One-dimensional fermions with δ-function repulsion in the Brueckner theory

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The single-particle energies for a one-dimensional system of interacting fermions via a repulsive δ-function potential are self-consistently calculated using the Brueckner and Gammel method. The breakdown of the usual Fermi liquid picture is signaled in this approach by the occurrence of a negative gap at $\pm k_F$ between the lowest energy of a particle state ($|k|>k_F$) and the highest energy of a hole state ($|k|<k_F$); a normal ground-state configuration with all the hole states double occupied is thus unstable. Two possible pictures getting account of the rearrangement of the particles in a stable configuration are discussed: (1) an effective potential simulating a normal Fermi liquid behavior; (2) a condensate of fermions and a reduced Fermi sea normally occupied. The ground-state energy computed within these schemes is in remarkable agreement with the Bethe-Ansatz results for all values of the model parameters. [S0163-1829(96)07048-8]

I. INTRODUCTION

It is well known that the Landau quasiparticle description of the Fermi liquids breaks down in one dimension (1D).[1] An alternative theory, based on an exactly soluble model[2] by the bosonization technique,[3] leads to the concept of the Luttinger liquid,[4] with distinct properties.[5] A quite rich picture of the 1D systems can be obtained in the renormalization group approach.[6] Exact results are available for certain 1D models by the Bethe-Ansatz method,[7] the model considered here —1D fermions with δ-function repulsion, belonging to this class.[8,9] Nevertheless, the use of the traditional methods of the many-body theory in 1D can be interesting for at least two reasons: to determine the range of their applicability and, if possible, to get a more intuitive picture of the specific phenomena. A repulsive δ-function interaction between the fermions suggested to us the us of a well-known method introduced in nuclear physics many years ago in order to overcome the difficulties associated with the hard-core interactions: the *Brueckner theory*.[10–14] We shall apply here the Brueckner and Gammel method[15]— slightly improved by an additional self-consistency requirement, to determine self-consistently the single-particle energies (SPE's) for the 1D repulsive δ model (RDM). The numerical solution of the corresponding integral equations for the SPE's indicates the breakdown of the usual Fermi liquid picture: the occurrence of a negative gap at $\pm k_F$ makes the presumed Fermi sea unstable; it determines a finite phase shift in the forward scattering of two particles — an argument used by Anderson[16] to conjecture Luttinger liquid properties for the 2D Hubbard model. A stable configuration can be reached only by assuming an abnormal occupation of the k states. We discuss at first how the redistribution of the particles can be included in an *effective potential picture*: a normal behavior is simulated by a reduced effective repulsion between the particles, a picture that should be viewed merely as a suitable scheme to compute the ground-state energy (GSE) of the system. The instability of the normal configuration is interpreted after that within a *fermion condensation picture*: the particles from the top of the unstable Fermi sea have the tendency to condense at $\pm k_F$, the rest form a reduced Fermi sea with an effective Fermi momentum. The GSE of the 1D RDM calculated within these two pictures is in excellent agreement with the numerical Bethe-Ansatz results[17] for all values of the model parameters.

II. ONE-DIMENSIONAL REPULSIVE δ MODEL

The Hamiltonian for N particles confined to a chain of length L and interacting between them through a repulsive δ-function potential (in units where $\hbar^2/2m=1$) is

$$H=\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2}+2c\sum_{i<j}\delta(x_i-x_j),\quad c>0. \tag{1}$$

For spin-1/2 particles, the interaction is only between those with opposite spin; the particles with the same spin never meet due to the exclusion principle. In this case the model can be solved by Bethe-Ansatz.[8,9] In particular, the GSE E_g can be found by solving an integral equation.[9] The numerical results for the dimensionless quantity $E\equiv(2/3\pi)E_g/E_{0g}$ (the lower index 0 corresponding to the noninteracting case) in terms of the dimensionless coupling constant $C\equiv c/k_F$ [$k_F=(\pi/2)N/L$] have been compared with other approximate schemes,[17,18] the T-matrix approximation[18] giving the best agreement with the Bethe-Ansatz solution.

The second quantized form of the Hamiltonian (1) in momentum representation reads

$$\mathcal{H}=\sum_{k,\sigma}\varepsilon_0(k)c^{\dagger}_{k,\sigma}c_{k,\sigma}+C\sum_{k_{1-3},\sigma}c^{\dagger}_{k_1+k_3,\sigma}c^{\dagger}_{k_2-k_3,-\sigma}c_{k_2,-\sigma}c_{k_1,\sigma}, \tag{2}$$

where, in the chosen units,

$$\varepsilon_0(k)=k^2,\quad k\in(-\infty,+\infty) \tag{3}$$

and $c^{\dagger}(c)_{k,\sigma}$ denotes the usual Fermi creation (annihilation) operator of a one-particle state with a given momentum k (in units of the Fermi momentum k_F) and spin $\sigma=\pm 1/2$. Al-

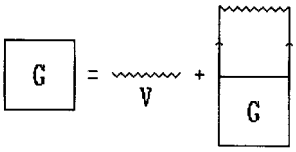

FIG. 1. Diagrammatic representation of the reaction matrix equation.

though it was assumed in (1) and (2) that the system is enclosed in a box of finite length, we shall consider only the thermodynamic limit ($N\to\infty$, $L\to\infty$, $N/L=$ finite).

