

## Tunneling into a periodically modulated Luttinger liquid

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We study the tunneling into the edge of the Luttinger liquid with a periodically modulated concentration of electrons. It is shown that the modulation, by causing a gap in the spectrum of plasmons, leads to an additional anomaly in the density of states at a frequency corresponding to the center of the gap. The shape of the anomaly depends strongly on the *phase* of the modulation. The sensitivity to the phase is related to the plasmon mode, localized at the edge, its frequency lying within the gap (analog of the Tamm state for an electron in a periodic potential). [S0163-1829(97)00304-4]

### I. INTRODUCTION

The explicit solution of the model of a one-dimensional interacting electron gas<sup>1</sup> (Luttinger liquid) allows one to describe analytically the gap in the tunnel density of states for the region of energies close to the Fermi level. The physical origin of the gap is that, in order to add an electron to the interacting system, a shift, vacating an extra space, should be created. Then the gap reveals the smallness of the overlap of many-electron wave functions with and without such a shift. The bosonization transformation<sup>1-3</sup> permits one to present the formation of the shift as a creation of plasmon modes in the system. The fact that the dispersion law of plasmons is linear in one dimension,  $\omega = sk$ , results in a power-law behavior of the density of states  $\nu(\omega)$ . For tunneling into the edge of a single-channel liquid the density of states has a form<sup>4,5</sup>

$$\nu(\omega) \propto \omega^{(s/v_F)-1}, \quad (1)$$

where  $v_F$  is the Fermi velocity. Matveev and Glazman<sup>5</sup> have traced the evolution of the exponent in (1) with increasing the number of channels.

The bosonization procedure can be generically extended to the case of longitudinally inhomogeneous Luttinger liquid. If the spacial scale of inhomogeneity is much larger than the Fermi wavelength,  $k_F^{-1}$  it can be considered as a ‘scattering potential’ for the plasmon modes, so that a plasmon gets partially reflected after propagation through the region of inhomogeneity. If the boundaries of this region are abrupt compared to the wavelength of a plasmon (but smooth on the scale of  $k_F^{-1}$ ), the reflection coefficient, as a function of frequency, exhibits the interference oscillations. Such oscillations were first considered in Ref. 6. In Ref. 7 it was demonstrated that the oscillations of the reflection coefficient lead to the oscillating structure in the density of states, superimposed on the power-law increase.

In the present paper we consider another realization of inhomogeneity. Namely, we assume that the density of the Luttinger liquid is periodically modulated with a period  $a \gg k_F^{-1}$ . We show that such a modulation results in additional singularity in the tunnel density of states. The reason for the singularity is a gap in the plasmon spectrum, which opens due to the periodic modulation. In complete analogy

with the motion of an electron in a periodic potential, the position of the center of the gap,  $\omega_0$ , is determined by the condition  $k(\omega_0) = \pi/a$ , so that  $\omega_0 = \pi s/a$ . We assume the modulation to be weak. Then the width of the gap,  $\Delta$ , is much smaller than  $\omega_0$ , the ratio  $\Delta/\omega_0$  being proportional to the relative amplitude of the modulation. It is also proportional to the interaction strength, since it is obvious that without interactions the gap is absent. In the vicinity of  $\omega_0$  the modulation-induced correction to the tunnel density of states,  $\delta\nu$ , is a function of  $\bar{\omega} = 2(\omega - \omega_0)/\Delta$ . For  $|\omega - \omega_0| \gg \Delta$ ,  $\delta\nu(\bar{\omega})$  behaves as a power law with the same exponent as in (1). Since we study the tunneling into the edge, the actual shape of  $\delta\nu(\bar{\omega})$  appears to depend strongly on the *phase* of the modulation. This is because for plasmons with frequencies close to  $\omega_0$  the behavior of the field near the edge is determined by this phase. The other reason is that, with periodic modulation, the presence of an edge leads to the formation of a localized plasmon mode with the frequency lying within the gap. For an electron in a periodic potential this fact was established more than 60 years ago.<sup>8</sup> The frequency of the localized mode sweeps through the gap as the phase of modulation changes. We show that the localized plasmon provides a contribution to  $\delta\nu(\bar{\omega})$  comparable to that from the extended modes.

The paper is organized as follows. In the next section the form of the Hamiltonian for an inhomogeneous Luttinger liquid is established. In Sec. III we derive a general formula for inhomogeneity-induced correction to the tunnel density of states. In Sec. IV the plasmon modes for the case of a weak sinusoidal modulation of the electron concentration are found. The results for  $\delta\nu(\bar{\omega})$  are analyzed in Sec. V. Section VI concludes the paper.

