Many-electron transport in strongly correlated nondegenerate two-dimensional electron systems

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We consider static conductivity and cyclotron resonance in a two-dimensional electron fluid and Wigner crystal. The theory is nonperturbative in the electron-electron interaction. It is formulated in terms of a Coulomb force that drives an electron due to thermal fluctuations of electron density. This force is used to describe the effect of electron-electron interaction on short-wavelength electron scattering by defects, phonons, and ripplons, and thus on electron transport. In a broad parameter range the force is uniform over the electron wavelength, and therefore the motion of an electron in the field of other electrons is semiclassical. In this range we derive the many-electron quantum transport equation and develop techniques for solving it. We find the static conductivity σ. Many-electron effects may ''restore'' Drude-type behavior of σ in the range from zero to moderate classically strong magnetic fields B, whereas in quantizing fields σ increases with B, i.e., the conductivity is a nonmonotonous function of B. Many-electron effects give rise also to a substantial narrowing of the cyclotron resonance absorption peak compared to what follows from the single-electron theory. The shape of the peak is found for both fast and slow rate of interelectron momentum exchange as compared with the relaxation rate. We apply the results to electrons on helium and explain why different types of B dependence of σ are observed. [S0163-1829(97)05924-9]

I. INTRODUCTION

Nondegenerate two-dimensional (2D) electron systems provide an important class of strongly correlated systems, in which electrons may form a normal fluid (to be distinguished from a Fermi liquid and other quantum electron liquids) or a Wigner crystal. The best known (but not at all the only) example is the 2D electron system on the surface of liquid helium[1,2] where mobilities higher than in any solid state conductors have been observed. In a nondegenerate system the interelectron distance $\sim n_s^{-1/2}$ greatly exceeds the de Broglie wavelength $\lambda_T=\hbar/(2mT)^{1/2}$ (where n_s is the electron density, and temperature is measured in the units of energy, $k=1$). Although the system is nondegenerate, the ratio of the characteristic Coulomb energy of electron-electron interaction to the kinetic energy, the plasma parameter

$$\Gamma=e^2(\pi n_s)^{1/2}/T \tag{1}$$

is usually large, $\Gamma\gtrsim 10$. Therefore the system is a normal fluid or, if $\Gamma\gtrsim 127$ (lower T), a Wigner crystal.[3–6]

An electron is not a ''good'' quasiparticle for a normal electron fluid, and its motion is very different from that in the much better understood Fermi liquid or low-density electron gas. Electron scattering by defects, phonons, or ripplons may also be substantially different. As a consequence, one may expect electron transport in a normal fluid to have distinctive features, and new physical effects to occur. Not only is the analysis of electron dynamics and transport phenomena interesting from the theoretical point of view, but such analysis, complemented with that of transport for a Wigner crystal, is necessary for the understanding of a large body of experimental data accumulated over the last few years (cf. Refs. 6–18) on mobility, magnetoconductivity, resonant absorption, and tunneling from nearly ideal nondegenerate electron layers. It should also help to understand transport phenomena in strongly correlated low-density electron systems in semiconductor heterostructures.[19]

In the investigation of many-electron effects in nondegenerate 2D systems the emphasis has traditionally been placed on plasma waves[20] (including edge plasmons[21,22]) and Wigner crystallization.[3,4,23] The analyses of transport phenomena for a plasma and a Wigner crystal are conducted quite differently. In the case of a plasma the basic transport coefficients like conductivity and magnetoconductivity are often considered in the effectively single-electron approximation. In this approximation the effect of the electron-electron interaction is described in terms of screening of the random potential that scatters individual electrons.[2(a)] In contrast, for a Wigner crystal the electron relaxation is described in terms of the decay of the collective excitations of the many-electron system, i.e., phonons.[4,24–28] In between these two models there lies one in which the effect of electron-electron interaction is described in terms of pair collisions that may occur more often than collisions with defects or the emission of phonons/ripplons. This approximation is well known in the physics of semiconductors;[29] it was used for nondegenerate 2D systems in.[10] However, in contrast to the low-density plasma in semiconductors where often $e^2 n_{3D}^{1/3}/\epsilon T\ll 1$, the 2D electron fluid is strongly correlated, and therefore the approximation of pair collisions does not apply. The effect of viscoelastic shear modes in the electron fluid on the mobility was considered in Ref. 30.

Electron-electron interaction would be expected to affect transport particularly strongly when a 2D electron system is placed into a magnetic field B perpendicular to the electron

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layer. In the single-electron approximation the electron energy spectrum in the magnetic field is a set of discrete Landau levels, with separation $\hbar\omega_c$ (where $\omega_c=|eB|/m$ is cyclotron frequency). Electrons do not have a finite group velocity. Therefore the standard Drude picture of well separated in time elastic or quasielastic collisions of a moving electron with defects, phonons, or ripplons does not apply. As a consequence, the (quasi)elastic scattering is always strong, irrespective of the strength of coupling to the scatterers, with random potential of the scatterers being the only reason for the centers of the cyclotron orbits to move.

In contrast, the energy spectrum of a system of interacting electrons is continuous even in the absence of scatterers. Therefore, although electron-electron interaction does not change the total momentum of the electron system, it may mediate the momentum transfer to the scatterers, and thus strongly affect the long-wavelength conductivity.

It was suggested in Ref. 31 that, for quantizing magnetic fields $\hbar\omega_c \gg T$ and yet not too low temperatures, one may describe many-electron transport of a nondegenerate electron fluid in terms of the fluctuational field $\mathbf{E}_f$ that drives each electron. Unlike the long-wavelength fluctuational electric field known in plasma physics,[32] the field $\mathbf{E}_f$, although also of fluctuational origin, determines the force driving an individual particle.

A special significance of the field $\mathbf{E}_f$ for a 2D electron system in a magnetic field stems from the fact that it causes the cyclotron orbit centers to drift. Thus it may ''restore'' the Drude picture of electron scattering in the sense that collisions with scatterers are short and well separated in time. The effect of electron-electron interaction on cyclotron resonance was observed in Ref. 8.

Recently it was outlined theoretically and showed experimentally[13,14] that, in the case of scattering by a δ-correlated random potential (pointlike defects), the field $\mathbf{E}_f$ may also strongly affect transport in classically strong magnetic fields, $\hbar\omega_c < T$, $\omega_c\tau_{B=0} \gg 1$ ($\tau_{B=0}$ is the momentum relaxation time for $B=0$). In particular, many-electron effects restore the Drude-type B^{-2} dependence of the magnetoconductivity for moderately strong B.

In the present paper we provide a theory of electron transport in strongly correlated electron systems. The theory applies for magnetic fields ranging from $B=0$ through classically strong up to quantizing fields provided the motion of an electron in the field of other electrons is semiclassical. It is clear from Fig. 1 that the motion is semiclassical if the characteristic electron wavelength (thermal, or quantum magnetic length) is small compared to the characteristic thermal displacement δ of an electron from its quasiequilibrium position in a normal electron liquid or the equilibrium position in a Wigner crystal:

$$\min(\lambda\!\!\!^{-}_T, l_B) \ll \delta,$$

$$\lambda\!\!\!^{-}_T = \hbar/(2mT)^{1/2}, \quad l_B = (\hbar/m\omega_c)^{1/2}. \tag{2}$$

An estimate of δ and the fluctuational field $\mathbf{E}_f$ can be obtained by linearizing the equations of motion of an nth electron about its equilibrium position (cf. Fig. 1) and by setting the potential energy of the fluctuational displacement equal to T (cf. Ref. 31):

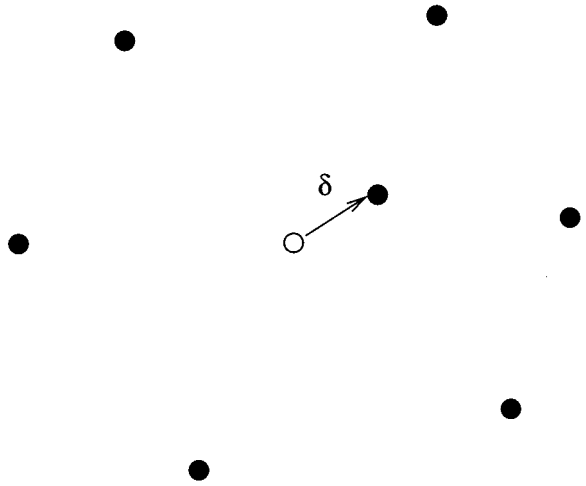

FIG. 1. Fluctuational electron displacement from a quasiequilibrium position (shown by an empty circle) in a strongly correlated system.

$$eE_f\delta \sim e^2\left|\frac{\partial^2}{\partial\mathbf{r}_n^2}\sum_m{}' \ |\mathbf{r}_n-\mathbf{r}_m|^{-1}\right|_{\rm eq}\delta^2 \sim T,$$

(the derivative is evaluated for the equilibrium electron positions; the characteristic values of E_f, δ are independent of n). This gives

$$\langle E_f^2\rangle \approx FTn_s^{3/2}, \quad \delta^2 \sim Tn_s^{-3/2}e^{-2}. \tag{3}$$

The coefficient F in (3) was found for a Wigner crystal in the harmonic approximation[25(b)] to be ≈ 8.9. A systematic Monte Carlo study of the fluctuational field $\mathbf{E}_f$ for a normal electron fluid and for a Wigner crystal shows that $F \equiv F(\Gamma)$ varies only slightly (by $\sim 10\%$) in the range of $\Gamma \gtrsim 10$.[33]

This paper is organized as follows. In Sec. II we provide a qualitative picture of many-electron transport and obtain an estimate for the conductivity in different ranges of the magnetic field. In Sec. III we derive the many-electron transport equation for the case of short-range scattering. This equation holds in the semiclassical range both in the absence and presence of a magnetic field. In Sec. IV we develop a technique for solving the transport equation in the Wigner representation, and obtain explicit solutions in the limiting cases where the rate at which electrons exchange momenta with each other is large or small compared to the momentum relaxation rate due to coupling with scatterers. In Sec. V we analyze the expression for the conductivity in classically weak and classically strong magnetic fields, and show when and how magnetoresistance arises in a 2D system of interacting electrons. In Sec. VI we analyze static magnetoconductivity and cyclotron resonance in quantizing magnetic fields. A solution of the many-electron transport equation is obtained using the separation of the fast oscillating and slowly varying in time parts of the electron coordinate operators. In Sec. VII magnetoconductivity as a function of B is analyzed in the cases of electron scattering by a δ-correlated random potential and by ripplons. Sec. VIII contains a brief discussion of the results. In the Appendix we analyze quantum corrections to the classical many-electron relaxation rate.

II. QUALITATIVE PICTURE OF MANY-ELECTRON TRANSPORT

A. The domains of classical and semiclassical dynamics

Depending on the electron density n_s, the 2D electron fluid may be a classical or a non-classical fluid or, in the presence of a magnetic field B transverse to the layer, a semiclassical fluid. The type of behavior is determined by the interrelation between temperature T, cyclotron frequency ω_c, and the characteristic frequency ω_p of short-wavelength vibrations in the system for $B=0$ (ω_p can be estimated from Fig. 1). For

$$T \gg \hbar\omega_p, \quad \omega_p=(2\pi e^2 n_s^{3/2}/m)^{1/2} \tag{4}$$

the fluid is classical for $B=0$. If, on the other hand, $T<\hbar\omega_p$, then quantum effects come into play. These effects are *not* related to overlapping of the wave functions of different electrons: it is the motion of an electron in the field created by other electrons (e.g., vibrations about a quasiequilibrium position) that becomes quantized.

In a nonquantizing field B, $\hbar\omega_c<T$, the electron fluid remains classical if (4) is fulfilled. For $\hbar\omega_c>T$ the fluid becomes semiclassical: the motion of an electron in the field $\mathbf{E}_f$ is a superposition of a quantum cyclotron motion with frequencies $\sim\omega_c$ and a semiclassical drift of the center of the cyclotron orbit. The frequency Ω that characterizes the drift can be estimated from Fig. 1 if one assumes that the field $\mathbf{E}_f$ is pointing towards the equilibrium position. Then the ''displaced'' electron drifts transverse to this field, with a velocity $eE_f/m\omega_c$, along a circle of radius δ. The frequency Ω gives the reciprocal period of this motion. For

$$T \gg \hbar\Omega, \quad \Omega=\omega_p^2/\omega_c \quad (\omega_c \gg \omega_p) \tag{5}$$

the drift (translational motion) is semiclassical. We note that the condition (5) may be fulfilled in a sufficiently strong magnetic field $\omega_c \gg \omega_p$ even if $T<\hbar\omega_p$, i.e., even if the fluid is nonclassical for $B=0$. Since

$$e\langle E_f^2\rangle^{1/2}\lambda\!\!\!^{-}_T \sim \hbar\omega_p, \quad \delta \sim \lambda\!\!\!^{-}_T T/\hbar\omega_p,$$

the conditions (4), (5), which are formulated in terms of energies, coincide with the condition (2) formulated in terms of lengths.

The conditions (4) and (5) apply also to the dynamics of a Wigner crystal and show where it is classical and semiclassical, respectively. The spectrum of phonons of a crystal was analyzed in Ref. 34; ω_p is the characteristic Debye frequency of the crystal for $B=0$. For $\omega_c \gg \omega_p$ the spectrum consists of the optical branch (that starts at ω_c) and a low frequency branch; the widths of the branches are $\sim\Omega$, and (5) means that the low-frequency vibrations are classical.

We note that the melting temperature of the crystal T_m as given by the condition $\Gamma\approx 127$ may be greater or less than $\hbar\omega_p$ depending on the electron density ($T_m \propto n_s^{1/2}$, $\omega_p \propto n_s^{3/4}$; for electrons on helium $\hbar\omega_p/T_m \approx 1.3$ when $n_s=10^8$ cm^{-2}). From this perspective it is particularly important that the magnetic field can be used to ''switch'' the 2D system, either a fluid or a crystal, from the domain of quantum dynamics, $\hbar\omega_p \gg T$, to the semiclassical domain, $T \gg \hbar\Omega$.

The long-wavelength conductivity $\sigma(\omega)$ depends on both the many-electron dynamics and the mechanism of electron scattering. We will consider scattering by short-range scatterers which include neutral point defects, acoustic phonons, and for electrons on liquid helium surface, helium vapor atoms and ripplons (the effects of long-range scattering by ripplons, including onset of coupled plasmon-ripplon modes,[4] will not be discussed in this paper). In most cases the corresponding scattering is elastic or quasielastic.

We will assume coupling to the scatterers to be weak enough that the characteristic scattering rate τ^{-1} is small compared to the reciprocal characteristic duration of a collision $t_{\rm coll}^{-1}$,

$$t_{\rm coll} \ll \tau. \tag{6}$$

The actual conditions that have to be fulfilled for (6) to hold true depend on the magnetic field and will be specified below. We notice that (6) may apply in the range of strong magnetic fields, $\omega_c\tau \gg 1$, only because of many-electron effects; in the single-electron approximation one should speak of lifting the degeneracy of Landau levels rather than of occasional collisions with the scatterers.

B. The conductivity for weak to moderately strong magnetic fields

We will first analyze the effect of the field $\mathbf{E}_f$ on the collisions with short-range scatterers for not too strong magnetic fields where

$$T \gg e\langle E_f^2\rangle^{1/2}\lambda\!\!\!^{-}_T \sim \hbar\omega_p \gg \hbar\omega_c. \tag{7}$$

The condition (7) does not mean that the magnetic field is weak. The field may well be classically strong, i.e., there may hold the inequality $\omega_c\tau \gg 1$, where τ^{-1} is the scattering rate. In what follows we use the term ''moderately strong fields'' for classically strong magnetic fields that satisfy condition (7).

In the range (7) an electron moves classically and has a well-defined kinetic energy $p^2/2m \sim T$ and a well-defined potential energy in the field of other electrons. Uncertainty of each of these energies is determined by smearing of the electron wave packet. For an electron in an electric field $\mathbf{E}_f$ this uncertainty is characterized by $eE_f\lambda\!\!\!^{-}_T$ and is small compared to T. This means that, in spite of the electron system being strongly correlated, the electron-electron interaction has little effect on collisions with short-range scatterers in the absence of a magnetic field. One can also see this from the following arguments. The duration of a collision is determined by the time it takes an electron to fly past the scatterer. For short-range scatterers and for electrons with thermal velocities $v_T=(2T/m)^{1/2}$ this time is $t_{\rm coll} \sim \lambda\!\!\!^{-}_T/v_T \sim \hbar/T$. The acceleration of the electron in the field E_f over this time is $\sim eE_f\lambda\!\!\!^{-}_T v_T/T \ll v_T$.

The role of the field $\mathbf{E}_f$ becomes very different in the presence of a magnetic field, since the field $\mathbf{E}_f$ tilts Landau levels and makes the electron energy spectrum continuous. It is clear from Fig. 2 that for an electron wave packet of size $\lambda\!\!\!^{-}_T$ the discreteness of the one-electron energy spectrum due to Landau quantization is washed out by many-electron effects if $eE_f\lambda\!\!\!^{-}_T \gg \hbar\omega_c$.[13] One would therefore expect that even

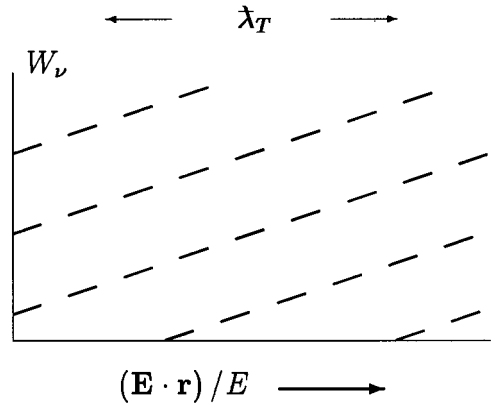

FIG. 2. Single-electron energy levels W_ν in the electric field $\mathbf{E}$ and transverse magnetic field (tilted Landau levels). Uncertainty of the electron kinetic energy exceeds $\hbar\omega_c$ for the shown size of the electron wave packet λ_T.