III. OVERVIEW OF THE BRUECKNER THEORY

The idea of the Brueckner theory can be easily understood by comparing it with the Hartree-Fock method:[12,13] if the last one considers the motion of a particle in the average field produced by the others (independent-particle approximation), in the Brueckner's approach the interaction of any two particles is treated exactly and the effect of the rest of the particles on the interacting pair is replaced by an average (independent-pair approximation). The main ingredient of the Brueckner theory is the *reaction matrix G*, describing the effective interaction between two particles: it replaces the bare potential V by an infinite series which takes into account the two-body interaction to all orders of the potential. Finding the reaction matrix is equivalent to solving a Schrödinger equation which describes the scattering of two particles in the presence of all the others. However, by neglecting the three-body scatterings, the Brueckner theory can give confident results only when the range of the potential is much less than the particle separation.

The reaction matrix obeys an integral equation written formally as

$$G[W]=V-V\frac{1}{e}PQG[W], \tag{4}$$

and is pictured in Fig. 1. All the operators G, P, Q, V, e in Eq. (4) act in the two-particle space spanned by the *unperturbed* two-body wave functions $\Phi_{k_1,k_2}\equiv|k_1,k_2\rangle$. The e denominator in Eq. (4) is defined by

$$e|k_1,k_2\rangle=[\varepsilon(k_1)+\varepsilon(k_2)-W]|k_1,k_2\rangle, \tag{5}$$

where ε denotes the (still undetermined) SPE (Ref. 19) and W represents the ''starting energy,''[14] i.e., the sum of the hole energies ($|k|<k_F$) minus the sum of the particle energies ($|k|>k_F$) in a given configuration, excepting the pair under consideration (examples are given in the next section). The reaction matrix depends parametrically on the starting energy: when W coincides with the total energy Ω of the initial (final) two-particle state, we say that the G matrix is calculated on-energy-shell (OnES), otherwise G is said to be calculated off-energy-shell (OffES).

In the Brueckner theory, the e denominators never vanish: P in Eq. (4) means the principal part operator. If the Pauli principle is taken into account, then the intermediate states below the Fermi level must be excluded and consequently the Pauli operator Q has been introduced[11,12,20]

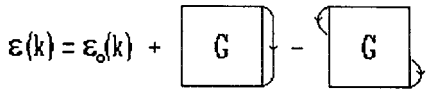

FIG. 2. The energy of a hole or a particle state in terms of the reaction matrix; this Hartree-Fock expression for the SPE together with the G-matrix equation pictured in Fig. 1 should determine, in principle, both the G matrix and the SPE.

$$Q|k_1,k_2\rangle=\begin{cases}|k_1,k_2\rangle & \text{if } |k_1|,|k_2|>k_F\\ 0 & \text{otherwise.}\end{cases} \tag{6}$$

In the initial formulation of the theory,[10] the Q operator did not appear. Usually, the Pauli operator makes superfluous the condition of principal part: if the particle energies are greater than the hole energies (a natural assumption for a normal Fermi liquid), then the e denominators are always positive quantities (evident for OnES propagation and also true for OffES one[12]). But one cannot say *a priori* if it is so or not, the SPE's are still not determined; it is the task of the theory to find them and only after that we can conclude if the system behaves as a normal Fermi liquid or not (provided a solution can be found). Consequently, the action of the P operator may be no superfluous in general.

The connection between the reaction matrix and the two-particle scattering can be seen by defining a *correlated* two-body wave function Ψ (Ref. 20)

$$\Psi_{k_1,k_2}=\Phi_{k_1,k_2}-\frac{1}{e}PQG\Phi_{k_1,k_2}. \tag{7}$$

From Eqs. (4) and (7) it follows $G\Phi=V\Psi$ and replacing it back in Eq. (7) we get

$$\Psi_{k_1,k_2}=\Phi_{k_1,k_2}-\frac{1}{e}PQV\Psi_{k_1,k_2}. \tag{8}$$

Equation (8) computed OnES is called the *Bethe-Goldstone equation*.[20] It becomes identical with the integral equation for the two-particle scattering in empty space by replacing the PQ operator with unity and taking the energies ε in the e denominator to be the kinetic energies. The scattering states, given by the zeros of the e operator and expressing the energy conservation in the two-body scattering, are excluded in the Brueckner theory by the PQ operator. The consequence for a normal Fermi liquid is the absence of any scattered wave at large distances: Ψ approaches Φ so rapidly that all phase shifts are zero,[20] a phenomenon known as the ''healing'' of the two-body wave function.[21] The healing occurs when the $1/e$ operator has no singularities in the corresponding domain determined by the Q operator (as for a normal Fermi liquid), i.e., no energy-conserving momentum states are available for scattering;[14] the presence of singularities in the $1/e$ operator gives rise to a finite phase shift and indicates the breakdown of the usual Fermi liquid picture.