### II. HAMILTONIAN

In this section we will establish the form of the Hamiltonian for the Luttinger liquid with spatially varying concentration of electrons  $n_0(x)$ . We start from the classical equation of motion

$$mn \frac{d^2 u}{dt^2} = -en\mathcal{E} - \frac{dP}{dx}, \quad (2)$$

where  $m$  is the electron mass;  $u(x,t)$  is the displacement;  $n(x,t)$  is the electron density:  $n(x,t) = n_0(x) + n_1(x,t)$ , with  $n_1$  describing the temporal fluctuations;  $\mathcal{E}(x,t) = \mathcal{E}_0(x) + \mathcal{E}_1(x,t)$  is the actual electric field consisting of static and fluctuating parts;  $P = \pi^2 \hbar^2 n^3 / 3m$  is the hydrostatic pressure. The static electric field can be eliminated from Eq. (1) using the equilibrium condition

$$e\mathcal{E}_0 n_0 = - \left. \frac{dP}{dx} \right|_{n=n_0(x)}, \quad (3)$$

which gives  $\mathcal{E}_0(x) = -(\pi^2 \hbar^2 n_0 / em)(dn_0/dx)$ . Substituting this value into (2) and linearizing (2) with respect to  $u$ ,  $\mathcal{E}_1$ , and  $n_1$ , we obtain

$$mn_0 \frac{d^2 u}{dt^2} = -en_0 \mathcal{E}_1 - \frac{\pi^2 \hbar^2}{m} \left( n_0^2 \frac{dn_1}{dx} + n_0 n_1 \frac{dn_0}{dx} \right). \quad (4)$$

Let  $V(x)$  be the potential of electron-electron interactions, screened by the presence of a gate electrode. Then  $\mathcal{E}_1$  can be presented as a field created by the density fluctuations  $n_1(x,t)$ ,

$$e\mathcal{E}_1 = - \frac{d}{dx} \int dx' V(x-x') n_1(x',t). \quad (5)$$

As usually we will assume that the screening radius is much smaller than the characteristic spatial scale of the fluctuations. Then Eq. (5) reduces to  $e\mathcal{E}_1 = -V_0(dn_1(x,t)/dx)$ , where  $V_0 = \int dx V(x)$ .

As a next step  $n_1$  is expressed through  $u$  with the help of the continuity equation:  $n_1 = -d(n_0 u)/dx$ . To derive the energy conservation law we multiply Eq. (4) by  $du/dt$ . Then, using (5), the first term in the right-hand side can be rewritten as

$$- \frac{V_0}{2} \frac{d}{dt} \left[ \frac{d(n_0 u)}{dx} \right]^2 + V_0 \left( \frac{d}{dx} \right) \left[ \frac{d(n_0 u)}{dx} \frac{d(n_0 u)}{dt} \right].$$

The two remaining terms can be combined into

$$\frac{\pi^2 \hbar^2}{2m} \frac{d}{dt} \left\{ \frac{d}{dx} \left[ n_0 \frac{d(n_0 u)^2}{dx} \right] - n_0 \left[ \frac{d(n_0 u)}{dx} \right]^2 \right\}.$$

After bringing all these terms into the left-hand side and integrating over  $x$  from 0 to  $\infty$  [taking into account that  $u(0,t) = 0$ ], Eq. (4) reduces to  $dE/dt = 0$ , where the energy  $E$  is given by

$$E = \int_0^\infty dx \left[ \frac{mn_0(x)}{2} \left( \frac{du}{dt} \right)^2 + \frac{1}{2} \left( V_0 + \frac{\pi^2 \hbar^2}{m} n_0(x) \right) \times \left( \frac{d(n_0 u)}{dx} \right)^2 \right]. \quad (6)$$

The expression (6) for the energy allows one to write down the corresponding Hamiltonian. Treating the displacement  $u(x)$  as an operator  $\hat{u}(x)$  and introducing the conjugate momentum operator  $\hat{p}(x)$ ,  $[\hat{u}(x), \hat{p}(x')] = i\hbar \delta(x-x')$ , one gets

$$\hat{\mathcal{H}} = \int_0^\infty dx \left[ \frac{\hat{p}^2(x)}{2mn_0(x)} + \frac{1}{2} \left( V_0 + \frac{\pi^2 \hbar^2}{m} n_0(x) \right) \left( \frac{d(n_0 \hat{u})}{dx} \right)^2 \right]. \quad (7)$$

If the concentration is constant, the last term in the integrand takes the standard form<sup>5</sup>  $\frac{1}{2} mn_0 v_F^2 (du/dx)^2$ , where  $v_F = \pi n_0 \hbar / m$  is the Fermi velocity.

