large parameter $kv/(k_x s + i\nu)$:

$$\frac{1}{\sigma_\parallel(\infty)} \approx \frac{ik_x s-\nu}{3\sigma\nu}\left[\frac{(kv)^2}{(k_x s+i\nu)^2} - i\pi\frac{kv}{2(k_x s+i\nu)} - \left(\frac{\pi^2}{4}-1\right)\right]. \tag{50}$$

The first (quadratic) term of this expansion gives a singular contribution to the electric field which is nonzero only at the site of the test particle [see Eq. (18)]. A smooth contribution to the electric field comes from the last two terms in the expansion (50). Substituting these two terms in the integral (7) and performing the integration we obtain

$$E_x^{(1)}(x-st,r_\perp) = -\frac{qsv}{6\pi\sigma\nu}\frac{\partial}{\partial x}\left\{\frac{x-st}{[(x-st)^2+r_\perp^2]}\right\} + \left(\frac{\pi^2}{4}-1\right)\frac{qs^2}{12\pi\sigma\nu} sng(x-st) \times \frac{\partial}{\partial x}\left\{[(x-st)^2+r_\perp^2]^{-3/2}\left[\frac{\nu}{s}(x-st)+\frac{r_\perp^2-2(x-st)^2}{(x-st)^2+r_\perp^2}\right]\right\}. \tag{51}$$

Now the electric field $E_x$, Eq. (7), can be written as a sum of three nonsingular terms:

$$E_x(x-st,r_\perp) = E_x^{(1)} - \frac{qs}{(2\pi)^2}\int_0^\infty k_\perp J_0(k_\perp r_\perp) dk_\perp \int_{-\infty}^{\infty}\left[\frac{1}{\sigma_\parallel(k_\perp,k_x s)} - \frac{1}{\sigma_\parallel(\infty)}\right]\frac{k_x^2 \exp[ik_x(x-st)]}{k_x^2+k_\perp^2} dk_x + \frac{iqs^2}{\pi}\int_0^\infty k_\perp^3 J_0(k_\perp r_\perp) dk_\perp \int_{-\infty}^{\infty}\frac{k_x dk_x}{k_x^2+k_\perp^2}\,\frac{\exp[ik_x(x-st)]}{k^2c^2-4\pi i k_x s\sigma_\perp(k_\perp,k_x s)}. \tag{52}$$

This is the equation that is to be calculated numerically.

We consider degenerate metallic plasma with typical parameters $N=10^{23}$ cm$^{-3}$, and $v=10^8$ cm/s. The charge and the mass of the test particle are those of the free electron. The velocity $s=10^5$ cm/s, which is of the order of sound velocity in metals. In this case the screening length, Eq. (25), is equal to $\delta_s = 3.46\times10^{-5}$ cm. It is interesting to compare $\delta_s$ with the skin depth $\delta_{ir}=c/\omega_p$ ($\omega_p$ is the plasma frequency) in the infrared region. For the selected parameters of metal, $\delta_{ir}=0.17\times10^{-5}$ cm; i.e., it is less, but of the same order of magnitude, than the screening length $\delta_s$. Hence $\delta_{ir}$ is the minimum value of the skin depth in metal,[6,7] and it is clear that the limiting value of screening length $\delta_s$ cannot be less than $c/\omega_p$. This minimum value is realized for the test particle moving with Fermi velocity $v$ [see Eq. (25)]. For higher velocities the plasma effects (excitation of plasmons) become important and thus again we see that our consideration is correct if the velocity of the test particle does not exceed the Fermi velocity.