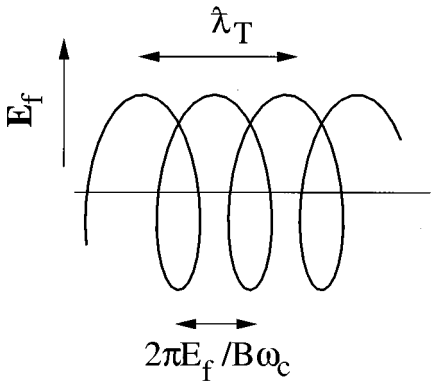

FIG. 3. Classical electron trajectory in the fluctuational electric field $\mathbf{E}_f$ and transverse magnetic field B. The characteristic radius of the spiral $R_B=(T/\hbar\omega_c)\lambda_T$.

in classically strong magnetic fields, $\omega_c\tau\gg1$, collisions with scatterers will occur nearly as if there were no magnetic field at all. Then the many-electron system should not display magnetoresistance, and in the whole range (7) the static conductivity σ is given by a simple expression

$$\sigma\equiv\sigma_{xx}(\omega=0)=\frac{e^2n_s}{m}\frac{\tau_{B=0}}{1+\omega_c^2\tau_{B=0}^2},\qquad e\langle E_f^2\rangle^{1/2}\lambda_T\gg\hbar\omega_c,\hbar\tau_{B=0}^{-1}, \tag{8}$$

where $\tau_{B=0}^{-1}$ is the scattering rate calculated for $B=0$ in the approximation where the effect of the electron-electron interaction on collisions with scatterers is ignored.

If the scattering remains the same as in the absence of the magnetic field, the off-diagonal component of the conductivity should be given by the expression $|\sigma_{xy}(\omega=0)|=\sigma\omega_c\tau_{B=0}$. In this case there is no magnetoresistance: the resistivity $\rho(B)=\sigma_{xx}/[\sigma_{xx}^2+\sigma_{xy}^2]=\rho(0)$.

We emphasize that the absence of magnetoresistance in the range (7) for *classically strong magnetic fields*, known experimentally since Ref. 7, is a purely many-electron effect.

C. The conductivity for ''strong'' strong magnetic fields

Onset of magnetoresistance in classically strong magnetic fields, $T>\hbar\omega_c>e\langle E_f^2\rangle^{1/2}\lambda_T$, can be qualitatively understood in the following way. If there were no fluctuational electric field, an electron in the magnetic field would be moving along a trajectory of the shape of a rosette: it is a nearly closed circle, with its center slowly rotating around the defect, so that the electron is coming back to the defect, over and over again, with period $2\pi/\omega_c$.[35] In the presence of the field $\mathbf{E}_f$ the center of the electron cyclotron orbit drifts with a velocity E_f/B. Therefore the number of times the scatterer is encountered is finite. It is clear from Fig. 3 that in order of magnitude, this number is $\zeta=\lambda_T(2\pi E_f/B\omega_c)^{-1}$ for a point-like scatterer. One would expect classical magnetoresistance to arise in the many-electron system for $\zeta>1$.

The magnetoconductivity σ can be estimated using the Einstein relation between the conductivity and the diffusion coefficient D, $\sigma=e^2n_sD/T$. It is seen from Fig. 3 that scattering results in a shift of the electron orbit by the cyclotron radius R_B. Therefore $R_B^2/2$ may be associated with the squared diffusion length, and then $D=R_B^2/2\tau$. The scattering rate τ^{-1} is proportional to the encountering factor ζ,[36] and the expression for σ takes on the form

$$\sigma=\frac{e^2n_s}{2T}R_B^2\tau^{-1},\quad \tau^{-1}\sim\zeta\tau_{B=0}^{-1},\qquad \zeta=\lambda\omega_cB/2\pi\langle E_f^2\rangle^{1/2},\quad \lambda=l_B[\tanh(\hbar\omega_c/2T)]^{1/2}. \tag{9}$$

(Here, λ is the characteristic scale over which the electron wave function varies; $\lambda=\lambda_T$ for $\hbar\omega_c\ll T$, and $\lambda=l_B$ for $\hbar\omega_c\gg T$.)

A distinctive feature of the many-electron magnetoconductivity (9) is its *independence* of the field B for classically strong fields where $R_B\sim(T/\hbar\omega_c)\lambda_T\propto B^{-1}$ and $\zeta\propto B^2$.

The arguments used to obtain an estimate of σ apply also if the electron fluid is in a quantizing magnetic field. For $\hbar\omega_c\gg T$ an electron is a ''hard disk'' with characteristic size $l_B=(\hbar/m\omega_c)^{1/2}$. It drifts transverse to the magnetic field with a velocity E_f/B, and the characteristic duration of a collision is (cf. Ref. 31)

$$t_e=l_BB\langle E_f^{-1}\rangle. \tag{10}$$

The scattering rate is increased relative to $\tau_{B=0}^{-1}$ by the encountering factor $\zeta\sim\omega_ct_e\propto B^{3/2}$. [This estimate can be also obtained using density-of-states arguments: the single-electron energies are ''squeezed'' into Landau bands with spacing $\hbar\omega_c$; the potential (and thus also kinetic) energy uncertainty of an electron wave packet of a size l_B in the field E_f is $\sim eE_fl_B$, and therefore the overall density of states into which the electron may be scattered is increased by a factor $\zeta\sim\hbar\omega_c/eE_fl_B$ compared to the single-electron density of states for $B=0$.] The value of R_B in the domain $\hbar\omega_c\gtrsim T$ is given by the characteristic radius of the electron wave function, whereas $t_{\rm coll}$ is given by the time of flight over the wavelength λ,

$$R_B=l_B[\coth(\hbar\omega_c/2T)]^{1/2},\quad t_{\rm coll}=\lambda B\langle E_f^{-1}\rangle. \tag{11}$$

It follows from (8), (9) that the magnetoconductivity σ is nonmonotonous as a function of B. It decreases in the range (8), reaches a minimum for ''strong'' classically strong fields where $\zeta \gg 1$, and then, for pointlike scatterers, increases as $B^{1/2}$ in the range $\hbar\omega_c \gg T$ (see Fig. 6 in Sec. VII A).

Equation (9) gives also the characteristic value of the halfwidth $\gamma \sim \tau^{-1}$ of the peak of cyclotron resonance of a many-electron system in a strong magnetic field. We note that in the classical range $T \gg \hbar\omega_c$ the expressions for γ and for the relaxation rate in Eqs. (8), (9) for the static conductivity coincide with each other. This is no longer true in the quantum range (see Sec. V).

D. Interelectron momentum exchange

The exchange of momentum between electrons does not affect the long-wavelength conductivity directly,[37] since it does not change the total momentum of the electron system. However, its role in the transport may be substantial. This is well-known in the theory of low-density electron plasma in semiconductors[29] from the analysis of the case where the single-electron rate of collisions with scatterers $\tau_s^{-1}(\epsilon)$ depends on the electron energy ϵ. In the single-electron approximation the static conductivity σ (for $B=0$) is a sum of the conductivities of electrons with different energies and thus different scattering rates. Therefore it is given by the appropriately averaged (over ϵ) *reciprocal* scattering rate, $\sigma = e^2 n_s \overline{\tau_s(\epsilon)}/m$. The interelectron momentum exchange occurs via pair electron-electron collisions. If their frequency greatly exceeds $\tau_s^{-1}(T)$, then the electron energy varies substantially between collisions with the scatterers, and relaxation of the total momentum of the electron system is characterized by the average collision rate $\overline{\tau_s^{-1}(\epsilon)}$, so that $\sigma = e^2 n_s / m \overline{\tau_s^{-1}(\epsilon)}$.

From the discussion in Sec. II B one would expect that similar arguments apply to the static conductivity of a *strongly correlated* classical electron fluid for weak magnetic fields. Here, an electron exchanges its momentum with other electrons not via pair collisions but by being accelerated by the Coulomb force from these electrons. The rate of interelectron momentum exchange $\tau_{\rm ex}^{-1}$ is given by the frequency of the electron vibrations ω_p, as it is clear from Fig. 1 (this frequency also characterizes time evolution of the velocity autocorrelation function in the electron system.[38]) If

$$\tau_{\rm ex}^{-1} = \omega_p \gg \tau^{-1} \quad (\omega_p \gg \omega_c), \tag{12}$$

as it was assumed in Eq. (8), the conductivity is determined by the average rate $\overline{\tau_s^{-1}(\epsilon)}$.

The role of interelectron momentum exchange in strong fields B, where collisions with scatterers are mediated by the electron-electron interaction, is clear from the analysis of cyclotron resonance. Resonant absorption at frequency ω_c is due to transitions between neighboring Landau levels, $|\nu\rangle \to |\nu+1\rangle$. ''Partial spectra'' which correspond to different transitions are broadened because of collisions with scatterers (the collision probabilities are determined by the fluctuational field $\mathbf{E}_f$). Prior to averaging over the many-electron ensemble the broadening of a spectrum $\gamma_\nu(\mathbf{E}_f)$ depends both on the level number ν and $\mathbf{E}_f$. Even if all partial spectra are Lorentzian, but with different widths, the total spectrum may be non-Lorentzian.[39]

Interelectron momentum exchange gives rise to transitions between the Landau levels of individual electrons. In a transition one electron ''jumps'' up and another electron ''jumps'' down by one Landau level (we neglect processes where the quantized cyclotron motion with the frequency ω_c is transformed into low-frequency motion of the centers of the electron wave packets). The transition probability can be estimated by separating fast-oscillating and slowly varying terms in electron coordinates and momenta, as described in Sec. V. For $\hbar\omega_c \gtrsim T$ this probability is $\sim \Omega \equiv \omega_p^2/\omega_c$. The frequency Ω gives also the reciprocal time over which the fluctuational field on an electron is averaged, as it is clear from Fig. 1. The condition for the interelectron momentum exchange to be faster than the momentum exchange with the scatterers is then of the form

$$\tau_{\rm ex}^{-1} = \omega_p^2 \omega_c^{-1} \sim e^2 \langle E_f^2 \rangle l_B^2/\hbar T^{-1} \gg \tau^{-1}. \tag{13}$$

For fast interelectron momentum exchange this is relaxation of the total momentum of the electron system that determines the shape of the cyclotron resonance spectrum, and the spectrum is Lorentzian with a width given by the appropriately averaged $\gamma_\nu(\mathbf{E}_f)$ (see Sec. V).

In the opposite case, $\tau_{\rm ex}^{-1} \ll \tau^{-1}$, the cyclotron resonance spectrum is non-Lorentzian. For $T \ll \hbar\omega_c$ the conductivity is determined by the transitions from the lowest Landau level ($\nu=0$). The explicit form of the spectrum in this case for Gaussian distribution of the fluctuational field $\mathbf{E}_f$ is obtained in Sec. VII (see Fig. 5).

III. MANY-ELECTRON QUANTUM TRANSPORT EQUATION

We will initially formulate the many-electron transport equation for the case of electrons coupled to (and quasielastically scattered by) 2D vibrations of the bath (phonons or ripplons). The Hamiltonian of the system is of the form (from now on we set $\hbar = 1$)

$$\hat{H} = \hat{H}_0 + \hat{H}_b + \hat{H}_i, \quad \hat{H}_0 = \frac{1}{2m}\sum_n \hat{\mathbf{p}}_n^2 + \hat{H}_{\rm ee}; \tag{14}$$

$$\hat{H}_b = \sum_{\mathbf{q}} \omega_q \hat{b}_{\mathbf{q}}^+ \hat{b}_{\mathbf{q}}; \quad \hat{H}_i = \sum_{\mathbf{q}} \sum_n V_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}_n} (\hat{b}_{\mathbf{q}} + \hat{b}_{-\mathbf{q}}^+).$$

Here, $\hat{b}_{\mathbf{q}}^+$, $\hat{b}_{\mathbf{q}}$ are creation and annihilation operators of the vibrations, $\hat{\mathbf{p}}_n = -i\nabla_n - e\mathbf{A}(\mathbf{r}_n)$ is the electron momentum, $\mathbf{A}(\mathbf{r})$ is the vector-potential of the magnetic field transverse to the electron layer, and

$$\hat{H}_{\rm ee} = \frac{1}{2} e^2 \sum_{n,n'}{}' |\mathbf{r}_n - \mathbf{r}_{n'}|^{-1}. \tag{15}$$

The wave vectors $\mathbf{q}$ of the vibrations as well as the vectors $\mathbf{r}_n, \mathbf{p}_n$ are 2D vectors. In the equations of motion, $e = -|e|$ is the ''true'' electron charge.

The real part of the long-wavelength conductivity is expressed in terms of the correlation function of the total momentum of the electron system $\hat{\mathbf{P}}$ in a standard way as

$$\sigma(\omega)\equiv\sigma_{xx}(\omega)=\sigma_{yy}(\omega)=(e^2/m^2\omega S)(1-e^{-\beta\omega})$$
$$\times\mathrm{Re}\int_0^\infty dt\; e^{i\omega t}\langle\hat{P}_x^H(t)\hat{P}_x^H(0)\rangle,\quad \hat{\mathbf{P}}=\sum_n\hat{\mathbf{p}}_n. \tag{16}$$

Here, S is the area of the system, $\beta=1/T$, and the superscript H means that the operators are evaluated in Heisenberg representation with a complete Hamiltonian $\hat{H}$ (14). It is convenient to write the momentum correlator in the interaction representation:

$$\langle\hat{P}_x^H(t)\hat{P}_x^H(0)\rangle=\mathrm{Tr}_e[e^{i\hat{H}_0t}\hat{P}_xe^{-i\hat{H}_0t}\hat{\mathcal{G}}_x(t)],$$
$$\hat{\mathcal{G}}_x(t)=Z^{-1}\mathrm{Tr}_b[\hat{S}(t)\hat{P}_xe^{-\beta\hat{H}}\hat{S}^+(t)], \tag{17}$$
$$\hat{S}(t)=e^{i\hat{H}_0t}e^{-i\hat{H}t},$$

where Tr_e and Tr_b are the traces over the wave functions of the electron system and of the vibrations, and $Z=\mathrm{Tr}_e\mathrm{Tr}_b\exp(-\beta\hat{H})$ is the partition function.

A. Transport equation in operator form

In Eq. (17) the coupling to the scatterers has been moved into the operator $\hat{\mathcal{G}}_x(t)$. In the parameter range (6) where collisions with the vibrations are short compared to the intercollision intervals, i.e., the interaction $\hat{H}_i$ is small enough, $\hat{\mathcal{G}}_x(t)$ may be evaluated by perturbation theory in $\hat{H}_i$ where in each order of $\hat{H}_i$ account is taken only of the terms that most strongly diverge when $t\to\infty$. This is the quantum transport equation approximation. In the single-electron problem this approximation corresponds, in terms of Feynman diagrams, to the neglect of nested diagrams and diagrams with intersecting lines.

The many-electron transport equation may be written in the operator form as

$$\frac{\partial\hat{\mathcal{G}}_x(t)}{\partial t}=-\mathrm{Tr}_b\int_0^t dt'[\hat{H}_i(t),[\hat{H}_i(t'),\hat{\rho}_b\hat{\mathcal{G}}_x(t)]], \tag{18}$$
$$\hat{H}_i(t)=e^{i(\hat{H}_0+\hat{H}_b)t}\hat{H}_ie^{-i(\hat{H}_0+\hat{H}_b)t};\quad \hat{\rho}_b=Z_b^{-1}\exp(-\beta\hat{H}_b),$$

where $Z_b=\mathrm{Tr}_b\exp(-\beta\hat{H}_b)$ is the partition function of the bath.

The most substantial assumptions made in deriving (18) are that $t,\tau\gg t_{\mathrm{coll}},T^{-1}$. The quantity t_{coll} characterizes the width of the interval $t-t'$ that contributes to the integral in (18): this interval is supposed to be small compared to t and to the relaxation time τ over which $\hat{\mathcal{G}}_x(t)$ varies.

In what follows we consider short-range scattering and assume it to be quasielastic. The latter means that the characteristic frequencies ω_q of the vibrations of the bath are small:

$$\omega_qt_{\mathrm{coll}}\ll1\quad\text{for}\quad q\lesssim q_{\max}=\max(\lambda_T^{-1},l_B^{-1}). \tag{19}$$

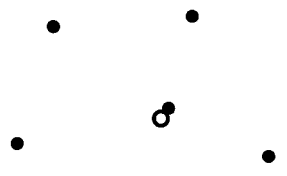

FIG. 4. An electron-scatterer collision. At most one electron (filled circle) collides with a short-range scatterer (open circle) at a time.

Here we have taken into account that the characteristic electron momentum that may be transferred to vibrations, and thus the characteristic values of q, are determined by the thermal wavelength λ_T (2) or the quantum magnetic length l_B (for $\lambda_T>l_B$).

The condition (19) means also that the polaronic renormalization of the electron energy is small compared to electron damping. In what follows we ignore polaron effects [the shift of the cyclotron resonance peak in quantizing magnetic fields was analyzed in Ref. 25(b)]. In this approximation the vibrations of the bath may be considered as creating a quasistationary classical zero-mean Gaussian field.