### III. GENERAL FORM OF THE CORRECTION TO THE DENSITY OF STATES

In this section we will derive a general expression for the correction, caused by the variation of the electron concentration  $n_0(x)$ , to the tunnel density of states at the edge of the Luttinger liquid. The Hamiltonian (7) can be diagonalized by the transformation

$$\begin{aligned} \hat{u}(x) &= \sum_\mu \frac{1}{\sqrt{n_0(x)}} \Phi_\mu(x) \hat{Q}_\mu, \\ \hat{p}(x) &= \sum_\mu \sqrt{n_0(x)} \Phi_\mu(x) \hat{P}_\mu, \end{aligned} \quad (8)$$

where  $\Phi_\mu(x)$  are the eigenfunctions of the equation

$$\begin{aligned} -\sqrt{n_0(x)} \frac{d}{dx} \left\{ \left[ \frac{V_0}{m} + \left( \frac{\pi \hbar}{m} \right)^2 n_0(x) \right] \frac{d}{dx} (\sqrt{n_0(x)} \Phi_\mu) \right\} \\ = \Omega_\mu^2 \Phi_\mu, \end{aligned} \quad (9)$$

and  $\Omega_\mu^2$  are the eigenvalues. We assume that  $\Phi_\mu$  are normalized:  $\int_0^\infty dx \Phi_\mu(x) \Phi_{\mu'}(x) = \delta_{\mu,\mu'}$ , and turn to zero at the edge,  $\Phi_\mu(0) = 0$ . As a result of the transformation (8) the Hamiltonian (7) reduces to the system of harmonic oscillators,

$$\hat{\mathcal{H}} = \sum_\mu \left[ \frac{\hat{P}_\mu^2}{2m} + \frac{m\Omega_\mu^2}{2} \hat{Q}_\mu^2 \right], \quad (10)$$

with frequencies  $\Omega_\mu$ . Next, the transformation (8) is applied to the operator  $\Psi^\dagger$  which creates an electron at the edge  $x=0$ ,

$$\Psi^\dagger = \exp \left( - \frac{i}{\hbar} \int_0^\infty dx \frac{\hat{p}(x)}{n_0(x)} \right) = \exp \left( - \frac{i}{\hbar} \sum_\mu \alpha_\mu \hat{P}_\mu \right), \quad (11)$$

where the coefficients  $\alpha_\mu$  are defined as

$$\alpha_\mu = \int_0^\infty dx \frac{\Phi_\mu(x)}{\sqrt{n_0(x)}}. \quad (12)$$

One can also express  $\alpha_\mu$  explicitly through  $(d\Phi_\mu/dx)|_{x=0}$ . Dividing Eq. (9) by  $\sqrt{n_0(x)}$  and integrating, one gets

$$\alpha_\mu = - \frac{1}{\Omega_\mu^2} \left[ \frac{V_0}{m} + \left( \frac{\pi \hbar}{m} \right)^2 n_0(0) \right] \sqrt{n_0(0)} \left( \frac{d}{dx} \Phi_\mu \right) \Big|_{x=0}. \quad (13)$$

With the Hamiltonian and  $\Psi^\dagger$  having the form (10) and (11), the calculation of the density of states  $\nu(\omega)$  reduces to the standard procedure<sup>9</sup> and results in

$$\nu(\omega) = \frac{1}{\pi} \text{Re} \int_0^\infty dt e^{i\omega t} \langle \Psi(t) \Psi^\dagger(0) \rangle = \frac{1}{\pi} \text{Re} \int_0^\infty dt e^{i\omega t} e^{-W(t)}, \quad (14)$$

where the function  $W(t)$  is the sum over eigenmodes

$$W(t) = \sum_{\mu} \frac{m\alpha_{\mu}^2 \Omega_{\mu}}{2\hbar} (1 - e^{-i\Omega_{\mu}t}). \quad (15)$$

At this point we will make use of the assumption that the relative variation of the concentration  $n_0(x)$  is small:  $n_0(x) = \bar{n} + \delta n(x)$ ;  $\delta n \ll \bar{n}$ . For the average concentration,  $\bar{n}$ , the eigenfunctions of Eq. (9), which satisfy the boundary condition, are  $\propto \sin kx$ , the corresponding eigenvalues being  $s^2 k^2$ , where

$$s = v_F \sqrt{1 + \frac{V_0 \bar{n}}{m v_F^2}} \quad (16)$$

is the plasmon velocity. Then the function  $W(t)$  takes the standard form

$$W_0(t) = \frac{s}{v_F} \int_0^{\infty} \frac{dk}{k} (1 - e^{-isk t}) e^{-r_0 k} = \frac{s}{v_F} \ln \left( \frac{r_0 + ist}{r_0} \right), \quad (17)$$

where, as usual, the cutoff parameter  $e^{-r_0 k}$  is introduced to ensure the convergence of the integral at  $k \rightarrow \infty$ . With  $W_0(t)$  given by (17), Eq. (14) reproduces the result (1). Treating the difference  $W(t) - W_0(t)$  as a perturbation and expanding the exponent in (14) to the first order, one gets the following correction to the density of states:

$$\delta\nu = -\frac{1}{\pi} \text{Re} \int_0^{\infty} dt e^{i\omega t} \left( \frac{r_0}{r_0 + ist} \right)^{s/v_F} [W(t) - W_0(t)]. \quad (18)$$