Two different values for the parameter $k_w l$ were used in

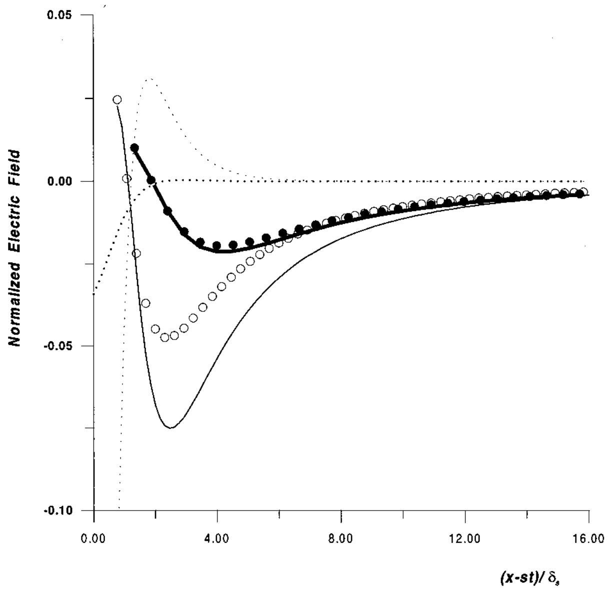


FIG. 2. Normalized electric field in front of the charge as a function of dimensionless distance for $k_w l=1$ and $r_\perp/\delta_s=1.65$ (thin lines) and for $k_w l=3$ and $r_\perp/\delta_s=2.67$ (thick lines). Solid lines are for the electric field, Eq. (52), dashed lines are for the limit of weak spatial dispersion, Eq. (12), and circles are for the transversal part [the last term in Eq. (52)].

our calculations: $k_w l=1$ and $k_w l=3$. They correspond, respectively, to the relaxation frequencies $\nu_1=1.88\times10^{12}$ s$^{-1}$ and $\nu_2=1.08\times10^{12}$ s$^{-1}$, and screening lengths $\delta_w^{(1)}=5.71\times10^{-5}$ cm and $\delta_w^{(2)}=3.08\times10^{-5}$ cm. Such relaxation frequencies are typical for nitrogen temperatures. Note that condition (49) is satisfied.

Figure 2 shows the normalized electric field $E_x/E_0$ in front of the charges a function of the dimensionless distance $(x-st)/\delta_s$. The electric field is normalized to the value of $E_0=(s^2/c^2)e/\delta_s^2\approx4.45\times10^{-12}$ (cgs units). Solid lines are for the electric field given by Eq. (52) and dashed lines are for the electric field calculated in the limit of weak spatial dispersion.[15] The difference between solid and dashed lines results from the effects of spatial dispersion. Even for $k_w l=1$ (thin lines) spatial dispersion is important and it becomes more so with the increase of the parameter $k_w l$ (thick lines). In Fig. 2 the solid and open circles represent the contribution of the transversal part to the electric field [the last term in Eq. (52)] for $k_w l=1$ and $k_w l=3$, respectively. It can be readily seen that the transversal part gives a good approximation for the electric field. When parameter $k_w l$ increases the contribution of the longitudinal part vanishes. Therefore in the limit of strong spatial dispersion, Eq. (16), this part can be neglected (see Sec. IV).

The electric field behind the test particle for the same values of the parameters $k_w l$ and $r/\delta_s$ is shown in Fig. 3. In this region the contribution of the longitudinal part to the electric field becomes even less than in the region in front of the particle. Thus, this part can be neglected even for values of parameters $k_w l$ as large as 1. Behind the test particle, the electric field decays slowly, in qualitative agreement with the power law in Eq. (37). The domain which appears in Fig. 3 lies inside the parabola, Eq. (43), where we predict nonex-

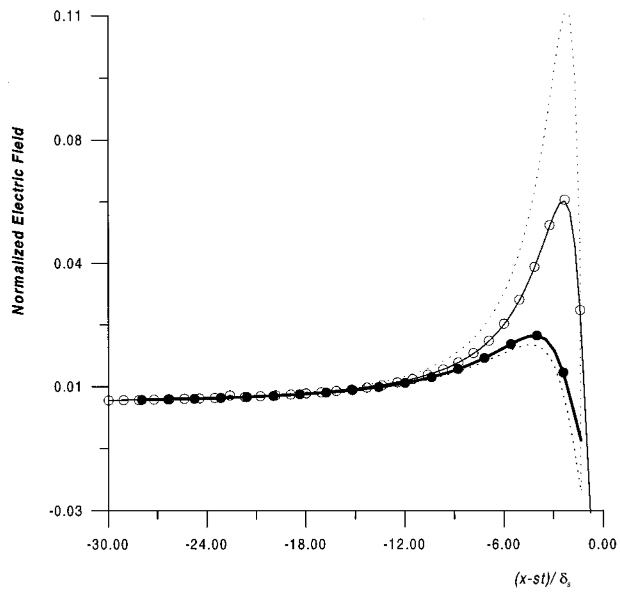


FIG. 3. Normalized electric field in the wake behind the particle. The parameters are the same as in Fig. 2.