B. ''Single-site'' approximation

In a strongly correlated electron system at most one electron at a time may collide with a given short-range scatterer (see Fig. 4; we notice that the colliding electron is driven by the field from other electrons). Therefore short-range scattering may be described in the ''single-site'' approximation (cf. Ref. 25). In this approximation only diagonal terms are retained in the double sum over the electrons that enters the product $\hat{H}_i(t)\hat{H}_i(t')$ in (18). Equation (18) may then be written in the form

$$\frac{\partial\hat{\mathcal{G}}_x}{\partial t}=-\sum_{\mathbf{q}}\overline{|V_{\mathbf{q}}|^2}\sum_n\int_0^t dt'$$
$$\times[\exp(i\mathbf{q}\hat{\mathbf{r}}_n(t)),[\exp(-i\mathbf{q}\hat{\mathbf{r}}_n(t')),\hat{\mathcal{G}}_x(t)]],$$
$$\hat{\mathbf{r}}_n(t)=e^{i\hat{H}_0t}\hat{\mathbf{r}}_ne^{-i\hat{H}_0t};\quad \hat{\mathcal{G}}_x(0)=Z_e^{-1}\hat{P}_xe^{-\beta\hat{H}_0},$$
$$\overline{|V_{\mathbf{q}}|^2}=2T\omega_q^{-1}|V_{\mathbf{q}}|^2;\quad Z_e=\mathrm{Tr}_e\exp(-\beta\hat{H}_0). \tag{20}$$

Equation (20) applies also if electrons are scattered by defects or helium vapor atoms. In this case $\overline{|V_{\mathbf{q}}|^2}$ should be replaced by the mean squared Fourier component of the random potential of the defects.

The first step towards solution of the operator equation (20) is transformation of this equation into a set of equations for the matrix elements of $\hat{\mathcal{G}}_x$. It follows from (17), (20) that it is convenient to evaluate these matrix elements on the wave functions of the many-electron system at $t=0$.

It is a distinctive feature of the transport equation (20) that the time evolution of the operators $\hat{\mathbf{r}}_n(t)$ is given by the solution of a problem of many-electron dynamics which is not known. Therefore the matrix elements of the operators

$\exp(i\mathbf{q}\hat{\mathbf{r}}_n(t))$ in (20) for actual $t\sim\tau\gg t_{\rm coll}$ on the wave functions at $t=0$ are also not known. This is in contrast to what is the case for simple systems described by a transport equation, like a single electron or an oscillator, where the evolution of the dynamical variables of the system in the absence of coupling to the scatterers can be found explicitly. It is convenient therefore to change from the operator $\hat{\mathcal{G}}_x$ to the operator $\hat{G}_x$,

$$\hat{G}_x(t)\equiv\exp(-i\hat{H}_0t)\hat{\mathcal{G}}_x(t)\exp(i\hat{H}_0t),$$
$$\frac{\partial G_x}{\partial t}=i[G_x(t),H_0]+\left[\frac{\partial G_x}{\partial t}\right]_{\rm coll},$$
$$\left[\frac{\partial\hat{G}_x}{\partial t}\right]_{\rm coll}=\exp(-i\hat{H}_0t)\left(\frac{\partial\hat{\mathcal{G}}_x}{\partial t}\right)\exp(i\hat{H}_0t). \tag{21}$$

It is seen from (20) that the collision term $[\partial G_x/\partial t]_{\rm coll}$ contains the operators $\exp(-i\hat{H}_0t)\hat{\mathbf{r}}_n(t)\exp(i\hat{H}_0t)\equiv\hat{\mathbf{r}}_n(0)$, $\exp(-i\hat{H}_0t)\hat{\mathbf{r}}_n(t')\exp(i\hat{H}_0t)\equiv\hat{\mathbf{r}}_n(t'-t)$. The matrix elements of the latter operators on the wave functions of the many-electron system at $t=0$ can be evaluated taking into account that the instants of time t and t' in (20), (21) are close to each other, $t-t'\sim t_{\rm coll}\ll\tau$. In what follows we will analyze the solution of Eq. (21) in different ranges of the parameters of the system.

IV. TRANSPORT EQUATION FOR CLASSICAL MAGNETIC FIELDS

Equations (16), (17), (20), (21) reduce the calculation of the conductivity to evaluation of the expressions that are determined by dynamics of the isolated many-electron system. In transforming Eq. (21) into a set of equations for matrix elements of the operator $\hat{G}_x$ it is convenient to use different wave functions for different ranges of the magnetic field. In the present subsection we investigate the range of B where

$$T\gg\omega_p,\omega_c,\ \text{ or }\ T\gg eE_f\lambda_T,\omega_c. \tag{22}$$

When (22) holds an electron has a well-defined kinetic energy $p^2/2m\sim T$ and a well-defined potential energy in the field of other electrons. Uncertainty of each of these energies is determined by the smearing of the electron wave packet λ_T. For an electron in an electric field $\mathbf{E}_f$ this uncertainty is given by $eE_f\lambda_T$, and it is small compared to T. Although the field $\mathbf{E}_f$ is small in a certain sense, it may still dramatically affect magnetotransport, as explained in Secs. II B and II C, and the transport is qualitatively different depending on the relation between $eE_f\lambda_T$ and the Landau level spacing ω_c.

A. Wigner representation of the transport equation

In the domain (22) the electron dynamics are nearly classical. Therefore an appropriate set of wave functions of the many-electron system are plane waves,

$$|\{\mathbf{k}_n\}\rangle\equiv\prod_n(2\pi)^{-1}\exp(i\mathbf{k}_n\mathbf{r}_n), \tag{23}$$

and it is convenient to use the Wigner representation for the electron operators,

$$K(\{\mathbf{p}_n\},\{\mathbf{r}_n\})=\int\left[\prod_n d\boldsymbol{\zeta}_n\exp(i\boldsymbol{\zeta}_n\mathbf{r}_n)\right]\times\left\langle\left\{\mathbf{k}_n+\frac{1}{2}\boldsymbol{\zeta}_n\right\}\middle|\hat{K}\middle|\left\{\mathbf{k}_n-\frac{1}{2}\boldsymbol{\zeta}_n\right\}\right\rangle,$$
$$\mathbf{p}_n\equiv\mathbf{k}_n-e\mathbf{A}(\mathbf{r}_n). \tag{24}$$

It follows from (24) that the correlator (17) that determines the conductivity $\sigma_{xx}(\omega)$ can be written in the form

$$\langle\hat{P}_x^H(t)\hat{P}_x^H(0)\rangle=\iint\left[\prod_n(2\pi)^{-2}d\mathbf{p}_nd\mathbf{r}_n\right]P_x(\{\mathbf{p}_n\},\{\mathbf{r}_n\})\times G_x(t;\{\mathbf{p}_n\},\{\mathbf{r}_n\}), \tag{25}$$

where $G_x(t;\{\mathbf{p}_n\},\{\mathbf{r}_n\})$ is the matrix element of the operator $\hat{G}_x(t)$.

The equation for $G_x(t;\{\mathbf{p}_n\},\{\mathbf{r}_n\})$ follows from (21). In writing this equation we will take into account that the characteristic values of p_n are $\sim(mT)^{1/2}$, and that the scale of $\mathbf{r}_n$ on which $G_x(t;\{\mathbf{p}_n\},\{\mathbf{r}_n\})$ varies is given by the electron mean free path L [$L\sim(T/m)^{1/2}\tau$ in the range (22)] and the characteristic displacement δ of an electron from its quasiequilibrium position (cf. Fig. 1). To lowest order in λ_T/δ, λ_T/L we have

$$\frac{\partial G_x}{\partial t}=\{G_xH_0\}+\left[\frac{\partial G_x}{\partial t}\right]_{\rm coll}, \tag{26}$$

where $G_x\equiv G_x(t;\{\mathbf{p}_n\},\{\mathbf{r}_n\})$.

The first term in (26) is the Poisson bracket of the matrix elements $G_x(t;\{\mathbf{p}_n\},\{\mathbf{r}_n\}),H_0(\{\mathbf{p}_n\},\{\mathbf{r}_n\})$. It describes evolution of the classical many-electron distribution function in the absence of scatterers. In deriving the expression for $\{G_xH_0\}$ from Eqs. (21), (24) it is convenient to write it first in terms of the derivatives over $\mathbf{k}_n,\mathbf{r}_n$ (in these variables the expression has a standard form[40]), and then go over to derivatives over $\mathbf{p}_n,\mathbf{r}_n$. The matrix elements $H_0(\{\mathbf{p}_n\},\{\mathbf{r}_n\})$ of the Hamiltonian H_0 are given by the corresponding terms in (14) with the operators $\hat{\mathbf{p}}_n$ replaced by numbers $\mathbf{p}_n$. Finally we obtain

$$\{G_xH_0\}=-\sum_n\left[e\left(\mathbf{E}_n+\frac{\mathbf{p}_n\times\mathbf{B}}{m}\right)\frac{\partial G_x}{\partial\mathbf{p}_n}+\frac{\mathbf{p}_n}{m}\frac{\partial G_x}{\partial\mathbf{r}_n}\right],$$
$$\mathbf{E}_n\equiv-e^{-1}\frac{\partial H_{\rm ee}}{\partial\mathbf{r}_n}=e\sum_{n'}{}'\frac{\mathbf{r}_n-\mathbf{r}_{n'}}{|\mathbf{r}_n-\mathbf{r}_{n'}|^3}. \tag{27}$$

Here, $\mathbf{E}_n$ is the electric field that drives the nth electron because of its interaction with other electrons.

The collision term

To find the collision term in (26) we have to perform integration over t' in (20), (21). The characteristic range of t' that contributes to the integral is given by $t_{\rm coll}$. We will see that $t_{\rm coll}$ is small compared to the time during which an electron moves by the distance $\sim\delta$ (see Fig. 1) and the fluc-

tuational electric field varies substantially. Therefore (except when analyzing corrections) we will assume the field $\mathbf{E}_n$ to be independent of time when evaluating

$$\hat{\mathbf{r}}_n(t') = \hat{\mathbf{r}}_n(t) + \frac{1}{m}\int_t^{t'} dt_1 \hat{\mathbf{p}}_n(t_1),$$

and we will use for $\hat{\mathbf{p}}_n(t')$ the solution of the equation of motion $d\hat{\mathbf{p}}_n/dt = e\mathbf{E}_n + (e/m)\hat{\mathbf{p}}_n \times \mathbf{B}$ in a uniform time-independent electric field $\mathbf{E}_n$ and transverse magnetic field $\mathbf{B}$,

$$\exp[-i\mathbf{q}\hat{\mathbf{r}}_n(t')] \approx \exp[-i\mathbf{q}\hat{\mathbf{r}}_n(t)]\exp[-i\mathbf{q}\mathbf{F}(t'-t,\hat{\mathbf{p}}_n(t))] \times \exp\left[i\frac{q^2}{2m\omega_c}\sin\omega_c(t'-t)\right], \tag{28}$$

where

$$\mathbf{F}(t,\hat{\mathbf{p}}_n) = \mathbf{f}(t,\hat{\mathbf{p}}_n) - \mathbf{f}(t,m\mathbf{v}_n^{(d)}) + \mathbf{v}_n^{(d)}t,$$

$$\mathbf{f}(t,\hat{\mathbf{p}}_n) = \frac{\hat{\mathbf{p}}_n}{m\omega_c}\sin\omega_c t + e\frac{\hat{\mathbf{p}}_n \times \mathbf{B}}{m^2\omega_c^2}(1-\cos\omega_c t), \tag{29}$$

$$\mathbf{v}_n^{(d)} = (\mathbf{E}_n \times \mathbf{B})/B^2.$$

Although the operators $\hat{\mathbf{p}}_n$ and $\hat{\mathbf{p}}_n \times \mathbf{B}$ do not commute, the commutator of the respective terms in (29) is small in the range (22). It is seen from (21), (26) that to find G_x we need the matrix elements of the operators $\exp(-i\hat{H}_0 t)\exp[-i\mathbf{q}\mathbf{F}(t'-t,\hat{\mathbf{p}}_n(t))]\exp(i\hat{H}_0 t)$ on the wave functions (23). They can be obtained in the WKB approximation simply by replacing the operators $\hat{\mathbf{r}}_n, \hat{\mathbf{p}}_n$ by the numbers $\mathbf{r}_n, \mathbf{p}_n = \mathbf{k}_n - e\mathbf{A}(\mathbf{r}_n)$. Then the collision term in (26) takes on the form

$$\left[\frac{\partial G_x(t;\{\mathbf{p}_n\},\{\mathbf{r}_n\})}{\partial t}\right]_{\text{coll}} = -\sum_{\mathbf{q}} \overline{|V_{\mathbf{q}}|^2} \sum_{n'} \xi_{n'}(\mathbf{q},\mathbf{p}_{n'})[G_x(t;\{\mathbf{p}_n\},\{\mathbf{r}_n\}) - G_x(t;\{\mathbf{p}_n + \mathbf{q}\delta_{nn'}\},\{\mathbf{r}_n\})], \tag{30}$$

$$\xi_n(\mathbf{q},\mathbf{p}_n) = 2\,\mathrm{Re}\int_0^t dt' \exp[-i\mathbf{q}\mathbf{F}(t'-t,\mathbf{p}_n) + i(q^2/2m\omega_c)\sin\omega_c(t'-t)].$$

In (30) we have assumed that $\overline{|V_{\mathbf{q}}|^2}$ is independent of the direction of $\mathbf{q}$.

B. Single-electron approximation for B=0

Equations (16), (25)–(30) give a well-known result in the absence of a magnetic field and in the single-electron approximation, i.e., in the neglect of the electron-electron interaction H_{ee} in (14). In this case, for characteristic $q \sim p_n \sim (mT)^{1/2}$ and for time $t \gg t_{\text{coll}} = 1/T$ the function $\xi_n \equiv \xi_n^{(s)}$ in (30) becomes a δ function of the energy conservation law:

$$\xi_n^{(s)}(\mathbf{q},\mathbf{p}_n) = 2\pi\delta\left(\frac{\mathbf{q}\mathbf{p}_n}{m} + \frac{q^2}{2m}\right). \tag{31}$$

The solution of Eq. (26) in the single-electron zero-B approximation is given by

$$G_x^{(s)}(t;\{\mathbf{p}_n\},\{\mathbf{r}_n\}) = Z_e^{-1}\sum_n \exp[-t/\tau^{(s)}(\epsilon_n)]p_{nx} \times \exp\left[-(\beta/2m)\sum_{n'}\mathbf{p}_{n'}^2\right],$$

$$(\tau^{(s)}(\epsilon))^{-1} = \frac{\pi}{p^2}\sum_{\mathbf{q}} q^2\overline{|V_{\mathbf{q}}|^2}\,\delta\left(\frac{\mathbf{q}\mathbf{p}}{m} + \frac{q^2}{2m}\right), \tag{32}$$

$$\epsilon(p) = \frac{p^2}{2m}.$$

Equations (16),(25),(32) result in a standard expression for single-electron conductivity in the absence of a magnetic field, with a frequency-dependent relaxation rate,

$$\sigma^{(s)}(\omega) = \frac{e^2 n_s}{m}\overline{\tau^{(s)}(\epsilon)/[1+\omega^2(\tau^{(s)}(\epsilon))^2]},$$

where averaging over ϵ is performed with the weighting factor $\propto \epsilon\exp(-\beta\epsilon)$. In particular the low-frequency ($\omega\tau \ll 1$) conductivity $\sigma^{(s)}(\omega)$ is determined by $\overline{\tau^{(s)}(\epsilon)}$, whereas the high-frequency conductivity is determined by the average collision frequency $\overline{1/\tau^{(s)}(\epsilon)}$.

In the presence of a magnetic field the structure of the time dependence of the exponential in the expressions (28)–(30) for the kernel $\xi_n(\mathbf{q},\mathbf{p}_n)$ is completely changed: the function $\mathbf{F}$ becomes periodically oscillating in time, with a frequency ω_c. Therefore integration over t' in (30) does not give a δ function of the type (31). In fact, the integral over t' explicitly depends on t and diverges with increasing t (the orbit of an electron is a closed circle, and therefore the electron encounters a scatterer infinitely many times). This is an indication of the inapplicability of the transport equation in the single-electron approximation.

C. Many-electron theory

1. General form of the solution of the transport equation for strong electron-electron interaction

The interelectron momentum exchange is described by the terms $e\mathbf{E}_n\partial G_x/\partial\mathbf{p}_n$ and $m^{-1}\mathbf{p}_n\partial G_x/\partial\mathbf{r}_n$ in Eqs. (26), (27). The former terms are $\sim eE_f\chi_T G_x$, and so are the latter as is clear from (32) if one uses the full Boltzmann factor $\exp(-\beta H_0)$ in $G_x^{(s)}$ (instead of retaining only kinetic energy in H_0). Therefore the interelectron momentum exchange may substantially affect the conductivity if $eE_f\chi_T \gtrsim \tau^{-1}$.

The analysis of many-electron transport is simplified if the interelectron momentum exchange rate $\tau_{\text{ex}}^{-1} \sim eE_f\chi_T \gg \tau^{-1}$, or equivalently $\omega_p\tau \gg 1$ [cf. (12)]. This condition may also be understood as the condition for the uncertainty of the kinetic energy of an electron due to interaction with other electrons to be much larger than the uncertainty due to collisions with scatterers. In the corresponding parameter range

the solution for G_x may be sought in the form

$$G_x(t;\{\mathbf{p}_n\},\{\mathbf{r}_n\}) \approx g_x(t;\hat{\Omega}_c\mathbf{P}(\{\mathbf{p}_n\}),H_0(\{\mathbf{p}_n\},\{\mathbf{r}_n\})),$$

$$P=\sum_n \mathbf{p}_n, \quad \hat{\Omega}_c \equiv \hat{\Omega}_c(t)=\begin{pmatrix} \cos\omega_c t & \sin\omega_c t \\ -\sin\omega_c t & \cos\omega_c t \end{pmatrix}. \quad (33)$$

The sign of the off-diagonal terms of the matrix $\hat{\Omega}_c(t)$ corresponds to **B** pointing in the positive direction of the z axis and allows for the sign of the electron charge.