The self-consistency is treated in the Brueckner theory by assuming the SPE's in Eq. (5) to have the Hartree-Fock form, with the one-particle potential given in terms of the G-matrix elements as (see Fig. 2):

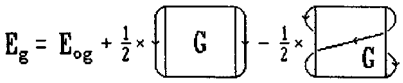

FIG. 3. The ground-state energy as computed in the Brueckner theory from the reaction matrix.

$$\varepsilon(k)=\varepsilon_0(k)+\sum_{|k'|<k_F}(\langle k,k'|G|k,k'\rangle-\langle k,k'|G|k',k\rangle). \tag{9}$$

The GSE is computed from (see Fig. 3)

$$E_g=E_{0g}+\frac{1}{2}\sum_{|k|,|k'|<k_F}(\langle k,k'|G|k,k'\rangle-\langle k,k'|G|k',k\rangle). \tag{10}$$

The diagonal part of the reaction matrix is thus related to the forward scattering from the potential V; when the interaction is only between particles with opposite spin, as for our model, the exchange terms [the last ones in Eqs. (9) and (10)] do not appear.

Formally, the problem reduces to finding a solution for the SPE from the coupled equations (4) and (9); however, while there is a unique prescription to calculate the energy ε of a hole state, the energy ϵ of a particle state (and beginning now we adopt a distinct notation for this case) cannot be uniquely defined and this is a major difficulty in the Brueckner theory.

IV. BRUECKNER AND GAMMEL METHOD

In order to see where the difficulty comes from, let us suppose we wish to calculate the GSE from Eq. (10) and for simplicity let us consider only the direct term [Fig. 4(a)]: for this, we need the matrix elements $\langle l_1,l_2|G|l_1,l_2\rangle$ with $|l_1|,|l_2|<k_F$. The G-matrix corresponding to this process can be found in principle from Eq. (4) with $G[W]$ computed OnES: $W=\Omega\equiv\varepsilon(l_1)+\varepsilon(l_2)$, as can be seen from the ground-state energy insertion considered in Fig. 4(b) (obtained by replacing the G matrix from Fig. 4(a) with the last term from Fig. 1). However, in order to find $G[W]$ we need not only the energies $\varepsilon(l_1)$ and $\varepsilon(l_2)$ – defined themselves in terms of the same G matrix through Eq. (9) (see Fig. 2), but also the energies $\epsilon(h_1;h_2,\Omega)$ and $\epsilon(h_2;h_1,\Omega)$ with $|h_1|,|h_2|>k_F$, depending on the actual configuration of the particles; if we consider the propagation in particle states to be the same as in hole states, i.e., $\epsilon\equiv\varepsilon$, the problem is already solved (and only OnES propagation occurs). An improved approximation is to consider distinct propagation in particle and hole states. For example, $\epsilon(h_1;h_2,\Omega)$, assumed to have the same Hartree-Fock form (9), can be found, in principle, from another reaction matrix $G'[W']$ obeying the same Eq. (4) but computed OffES, as can be understood from the diagram considered in Fig. 5(a) (obtained by assuming that the h_1 line has a ''history''): $W'=\Omega+\varepsilon(l_3)-\epsilon(h_2;h_1,\Omega)$. Now, energies of excited particles with a more complicated history are required, as $\epsilon(h_1';h_3,W')$, and so on endless [see Fig. 6(a)]. Brueckner and Gammel[15] approximated the difference $\epsilon(h_2;h_1,\Omega)-\varepsilon(l_3)$ by a constant quantity Λ, i.e., $W'\simeq W_{\mathrm{BG}}\equiv\Omega-\Lambda$. A similar approximation is made in any order, where the difference between the energy of a given

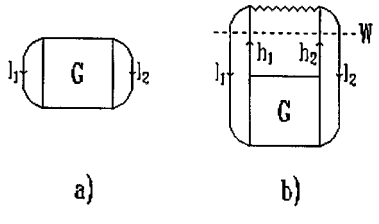

FIG. 4. (a) The contribution of the direct term to the interaction energy; (b) a first order ground-state energy insertion obtained by replacing the G matrix from (a) with the second term from Fig. 1. The starting energy W equals the total energy of the initial two-particle state $\Omega\equiv\varepsilon(l_1)+\varepsilon(l_2)$ and thus the G matrix is computed OnES.

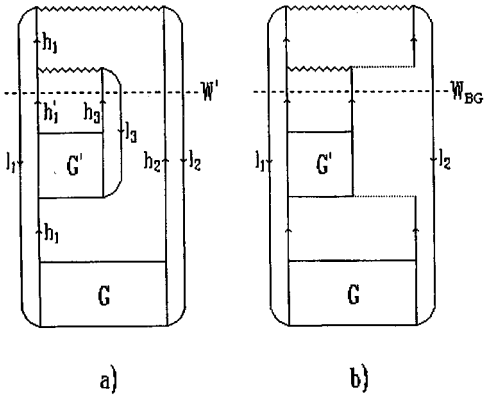

FIG. 5. (a) A second order ground-state energy insertion [obtained by assuming the particle h_1 from Fig. 4(b) already experienced an interaction with the medium—see Figs. 2 and 1], where the G' matrix is computed OffES because $W'=\Omega+\varepsilon(l_3)-\epsilon(h_2)$; (b) Brueckner and Gammel approximation: $W'\simeq W_{\mathrm{BG}}=\Omega-\Lambda$, where Λ denotes an average excitation energy and should be a small parameter.