This expression can be simplified if one uses the identity

$$(r_0 + ist)^{-s/v_F} = \frac{1}{\Gamma(s/v_F)} \int_0^{\infty} dz z^{(s/v_F)-1} e^{-z(r_0 + ist)}. \quad (19)$$

Substituting (19) into (18) and performing the integration first over  $t$  and then over  $z$ , one gets

$$\delta\nu = \frac{1}{\Gamma(s/v_F)} \left( \frac{r_0}{s} \right)^{s/v_F} \left[ \sum_{\mu} \frac{m\alpha_{\mu}^2 \Omega_{\mu}}{2\hbar} (\omega - \Omega_{\mu})^{(s/v_F)-1} - \frac{s}{v_F} \int_0^{\omega/s} \frac{dk}{k} (\omega - sk)^{(s/v_F)-1} \right]. \quad (20)$$

The divergence of the integral at small  $k$  is canceled by the similar divergence in the first term. Further calculations require the knowledge of the eigenfrequencies  $\Omega_{\mu}$ . For a weak periodic modulation they are found in the next section.

#### IV. PLASMON SPECTRUM

To find the eigenvalues of Eq. (9) it is convenient to introduce a function

$$\tilde{\Phi}_{\mu} = \Phi_{\mu} \sqrt{n_0(x) [n_0(x) + V_0 m / \pi^2 \hbar^2]}. \quad (21)$$

Then the equation for  $\tilde{\Phi}_{\mu}$  takes the form

$$\frac{d^2 \tilde{\Phi}_{\mu}}{dx^2} = \left\{ \frac{1}{4} \left( \frac{dn_0/dx}{n_0(x) + V_0 m / \pi^2 \hbar^2} \right)^2 - \frac{n_0(x)}{[n_0(x) + V_0 m / \pi^2 \hbar^2]} \left[ \left( \frac{\Omega_{\mu} m}{\pi \hbar n_0(x)} \right)^2 - \frac{d^2 n_0 / dx^2}{2n_0(x)} \right] \right\} \tilde{\Phi}_{\mu}. \quad (22)$$

Let us specify the modulation profile:  $\delta n(x) = -n_M \cos(\sigma x + \varphi)$ , where  $\varphi$  is the phase and  $\sigma = 2\pi/a$ . For such a modulation and  $n_M \sim \bar{n}$  the plasmon spectrum was studied numerically in Ref. 10. If  $n_M \ll \bar{n}$  the analytical solution can be obtained. After expanding the ‘‘potential energy’’ in (22) to the first order in  $n_M/\bar{n}$ , the equation reduces to the conventional Mathieu equation<sup>11</sup>

$$\frac{d^2 \tilde{\Phi}_{\mu}}{dx^2} + \left[ \frac{\sigma^2 \theta}{2} \cos(\sigma x + \varphi) + \left( \frac{\Omega_{\mu}}{s} \right)^2 \right] \tilde{\Phi}_{\mu} = 0, \quad (23)$$

where the dimensionless parameter  $\theta$  is defined as

$$\theta = \frac{2n_M}{\bar{n}} \left[ \left( \frac{\Omega_{\mu}}{\sigma s} \right)^2 \left( \frac{v_F^2}{s^2} + 1 \right) - \frac{v_F^2}{2s^2} \right]. \quad (24)$$

Since  $\theta$  is proportional to the relative modulation, we have  $\theta \ll 1$ , so that the gap in the spectrum of eigenfrequencies  $\Omega_{\mu}$  is narrow. The center of the gap is determined by the condition  $\Omega_{\mu}/s = \sigma/2$ . This allows us to simplify the expression for  $\theta$  by the substitution  $\Omega_{\mu} = \sigma s/2$ . Then (24) takes the form

$$\theta = \frac{n_M}{2\bar{n}} \left( 1 - \frac{v_F^2}{s^2} \right) = \left( \frac{n_M}{2\bar{n}} \right) \frac{V_0 m / \pi^2 \hbar^2}{\bar{n} + V_0 m / \pi^2 \hbar^2}. \quad (25)$$

We emphasize again that  $\theta$  is proportional to the interaction strength  $V_0$ ; this supports the obvious observation that there is no gap for noninteracting electrons. Note also that, in principle, there are higher-order gaps in the spectrum of Eq. (23). They are centered at  $\Omega_{\mu} = p\sigma s/2$  ( $p=2,3,\dots$ ) with the widths proportional to  $(n_M/\bar{n})^p$ . However, they do not describe the higher-order gaps in the spectrum of Eq. (22), since (23) was derived from (22) using the first-order expansion. As it was mentioned in the Introduction, the eigenmodes of Eq. (23), satisfying the condition  $\tilde{\Phi}_{\mu}(0) = 0$ , can be of two types: localized and extended.