The function g_x depends on the coordinates and momenta of individual electrons only in terms of the total momentum and energy of the whole system. For G_x given by (33) the sum of the terms that contain $\mathbf{E}_n$ and $\partial G_x/\partial \mathbf{r}_n$ in (27) is equal to zero (because $\Sigma_n \mathbf{E}_n=0$). Qualitatively, Eq. (33) means that, for fast interelectron momentum exchange, the change of the momentum of an nth electron due to a collision with a scatterer is ''shared'' by other electrons before the electron is scattered again.

In view of the initial conditions for G_x that follow from (17), (21), and allowing for symmetry arguments (G_x is the x component of a vector) we will assume that g_x is the x component of a vector $\mathbf{g}(t;\hat{\Omega}_c\mathbf{P},H_0)$, and we will seek this vector in the form

$$\mathbf{g}(t;\hat{\Omega}_c\mathbf{P},H_0)=\tilde{g}(t)\hat{\Omega}_c(t)\mathbf{P}Z_e^{-1}\exp(-\beta H_0) \quad (34)$$

with the initial condition $\tilde{g}(0)=1$. In fact, we could seek $\tilde{g}$ in a more general form of a function of t and H_0, but in the case of elastic scattering the energy of a colliding electron, and thus the energy of the electron system as a whole, is conserved, and therefore the dependence of **g** on H_0 does not vary in time and is determined by the initial conditions.

2. Many-electron collision term

The collision integral $[\partial \mathbf{g}/\partial t]_{\text{coll}}$ for the solution of the kinetic equation of the form (33), (34) is given by (30) with G_x replaced by $\mathbf{g}(t;\hat{\Omega}_c\mathbf{P},H_0)$. Since the value of H_0 is not changed in a collision, we have

$$\begin{aligned} &g_i(t;\hat{\Omega}_c\mathbf{P}(\{\mathbf{p}_n\}),H_0(\{\mathbf{p}_n\},\{\mathbf{r}_n\})) \\ &\quad -g_i(t;\hat{\Omega}_c\mathbf{P}(\{\mathbf{p}_n+\mathbf{q}\delta_{nn'}\}),H_0(\{\mathbf{p}_n+\mathbf{q}\delta_{nn'}\},\{\mathbf{r}_n\})) \\ &\quad =-(\hat{\Omega}_c(t)\mathbf{q})_i\tilde{g}(t)Z_e^{-1}\exp(-\beta H_0) \quad (i=x,y). \end{aligned}$$

The only singled out direction of the transferred momentum for the many-electron system is the direction of the total momentum **P**. Therefore in the last line of the above equation one may replace

$$\mathbf{q} \Rightarrow \frac{(\mathbf{q}\cdot\mathbf{P})\mathbf{P}}{\mathbf{P}^2}. \quad (35)$$

The characteristic values of **P** we are interested in are the fluctuational ones,

$$|\mathbf{P}| \sim [\langle \mathbf{P}^2 \rangle]^{1/2}=(2NmT)^{1/2} \quad (N=n_sS). \quad (36)$$

The momenta of different electrons subject to the condition that the total momentum be equal to **P** are basically uncorrelated for **P** of the order of (36), $\langle \mathbf{p}_n\mathbf{p}_{n'}\rangle \sim N^{-3/2}$ for $n \neq n'$ [to show this one may write $\delta(\mathbf{P}-\Sigma_n\mathbf{p}_n)$ in the form of a Fourier integral and then perform averaging over all momenta $\mathbf{p}_n$ with the Boltzmann weighting factor]. Therefore to the lowest order in the number of electrons

$$\mathbf{P}^2 \approx \sum_n \mathbf{p}_n^2=2NmT.$$

It follows from (30), (35) that in the collision integral $[\partial \mathbf{g}/\partial t]_{\text{coll}}$ the term that depends on the direction of **q** is proportional to the expression $\xi_n(\mathbf{q},\mathbf{p}_n)\Sigma_{n'}(\mathbf{q}\cdot\mathbf{p}_{n'})$. This term should be averaged over **q** directions [this is a part of the summation over **q** in (30)]. Since the momenta of different electrons are approximately independent from each other, the major contribution to the average comes from the term in the sum over n' with $n'=n$. According to (31) $\mathbf{q}\mathbf{p}_n=-\frac{1}{2}q^2$. Therefore upon averaging over the directions of **q** (denoted by the subscript $\mathbf{q}/q$) we obtain

$$\left[(\mathbf{q}\mathbf{P})\sum_n \xi_n(\mathbf{q},\mathbf{p}_n)\right]_{\mathbf{q}/q} \approx -\frac{1}{2}q^2N\langle \xi_n(\mathbf{q},\mathbf{p}_n)\rangle, \quad (37)$$

$$\langle \xi_n(\mathbf{q},\mathbf{p}_n)\rangle \approx \xi(\mathbf{q}) \equiv \int_{-\infty}^{\infty} dt\ \langle e^{i\mathbf{q}\hat{\mathbf{r}}_n(t)}e^{-i\mathbf{q}\hat{\mathbf{r}}_n(0)}\rangle.$$

Here, we have set the limits of integration over time to be infinite; this can be done if the duration of a collision (the actual range of time that contributes to the integral over t) is much smaller than the relaxation time [which determines the characteristic limit of the integral over time in the expression (30) for $\xi_n(\mathbf{q},\mathbf{p}_n)$]. The statistical averaging in (37) is performed to zeroth order in the coupling to the scatterers. Clearly, $\xi(\mathbf{q})$ in (37) is a dynamical structure factor of the electron system at zero frequency evaluated in the single-site approximation (it should not be confused with a static structure factor which is the integral over the frequency).

The above expressions result in the following simple form of the collision term for the function **g**:

$$\left[\frac{\partial \mathbf{g}}{\partial t}\right]_{\text{coll}}=-\tau^{-1}\mathbf{g}, \quad \tau^{-1}=\frac{1}{4mT}\sum_{\mathbf{q}} q^2\overline{|V_{\mathbf{q}}|^2}\xi(\mathbf{q}). \quad (38)$$

For zero magnetic field Eq. (38) was derived in Ref. 25(c) assuming that electrons form a Wigner crystal. The relation between losses of an electron system moving above the helium surface and the structure factor was considered for an electron fluid at $B=0$ in Ref. 41 [in the case of strong magnetic fields this relation was also considered in Ref. 31(a)], and the problem of corrections due to simultaneous scattering of several electrons by one ripplon was addressed there.

The solution of the kinetic equation for the function $\tilde{g}(t)$ in (34) is exponential, $\tilde{g}(t)=\exp(-t/\tau)$.

V. CLASSICAL MANY-ELECTRON CONDUCTIVITY

Equations (16), (25), (33), (38) provide a simple expression for the frequency-dependent conductivity of the many-electron system. In particular the static conductivity is of the Drude type,

$$\sigma \equiv \sigma(0) = \frac{e^2 n_s}{m} \frac{\tau}{1+\omega_c^2 \tau^2}. \tag{39}$$

For $\omega_c \tau \gg 1$ (classically strong magnetic fields) the conductivity as a function of frequency ω has a sharp peak at $\omega = \omega_c$. This peak corresponds to cyclotron resonance,

$$\sigma(\omega) = \frac{e^2 n_s}{2m} \frac{\tau}{1+(\omega-\omega_c)^2 \tau^2}, \quad |\omega-\omega_c| \ll \omega_c, \quad \omega_c \tau \gg 1. \tag{40}$$

The parameter τ^{-1} in (39), (40) is the collision frequency calculated in the many-electron theory and given by Eq. (38). We note that one and the same collision frequency determines static conductivity and cyclotron resonance in the classical theory (it is no longer true in quantizing magnetic fields). It is expressed in terms of the (Fourier transformed) short-wavelength electron density correlator $\xi(\mathbf{q})$ (37), and it depends both on the magnetic field and the fluctuational electric field in the system. It is τ^{-1} that describes onset of magnetoresistance, the dependence of mobility on electron density, as well as the density and temperature dependence of the width of the cyclotron resonance peak in classically strong magnetic fields.

In the classical limit we are considering in this section the statistical averaging for the isolated electron system in Eq. (37) for $\xi(\mathbf{q})$ is reduced to integration over electron coordinates and over electron momenta with the weight $\exp[-\beta H_0(\{\mathbf{p}_n\},\{\mathbf{r}_n\})]$ (quantum corrections are discussed in the Appendix). The averaging of $\xi_n(\mathbf{q},\mathbf{p}_n)$ over $\mathbf{p}_n$ is straightforward with account taken of the explicit form of the function $\mathbf{F}(t,\mathbf{p}_n)$ (29), and the resulting expression for $\xi(\mathbf{q})$ contains only configuration averaging which comes to the averaging over the fluctuational field $\mathbf{E}_f$:

$$\xi(\mathbf{q}) = \int_{-\infty}^{\infty} dt\, \phi(t) \Phi_E(t),$$

$$\phi(t) = \exp\left[-\frac{q^2 T}{m\omega_c^2}(1-\cos\omega_c t) - i\frac{q^2}{2m\omega_c}\sin\omega_c t \right], \tag{41}$$

$$\Phi_E(t) = \left\langle \exp\left[i\frac{\mathbf{q}\mathbf{v}_f^{(d)}}{\omega_c}(\omega_c t - \sin\omega_c t) - ie\frac{\mathbf{B}}{m\omega_c^2} \times \mathbf{q}\times\mathbf{v}_f^{(d)}(1-\cos\omega_c t)\right]\right\rangle, \tag{42}$$

$$\mathbf{v}_f^{(d)} = \frac{\mathbf{E}_f \times \mathbf{B}}{B^2}.$$

The probability density distribution of the fluctuational field $\rho(\mathbf{E}_f)$ to be used for the averaging in $\Phi_E(t)$ is the probability density of the field $\mathbf{E}_n$ on an nth electron:

$$\rho(\mathbf{E}_f) \equiv Z_{\rm conf}^{-1} \int \left[\prod_{n'} d\mathbf{r}_{n'}\right] \delta(\mathbf{E}_f - \mathbf{E}_n) e^{-\beta H_{\rm ee}(\{\mathbf{r}_{n'}\})},$$

$$\mathbf{E}_n = -\frac{1}{e}\frac{\partial H_{\rm ee}}{\partial \mathbf{r}_n}, \quad Z_{\rm conf} = \int \left[\prod_{n'} d\mathbf{r}_{n'}\right] e^{-\beta H_{\rm ee}(\{\mathbf{r}_{n'}\})} \tag{43}$$

[clearly, the distribution (43) is independent of n]. Detailed results on the probability distribution (42) are presented in Ref. 33(b).

In the rest of this section we consider the explicit form of the correlator $\xi(\mathbf{q})$ (41) in the two interesting limiting cases.

A. Weak to moderately strong magnetic fields

The expression for $\xi(\mathbf{q})$ and thus for the collision frequency τ^{-1} (38) is simplified in the range of weak to moderately strong magnetic fields, $T \gg e\langle E_f^2\rangle^{1/2} \lambda\!\!\!^{-}_T \gg \omega_c$ (7). As discussed in Sec. II B, in this range the effects of magnetic field on the electron energy spectrum, as well as on the electron collisions with scatterers, are washed out by the fluctuational field. Mathematically this is immediately seen from Eq. (42) if one notices that in $\Phi_E(t)$, for characteristic $q \sim (mT)^{1/2}$ [cf. Eq. (45) below], the parameter $q v_n^{(d)} \sim (e\langle E_f^2\rangle^{1/2} \lambda\!\!\!^{-}_T/\omega_c) T \gg T \gg \omega_c$. Therefore $\Phi_E(t)$ is a rapidly oscillating function of time if $t \gtrsim \omega_c^{-1}$, and the contribution of this time domain to the integral over time (41) is negligibly small.

The major contribution to $\xi(\mathbf{q})$ in the range (7) comes from the domain $\omega_c t \ll 1$. To lowest order in $\omega_c t$

$$\phi(t) \approx \exp\left[-\frac{q^2}{2m}(Tt^2 + it)\right], \quad \Phi_E(t) \approx 1, \quad t \ll \omega_c^{-1}.$$

The characteristic time that contributes to the integral of $\phi(t)\Phi_E(t)$ (the collision time) is seen to be equal to

$$t_{\rm coll} = T^{-1}, \quad T \gg e\langle E_f^2\rangle^{1/2} \lambda\!\!\!^{-}_T \gg \omega_c. \tag{44a}$$

This can be easily understood, since for $e\langle E_f^2\rangle^{1/2} \lambda\!\!\!^{-}_T \gg \omega_c$ an electron has a continuous spectrum and moves with a thermal velocity $(T/m)^{1/2}$. Therefore a collision with a short-range scatterer ''lasts'' for the time it takes an electron to fly over the thermal wavelength $\lambda\!\!\!^{-}_T$. One may also say that an electron is ''blown away'' by the fluctuational field once it has collided with a scatterer.

Both the magnetic field *and* the fluctuational electric field drop out from $\xi(\mathbf{q})$ in the above approximation. They give rise only to quantum corrections. These corrections are found in the Appendix. With account taken of them the expression for $\xi(\mathbf{q})$ takes on the form

$$\xi(\mathbf{q}) = \left(\frac{2\pi m}{Tq^2}\right)^{1/2} (1+\mathcal{F}) \exp\left[-\frac{q^2}{8mT}(1-\mathcal{F})\right],$$

$$\mathcal{F} = \frac{\hbar^2}{48T^2}\left(\omega_c^2 + \frac{e^2}{2mT}\langle E_f^2\rangle\right). \tag{45}$$

It is clear from Eq. (45) that not only are the quantum corrections parametrically small, but that they also contain a small numerical factor. This means that in the range (7), although the electron system is strongly correlated, the electron-electron interaction only weakly affects the rate of short-range scattering τ^{-1},

$$\tau^{-1} \approx \tau_{B=0}^{-1}, \quad \text{for} \quad T \gg e\langle E_f^2\rangle^{1/2} \lambda\!\!\!^{-}_T \gg \omega_c. \tag{46}$$

The fact that the correlator $\xi(\mathbf{q})$, and thus τ^{-1}, are nearly independent of the magnetic field, is an indication of the extremely important role of electron-electron interaction: it is because of this interaction, and only in the range (7) where this interaction is in a certain sense *stronger* than the magnetic field, that the magnetic field just drops out of the expression for a static conductivity, even when the field is classically strong, $\omega_c\tau \gg 1$. As explained in Sec. II B, in the range (7) electron-electron interaction ''restores'' a simple-minded Drude model of conductivity which shows no magnetoresistance. We notice that the peak of the cyclotron resonance may be very sharp in the range (7), and its halfwidth is approximately given by the scattering rate τ^{-1} calculated for $B=0$ and in the neglect of the effect of the fluctuational field.

B. ''Strong'' classically strong magnetic fields

It follows from the qualitative arguments given in Sec. II C that scattering by short-range scatterers should change and magnetoresistance in classical magnetic fields should arise when the displacement of cyclotron orbit center over the time $2\pi/\omega_c$ due to the electron drift in fluctuational field becomes smaller than the thermal wavelength,

$$\omega_c \gtrsim (2\pi e\langle E_f^2\rangle^{1/2}\varkappa_T T)^{1/2}. \tag{47}$$

In this case an electron collides with the same scatterers several times [the encountering factor ζ is estimated in Eq. (9)]. We note that the occurrence of magnetoresistance in the range (47), predicted based on the picture of an electron spiralling along a semiclassical orbit, is consistent with the quantum picture. Indeed, it follows from the condition $e\langle E_f^2\rangle^{1/2}\varkappa_T \ll T$ [which, in turn, follows from (2)] that in the range (47) we have

$$\omega_c \gg e\langle E_f^2\rangle^{1/2}\varkappa_T,$$

and therefore the Landau level spacing exceeds the uncertainty of the kinetic energy of an electron wave packet in the fluctuational field.

To calculate the correlator $\xi(\mathbf{q})$ and thus the relaxation rate in the domain (47) we will evaluate the integral over time in (41) by the steepest descent method. This is justified, since for characteristic $q\sim(mT)^{1/2}$ the exponent in $\phi(t)$ (41) is a large negative number $\sim(T/\omega_c)^2$ everywhere except for comparatively narrow ($\sim T^{-1}$) time intervals around the points $\omega_c t = 2\pi s$ with integer s. For the same q the parameter $\mathbf{q}\mathbf{v}_f^{(d)}/\omega_c$ in $\Phi_E(t)$ is $\sim eE_f\varkappa_T T/\omega_c^2 \lesssim 1$. Therefore the positions of the saddle points of the integrand $\phi(t)\Phi_E(t)$ are determined by the function $\phi(t)$ and are given by $t_s = 2\pi s\omega_c^{-1} - i(2T)^{-1}$, and the result of the integration over t in (41) reads

$$\xi(\mathbf{q}) = \left(\frac{2\pi m}{Tq^2}\right)^{1/2} \exp\left[-\frac{q^2}{8mT}\right] \sum_{s=-\infty}^{\infty} \left\langle \exp\left[i\mathbf{q}\mathbf{E}_f \frac{2\pi s}{\omega_c B}\right]\right\rangle \tag{48}$$

[averaging over $\mathbf{E}_f$ is done with the probability distribution $\rho(\mathbf{E}_f)$ defined in (43)].