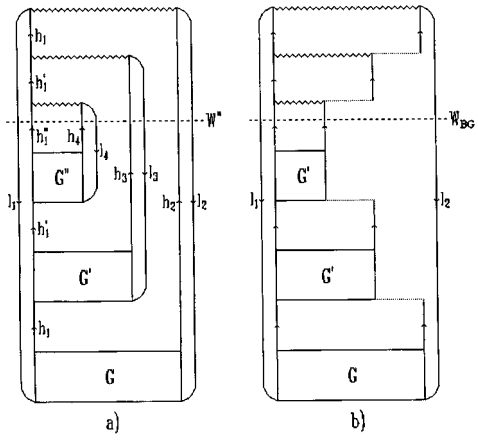

FIG. 6. (a) A third order ground-state energy insertion, and (b) the corresponding Brueckner and Gammel approximation. The diagram (b) here is equivalent to the diagram from Fig. 5(b) by virtue of the integral equation satisfied by G' matrix (see Fig. 1); consequently, the ladder is terminated in the second order.

number of particle lines and the same number of hole lines is replaced also by Λ. The consequence of this approximation is that all particle states remember from the history only the energy Ω of the initial state. Now the OffES propagation can be solved: we can find at first G' in terms of ϵ from the reaction matrix equation and then ϵ from its Hartree-Fock form. We get thus a family of curves $\epsilon(h;\Omega)$ labeled by the total energy Ω of the initial two-hole state (Λ enters as a parameter in the whole calculation of both ϵ and ε, a dependence which will be not explicitly specified). Once the OffES propagation is determined, we can find the energy $\varepsilon(l)$ of a hole state in a similar manner.

Let us discuss now when the approximation is justified. Of course the difference between the energy of a particle state and the energy of a hole state, i.e., the excitation energy, can be arbitrary big (depending on the values of the momenta) and there seems to be no reason to consider it as constant; however, the results should depend only on some average of these differences because we have to sum over all the intermediate states. If the main effects come from the correlations of the (real) particles in states around the Fermi level (as it is assumed also, for example, in the bosonization method), then we expect small values for the average excitation energy parameter Λ; in this case it could be assumed that the hole line l_3 in Fig. 5(a) almost compensates the contribution of the particle line h_2 and the diagram changes as in Fig. 5(b); similarly in higher orders (see Fig. 6). Physically it means that we include the effect of the third particle on the interacting pair in the propagation of the partners; the problem reduces thus to the study of only one pair of particles, with a different propagation in particle and hole states (unlike the simplest version of the independent-pair model). The effective interaction between the partners in particle states is given by a reaction matrix computed slightly OffES and depending on the initial hole states left by the particles; the total momentum of the interacting pair is (in absolute value) less than $2k_F$, as the particles were initially below the Fermi level. The approximations made in the Brueckner and Gammel method are thus justified for small values of Λ parameter. The problem is this parameter is not determined by the theory; Brueckner and Gammel[15] defined Λ as taking values between two limits: the energy required to bring one particle on the first excited (particle) state from the top of the Fermi sea and from its bottom, respectively. We shall come back to this problem in Sec. VI.

V. THE SPE EQUATIONS FOR THE 1D RDM

In the particular case of a constant interaction in the k space, the G-matrix equation is solved immediately independent on the SPE form. The resulting integral equations for ε and ϵ corresponding to the 1D RDM in the Brueckner and Gammel method are

$$\varepsilon(l)=\varepsilon_0(l)+U(l), \quad |l|<1,$$

$$\epsilon(h;\Omega)=\varepsilon_0(h)+\mathcal{U}(h;\Omega), \quad |h|>1, \tag{11}$$

where the one-particle potentials U and $\mathcal{U}$ are given by

$$U(l)=\frac{C}{\pi}\int_{-1}^{1}\left[1+\frac{C}{\pi}\int_{\mathcal{D}_K}\frac{dq}{e(q;l,l')}\right]^{-1}dl', \quad K=l+l',$$

$$\mathcal{U}(h;\Omega)=\frac{C}{\pi}\int_{|K|<2}^{1}\left[1+\frac{C}{\pi}\int_{\mathcal{D}_K}\frac{dq}{e(q;K,\Omega)}\right]^{-1}dl, \quad K=l+h, \tag{12}$$

with the notations

$$e(q;l,l')=\epsilon[K/2+q;\Omega(l,l')]+\epsilon[K/2-q;\Omega(l,l')]-\Omega(l,l'),$$

$$e(q;K,\Omega)=\epsilon(K/2+q;\Omega)+\epsilon(K/2-q;\Omega)-\Omega+\Lambda. \tag{13}$$

In the first (second) line of Eq. (13), $K=l+l'$ ($K=h+l$) and

$$\Omega(l,l')=\varepsilon(l)+\varepsilon(l'),$$

$$\Omega\in[2\varepsilon(0),2\varepsilon(1)]. \tag{14}$$

The range $\mathcal{D}_K$ in the second integrals from Eq. (12), defined by

$$\mathcal{D}_K=(-\infty,-|K|/2-1)\cup(|K|/2+1,+\infty), \tag{15}$$

follows from the action of the Pauli operator Q only; it restricts all the intermediate scatterings to the states above the Fermi level. The principal part operator P has been omitted in Eq. (12); if it is the case, only the principal part of the corresponding q integrals has to be taken.