#### A. Localized mode

Substituting into (23)

$$\tilde{\Phi} = 2\bar{n} \frac{s}{v_F} \sqrt{\gamma} e^{-\gamma x} \sin \frac{\sigma x}{2}, \quad (26)$$

where the prefactor ensures the normalization, and neglecting the ‘‘nonresonant’’ terms  $\propto \sin[(3\sigma x/2) + \varphi]$ , we get the following expressions for the frequency and the decay constant of a localized mode:

$$\Omega = \frac{s\sigma}{2} \sqrt{1 + \theta \cos \varphi} \approx \frac{s\sigma}{2} \left( 1 + \frac{\theta}{2} \cos \varphi \right), \quad \gamma = -\frac{\sigma\theta}{4} \sin \varphi. \quad (27)$$

We see that the localized mode exists only if  $\sin\varphi < 0$ , when  $\gamma$  is positive. As  $\varphi$  changes within the interval  $[\pi, 2\pi]$ , the frequency  $\Omega$  moves from the lower to the upper boundary of the gap. The decay constant of the localized mode turns to zero as its frequency merges with the continuum. To calculate the contribution of the localized mode to  $\delta\nu$ , we need the constant  $\alpha$ . Substituting (27) into (13), we obtain

$$\alpha = -\frac{4}{\sigma} \sqrt{\frac{\gamma}{\bar{n}}}. \quad (28)$$

### B. Extended modes

Let  $L$  be the normalization length. We search for the solution of (23) in the form

$$\begin{aligned} \tilde{\Phi}_\mu = \bar{n} \frac{s}{v_F} \sqrt{\frac{2}{L(\beta_1^2 + \beta_2^2)}} & \left\{ \beta_1 \cos \left[ \left( \frac{\sigma}{2} + \kappa_\mu \right) x + \psi_1 \right] \right. \\ & \left. + \beta_2 \cos \left[ \left( \frac{\sigma}{2} - \kappa_\mu \right) x + \psi_2 \right] \right\}. \end{aligned} \quad (29)$$

The boundary condition requires that

$$\beta_1 \cos \psi_1 + \beta_2 \cos \psi_2 = 0. \quad (30)$$

Substituting (29) into (23), we establish the following relation between the phases  $\psi_1$  and  $\psi_2$ :

$$\psi_1 + \psi_2 = \varphi. \quad (31)$$

The system of coupled equations for coefficients  $\beta_1$ ,  $\beta_2$  takes the form

$$\begin{aligned} \left[ \left( \frac{\Omega_\mu}{s} \right)^2 - \left( \frac{\sigma}{2} + \kappa_\mu \right)^2 \right] \beta_1 + \frac{\sigma^2 \theta}{4} \beta_2 &= 0, \\ \left[ \left( \frac{\Omega_\mu}{s} \right)^2 - \left( \frac{\sigma}{2} - \kappa_\mu \right)^2 \right] \beta_2 + \frac{\sigma^2 \theta}{4} \beta_1 &= 0. \end{aligned} \quad (32)$$

The solution of the system yields the plasmon spectrum

$$\Omega_\mu = \omega_0 + \lambda \sqrt{\kappa_\mu^2 s^2 + \frac{\Delta^2}{4}}, \quad (33)$$

where  $\omega_0 = s\sigma/2$ , and the width of the gap,  $\Delta$ , is given by

$$\Delta = \frac{s\sigma\theta}{2} = \theta\omega_0, \quad (34)$$

so that the smallness of  $\theta$  guarantees that  $\Delta/\omega_0 \ll 1$ . Parameter  $\lambda$  takes the values  $\pm 1$  depending on the sign of  $\kappa_\mu$ ,

$$\lambda = \text{sign}(\kappa_\mu). \quad (35)$$

The plasmon spectrum (33) is shown in Fig. 1. The phase of modulation  $\varphi$  does not affect the spectrum, but it reveals itself in the parameter  $\alpha_\mu$ , which for  $\tilde{\Phi}_\mu$ , given by (29), takes the form

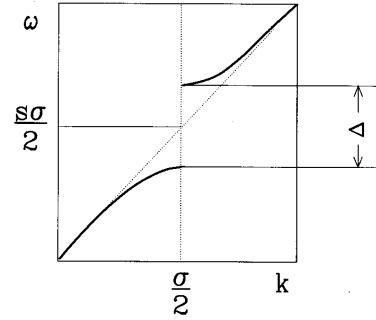


FIG. 1. Plasmon spectrum of a periodically modulated Luttinger liquid.