It follows from (48) that in the range of comparatively strong (but still classical) magnetic fields (47) the electron relaxation rate depends explicitly on the many-electron fluctuational field. This dependence becomes particularly simple in sufficiently high magnetic fields where the inequality (47) is strong. In this case the exponent in (48) varies only slightly when s is changed by 1, and therefore one may go from the sum over s to the integral,

$$\xi(\mathbf{q}) = \left(\frac{2\pi m}{Tq^2}\right)^{1/2} \exp\left[-\frac{q^2}{8mT}\right] \frac{\omega_c B}{\pi q}\langle E_f^{-1}\rangle, \tag{49}$$

$$\langle E_f^{-1}\rangle \equiv \int E_f^{-1} d\mathbf{E}_f\, \rho(\mathbf{E}_f), \quad \omega_c \gg (2\pi e\langle E_f^2\rangle^{1/2}\varkappa_T T)^{1/2}.$$

Equation (49) corresponds to the case where an electron collides with one and the same scatterer many times. The encountering factor ζ is given by the coefficient $(\omega_c B/\pi q)\langle E_f^{-1}\rangle$ in (49) for characteristic $q^{-1}\sim\varkappa_T$, and this factor coincides with the estimate of ζ in Eq. (9). We notice that each ''individual'' collision event is an elastic collision, and in this collision the electron kinetic energy is conserved: $\mathbf{q}\mathbf{p}_n = -q^2/2$ where $\mathbf{q}$ is the transferred momentum. This can be seen from Eq. (30) for $\xi(\mathbf{q},\mathbf{p}_n)$ if instead of averaging over $\mathbf{p}_n$ [made to obtain (41)] one first performed integration over time. In the range (47) the major contribution to the integral over time comes from the integrals over the intervals which are centered at $t = 2\pi s/\omega_c$ and have characteristic widths that exceed T^{-1} but are small compared to ω_c^{-1}. Each of these integrals gives the δ function of the energy conservation (31). The total duration of a collision in the range (47) is

$$t_{\text{coll}} = B\varkappa_T\langle E_f^{-1}\rangle \sim \omega_c/\omega_p T. \tag{44b}$$

It is small compared to the time Ω^{-1} (5) over which the fluctuational field driving an electron noticeably varies, which provides justification of the approach in which this field is assumed to be time independent.

We notice also that, in contrast to the case of moderately strong magnetic fields (7) where the E_f-dependent correction to the relaxation rate in (45) contains $\langle E_f^2\rangle$, Eq. (49) contains a different moment of the probability density distribution of the fluctuational field, the mean reciprocal fluctuational field $\langle E_f^{-1}\rangle$.

VI. MANY-ELECTRON CONDUCTIVITY AND CYCLOTRON RESONANCE IN QUANTIZING MAGNETIC FIELDS

In quantizing magnetic fields,

$$\omega_c \gtrsim T, \tag{50}$$

the band structure of the electron energy spectrum should be taken into account explicitly. The qualitative picture of electron scattering in this case was described in Sec. II C.

For $\omega_c \gg T$ electrons occupy the lowest Landau level, whereas for higher T higher Landau levels are occupied. The characteristic wavelength of an electron (the distance between the nodes of the wave functions) is given in order of magnitude by the expression

$$\lambda\!\!\!^{-}=[m\omega_c(2\overline{n}+1)]^{-1/2},\quad \overline{n}=[\exp(\omega_c/T)-1]^{-1} \tag{51}$$

[cf. (9)]. The value of $\lambda\!\!\!^{-1}$ determines the momentum that may be transferred to a short-range scatterer in the collision. The total scattering probability would be expected to be proportional to the ''encountering factor'' discussed in Sec. II C and given by Eq. (9).

The quantitative many-electron theory in quantizing magnetic fields should be formulated in a different way for static conductivity and for cyclotron resonance. This is clear from the fact that, e.g., for $T\ll\omega_c$ static conductivity is determined by the scattering within the lowest Landau level, whereas broadening of the cyclotron resonance peak is determined by the scattering both in the lowest and first excited levels. In more formal terms, the conductivity $\sigma(\omega)$ is determined by the polarization operator $\Pi(\omega)$. The difference of $\Pi(\omega)$ for $\omega=0$ and $\omega=\omega_c$ becomes substantial when the duration of a collision exceeds ω_c^{-1}, as it does in the range (50).

A. Static conductivity

In the range of strong magnetic fields, $\omega_c\tau\gg1$, it is convenient to transform Eq. (16) when evaluating static conductivity. One may first multiply the Heisenberg equation of motion

$$\frac{d}{dt}\hat{P}_y^H(t)=-\kappa\omega_c\hat{P}_x^H(t)-\sum_n\frac{\partial\hat{H}_i^H(t)}{\partial y_n},\qquad(\kappa=eB_z/m\omega_c,\ \ |\kappa|=1) \tag{52}$$

by $\hat{P}_x^H(0)$ from the right and perform statistical averaging and a Fourier transform over time. Then Eq. (52) may be multiplied by $\Sigma_n\partial\hat{H}_i^H(0)/\partial y_n$ from the left, and again statistical averaging and a Fourier transform over time (at frequency $-\omega$) should be performed. Neglecting the terms proportional to ω in the resulting two equations and allowing for the system to be isotropic one then arrives at the expression

$$\sigma(\omega)=\frac{e^2}{2m^2\omega_c^2TS}\mathrm{Re}\int_0^\infty dt e^{i\omega t}\times\sum_{nn'}\langle(\nabla_n\hat{H}_i^H(t))\cdot(\nabla_{n'}\hat{H}_i^H(0))\rangle,\quad\omega\ll T,\omega_c. \tag{53}$$

To lowest order in $(\omega_c\tau)^{-1}$ the correlation function of the operators $\nabla_n\hat{H}_i^H$ can be calculated in the neglect of interaction between the electrons and the scatterers, i.e., one can replace

$$\hat{H}_i^H(t)\Rightarrow e^{i(\hat{H}_0+\hat{H}_b)t}\hat{H}_ie^{-i(\hat{H}_0+\hat{H}_b)t}\equiv\hat{H}_i(t).$$

In the case of short-range scattering, as is clear from Fig. 4, one should keep only diagonal terms with $n=n'$ in the double sum in (53). If one further assumes that electrons are scattered by defects or by 2D vibrations of the bath (phonons or ripplons) with typical frequencies small compared to T, t_{coll}^{-1}, the expression for the static conductivity can be written in the form

$$\sigma\equiv\sigma_{xx}(0)=\frac{\hbar e^2n_s}{2mT\omega_c}(2\overline{n}+1)\tau^{-1},\quad\omega_c\tau\gg1, \tag{54}$$

$$\tau^{-1}=\frac{1}{2}\lambda\!\!\!^{-2}\hbar^{-2}\sum_{\mathbf{q}}q^2\overline{|V_{\mathbf{q}}|^2}\xi(\mathbf{q})$$

(for clarity, we have explicitly incorporated $\hbar$). Here, $\overline{|V_{\mathbf{q}}|^2}$ is the mean square Fourier component of the potential of the scatterers, and $\xi(\mathbf{q})$ is the (Fourier transformed) electron density correlator defined in Eq. (37).

Equation (54) has the form of Eq. (9) which gives the conductivity in terms of phenomenologically introduced diffusion length R_B and scattering rate τ^{-1}, with R_B given by the estimate (11). In the limit of classically strong magnetic fields, $\omega_c\ll T$ but $\omega_c\gg\tau^{-1}$, Eq. (54) goes over into Eqs. (38), (39) obtained before in a different way.

1. Operators of the centers of the electron wave packets

To evaluate the electron density correlator $\xi(\mathbf{q})$ for a many-electron system in the quantum range (50) it is convenient to introduce the operators $\hat{\tilde{\mathbf{r}}}_n$ of the positions of the centers of the electron wave packets:

$$\hat{\tilde{\mathbf{r}}}_n=\hat{\mathbf{r}}_n+e\frac{\hat{\mathbf{p}}_n\times\mathbf{B}}{m^2\omega_c^2},\ \ [\hat{\tilde{\mathbf{r}}}_{ni},\hat{p}_{nj}]=0\ \ (i,j=x,y),\qquad[\hat{\tilde{x}}_n,\hat{\tilde{y}}_n]=-i\kappa/m\omega_c \tag{55}$$

[κ is defined in (52), $|\kappa|=1$].

In the semiclassical domain (4), (5) the characteristic values of the momenta p_n are $\sim\lambda\!\!\!^{-1}$, and they are very much smaller than the characteristic range δ (3) within which the centers of the wave packets $\tilde{r}_n$ vary. In the analysis of the dynamics of the centers of electron orbits, to the lowest order in $\lambda\!\!\!^{-}/\delta$ one can express the operator $\hat{H}_{\mathrm{ee}}(\{\hat{\mathbf{r}}_n\})$ in terms of the operators $\hat{\tilde{\mathbf{r}}}_n$ and $\hat{\mathbf{p}}_n$ and retain only the zeroth-order terms in $\hat{\mathbf{p}}_n$ in the expansion of $\hat{H}_{\mathrm{ee}}$:

$$\hat{H}_{\mathrm{ee}}(\{\hat{\mathbf{r}}_n\})\approx\hat{H}_{\mathrm{ee}}(\{\hat{\tilde{\mathbf{r}}}_n\}) \tag{56}$$

(cf. Ref. 31). It follows from (55) and also from (2), (3) that the terms dropped in (56) are

$$\sim eE_fp_n/m\omega_c\sim eE_fl_B^2/\lambda\!\!\!^{-}\ll eE_fl_B\ll T\lesssim\omega_c.$$

In the approximation (56) the electron motion is a superposition of quantum cyclotron motion and semiclassical drift of the orbit centers. The cyclotron motion has much in common with vibrations of a harmonic oscillator. It is described by the raising and lowering operators $\hat{p}_{n\alpha}$ that move the electron to an upper (for $\alpha=+$) or lower (for $\alpha=-$) Landau level, and by the wave functions $|\nu_n\rangle$ in the occupation number representation:

$$\hat{p}_{n\alpha}=(2m\omega_c)^{-1/2}(\hat{p}_{nx}-i\alpha\kappa p_{ny}),\quad \alpha=\pm,$$

$$[\hat{p}_{n-},\hat{p}_{n+}]=1\quad(|\kappa|=1),$$

$$\hat{p}_{n\pm}|\nu_n\rangle=\left(\nu_n+\frac{1}{2}\pm\frac{1}{2}\right)^{1/2}|\nu_n\pm1\rangle,\quad \nu_n=0,1,\ldots \quad (57)$$

(the functions $|\nu_n\rangle$ with different ν_n correspond to the same position of the center of the cyclotron orbit of the nth electron).

The operators $\hat{p}_{n\alpha}$ commute with the operators $\hat{\tilde{\mathbf{r}}}_n$, and the Hamiltonian of the electron system takes on the form

$$\hat{H}_0\approx\omega_c\sum_n\left(\hat{p}_{n+}\hat{p}_{n-}+\frac{1}{2}\right)+\hat{H}_{ee}(\{\hat{\tilde{\mathbf{r}}}_n\}). \quad (58)$$

2. Electron density correlator

Using (55), (57), (58) one can write the operator $\exp(i\mathbf{q}\hat{\mathbf{r}}_n(t))$ in $\xi(\mathbf{q})$ (37) in the form

$$\exp(i\mathbf{q}\hat{\mathbf{r}}_n(t))\approx\exp\left[\sum_\alpha \alpha l_B q_{-\alpha}\hat{p}_{n\alpha}(0)e^{i\alpha\omega_c t}\right]\times\exp[i\mathbf{q}\hat{\tilde{\mathbf{v}}}_n^{(d)}(0)t]\exp[i\mathbf{q}\hat{\tilde{\mathbf{r}}}_n(0)],$$

$$\hat{\tilde{\mathbf{v}}}_n^{(d)}\equiv\frac{d}{dt}\hat{\tilde{\mathbf{r}}}_n=\frac{\mathbf{E}_n(\{\hat{\tilde{\mathbf{r}}}_{n'}\})\times\mathbf{B}}{B^2},\quad q_\alpha=\frac{q_x-i\alpha\kappa q_y}{\sqrt{2}}. \quad (59)$$

The field $\mathbf{E}_n$ here is the fluctuational field driving the nth electron. It is given by Eq. (27), with $\mathbf{r}_{n'}$ replaced by $\hat{\tilde{\mathbf{r}}}_{n'}$. In deriving the expression for the drift velocity $d\hat{\tilde{\mathbf{r}}}_n/dt=-i[\hat{\tilde{\mathbf{r}}}_n,\hat{H}_{ee}]$ we used the commutation relations (55) and dropped the higher-order commutators $\propto[\hat{\tilde{\mathbf{r}}}_{n'},\tilde{\mathbf{E}}_n]$ (or $\propto[\hat{\tilde{\mathbf{r}}}_{n'},\mathbf{v}_n^{(d)}]$), so that, in fact, the operators $\hat{\tilde{\mathbf{r}}}_{n'}$ in (59) should be considered as c numbers. This is justified provided the field $\mathbf{E}_n$ is smooth on the characteristic wavelength χ (51), i.e.,

$$\chi|\langle\nabla_n\mathbf{E}_n\rangle|=e\langle E_f^2\rangle\chi/T\ll\langle E_f^2\rangle^{1/2}. \quad (60)$$

The latter inequality is the condition for the electron drift in the fluctuational field to be semiclassical [cf. (2), (3); in evaluating $\langle\nabla_n\mathbf{E}_n\rangle$ we used Eq. (A2)]. It follows from the estimate of the field E_f (3) that (60) and the condition (5) for the drift of the orbit centers to be semiclassical coincide with each other.

The other approximation made in (59) concerns the time t which was assumed comparatively small so that the variation of the field $\mathbf{E}_n$ could be ignored and the drift velocity could be assumed time independent. Since $\dot{E}_n\sim(\nabla_{n'}\mathbf{E}_n)\tilde{v}_{n'}^{(d)}$, the condition (60) justifies this approximation for $t\lesssim t_{coll}\sim\chi B/E_f$ [t_{coll} is given by Eq. (65) below].

Equations (58), (59) make it straightforward to perform the averaging $\langle\exp(i\mathbf{q}\hat{\mathbf{r}}_n(t))\exp(-i\mathbf{q}\hat{\mathbf{r}}_n(0))\rangle$. Since the operators $\hat{\mathbf{p}}_n$ and $\hat{\tilde{\mathbf{r}}}_n$ commute, the trace over the electron states with the weight $Z_e^{-1}\exp(-\beta H_0)$ factors into the trace over the wave functions $|\nu_n\rangle$ and the integral over the positions of the centers $\tilde{\mathbf{r}}_n$ of all electrons (strictly speaking, the trace is taken over $|\nu_n\rangle$ and over the eigenfunctions of the operators $\hat{\tilde{y}}_n$ (or $\hat{\tilde{x}}_n$), but the latter is reduced to the integral over $\{\tilde{\mathbf{r}}_n\}$ in the semiclassical range). With the account taken of the commutation relations (57) for $\hat{p}_{n\pm}$ we obtain

$$\langle\exp(i\mathbf{q}\hat{\mathbf{r}}_n(t))\exp(-i\mathbf{q}\hat{\mathbf{r}}_n(0))\rangle=\langle\exp[i\mathbf{q}\hat{\tilde{\mathbf{v}}}_n^{(d)}t]\rangle\times\left\langle\exp\left[\sum_\alpha \alpha l_B q_{-\alpha}\hat{p}_{n\alpha}(0)(e^{i\alpha\omega_c t}-1)\right]\right\rangle\times\exp\left[-i\frac{1}{2}l_B^2q^2\sin\omega_c t\right]. \quad (61)$$

A simple (and useful for what will be done in the analysis of cyclotron resonance) way of finding the trace over $|\nu_n\rangle$ is to replace in the second line of (61)

$$\exp[A_-\hat{p}_{n+}(0)-A_+\hat{p}_{n-}(0)]\Rightarrow\hat{M}_n(A_+A_-;0)e^{-(1/2)A_+A_-},$$

$$\hat{M}_n(A_+A_-;0)=\sum_{s=0}^{\infty}\frac{(-A_+A_-)^s}{(s!)^2}\hat{p}_{n+}^s(0)\hat{p}_{n-}^s(0), \quad (62)$$

where $A_\alpha=l_Bq_\alpha[\exp(-i\alpha\omega_c t)-1]$. To perform averaging the matrix elements

$$\langle\nu_n|\hat{M}_n(|A|^2;0)|\nu_n\rangle=\sum_{s=0}^{\nu_n}\frac{(-|A|^2)^s}{s!}\binom{\nu_n}{s} \quad (62a)$$

should be multiplied by $\exp(-\beta\omega_c\nu_n)$, and then the summation over ν_n should be done [it is convenient to sum over ν_n prior to taking the sum over s in (62a)]. Finally we arrive at the expression

$$\left\langle\exp\left[\sum_\alpha \alpha l_B q_{-\alpha}\hat{p}_{n\alpha}(0)[\exp(i\alpha\omega_c t)-1]\right]\right\rangle=\exp\left[-\frac{1}{2}l_B^2q^2(2\overline{n}+1)(1-\cos\omega_c t)\right] \quad (63)$$

[the Planck number $\overline{n}$ is defined in (51)].