ponential decay. To check this, we plot in Fig. 4 the electric field evaluated at a distance far from the test particle ($|x-st|/\delta_s\sim10^2$) but still inside the wake. We see that the field fluctuates with $|x-st|/\delta_s$, but the amplitude does not decay exponentially. Moreover, for $|x-st|/\delta_s=200$ the amplitude noticeably increases. In these regions we were not able to calculate the asymptotic behavior and the nature of these oscillations is unknown.

Our analytical and numerical calculations show that in the limit of strong spatial dispersion the electromagnetic field has a complicated spatial distribution. The asymptotics of the distribution can be obtained for distances which are much larger than the screening length $\delta_s$ only. Within the sphere of radius $\delta_s$, where the electric field is the same order of mag-

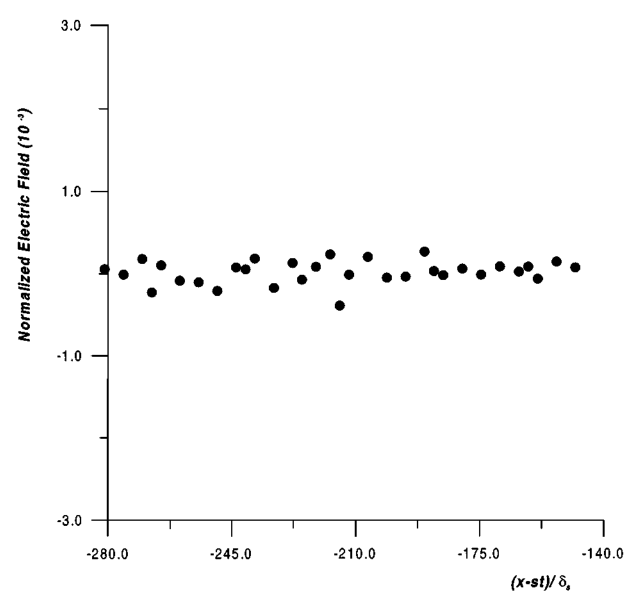


FIG. 4. Normalized electric field for $k_w l=3$ and $r_\perp/\delta_s=2.67$ in the wake far away from the particle.

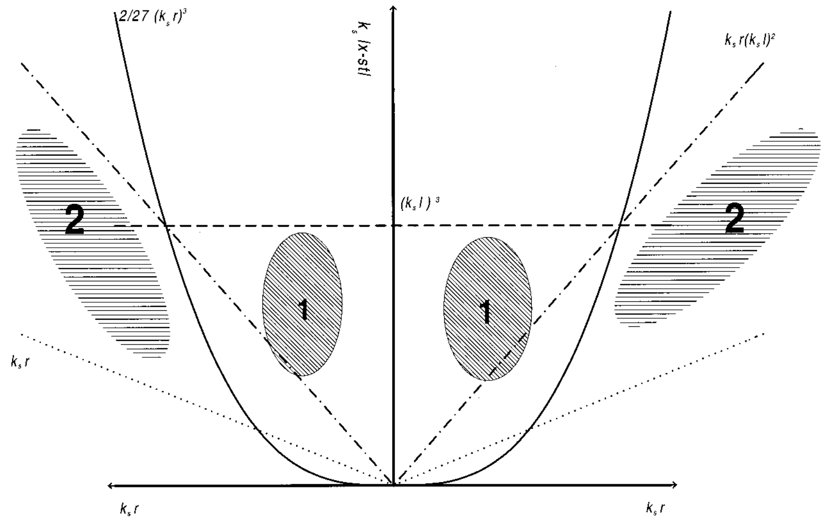


FIG. 5. The parabolic wake behind the test particle in the regime of strong spatial dispersion. In shaded region the analytical approach is valid. In region 1 the field decays by power law, Eq. (37), and in region 2 it decays exponentially, Eq. (41).

nitude as in vacuum, the screening effects start to manifest themselves. In the theory of an anomalous skin effect[13] the situation is very similar: Asymptotics of the field can be calculated far away from the metal surface at the distances which are larger than the skin depth $\delta_a$. However, similar though these two problems appear to be, there is an important difference.