$$\alpha_\mu = \frac{2}{\sigma} \sqrt{\frac{2}{L\bar{n}}} \frac{\beta_1 \cos \psi_1 + \beta_2 \cos \psi_2}{(\beta_1^2 + \beta_2^2)^{1/2}}. \quad (36)$$

The last factor in (36) can be expressed through  $\varphi$  using the relations (30) and (31)

$$\frac{\beta_1 \cos \psi_1 + \beta_2 \cos \psi_2}{(\beta_1^2 + \beta_2^2)^{1/2}} = \frac{\beta_1^2 - \beta_2^2}{(\beta_1^2 + \beta_2^2)^{1/2} (\beta_1^2 + \beta_2^2 + 2\beta_1\beta_2 \cos \varphi)^{1/2}}. \quad (37)$$

Expressing the ratio  $\beta_2/\beta_1$  with the use of Eqs. (32), (33), we finally obtain

$$\begin{aligned} \frac{\beta_1 \cos \psi_1 + \beta_2 \cos \psi_2}{(\beta_1^2 + \beta_2^2)^{1/2}} &= -\frac{s\kappa_\mu}{\sqrt{s^2\kappa_\mu^2 + \frac{\Delta^2}{4} \sin^2 \varphi}} \left[ 1 + \lambda \cos \varphi \frac{\Delta/2}{\sqrt{s^2\kappa_\mu^2 + \frac{\Delta^2}{4}}} \right]^{1/2}. \end{aligned} \quad (38)$$

The discreteness of  $\kappa_\mu$  is related to the final normalization length. The summation over  $\mu$  reduces to the integration with the help of the relation  $\delta\kappa_\mu = \pi/L$ .

### V. ASYMPTOTES AND NUMERICAL RESULTS

The resulting expression for  $\delta\nu$  emerges after the substitution of (27), (28), (33), and (36) into Eq. (20). It is convenient to present this expression by introducing the relative deviation of the frequency  $\omega$  from the center of the gap

$$\tilde{\omega} = \frac{2(\omega - \omega_0)}{\Delta}. \quad (39)$$

Then we obtain

$$\delta\nu(\tilde{\omega}) = \frac{1}{\Gamma(s/v_F)} \left( \frac{r_0\theta}{s} \right)^{s/v_F} \left( \frac{s}{v_F} \right) \omega_0^{(s/v_F)-1} F(\tilde{\omega}), \quad (40)$$

where the dimensionless function  $F(\tilde{\omega})$  has the form

$$F(\bar{\omega}) = \left[ -\pi \sin\varphi (\bar{\omega} - \cos\varphi)^{(s/v_F)-1} - \frac{1}{2} f_1(\bar{\omega}) + \frac{\cos\varphi}{2} f_2(\bar{\omega}) \right], \quad (41)$$

and the functions  $f_1$  and  $f_2$  are defined as

$$f_1(\bar{\omega}) = \int_{-\infty}^{\infty} dv \left[ (\bar{\omega} - v)^{(s/v_F)-1} - \frac{v^2}{v^2 + \sin^2\varphi} (\bar{\omega} - \lambda_v \sqrt{v^2 + 1})^{(s/v_F)-1} \right], \quad (42)$$

$$f_2(\bar{\omega}) = \int_{-\infty}^{\infty} dv \frac{\lambda_v v^2}{(v^2 + \sin^2\varphi) \sqrt{v^2 + 1}} (\bar{\omega} - \lambda_v \sqrt{v^2 + 1})^{(s/v_F)-1}. \quad (43)$$

Similar to (35), in (42), (43)  $\lambda_v = \text{sign}(v)$ ; the form  $(\dots)^{(s/v_F)-1}$  is defined only when the argument is positive and it is zero otherwise.

We remind you that the first term in  $F(\bar{\omega})$  is present only if  $\sin\varphi < 0$ . This term describes a power-law anomaly in the density of states originating from the localized plasmon. The second and the third terms represent the contribution from the continuous plasmon spectrum outside the gap.

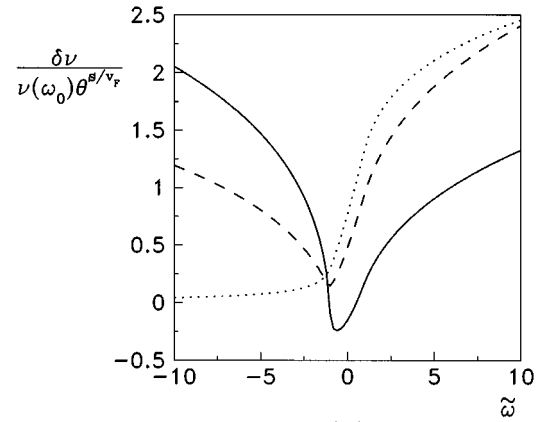
In the integral  $f_1$  two terms of the integrand almost cancel each other at large negative  $v$ , indicating that the modulation-induced correction to the spectrum is important only in the region of the gap. One can see that the integral  $f_2$  diverges as  $v \rightarrow -\infty$ , since for large negative  $v$  the integrand behaves as  $-(-v)^{(s/v_F)-2}$ . The origin of the divergence is that in (42), (43) we have used the expansion of the spectrum in the vicinity of the gap:  $|k - (\sigma/2)| \ll \sigma$ , so the integration should be cut at  $v \sim -1/\theta$ . Note however, that the divergent part is frequency independent and causes only a small correction to the background density of states  $\nu(\omega_0)$ . To study the frequency dependence, one should subtract this divergent part by considering the difference  $\tilde{f}_2 = f_2(\bar{\omega}) - f_2(0)$ , which converges.