The functions $\cos\omega_c t$, $\sin\omega_c t$ are fast oscillating (ω_c is the highest frequency in the problem for quantizing magnetic fields). Therefore when (61) is integrated over time (from $-\infty$ to ∞) to obtain $\xi(\mathbf{q})$ (37) one may expand the integrand in $\exp(\pm i\omega_c t)$ with account taken of (63) and retain the terms in which the exponents with the opposite signs cancel each other. Then the only term in (61) that remains t dependent is $\langle\exp[i\mathbf{q}\hat{\tilde{\mathbf{v}}}_n^{(d)}t]\rangle$, and we have

$$\int_{-\infty}^{\infty}dt\langle\exp[i\mathbf{q}\hat{\tilde{\mathbf{v}}}_n^{(d)}t]\rangle=2\pi\langle\delta(\mathbf{q}\hat{\tilde{\mathbf{v}}}_n^{(d)})\rangle=2q^{-1}B\langle E_f^{-1}\rangle. \quad (64)$$

Here, we took into account that the semiclassical averaging over the positions of the centers of electron wave packets $\hat{\tilde{\mathbf{r}}}_n$ comes to integrating over $\tilde{\mathbf{r}}_n$ with the weight $\propto\exp(-\beta H_{ee})$. Therefore $\langle E_f^{-1}\rangle$ can be evaluated using the classical distribution of the fluctuational field (43). We em-

phasize that it is only the drift of the centers of the wave packets that is classical: fast cyclotron motion of the electrons is quantized. We notice also that, although the distribution of $\mathbf{E}_f$ may be anisotropic for a Wigner monocrystal, the anisotropy dropped out of (64), since we consider scattering which is isotropic in $\mathbf{q}$, and we performed averaging over the directions of $\mathbf{q}$ in (64).

Equation (64) is the condition of energy conservation for elastic scattering: the scattered electron remains on the same Landau level, and the recoil is such that the cyclotron orbit center moves transverse to the fluctuational field $\mathbf{E}_f$.

The resulting expression for the correlator $\xi(\mathbf{q})$ is of the form

$$\xi(\mathbf{q})=2(l_Bq)^{-1}t_e\exp\left[-\frac{1}{2}l_B^2q^2(2\overline{n}+1)\right]\times\sum_{m=0}\left(\frac{1}{2}l_B^2q^2\right)^{2m}\frac{[\overline{n}(\overline{n}+1)]^m}{(m!)^2},\quad t_e=Bl_B\langle E_f^{-1}\rangle. \tag{65}$$

The quantity t_e here is the time during which an electron drifts, in the crossed fields $\mathbf{E}_f,\mathbf{B}$, over the quantum magnetic length l_B [cf. Eq. (10)]. Therefore t_e gives the characteristic duration of a collision $t_{\rm coll}$ for $T\ll\omega_c$. It follows from (51) that for higher T

$$t_{\rm coll}=(2\overline{n}+1)^{-1/2}t_e\equiv(2\overline{n}+1)^{-1/2}Bl_B\langle E_f^{-1}\rangle,\qquad \omega_ct_{\rm coll}\gg1. \tag{66}$$

Equations (54), (65) provide a simple explicit expression for the static many-electron conductivity $\sigma(0)$ in quantizing magnetic fields [the inequality (66) specifies the range of the fields where (65) applies]. For low temperatures, $\overline{n}\ll1$, the major contribution to $\xi(\mathbf{q})$ comes from the term in the sum (65) with $m=0$, and the expression for $\sigma(0)$ coincides with the result obtained earlier.[31] In the opposite limit of high T where $\overline{n}\gg1$, the sum in (65) can be replaced by an integral, and the latter can be evaluated by the steepest descent method. The result coincides with Eq. (49) obtained above by a completely different method.

B. Cyclotron resonance

1. General expression for the conductivity

For strong magnetic fields, $\omega_c\tau\gg1$, the resonant contribution to the many-electron conductivity $\sigma(\omega)$ (16) at the cyclotron resonance frequency $\omega=\omega_c$ comes from the term in the correlation function of the momentum $\langle\hat{P}_x^H(t)\hat{P}_x^H(0)\rangle$ which, in the absence of scattering, oscillates as $\exp(-i\omega_ct)$. Keeping this term only and expressing it in terms of the raising and lowering operators $p_{n\alpha}$ (57) we get

$$\sigma(\omega)\approx\frac{e^2(\overline{n}+1)^{-1}}{2mS}\mathrm{Re}\int_0^\infty dte^{i\omega t}\langle\hat{P}_-^H(t)\hat{P}_+^H(0)\rangle,\qquad (|\omega-\omega_c|\ll\omega_c),\ \ \hat{P}_\alpha\equiv\sum_n\hat{p}_{n\alpha}. \tag{67}$$

Similar to (17) we may write

$$\langle\hat{P}_-^H(t)\hat{P}_+^H(0)\rangle=\mathrm{Tr}_e[\hat{P}_-(0)\hat{G}_+(t)],\qquad \hat{G}_+(t)=e^{-i\hat{H}_0t}\hat{\mathcal{G}}_+(t)e^{i\hat{H}_0t},\ \ \hat{\mathcal{G}}_+(0)=Z_e^{-1}\hat{P}_+e^{-\beta\hat{H}_0}, \tag{68}$$

where the operator $\hat{\mathcal{G}}_+(t)$ satisfies the kinetic equation (20) with the initial conditions specified in (68). As in Sec. III B, in Eq. (68) we introduced the operator $\hat{G}_+(t)$ instead of $\hat{\mathcal{G}}_+(t)$, because the matrix elements of the operators in the collision integral for $\hat{G}_+(t)$ on the wave functions of the many-electron system are determined by the evolution of the system during the time $\sim t_{\rm coll}$. This evolution can be described explicitly in the range where the drift of the cyclotron orbit centers is semiclassical [in contrast, the collision integral for $\hat{\mathcal{G}}_+(t)$ is determined by the evolution of the electron variables during the time $\sim\tau$ which is not known].

2. The solution of the kinetic equation for fast interelectron momentum exchange

Electron-electron interaction affects the shape of the peak of cyclotron resonance (67) through its effect on the decay of the operator $\hat{G}_+(t)$. As explained in Sec. II D, this effect is twofold: (i) electron-electron interaction defines the mechanism of collisions with the scatterers in a strong magnetic field, and (ii) if the interaction is strong enough so that the rate of interelectron momentum exchange $\tau_{\rm ex}^{-1}$ exceeds the collision rate τ^{-1}, it defines the functional form of the operator $\hat{G}_+(t)$, and thus not only the broadening, but also the shape of the absorption spectrum (e.g., Lorentzian vs non-Lorentzian).

Resonant absorption at cyclotron frequency corresponds to the transitions between the Landau levels, $|\nu\rangle\to|\nu+1\rangle$. We note that broadening of the absorption line is due not to very occasional collision-induced transitions between the levels (''longitudinal relaxation,'' in spectroscopic terms). The actual mechanism is random modulation of the difference of phases of the wave functions of adjacent Landau levels, which is induced by electron collisions with scatterers in the fluctuational field. This modulation is also the modulation of the transition frequency. Modulational broadening of resonant absorption lines is well known in different contexts in solid state spectroscopy (cf. Ref. 42).

Although electron-electron interaction does not give rise to the spectrum broadening, it may cause transitions between the Landau levels of individual electrons. The transition probability is given by the rate at which the amplitude $\hat{p}_{n\alpha}\exp(-i\alpha\omega_ct)$ of the quantized electron momentum is changed. For an nth electron this rate can be estimated from the equation

$$d\hat{\mathbf{p}}_n/dt=e\mathbf{E}_n(\{\hat{\mathbf{r}}_{n'}\})+\frac{e}{m}\hat{\mathbf{p}}\times\mathbf{B}.$$

If one expands $\hat{\mathbf{r}}_{n'}$ in $l_B^2\hat{\mathbf{p}}_{n'}$ using (55), one finds that the rate of the momentum amplitude change is $\sim e(\nabla_n\mathbf{E}_n)l_B^2$ (for the mean occupation number of the Landau levels $\overline{n}\lesssim1$). Therefore it follows from (A2) that the interlevel transitions occur more frequently than collisions with scatterers provided

$$\tau_{\rm ex}^{-1}=(\omega_{p'}^2/\omega_c)\sim e^2\langle E_f^2\rangle l_B^2/T\gg\tau^{-1} \tag{69}$$

[we have used here the estimate of the fluctuational field (3) and the expression for the characteristic plasma frequency ω_p (4)]. The criterion (69) justifies the condition (13) and, as explained in the discussion of (13), is also sufficient for the fluctuational field that drives an electron to be randomized between successive collisions with scatterers.

We notice that (69) is not necessary for applicability of the quantum transport equation and of the decoupling used in the transition from (53) to (54), this decoupling requires a weaker inequality $t_{\rm coll} \ll \tau$ [$t_{\rm coll}$ is defined in (66)].

In the range (69) the dominating term in the equation of motion for the operator $\hat{G}_+(t)$,

$$\frac{\partial \hat{G}_+}{\partial t} = i[\hat{G}_+(t), \hat{H}_0] + \left[\frac{\partial \hat{G}_+}{\partial t}\right]_{\rm coll},$$

$$\left[\frac{\partial \hat{G}_+}{\partial t}\right]_{\rm coll} = -\sum_{\mathbf{q}} \overline{|V_{\mathbf{q}}|^2} \sum_n \int_0^t dt' e^{-iH_0 t} \times [\exp(i\mathbf{q}\hat{\mathbf{r}}_n(t)), [\exp(-i\mathbf{q}\hat{\mathbf{r}}_n(t')), \hat{\mathcal{G}}_+(t)]] e^{iH_0 t} \tag{21a}$$

is the first term which is linear in the electron-electron interaction $H_{\rm ee}$ [Eq. (21a) is similar to Eqs. (20), (21)]. Therefore the major term in the solution of (21a) is a function of the total electron momentum $\hat{P}_\pm(t) \equiv \exp[i\hat{H}_0 t]\hat{P}_\pm(0)\exp[-i\hat{H}_0 t]$ and of the energy $\hat{H}_0$ [cf. Eq. (33)]. In view of the initial condition (68) and taking into account that (i) the operator $\hat{G}_+(t)$ has a symmetry of the momentum operator, and (ii) we are considering elastic scattering, and therefore the total electron energy and distribution over the energy do not change, we will seek $\hat{G}_+(t)$ in the form

$$\hat{G}_+(t) = \tilde{g}_+(t) Z_e^{-1} e^{-i\omega_c t} \hat{P}_+(0) \exp[-\beta \hat{H}_0]. \tag{70}$$

The function $\tilde{g}_+(t)$ is slowly varying; it accounts for relaxation. Fast oscillating terms in $\tilde{g}_+$ are small and nonresonant, and they have been dropped. In fact, even in the neglect of fast oscillating terms $\tilde{g}_+$ still may be an arbitrary function of the operator $\hat{P}_+(0)\hat{P}_-(0) \equiv \hat{P}_+(t)\hat{P}_-(t)$; however, it follows from the form of the collision integral derived below [see Eq. (71)] that the terms $(\hat{P}_+(0)\hat{P}_-(0))^m$ in $\tilde{g}_+$ that have different m are decoupled from each other in the statistical limit of the large number of electrons. Therefore from the initial condition (68) it follows that $\tilde{g}_+(t)$ is a c number.

3. The collision term

Time evolution of the function $\tilde{g}_+(t)$ in (70) is determined by the collision term in (21a). To find it we notice that the duration of a collision is given by the time of flight $t_{\rm coll}$ (66) of an electron past a scatterer in the fluctuational field. Therefore, in the collision integral in (21a) the actual time difference $t - t' \sim t_{\rm coll}$. Although $t - t'$ largely exceeds ω_c^{-1}, it is still small compared to the time $\sim \omega_c/\omega_p^2$ over which the electric field driving an electron varies substantially. Consequently we may assume the field to be constant, and then write the operators in (21a) in the form similar to that used in Eq. (59):

$$\exp[i\mathbf{q}\hat{\mathbf{r}}_n(t)] = \exp\left[\sum_{\alpha=\pm} \alpha l_B q_{-\alpha} \hat{p}_{n\alpha}(t)\right] \exp[i\mathbf{q}\hat{\tilde{\mathbf{r}}}_n(t)],$$

$$\exp[-i\mathbf{q}\hat{\mathbf{r}}_n(t')] \approx \exp\left[-\sum_{\alpha=\pm} \alpha l_B q_{-\alpha} \hat{p}_{n\alpha}(t) e^{i\alpha\omega_c(t'-t)}\right] \times \exp[-i\mathbf{q}\hat{\tilde{\mathbf{v}}}_n^{(d)}(t)(t'-t)] \exp[-i\mathbf{q}\hat{\tilde{\mathbf{r}}}_n(t)].$$

The exponentials $\exp[\pm i\omega_c(t'-t)]$ in the term $\exp[-i\mathbf{q}\hat{\mathbf{r}}_n(t')]$ are fast oscillating for $t - t' \sim t_{\rm coll} \gg \omega_c^{-1}$. Therefore one should keep only diagonal terms in the expansion of $\exp[-i\mathbf{q}\hat{\mathbf{r}}_n(t')]$ in $\exp[\pm i\omega_c(t'-t)]$, which means that the corresponding operator may be expressed in terms of the operator $\hat{M}_n$ introduced in (62) [with $A_\alpha = l_B q_\alpha \exp(-i\alpha\omega_c(t'-t))$]. At the same time, in the derivation of the kinetic equation (20) it has been assumed that both the operator $\hat{\mathcal{G}}$ itself and the collision term as a whole are smooth functions of time [it is seen from (68), (70) that time evolution of $\hat{\mathcal{G}}_+(t)$ is given by that of $\tilde{g}_+(t)$, i.e., $\hat{\mathcal{G}}_+(t)$ is indeed smooth]. It follows from these arguments, with account taken of the fact that the operators $\hat{p}_{n\alpha}(t)$ oscillate approximately as $\exp(i\alpha\omega_c t)$, that the substitution (62) should be applied to the $\hat{p}_{n\pm}$-dependent terms not only in $\exp[-i\mathbf{q}\hat{\mathbf{r}}_n(t')]$, but also in $\exp[i\mathbf{q}\hat{\mathbf{r}}_n(t)]$ (in the latter case $A_\alpha = -l_B q_\alpha$).

With Eq. (62) taken into account we can rewrite the collision term in (21a) in the form

$$\left[\frac{\partial \hat{G}_+}{\partial t}\right]_{\rm coll} \approx -\pi \sum_{\mathbf{q}} \overline{|V_{\mathbf{q}}|^2} e^{-(1/2) l_B^2 q^2} \sum_n \delta(\mathbf{q}\hat{\tilde{\mathbf{v}}}_n^{(d)}(0)) \times \left[\hat{M}_n\left(\frac{1}{2} l_B^2 q^2; 0\right), \left[\hat{M}_n\left(\frac{1}{2} l_B^2 q^2; 0\right), \hat{G}_+(t)\right]\right]. \tag{71}$$

We notice that all operators here [including the ones in $\hat{G}_+(t)$, cf. (70)] are evaluated at the same time $t = 0$.

In deriving (71) we took into account that in the semiclassical domain (5), different components of $\hat{\tilde{\mathbf{r}}}_n, \hat{\tilde{\mathbf{v}}}_n^{(d)}$ commute not only with $\hat{p}_{n\pm}$ but also with each other, and hence with $\exp(-\beta H_0)$ and with $\hat{G}_+(t)$ as a whole. In the same semiclassical approximation the operator $\hat{H}_0$ in Eq. (70) for $\hat{G}_+(t)$ can be written in the form (58) of the sum of the terms that correspond to quantized cyclotron motion and to semiclassical drift of the centers of electron orbits $\hat{\tilde{\mathbf{r}}}_n$. Therefore the operators $\hat{M}_n$ in (71) commute with $\exp(-\beta H_0)$ in $\hat{G}_+(t)$, and the only term in $\hat{G}_+(t)$ they do not commute with is $\hat{P}_+(0)$.

It follows from the above arguments that the right-hand side of Eq. (71) is a sum of collision probabilities for individual electrons,

$$\sum_n \delta(\mathbf{q}\hat{\tilde{\mathbf{v}}}_n^{(d)})[\hat{M}_n,[\hat{M}_n,\hat{p}_{n+}]]\exp(-\beta H_0)$$

(this expression should be further summed over the momentum transfer $\mathbf{q}$). The above form is not the same as the form of the expression (70) for $\hat{G}_+(t)$, which is a sum of $\hat{p}_{n+}$ multiplied by an operator independent of n. This is a consequence of the collision probability being dependent on the state of the electron, as explained in Sec. II D. In the range (69), where the interlevel transitions due to electron-electron interaction are comparatively frequent, the relaxation rate is determined by the collision probabilities averaged over the electron states, and therefore the above expression has to be appropriately averaged. Formally, the averaging can be done and the equation for the function $\tilde{g}_+(t)$ in (70) can be obtained by substituting (70), (71) into the kinetic equation (21a), multiplying by $\hat{P}_-(0)$ from the left and taking trace over the states of the many-electron system. With account taken of the expression

$$\mathrm{Tr}_e[\hat{P}_-(0)\hat{G}_+(t)]\approx n_s S(\overline{n}+1)\tilde{g}_+(t)e^{-i\omega_c t},$$

one arrives at the following equation for $\tilde{g}_+(t)$:

$$\frac{\partial\tilde{g}_+(t)}{\partial t}=-\gamma\tilde{g}_+(t),$$

$$\gamma=\frac{1}{4}t_e\sum_{\mathbf{q}}(l_B q)^3\overline{|V_{\mathbf{q}}|^2}\exp\left[-\frac{1}{2}l_B^2q^2(2\overline{n}+1)\right] \quad (72)$$

$$\times\sum_{m=0}\left(\frac{1}{2}l_B^2q^2\right)^{2m}\frac{[\overline{n}(\overline{n}+1)]^m}{m!(m+1)!}.$$

Both $\hat{P}_-(0)$ and the collision term (71) are sums over the electrons; the contribution to (72) comes from the diagonal terms in the trace of their product (the terms that refer to the same electrons). Averaging over the occupation numbers of the Landau levels ν_n of the term in the double commutator in (71) with a given n multiplied by $\hat{p}_{n-}(0)$ may be performed using Eq. (62) (in fact, it can be simplified using some operator identities, but the details go beyond the scope of this paper).