VI. AVERAGE EXCITATION ENERGY

The parameter Λ has been introduced as an average excitation energy of a particle. In the nuclear matter calculations it was considered an arbitrary quantity taking values from zero to $\varepsilon(1)-\varepsilon(0)$, limits corresponding to the minimum energy required to excite a particle from the top and the bottom of the Fermi sea, respectively. Let us note that the above range of Λ implicitly assumes that the energy of the

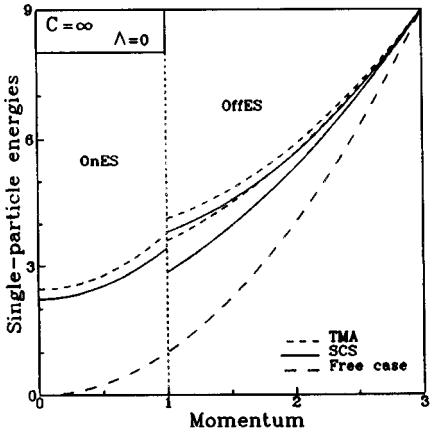

FIG. 7. The SPE versus momentum (in units of k_F) for the 1D RDM with an infinite coupling constant C in the T matrix approximation (TMA) and the self-consistent solution (SCS).

first excited state (the lowest particle state) is almost equal to the energy of the highest unexcited (hole) state, i.e., there is no shift at the Fermi level between the OnES and OffES propagation. Brueckner and Gammel[15] worked with $\Lambda=0$ and found no significant gap, the results depending only weakly on Λ and justifying, in their opinion, the introduced approximations; later,[22] an appreciable upward shift of the SPE for states above the Fermi surface, coming from finite values of Λ, has been found. If we interpret Λ as an average of the minimum excitation energies for the particles in the Fermi sea, as it has been understood in the original work of Brueckner and Gammel,[15] the range of this parameter should be defined in general as

$$\Lambda \in [\epsilon(1;\Omega_{\max})-\varepsilon(1), \epsilon(1;\Omega_{\max})-\varepsilon(0)], \qquad (16)$$

where $\Omega_{\max}$ denotes the maximum value of Ω (we shall use also $\Omega_{\min}$ with a similar meaning). However, the definition of Λ is still ambiguous: the limits of the range (16) depend themselves on the initial value of Λ used in the determination of both ε and ϵ. In a full self-consistent treatment Λ should be uniquely determined from its definition; this can be done by averaging the momentum dependence of ε over the hole states. We considered Λ to be the arithmetical mean of the two limits of its range (16):

$$\Lambda = \epsilon(1;\Omega_{\max}) - \frac{1}{2}[\varepsilon(0)+\varepsilon(1)]. \qquad (17)$$

This choice corresponds to a minimum average excitation energy (assuming ε a convex function of its variable, as it follows from the numerical computations) and it is consistent with the way in which this quantity has been introduced. Equation (17) represents another condition on the SPE determining self-consistently the parameter Λ.

VII. BREAKDOWN OF FERMI LIQUID THEORY

The integral equations (11) can be solved numerically by iteration, starting with the free expression (3) for the SPE and with $\Lambda=0$; the first iteration results (T-matrix approximation) in the $C=\infty$ case are represented in Fig. 7 by dashed lines. For the OffES propagation only two curves have been drawn: the lower (upper) one corresponds to $\Omega=\Omega_{\max}$ ($\Omega_{\min}$). For $|k|>3$ the OffES curves join the free dispersion law $\varepsilon_0(k)$, a consequence of the restriction $|K|<2$ in intermediate states; the same happens in nuclear matter,[15] reflecting the fact that the high excited states (far from the Fermi level) are not altered by the interaction. But the most important remark is the shift between $\varepsilon(1)$ and $\epsilon(1;\Omega_{\max})$ at the Fermi momentum (1 in our units), corresponding to a *negative gap* and leading to singularities in the next iterations. In order to see if such a gap is a real effect or only an artifact of the starting conditions, the iteration procedure must be continued; only a convergent solution can have a physical meaning. The difficulties created by the negative gap can be avoided in two ways: either to adjust the value of Λ at every iteration step so that no shift occurs, or to take the principal value of the improper integrals, as discussed in the end of Sec. V. By proceeding in the first manner we can find a convergent solution, but the minimum value of Λ required to vanish the shift is much greater (at least for strong couplings) than the value indicated by the right side of Eq. (17), i.e., we cannot find a full self-consistent solution. It follows that we have to take only the principal part of the q integrals in Eq. (12), i.e., the P operator cannot be dropped. The solution of the coupled equations (11) and (17) in the $C=\infty$ case, represented in Fig. 7 by continuous lines, corresponds to $\Lambda=0$ and to a value of the gap $\Delta=-0.6$. For smaller values of the interaction constant C, both ε and ϵ get closer to the free dispersion law ε_0, concomitantly with a reduction of $|\Delta|$ and an increase of Λ; in the $C\to 0$ limit, $\Delta\to 0$ and $\Lambda\to 0.5$. From the results presented in Fig. 7 it follows also that the main effect of the interaction on the OnES propagation is to shift the values of ε_0 by a constant quantity, i.e., the average potential U experienced by a fermion in the ground-state depends very weak on its momentum; this is of course a consequence of the k independence of the bare potential in the momentum representation.