For large positive  $\bar{\omega}$  (outside the gap) all three contributions to  $F(\bar{\omega})$  behave as  $\bar{\omega}^{(s/v_F)-1}$  with prefactors depending on the phase of modulation  $\varphi$ . For integrals  $f_1$  and  $f_2$  the following asymptotic behavior in the limit  $\bar{\omega} \gg 1$  can be derived:

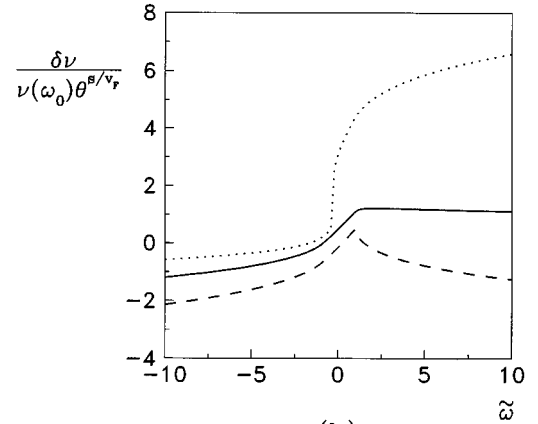
$$f_1 \approx \pi |\sin\varphi| \bar{\omega}^{(s/v_F)-1},$$

$$\tilde{f}_2 = f_2(\bar{\omega}) - f_2(0) \approx -\frac{\pi}{\tan\left(\frac{\pi s}{v_F}\right)} \bar{\omega}^{(s/v_F)-1}. \quad (44)$$

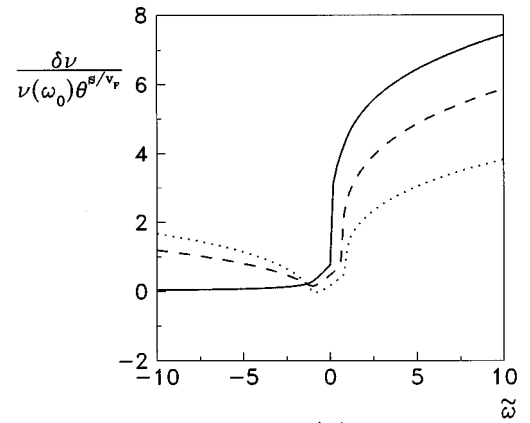
In the opposite limit  $\bar{\omega} < 0$ ,  $|\bar{\omega}| \gg 1$  the contribution from the localized plasmon is absent and the integral  $f_1$  falls off as  $|\bar{\omega}|^{(s/v_F)-2}$ . The only contribution to  $F(\bar{\omega})$  in this limit comes from  $\tilde{f}_2$  which has the following asymptotics:



(a)



(b)



(c)

FIG. 2. The normalized correction to the density of states,  $\delta\nu/\nu(\omega_0)\theta^{s/v_F}$ , is plotted as a function of the dimensionless frequency  $\bar{\omega}$  for  $(s/v_F)=1.2$  and different values of the phase of modulation  $\varphi$ . (a)  $\varphi=0$  (solid curve),  $\varphi=\pi/4$  (long-dashed curve),  $\varphi=\pi/2$  (dash-dotted curve). (b)  $\varphi=3\pi/4$  (solid curve),  $\varphi=\pi$  (long-dashed curve),  $\varphi=11\pi/8$  (dash-dotted curve). (c)  $\varphi=3\pi/2$  (solid curve),  $\varphi=7\pi/4$  (long-dashed curve),  $\varphi=15\pi/8$  (dash-dotted curve).

$$\tilde{f}_2 \approx -\frac{\pi}{\sin\left(\frac{\pi s}{v_F}\right)} |\bar{\omega}|^{(s/v_F)-1}. \quad (45)$$

The frequency dependence of the correction  $\delta\nu(\bar{\omega})$ , cal-

culated numerically from Eq. (41) for various values of the phase  $\varphi$ , is shown in Fig. 2. It can be seen that some of the curves have cusps at frequencies, corresponding to the boundaries of the gap  $\tilde{\omega} = \pm 1$ . The cusps in the curves  $\varphi = 11\pi/8$  and  $\varphi = 3\pi/2$  have their origin in the localized plasmon.