In the skin-effect geometry the field in the metal depends only on the distance from the surface, and the field generated by the test particle has a cylindrical symmetry; i.e., it depends on two coordinates ($x$-$st$ and $r_\perp$). Because of this complication, the analytical results for the induced field can be obtained only for two domains in space. In Fig. 5 these important domains are shaded. They are separated by the cubic parabola, Eq. (43). The location of domains 1 and 2 is given by inequalities (35) and (36), respectively. For example, domain 1 is located in the region bounded by the cubic parabola and the horizontal line $k_s|x-st|=(k_sl)^3$. Outside the shaded region the field should be calculated numerically. We mention here that the calculations are computationally very intensive if the parameter $k_sl$ is large. To calculate the field outside domain 2 for values of $k_sl$ larger than 10 or so, parallel computing would be desirable.

Fortunately, for physical applications these regions should not be important because the field there is rather weak. Therefore we hope that the asymptotics obtained in this paper are sufficient to estimate the induced field in the important region around the moving charge.

## VI. CONCLUSIONS

We have calculated the EM fields generated by a slowly moving ($s \ll v$) charged particle in a metal. Unlike the usual approach which considers only the energy loss, here we have calculated the whole distribution of the EM field, taking into account the effects of spatial dispersion. In the case of weak spatial dispersion we demonstrated that the EM field decays exponentially with the distance from the moving particle. This behavior is typical for weak spatial dispersion; for example, the EM field also decays exponentially in metals under normal skin-effect conditions. In contrast, we have found, for the case of strong spatial dispersion, that the distribution of the EM field has a much more complicated structure. The distribution of the EM field in front of the moving particle is different than behind the particle. Not only are the fields weaker in front of it but they also decay exponentially faster than those behind it. The fields behind the particle have a ''wake'' structure; the fields' intensity is stronger inside the wake. The equation of the profile of the wake is given by Eq. (43).

The electric field is oriented primarily along the direction of the particle motion ($E_x \gg E_\perp$). We remark that such a pattern is true for rather large distances behind the particle, at least larger than a scaling length $\delta_s$ which we have introduced here [cf. Eq. (25)]. $\delta_s$ plays the same role in the distribution of the EM field as does the skin depth in the theory of the skin effect in metals. For typical metals with concentration $N=10^{23}$ sec$^{-3}$, Fermi velocity $v \approx 10^8$ cm/sec, and particle speed $s \approx 10^5$ cm/sec (speed of sound in metals) the scaling length is $\delta_s \approx 10^{-5}$ cm. Since the electron mean free path $l$ in pure metals at low temperatures exceeds $10^{-2}$ cm, the condition of strong spatial dispersion, Eq. (16), is fulfilled.

## ACKNOWLEDGMENTS

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[14] Note that the singularity at $k^2=k_\perp^2+k_x^2=0$ appearing in each of the integrands in Eq. (7) cancel out. The same is true for Eq. (8). Therefore, the contribution of $k=0$ does not have to be calculated in Eq. (21). Moreover, $k=0$ belongs to the case of weak spatial dispersion where there is no singularity in the integrand.

[15] To calculate this limit we used Eq. (12) in which we substituted two values of screening length ($\delta_w^{(1)}$ and $\delta_w^{(2)}$) and relaxation frequencies ($\nu_1$ and $\nu_2$). Note that the decay length $\delta_w^{(2)}$ is almost twice less than $\delta_w^{(1)}$ therefore the electric field for $k_w l=3$ drops faster than for $k_w l=1$ (dash lines).