From the full self-consistent solution (convergence in both SPE's and Λ) it follows the negative gap is unavoidable and thus the original Fermi level becomes unstable: the highest hole states (assumed filled in a normal ground-state configuration) have greater energies than the lowest particle states. The occurrence of a negative gap at $\pm k_F$ in the excitation spectrum, a consequence of the kinetic restrictions imposed by the momentum conservation on the two-body scatterings, destroys the usual Fermi liquid picture in 1D; it determines a singular forward scattering in any order of the G-matrix expansion and gives rise to a finite phase shift in the scattered wave (see Sec. III), an argument used previously in predicting a similar breakdown in the 2D Hubbard model.[16]

VIII. POSSIBLE INTERPRETATIONS

The form of the SPE given by the Brueckner and Gammel method shows clearly that a ground-state configuration with all the hole states double occupied and the others empty, as for a normal Fermi liquid, is unstable, the fermions tending to spread over states outside the original Fermi surface. The obtained solution indicates also a possible interpretation: we can assume that beginning from the top of the presumed

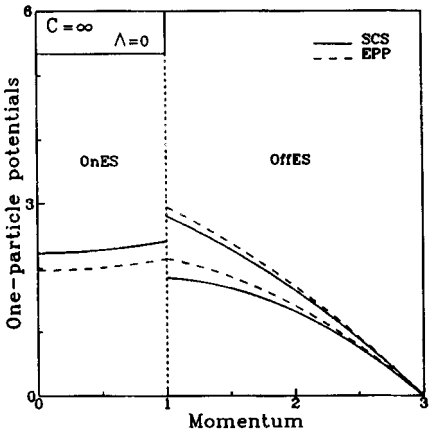

FIG. 8. The one-particle potentials as function of momentum for the 1D RDM with an infinite coupling constant C in the self-consistent solution (SCS) and in the effective potential picture (EPP).

Fermi sea, the unstable particles fall down on their lowest ''excited'' states. However, before discussing this scenario, we shall use the particular form of the determined SPE to introduce an effective one-particle potential simulating a normal Fermi liquid behavior, a picture suitable maybe only for GSE calculations.

Let us remark first that the obtained solution for the OffES propagation (coupled to the OnES one only through the parameter Ω, but not depending on the form of ε) indicates an upper limit $\Omega_{\rm lim}$ for the total energy of a stable two-particle state OnES: it is defined by the condition

$$\Omega_{\rm lim}=2\epsilon(1;\Omega_{\rm lim}) \tag{18}$$

and tells us that the highest occupied state in a stable ground-state configuration should have the energy equal to (or less than) its lowest excited state.

Effective potential picture. Let us consider that the most unstable particles, initially with the energy $\varepsilon(1)$, will go down on the $\Omega_{\rm lim}/2$ level at the same value of the momentum. Something similar happens with the rest of the particles which will loose energy not only due to their unstable positions, but also as an effect of the particles already fallen (the effective repulsion between the particles decreases). In other words, we introduce an *effective one-particle potential* $U_{\rm eff}$ instead of the original one U, describing the rearrangement of the particles induced by the occurrence of the negative gap. In the $|l|=1$ limit, the OnES potential $U_{\rm eff}$ is determined by the condition (18) of continuity with the OffES one $\mathcal{U}$ at the Fermi level; we can define it for $|l|<1$ as the shifted values of U with the same quantity as for $|l|=1$, i.e., by

$$U_{\rm eff}(l)=U(l)-\Delta U, \quad \Delta U\equiv(\Omega_{\rm max}-\Omega_{\rm lim})/2 \tag{19}$$

an assumption justified by the weak dependence of the original one-particle potential U on the momentum. The effective one-particle potentials, both OnES and OffES (only the delimiting curves), are represented in Fig. 8 by dashed lines, where the same quantities given by the self-consistent solution (the original one-particle potentials U and $\mathcal{U}$) are drawn by continuous lines.

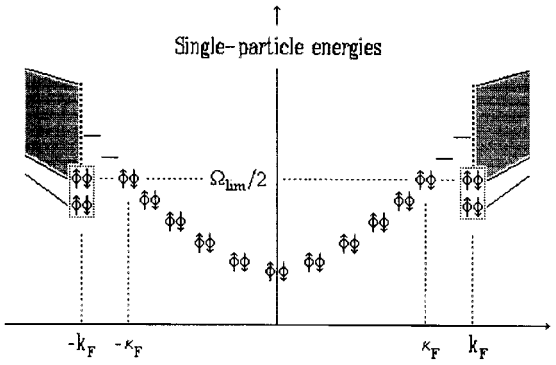

FIG. 9. Schematic representation of the ground-state configuration for the 1D RDM in the fermion condensation picture: the shaded regions correspond to the excited states of the particles from the reduced Fermi sea; the two dashed rectangles show the position of the condensate in the determined spectrum.