## VI. CONCLUSION

The singularity in the tunnel density of states, caused by the periodic modulation of the electron concentration, should reveal itself as an anomaly in the differential resistance  $dI/dV$  at voltage  $eV = \hbar\omega_0 = \pi\hbar s/a$ , which depends on the period of modulation and the plasmon velocity. We considered the case of a weak sinusoidal modulation. If higher harmonics with periods  $a/p$  are present in  $n_0(x)$ , the gaps in the plasmon spectrum at  $\omega = p\omega_0$  (and corresponding anomalies in  $dI/dV$  at  $eV = \pi p\hbar s/a$ ) should emerge.

The strength of the anomaly is governed by the dimensionless parameter  $\theta$  (24), which we assumed to be small. To sense the magnitude of the anomaly it is convenient to relate  $\delta\nu$  to the density of states at  $\omega = \omega_0$  in the absence of modulation:  $\delta\nu/\nu(\omega_0) = \theta^{s/v} F(\tilde{\omega})$ . In our calculations we assumed the interactions to be short range. This implies that the interaction radius  $\sim r_0$  should be much smaller than the period of modulation  $a$ .

By now, for the experimental realization of quantum wires, several different techniques were used (see, e.g., Refs.

12–16). The most suitable technique for the fabrication of the modulated quantum wire seems to be the one employed in Ref. 16. Actually, periodic modulation was achieved in Ref. 16 by confining electrons under the splitted gate with one of the two sides being periodically corrugated. Changing the bias applied to one of the sides (electrodes) allowed the authors to vary the amplitude of modulation of the electron density, but the phase of modulation remained fixed in their measurements. However the phase could be easily made tunable if *both* sides of the splitted gate were corrugated with a mutual phase shift of  $\pi$ . Then the actual phase of the modulation would be determined by the relation between the voltages applied to the two electrodes, and, thus, can be effectively tuned (without changing the amplitude) by adjusting these voltages.

As a final remark, we considered a single-channel Luttinger liquid. Increasing the number of channels would result in additional anomalies in the tunnel density of states, caused by resonances within each channel. Besides, modulation would couple the plasmons from different channels, thus causing additional anomalies.

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<sup>1</sup>D. C. Mattis and E. H. Lieb, *J. Math. Phys.* **6**, 304 (1965).

<sup>2</sup>A. Luther and I. Peschel, *Phys. Rev. B* **9**, 2911 (1974).

<sup>3</sup>F. D. M. Haldane, *J. Phys. C* **14**, 2585 (1981).

<sup>4</sup>C. L. Kane and M. P. A. Fisher, *Phys. Rev. Lett.* **68**, 1220 (1992).

<sup>5</sup>K. A. Matveev and L. I. Glazman, *Phys. Rev. Lett.* **70**, 990 (1993).

<sup>6</sup>Y. Oreg and A. M. Finkel'stein, *Phys. Rev. Lett.* **74**, 3668 (1995).

<sup>7</sup>Yu. V. Nazarov, A. A. Odintsov, and D. V. Averin, *Bull. Am. Phys. Soc.* **41**, 376 (1996).

<sup>8</sup>I. E. Tamm, *Z. Phys. Sowjetunion* **1**, 733 (1932).

<sup>9</sup>G. Mahan, *Many-Particle Physics*, 2nd ed. (Plenum, New York, 1990).

<sup>10</sup>B. S. Mendoza, M. del Castillo-Mussot, and W. L. Mochán, *Phys. Rev. B* **53**, 1026 (1996).

<sup>11</sup>*Higher Transcendental Functions*, edited by A. Erdélyi (McGraw-Hill, New York, 1953), Vol. 3.

<sup>12</sup>W. Hansen *et al.*, in *Nanostructure Systems*, edited by Mark Reed, Semiconductors and Semimetals Vol. 35 (Academic, San Diego, 1992), Chap. 5, pp. 279–380.

<sup>13</sup>K. Ismail, D. A. Antoniadis, and Henry I. Smith, *Appl. Phys. Lett.* **54**, 1130 (1989).

<sup>14</sup>T. Demel, D. Heitmann, P. Grambow, and K. Ploog, *Phys. Rev. Lett.* **66**, 2657 (1991).

<sup>15</sup>Seigo Tarucha, Takashi Honda, and Tadashi Saku, *Solid State Commun.* **94**, 413 (1995).

<sup>16</sup>L. P. Kouwenhoven, F. W. J. Hekking, B. J. van Wees, C. J. P. M. Harmans, C. E. Timmering, and C. T. Foxon, *Phys. Rev. Lett.* **65**, 361 (1990).