It follows from Eqs. (67), (68), (72) that in the range (69) the conductivity $\sigma(\omega)$ near the cyclotron frequency has a Lorentzian peak:

$$\sigma(\omega)\approx\frac{e^2n_s}{2m}\frac{\gamma}{(\omega-\omega_c)^2+\gamma^2}. \quad (73)$$

The halfwidth of the peak γ is determined by the rate of the collisions in the many-electron fluctuational field. For $\omega_c t_{\mathrm{coll}}\gg 1$ it is proportional to the time of flight t_e past a short-range scatterer (65).

We note that Eq. (72) for the relaxation rate γ can be written in the form in which γ is expressed in terms of the frequency-dependent electron structure factor $\xi(\mathbf{q},\omega)$,

$$\gamma\equiv\gamma(\omega_c),\quad \gamma(\omega)=\frac{1-\exp(-\beta\omega)}{4m\hbar\omega}\sum_{\mathbf{q}}q^2\overline{|V_{\mathbf{q}}|^2}\xi(\mathbf{q},\omega),$$

$$\xi(\mathbf{q},\omega)=\int_{-\infty}^{\infty}dt\ e^{i\omega t}\langle\rho_{\mathbf{q}}(t)\rho_{-\mathbf{q}}(0)\rangle, \quad (74)$$

$$\rho_{\mathbf{q}}=(n_sS)^{-1/2}\sum_n\exp(i\mathbf{q}\mathbf{r}_n).$$

This form is familiar from the memory function theory.[43] We emphasize that, in evaluating the structure factor, we allow explicitly for strong electron correlations. Also, in contrast to the memory function theory, our technique makes it possible to investigate the limits of large and small ratio τ_{ex}/τ.

In the ultraquantum limit $\omega_c\gg T$ Eqs. (72), (73) go over into the result of Ref. 31. In the opposite limit, $\omega_c\ll T$ [but $\omega_c\gg(2\pi e\langle E_f^2\rangle^{1/2}\chi_T T)^{1/2}$], Eqs. (72), (73) go over into the result of the strong-field classical theory given by Eqs. (38), (40), (49) [however, quantum corrections to (49) may be sometimes substantial even for $\omega_c\ll T$; see below]. We note that, as expected, in quantizing magnetic fields the frequency dispersion of the structure factor is substantial, and the relaxation rate τ^{-1} that determines the static conductivity and is expressed in terms of $\xi(\mathbf{q},\omega\to 0)$ [cf. (54), (65)], and the halfwidth of the cyclotron resonance peak γ (72), (74) do not coincide with each other.

4. Cyclotron resonance for slow interelectron momentum exchange

The shape of the cyclotron resonance peak differs from Lorentzian in the case where the duration of the collision t_{coll} is small compared to the reciprocal scattering rate τ, but the latter is small compared to the time τ_{ex} over which the correlations in the electron system decay,

$$\tau_{\mathrm{ex}}^{-1}=\omega_p^2/\omega_c\ll\tau^{-1}\ll t_{\mathrm{coll}}^{-1}. \quad (75)$$

In this case the electron momentum is randomized because of collisions with scatterers faster than it is exchanged with other electrons (cf. the discussion in Sec. II D).

Since the collisions are short compared to the intervals between successive collisions, the many-electron kinetic equation (21a) still applies, but now the term $i[\hat{G}_+,\hat{H}_{\mathrm{ee}}]$ in (21a) is small compared to the collision term, and to zeroth order in $\omega_p^2\tau/\omega_c$ it can be neglected. A solution of the kinetic equation can be obtained in the extreme quantum limit where electrons occupy only the lowest Landau level, $\overline{n}\ll 1$. One can seek it in a quasi-single-electron form (''quasi'' here means that we take into account that collisions with scatterers are strongly affected by the electron-electron interaction),

$$\hat{G}_+(t)=e^{-i\omega_c t}\sum_n\tilde{g}_{n+}(t)p_{n+}Z_e^{-1}\exp(-\beta\hat{H}_0).$$

One then finds from (71) that the functions $\tilde{g}_{n+}(t)$ exponentially decay in time, but the decrements for different electrons (different n) are determined by the ''instantaneous'' values of the reciprocal fluctuational field E_n^{-1}: this field is randomized over the time ω_c/ω_p^2 which exceeds the time over which the functions $\tilde{g}_{n+}(t)$ decay. The averaging over

the field has to be done in the final expression for the conductivity, and therefore we get

$$\sigma(\omega)\approx\frac{e^2n_s}{2m}\left\langle\frac{\gamma_0(\mathbf{E}_f)}{(\omega-\omega_c)^2+\gamma_0^2(\mathbf{E}_f)}\right\rangle,\quad \overline{n}\ll 1,$$

$$\gamma_0(\mathbf{E}_f)=\frac{1}{4}l_BBE_f^{-1}\sum_{\mathbf{q}}(l_Bq)^3\overline{|V_\mathbf{q}|^2}\exp(-l_B^2q^2/2). \tag{76}$$

The averaging over the field $\mathbf{E}_f$ is performed with the distribution (43).

It is seen from (76) that in its maximum ($\omega=\omega_c$) the conductivity is given by the average *reciprocal* scattering rate. This is similar to the static conductivity $\sigma(0)$ in the single-electron approximation for $B=0$. On the tails of the cyclotron resonance peak, $|\omega-\omega_c|\gg\gamma$, the expression (76) goes over into Eq. (73). This is again similar to what happens for low-frequency single-electron conductivity in the absence of the magnetic field: for $\omega\tau\gg 1$ it is given by the average single-electron scattering rate (not the average reciprocal rate).

VII. CONDUCTIVITY AND CYCLOTRON RESONANCE FOR SPECIFIC SCATTERING MECHANISMS

The expressions for the static conductivity σ and for the parameters of the cyclotron resonance spectrum are simplified for specific scattering mechanisms. In the analysis we will assume that the distribution of the fluctuational field $\mathbf{E}_f$ is Gaussian,

$$\rho(\mathbf{E}_f)=(\pi\langle E_f^2\rangle)^{-1}\exp(-E_f^2/\langle E_f^2\rangle). \tag{77}$$

Equation (77) has been shown[33(b)] to describe the substantial central part of the distribution $\rho(\mathbf{E}_f)$ in the broad range $20\lesssim\Gamma\lesssim 200$. In particular, to an accuracy better than 10% the mean reciprocal field,

$$\langle E_f^{-1}\rangle=\pi^{1/2}\langle E_f^2\rangle^{-1/2}.$$

Equation (77) makes it possible to characterize the effect of the electron-electron interaction on the conductivity and cyclotron resonance by one parameter, the mean square fluctuational field $\langle E_f^2\rangle$. The dependence of $\langle E_f^2\rangle$ on electron density and temperature is given by Eq. (3) with F being nearly a constant.[33(b)]

One of the results that immediately follows from (77) is the explicit shape of the cyclotron resonance peak (76) in the range of quantizing magnetic fields for the case where the relaxation rate exceeds the interelectron momentum exchange rate, $\tau_{\rm ex}^{-1}\ll\tau^{-1}\ll t_{\rm coll}^{-1}$. It is seen from Fig. 5 that this shape is noticeably different from Lorentzian.

In the range of fast interelectron momentum exchange, which is of central interest for the present paper, the dependence of the conductivity and cyclotron resonance on the magnetic field is characterized by two parameters, B/B_0 and B/B_T where

$$B_0=\left(\frac{2\pi m^3T}{\hbar^2e^2\langle E_f^{-1}\rangle^2}\right)^{1/4},\quad B_T=\frac{mT}{\hbar e}. \tag{78}$$

The parameter B_0 gives the magnetic field for which there arises magnetoresistance [the encountering factor ζ (9) discussed in Sec. III B is $\sim B^2/B_0^2$ in classical magnetic fields]. The field B_T ''separates'' the regions of quantizing and nonquantizing magnetic fields for a given temperature ($\hbar\omega_c=T$ for $B=B_T$).

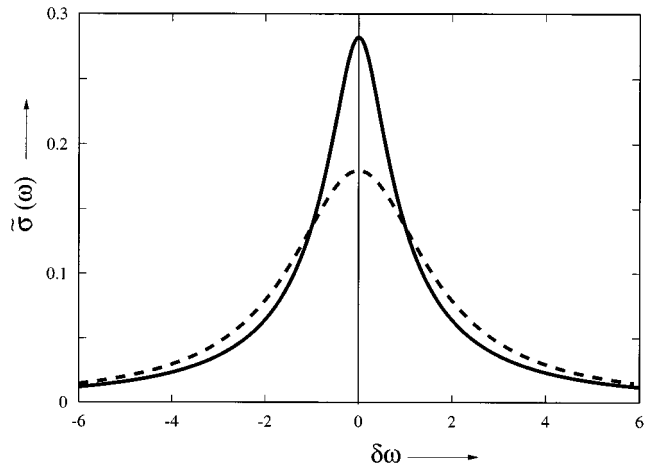

FIG. 5. Reduced high-frequency conductivity $\tilde{\sigma}(\omega)=2m\gamma_0\sigma(\omega)/\pi e^2n_s$ [$\gamma_0\equiv\gamma_0(\langle E_f^2\rangle^{1/2})$] near the cyclotron resonance peak as a function of the reduced frequency $\delta\omega=(\omega-\omega_c)/\gamma_0$ for slow interelectron momentum exchange (76) (solid line). Lorentzian distribution with the same area and with the halfwidth $\pi^{1/2}\gamma_0$ is shown with a dashed line.

The ratio $B_0/B_T=(2e\langle E_f^2\rangle^{1/2}\varkappa_T/T)^{1/2}$, and therefore if the electron motion is classical in the absence of magnetic field (see Sec. II A), then $B_0\ll B_T$. In this case the decrease of the static many-electron conductivity with increasing B, which is described by the Drude law (39) for $B\ll B_0$, saturates for $B_0\ll B\ll B_T$. The conductivity becomes nearly independent from magnetic field, and according to (39), (49) is given by the expression

$$\sigma_{\rm sat}=\frac{mn_s}{B_0^2}\frac{1}{2\hbar T}\sum_{\mathbf{q}}\overline{|V_\mathbf{q}|^2}\exp\left(\frac{-\hbar^2q^2}{8mT}\right),\qquad B_0\ll B\ll B_T. \tag{79}$$

The conductivity (79) is determined by the fluctuational electric field. Its dependence on the electron density is strongly sublinear and, in fact, very weak, $\sigma_{\rm sat}\propto n_s^{1/4}$, according to (3). The value (79) provides the scaling factor for the conductivity in strong magnetic fields.

In the range $B\ll B_T$ the expression for the scaled static conductivity $\tilde{\sigma}$ for Gaussian distribution of the field $\mathbf{E}_f$ is of the form

$$\tilde{\sigma}=\frac{\sigma}{\sigma_{\rm sat}},$$

$$\tilde{\sigma}=\pi^{1/2}\frac{B_0^2}{B^2}\varkappa_T\sum_{\mathbf{q}}q\overline{|V_\mathbf{q}|^2}\sum_{s=-\infty}^{\infty}\exp\left[\frac{-\hbar^2q^2}{8mT}\left(1+4\pi^2s^2\frac{B_0^4}{B^4}\right)\right]\times\left[\sum_{\mathbf{q}}\overline{|V_\mathbf{q}|^2}\exp\left(\frac{-\hbar^2q^2}{8mT}\right)\right]^{-1},\quad B\ll B_T \tag{80}$$

(clearly, $\tilde{\sigma}\to 1$ for $B/B_0\to\infty$). The explicit expression for the reduced conductivity $\tilde{\sigma}$ for quantizing fields, $B\gtrsim B_T$, follows from (54), (65), (79). We note that the fluctuational field drops out of $\tilde{\sigma}$ in quantizing fields.

A. Delta-correlated random potential

The analysis of the conductivity becomes particularly simple in the important case of a δ-correlated random potential,

$$\overline{|V_{\mathbf{q}}|^2}=\frac{\hbar^3}{m}S^{-1}\tau_{B=0}^{-1}. \tag{81}$$

Here, $\tau_{B=0}^{-1}$ is the scattering rate τ^{-1} (38) for $B=0$.

The value of σ at saturation (79) for δ-correlated potential is of the form

$$\sigma_{\rm sat}=(mn_s/\pi B_0^2)\tau_{\rm B=0}^{-1}. \tag{82}$$

In classically strong fields we obtain from (80)

$$\tilde{\sigma}=\pi\frac{B_0^2}{B^2}\sum_{s=-\infty}^{\infty}\left(1+4\pi^2 s^2\frac{B_0^4}{B^4}\right)^{-3/2},\quad B\ll B_T, \tag{83}$$

whereas in the range

$$\frac{B_0^4}{B^3B_T}\left(\overline{n}+\frac{1}{2}\right)\ll 1,\quad \overline{n}\equiv[\exp(B/B_T)-1]^{-1} \tag{84}$$

(which includes the range of quantizing fields) we obtain from (54), (65)

$$\tilde{\sigma}=\left[\frac{\pi B}{4B_T(2\overline{n}+1)^3}\right]^{1/2}\sum_{m=0}^{\infty}\frac{\left(2m+\frac{1}{2}\right)!}{(m!)^2}\left[\frac{\overline{n}(\overline{n}+1)}{(2\overline{n}+1)^2}\right]^m \tag{85}$$

[$\tilde{\sigma}$ as given by (85) approaches 1 for $\overline{n}\gg 1$; we note that the condition (84) coincides with the condition $\omega_c t_{\rm coll}\gg 1$ (66)].

It is seen from (83), (85) that the conductivity has a minimum as a function of B in the range $B\sim B_0$: it decays as B^{-2} for $B\ll B_0$, and for $B\gg B_T$ it increases as $B^{1/2}$. The dependence of the reduced conductivity on B is shown in Fig. 6.

The halfwidth of the cyclotron resonance absorption peak γ in the range of classically strong fields is seen from (40) to be given by $m\omega_c^2\sigma/e^2n_s$. It is independent of electron density and of B for $B\ll B_0$. As B approaches B_0 the halfwidth starts increasing with B; for $B_T\gg B\gg B_0$ we have $\gamma\propto B^2$. In the range (84) γ is given by Eq. (72). It can be easily evaluated for the δ-correlated potential (81):

$$\gamma=\tau_{B=0}^{-1}\left[\frac{B^3B_T}{4\pi B_0^4(2\overline{n}+1)^5}\right]^{1/2}\times\sum_{m=0}^{\infty}\frac{(2m+\frac{3}{2})!}{m!(m+1)!}\left[\frac{\overline{n}(\overline{n}+1)}{(2\overline{n}+1)^2}\right]^m. \tag{86}$$

It is seen from Eq. (86) that the halfwidth of the cyclotron resonance peak increases with the magnetic field as $B^{3/2}$ for $B\gg B_T$ (i.e., slower than in the range $B_0\gg B\gg B_T$). The parameter γ explicitly depends on electron density for $B\gtrsim B_0$.

B. Electrons on helium surface

For electrons on helium surface with a density $n_s=10^8$ cm^{-2} and $T=1$ K the fluctuational field is $\langle E_f^2\rangle^{1/2}=F^{1/2}(\Gamma)n_s^{3/4}T^{1/2}\approx 11$ V/cm, and the characteristic magnetic field for the onset of magnetoresistance is $B_0\approx(2m^3F/\hbar^2e^2)^{1/4}n_s^{3/8}T^{1/2}\approx 0.54$ T. Therefore the specific features of many-electron transport are accessible to experimental investigation.

Electrons on helium are scattered by helium vapor atoms and by capillary waves, ripplons. The advantageous feature of the system is that the interaction with the scatterers is known and can be easily controlled: the saturated vapor density can be changed by orders of magnitude in a comparatively narrow range around 1 K just by changing temperature, whereas the coupling to ripplons can be changed by varying the field $E_\perp$ that presses electrons against the helium surface.