Fermion condensation picture. We shall assume now that the unstable particles (above $\Omega_{\rm lim}/2$ level) make a transition to another phase — let us call it *condensate* — while the stable ones remain at the same positions as before. The transition to the condensed phase could be imagined as the fall of the unstable particles (through $K=0$ virtual scatterings) in their lowest ''excited'' states located at $\pm k_F$, as it is shown in Fig. 9. Let us note that such a configuration, with more particles at the same k, is, in principle, possible: in the linked cluster expansion one has to take into account processes violating the exclusion principle and consequently a state with $|k|>k_F$ can be occupied by more than two particles.[13] In a normal Fermi liquid, such a distribution of the particles can be realized only during virtual scatterings where both initial and final states are below the original Fermi surface; the obtained solution indicates a final configuration with a preferential occupation of certain particle states, a fact which can be used, in our opinion, to anticipate some properties of the new ground state. The particles from one component (left or right) of the condensate have all the same momentum, suggesting thus a possible description of the fermions close to the original Fermi level in terms of bosons, a fact already known from the bosonization theory; they can be viewed also as fermions with an additional quantum number (Ω).[23] The stable particles form a *reduced Fermi sea* with an *effective Fermi momentum* κ_F determined by

$$\Omega_{\rm lim}=2\varepsilon(\kappa_F). \tag{20}$$

They can be excited in states from the shaded regions of Fig. 9.

The two pictures presented above are based on the already obtained solution which shows, in fact, that the particles are too compressed in a normal configuration (in k space): some of them have to condense in a less energetic phase (fermion condensation picture); they should have been stable if the effective interparticle repulsion was smaller (effective potential picture). There is, in principle, another possible way to study the rearrangement of the particles in a stable configuration: to modify the original SPE equations such that an abnormal occupation of the k states are considered from the beginning. For example, we can assume the particles fill uni-

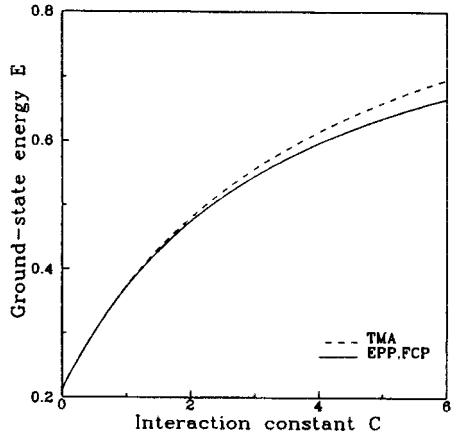

FIG. 10. The GSE of the 1D RDM for $C \equiv c/k_F \leqslant 6$ in the T-matrix approximation (TMA), effective potential picture (EPP), and fermion condensation picture (FCP); the last two schemes give practically the same results as the Bethe-Ansatz method at all values of the model parameters c and k_F.

formly a sea with an unknown Fermi momentum κ_F that could be determined from the condition of stability, i.e., the disappearance of the negative gap; a self-consistent solution with $\kappa_F > k_F$ is expected, by analogy with the Bethe-Ansatz results which indicate, for an infinite coupling constant, a GSE as for free spinless fermions.[17] This approach will be discussed in a forthcoming work.

IX. GROUND-STATE ENERGY

The GSE for the 1D RDM is computed from [see Eq. (10)],

$$E = \frac{2}{3\pi} + \frac{1}{2\pi}\int_{-1}^{1} U(l)dl, \tag{21}$$

where E represents the density of the GSE in units of $k_F\varepsilon_0(k_F)$.

The values of E for $C \leqslant 6$ in the T-matrix approximation, i.e., in the first iteration, are along the dashed line from Fig. 10; they coincide with the results obtained previously.[18] The maximum (relative) deviation from the Bethe-Ansatz values,[17] realized when $C=\infty$, is 20%. Using now in Eq. (21) the expression of U given by the self-consistent solution and ignoring the instability of the original Fermi level, the results for the GSE get closer to the exact ones: for an infinite coupling constant, the relative deviation decreases to 9%.

Within the effective potential picture, the GSE has been calculated from the same Eq. (21) but with U replaced by U_{eff} defined as in Eq. (19); for $C=\infty$ we obtained $E=0.856$, i.e., a relative difference from the exact result $E=8/(3\pi)$ less than 1%. For $C \leqslant 6$ the results are along the continuous line from Fig. 10; they cannot be distinguished from the Bethe-Ansatz ones at the figure scale.

In calculating the GSE in the fermion condensation picture, we divided the integral from Eq. (21) in two parts: over the reduced Fermi sea ($|l| < \kappa_F$) with U given by the self-consistent solution, and the rest ($\kappa_F < |l| < 1$). The contribution of the second term was assumed to change with some parameter $t \in [0,1]$ describing the continuous transformation from $\{l, U(l)\}$ at $t=0$ to $\{1, \mathcal{U}[1;2\varepsilon(l)]\}$ at $t=1$; the range $[\kappa_F,1]$ is mapped for any $t \neq 1$ in another one $[\kappa_t,1]$, with $\kappa_0 = \kappa_F$ and $\lim_{t\to 1}\kappa_t = 1$, while the transformed values of $U(l)$ are finite at any t. It follows that the second term goes to zero, i.e., the contribution of the condensate to the GSE is the same as that corresponding to the free particles above the κ_F level; if we imagine the condensate as composed of bosons, it means they can be considered as noninteracting in the ground state. In the $C=\infty$ case we obtained $E=0.849$ (with $\kappa_F=0.876$), i.e., a numerical coincidence with the Bethe-Ansatz result; at smaller couplings, the values of E are practically the same as those obtained in the effective potential picture (in the weak coupling regime, the negative gap becomes irrelevant for the GSE).

X. CONCLUSIONS

The results of this paper can be summarized as follows:

(i) One implicit assumption of the Brueckner theory, namely, all particle states have greater energies than the hole states, can be, in principle, relaxed by restoring the action of the principal part operator in the reaction matrix equation.

(ii) In terms of the Brueckner theory, the breakdown of the usual Fermi liquid picture occurs when the ''healing'' property of the correlated two-body wave function is destroyed, i.e., when a finite phase shift in the scattering of the two particles appears.

(iii) When applied to the 1D RDM, the Brueckner and Gammel method gives a solution for the SPE with a negative gap between the OffES propagation and OnES one, indicating the instability of a ground-state configuration with all states between $-k_F$ and $+k_F$ double occupied. The approximations introduced by Brueckner and Gammel are justified by the small values of the average excitation energy parameter Λ (determined by a natural self-consistency condition) in comparison with the values of the SPE around the Fermi level.

(iv) The solution given by the Brueckner and Gammel method has been used to put forward two possible pictures that could get account of the particle redistribution in a stable ground-state configuration: (1) an effective potential simulating a normal Fermi liquid behavior; (2) a condensate of particles in coexistence with a reduced Fermi sea normally occupied.

(v) The two pictures give practically the same results for the GSE of the 1D RDM, in remarkable agreement with the Bethe-Ansatz predictions at all values of the model parameters.

The same qualitative behavior of the SPE, as described here for the 1D RDM, has been noticed also in the case of the 1D repulsive Hubbard model;[24] the occurrence of a negative gap in the excitation spectrum of a Fermi liquid could be relevant in the dispute about a possible breakdown of the usual quasiparticle picture in 2D.

A fermion condensation mechanism has been recently discussed[25] for a class of Fermi systems with strongly repulsive interactions which do not obey the usual Landau theory; possible connections between this approach and our results could be an interesting subject for further investigations.

One might have expected perhaps to get reliable results

from the Brueckner theory even in 1D, as long as the system under consideration is a dilute Fermi liquid (short-range interactions); nevertheless, a comparison between the predictions of the independent-pair approximation and known results for other relevant quantities, such as the correlation functions, remains to certify its applicability to 1D systems.

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[1] For a discussion about the breakdown of the Landau's Fermi liquid theory in 1D see, for example, Sec. 2.2 from Ref. 5; see also Refs. 16 for a possible breakdown in higher dimensions.

[2] J.M. Luttinger, J. Math. Phys. **4**, 1154 (1963); D.C. Mattis and E.H. Lieb, *ibid.* **6**, 304 (1965).

[3] An introduction to the bosonization method can be found in V.J. Emery, in *Highly Conducting One-Dimensional Solids*, edited by J.T. Devreese, R.E. Evrard, and V.E. van Doren (Plenum Press, New York, 1979).

[4] F.D.M. Haldane, J. Phys. C **14**, 2585 (1981).

[5] For a recent review on the Luttinger liquid properties see J. Voit, Rep. Prog. Phys. **58**, 977 (1995).

[6] J. Sólyom, Adv. Phys. **28**, 201 (1979).

[7] For a review on the Bethe Ansatz method see B. Sutherland, in *Exactly Solvable Problems in Condensed Matter and Relativistic Field Theory*, edited by B. S. Shastry, S. S. Jha, and V. Singh, Lecture Notes in Physics Vol. 242 (Springer-Verlag, Berlin, 1985), p. 1.

[8] M. Gaudin, Phys. Lett. A **24**, 55 (1967).

[9] C.N. Yang, Phys. Rev. Lett. **19**, 1312 (1967).

[10] K.A. Brueckner and C.A. Levinson, Phys. Rev. **97**, 1344 (1955).

[11] R.J. Eden, Proc. R. Soc. London Ser. A **235**, 408 (1956).

[12] H.A. Bethe, Phys. Rev. **103**, 1353 (1956).

[13] J. Goldstone, Proc. R. Soc. London A **239**, 267 (1957).

[14] B.D. Day, Rev. Mod. Phys. **39**, 719 (1967).

[15] K.A. Brueckner and J.L. Gammel, Phys. Rev. **109**, 1023 (1958).

[16] P.W. Anderson, Phys. Rev. Lett. **64**, 1839 (1990); **65**, 2306 (1990).

[17] W.I. Friesen and B. Bergersen, J. Phys. C **13**, 6627 (1980).

[18] S. Nagano and K.S. Singwi, Phys. Rev. B **27**, 6732 (1983).

[19] The replacement of SPE ε by the kinetic energies ε_0 in Eq. (5) corresponds, in our terminology, to the T-matrix approximation.

[20] H.A. Bethe and J. Goldstone, Proc. R. Soc. London Ser. A **238**, 551 (1957).

[21] L.C. Gomes, J.D. Walecka, and V.F. Weisskopf, Ann. Phys. (Leipzig) **3**, 241 (1958).

[22] K.A. Brueckner and D.T. Goldman, Phys. Rev. **117**, 207 (1960).

[23] A fermion-boson duality has been previously emphasized in M. Apostol, V. Bârsan, and C. Mantea, Rev. Roum. Phys. **29**, 673 (1984).

[24] F.D. Buzatu, R. Hayn, and V. Oudovenko (unpublished).

[25] V.A. Khodel, V.R. Shaginyan, and V.V. Khodel, Phys. Rep. **249**, 1 (1994).