Since in the limit of weak coupling to the scatterers the scattering events are independent from each other, the relaxation rates for scattering by vapor atoms and by ripplons add up, and they will be analyzed separately. In the analysis we will imply the standard variational form of the wave function of the electron motion transverse to the surface (in the z direction)

$$\psi(z)=2\gamma_\perp^{3/2}z\exp(-\gamma_\perp z). \tag{87}$$

1. Vapor scattering

The cross section of a helium atom $b_{\rm He}^2\approx 5$ Å^2, and so helium vapor atoms create a nearly ideal δ-correlated potential. Therefore the magnetoconductivity and cyclotron resonance in the case of vapor scattering are described by the expressions (81)–(86). The explicit form of the squared matrix elements of the coupling to the vapor atoms is[44]

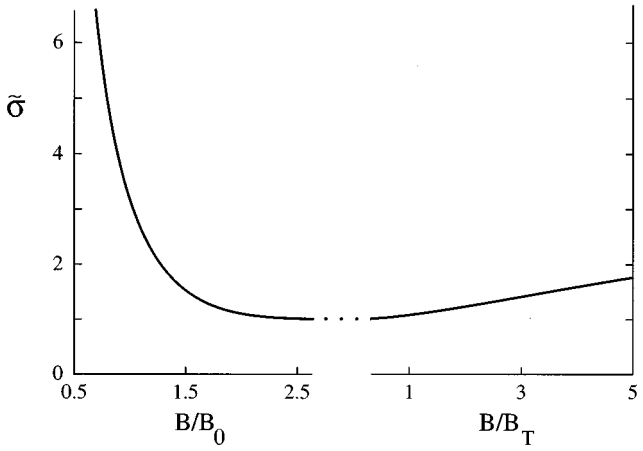

FIG. 6. Reduced conductivity $\tilde{\sigma}=\sigma/\sigma_{\rm sat}$ as a function of B/B_0 for classical fields and of $B/B_T\equiv\hbar\omega_c/T$ for $B^3\gg(B_0^4/B_T)(\overline{n}+1/2)$, for a δ-correlated random potential and for Gaussian distribution of the fluctuational field.

$$\overline{|V_{\mathbf{q}}|_v^2}=\frac{3\pi\hbar^4}{8m^2}\gamma_\perp b_{\mathrm{He}}^2 N_v S^{-1}, \tag{88}$$

where N_v is the (3D) vapor density. The corresponding value of the scattering rate for $B=0$ to be used in (82)–(86) is $\tau_{B=0}^{-1}=(3\pi\hbar/8m)\gamma_\perp b_{\mathrm{He}}^2 N_v$.

2. Ripplon scattering

The effective intensity of the random field of short-wavelength ripplons is of the form[44]

$$\overline{|V_{\mathbf{q}}|_r^2}=S^{-1}\frac{Te^2}{\alpha q^2}[E_\perp^2+2E_\perp E_{\mathrm{pol}}+E_{\mathrm{pol}}^2], \tag{89}$$

$$E_{\mathrm{pol}}\equiv E_{\mathrm{pol}}(q)=\frac{\hbar^2\gamma_\perp^{(0)}}{2me}q^2\varphi\left(\frac{q}{2\gamma_\perp}\right),$$

where α is the surface tension, $\gamma_\perp^{(0)}=(me^2/4\hbar^2)(\varepsilon-1)/(\varepsilon+1)$ is the value of the variational parameter $\gamma_\perp$ for $E_\perp=0$, ε is the dielectric constant of helium, and

$$\varphi(x)=(x^2-1)^{-1}+(1-x^2)^{-3/2}\ln\left[\frac{1+(1-x^2)^{1/2}}{x}\right],\quad x<1, \tag{90a}$$

$$\varphi(x)=(x^2-1)^{-1}-(x^2-1)^{-3/2}\tan^{-1}[(x^2-1)^{1/2}],\quad x>1. \tag{90b}$$

The first term in (89) describes coupling to ripplons due to the field $E_\perp$ that presses electrons against the surface, whereas the terms with E_{pol} are related to the change in the energy of the electron-induced polarization of helium due to ripplons. The terms with E_{pol} correspond to a random quasistatic Gaussian potential with a small correlation length. The term $\propto E_\perp^2$ diverges for small q (it has a cutoff at the reciprocal capillary length), i.e., the corresponding term describes a long-range potential. This divergence is seen from Eqs. (38) and (45), and (54) and (65) to be ''dangerous'' only in the range $B\sim B_0$: in both quantizing fields $B\gtrsim B_T$ and comparatively weak fields $B\ll B_0$ the sums over $\mathbf{q}$ contain weighting factors that fall down fast with the decreasing q.

It is convenient to evaluate separately the contributions to the magnetoconductivity of each of the three terms in (89). Respectively, we write the magnetoconductivity in the form

$$\sigma\equiv\sigma(0)=\sigma_{E_\perp}+\sigma_{\mathrm{cr}}+\sigma_{\mathrm{pol}}. \tag{91}$$

The dependence of the term $\sigma_{\mathcal{E}_\perp}\propto E_\perp^2$ on B in the range of strong fields (84) can be obtained from Eqs. (54), (65),

$$\sigma_{E_\perp}=\widetilde{\sigma}_{E_\perp}\sigma_{E_\perp}^{(0)},\quad \sigma_{E_\perp}^{(0)}=\frac{e^2E_\perp^2}{4\alpha\hbar}\frac{mn_s}{\pi B_0^2},$$

$$\widetilde{\sigma}_{E_\perp}=\left[\frac{\pi B_T}{B(2\overline{n}+1)}\right]^{1/2}\sum_{m=0}^{\infty}\left[\frac{\overline{n}(\overline{n}+1)}{(2\overline{n}+1)^2}\right]^m\frac{(2m-\frac{1}{2})!}{(m!)^2}. \tag{92}$$

It follows from (92) that $\sigma_{\mathcal{E}_\perp}$ does not display saturation with the increasing B. Its dependence on B is monotonous. In particular, $\widetilde{\sigma}_{E_\perp}\propto B^{-1/2}$ for $B\gg B_T$. In the opposite limit of large $\overline{n}$ (i.e., for $B\ll B_T$) the sum over m in (92) is close to $(2/\pi)^{1/2}\ln[B_T/B]$, and then $\sigma_{E_\perp}$ decreases with the increasing B as $\ln[B_T/B]$ [the expression for $\widetilde{\sigma}_{E_\perp}$ diverges logarithmically for $\hbar\to 0$; we note that the sum over $\mathbf{q}$ in the classical expression for σ_{sat} (79) logarithmically diverges at small q for $\overline{|V_{\mathbf{q}}|^2}$ being of the form of the term $\propto E_\perp^2$ in (89)].

To analyze $\widetilde{\sigma}_{E_\perp}$ in the intermediate range $B\sim B_0$ it is convenient to write the correlator $\xi(\mathbf{q})$ in the expression (54) for the conductivity in the form

$$\xi(\mathbf{q})=\int_{-\infty}^{\infty}dt\exp\left[-\frac{1}{2}l_B^2q^2W(\omega_c t)\right], \tag{93}$$

$$W(x)=(\overline{n}+1)(1-e^{-ix})+\overline{n}(1-e^{ix})+\frac{1}{4}\frac{B_0^4}{B^3B_T}x^2.$$

In deriving (93) we used Eqs. (37), (59), (61), (63); averaging over fluctuational field was done using (77). Equations (54), (93) make it possible to write the reduced conductivity $\widetilde{\sigma}_{E_\perp}$ in the form

$$\widetilde{\sigma}_{E_\perp}=\pi\frac{B_0^2}{B^2}\,\mathrm{Im}\sum_{s=-\infty}^{\infty}[W'(x_s)]^{-1},$$

where x_s are the roots of the equation

$$W(x_s)=0,\quad \mathrm{Im}x_s<0.$$

Numerical results for $\widetilde{\sigma}_{E_\perp}$ are shown in Fig. 7.

The analysis of the second and third terms in the expression for the ripplon conductivity (91), σ_{cr} and σ_{pol}, is similar to the analysis of the conductivity for a δ-correlated random potential. The dependence of the conductivities σ_{cr}, σ_{pol} on B is nonmonotonous, they reach minima in the range $B_0\ll B\ll B_T$ and then increase with the further increase of B. Their behavior for $B\ll B_T$ is described by Eqs. (79), (80), (89). In particular, the values of $\sigma_{\mathrm{cr}},\sigma_{\mathrm{pol}}$ at saturation are given by

$$\sigma_{\mathrm{cr,\,sat}}=\frac{eE_\perp\gamma_\perp^{(0)}T}{\hbar\alpha}\frac{mn_s}{\pi B_0^2}\varphi_1\left(\frac{2mT}{\hbar^2\gamma_\perp^2}\right),$$

$$\sigma_{\mathrm{pol,\,sat}}=\frac{2(\gamma_\perp^{(0)})^2T^2}{\hbar\alpha}\frac{mn_s}{\pi B_0^2}\varphi_2\left(\frac{2mT}{\hbar^2\gamma_\perp^2}\right), \tag{94}$$

where

$$\varphi_j(z)=\int_0^\infty dx\,x^{j-1}e^{-x}\varphi^j[(zx)^{1/2}]\quad (j=1,2). \tag{95}$$

The functions $\varphi_{1,2}$ depend on temperature and, through the variational parameter $\gamma_\perp$, on the pressing field $E_\perp$. In the actual case where the thermal wavelength λ_T is large compared to the localization length in the direction transverse to the helium surface we obtain from (89), (95)

$$\varphi_j(2mT/\hbar^2\gamma_\perp^2)\approx\left[\frac{1}{2}\ln(\hbar^2\gamma_\perp^2/mT)\right]^j\quad (j=1,2).$$

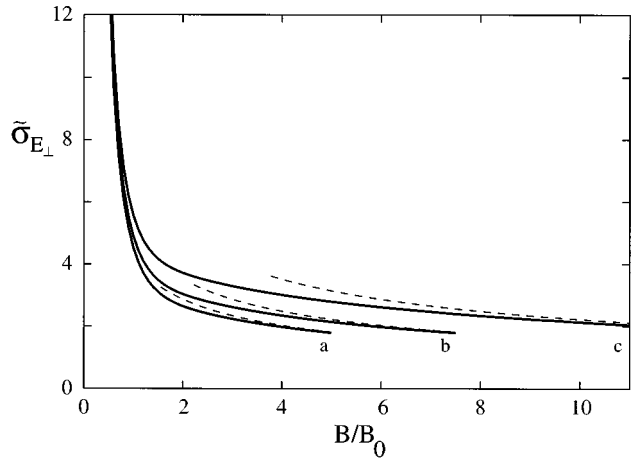

FIG. 7. Reduced conductivity $\tilde{\sigma}_{E_\perp}$ (92) as a function of B/B_0 for the contribution to the rate of ripplon scattering which is quadratic in the pressing field $E_\perp$. The curves a to c correspond to $B_0/B_T=0.6,0.4,0.2$. The dashed lines show the large B/B_T asymptotes of $\tilde{G}_{E_\perp}$.

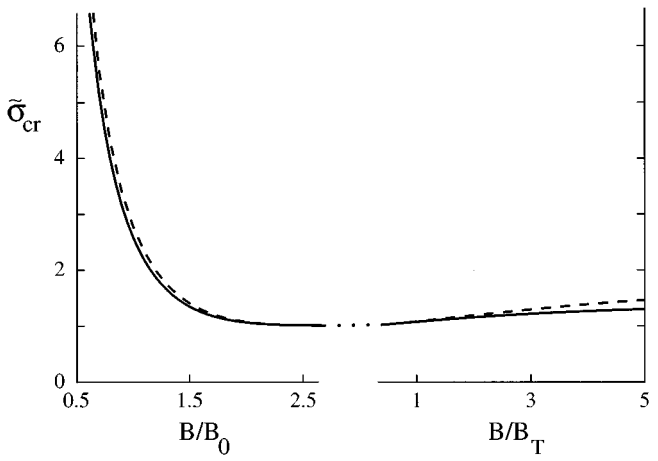

FIG. 8. Reduced conductivity $\tilde{\sigma}_{cr}=\sigma_{cr}/\sigma_{cr,\,sat}$, which is determined by the second (cross) term in the ripplon field intensity (89), as a function of B/B_0 for classical fields and of $B/B_T\equiv\hbar\omega_c/T$ for $B^3\gg(B_0^4/B_T)(\overline{n+\frac{1}{2}})$. The solid and dashed lines refer to $2mT/\hbar^2\gamma_\perp^2=0.1$ and 0.01, respectively.

In the range $B\gtrsim B_T$ the conductivities σ_{cr}, σ_{pol} increase with the magnetic field. In the limit $B\gg B_T$ we have $\sigma_{cr}\propto B^{1/2}$, $\sigma_{pol}\propto B^{3/2}$ to an accuracy of a factor that smoothly depends on B (approximately as a logarithm of $\gamma_\perp l_B$). The explicit expressions for the relaxation rate τ^{-1} in the case $\gamma_\perp l_B\gg 1$ are given in Ref. 45. The reduced conductivities σ_{cr}, σ_{pol} as functions of B obtained from (54), (65), (80), (89) are shown in Figs. 8 and 9.

The overall conductivity σ (91) due to the ripplon scattering has a minimum as a function of magnetic field. The position of the minimum depends on temperature, electron density (in terms of the mean square fluctuational field $\langle E_f^2\rangle$), and the pressing field $E_\perp$. The occurrence of the minimum is a many-electron effect. Figures 6–9 refer to the case where the saturation of σ^{-1} with the increasing B occurs in classical magnetic fields, i.e., for $B_0\ll B_T$. However, the theory applies for an arbitrary ratio between the field B_0 (78) and $B_T\equiv mT/\hbar e$ provided there holds the inequality

$$\left(\frac{B_0}{B_T}\right)^2\left[\frac{B_T}{B(2\overline{n}+1)}\right]^{1/2}\ll 1$$

which is equivalent to (2). Detailed comparison of the theory and the experiment for electrons on helium will be given in Ref. 45.

VIII. CONCLUSIONS

We have considered electron transport in classical and semiclassical strongly correlated electron systems for short-range electron scattering. The effects of electron-electron interaction on electron transport have been described in a nonperturbative way. The basic physical idea is that although relaxation of the total electron momentum occurs via individual electron collisions with scatterers, during the collision the electron is driven by a fluctuational field from other electrons, and this field may dramatically change the character of the collisions, particularly in a strong magnetic field.

We investigate the case in which the collisions are short, so that the fluctuational field does not vary in time during a collision and is uniform over the electron wavelength. Both these two conditions are met in the classical domain, $T\gg e\langle E_f^2\rangle^{1/2}\lambda_T$, $\hbar\omega_c$, and the specific for many-electron systems semiclassical domain, $\hbar\omega_c\gtrsim T\gg e\langle E_f^2\rangle^{1/2}l_B(2\overline{n}+1)^{-1/2}$, where the electron motion is a superposition of a quantized cyclotron motion and a nearly classical drift of the centers of cyclotron orbits.

The analysis is based on the many-electron quantum transport equation. We derive this equation and develop techniques for solving it, for classical and semiclassical domains. The solutions are obtained in the limiting cases of large and small ratios between the rate τ_{ex}^{-1} of interelectron momentum exchange due to electron-electron interaction and the relaxation rate τ^{-1} due to collisions with the scatterers. For $\tau_{ex}\ll\tau$ the general expression for the conductivity coin-

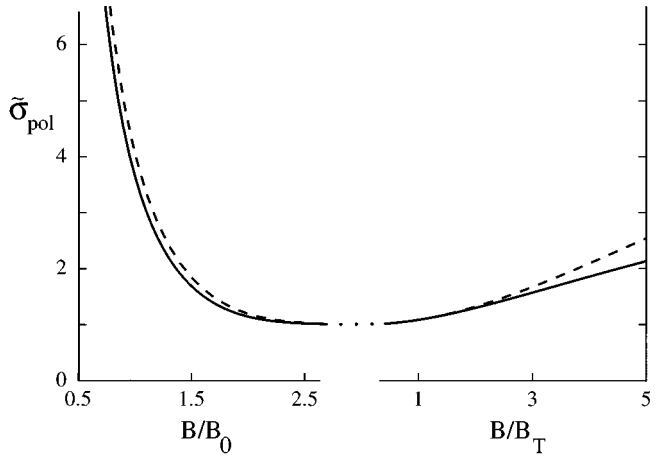

FIG. 9. Reduced conductivity $\tilde{\sigma}_{pol}=\sigma_{pol}/\sigma_{pol,\,sat}$, which is determined by the third (polarization) term in the ripplon field intensity (89), as a function of B/B_0 for classical fields and of $B/B_T\equiv\hbar\omega_c/T$ for $B^3\gg(B_0^4/B_T)(\overline{n+\frac{1}{2}})$. The solid and dashed lines refer to $2mT/\hbar^2\gamma_\perp^2=0.1$ and 0.01, respectively.

cides with the expression which can be formally written in terms of the memory function [cf. Eq. (74)]. We emphasize that, in evaluating the electron structure factor, we do not use the random phase approximation: the major effects come from strong electron correlations. It follows from the results that, in the classical and semiclassical domains, polaronic effects of the mass renormalization due to short-wavelength scattering are small.

If $\tau_{ex} \ll \tau$ then, *because of the many-electron effects*, up to moderately strong B (including classically strong fields, $\omega_c \tau \gg 1$, but $\hbar \omega_c < e\langle E_f^2 \rangle^{1/2} \chi_T$), the frequency-dependent conductivity $\sigma(\omega)$ is described by the single-electron Drude formula.

For higher magnetic fields the scattering rate starts to increase with B, and in quantizing fields the conductivity as a whole increases with B, i.e., σ is a nomonotonous function of B. The dependence of the conductivity on the magnetic field has a simple form for a δ-correlated random potential (cf. Fig. 6). A detailed comparison of the results on the static conductivity with the experiment is given in Ref. 45.

If coupling to the scatterers is comparatively strong, so that for $B=0$ there holds the inequality $\tau_{ex}^{-1} \lesssim \tau^{-1}$, the static conductivity of the classical strongly correlated many-electron system is still given by the single-electron theory provided $T \gg \hbar \tau^{-1}$. The many-electron effects may come into play in a certain range of classically strong magnetic fields, $\tau^{-1} \ll \omega_c \ll T/\hbar$, but the range of classically strong B where there is no magnetoresistance disappears. With further increase of B the relaxation rate τ^{-1} increases quickly, and the duration of a collision t_{coll} as determined by the time of flight past a defect becomes larger than τ, which means that the idea of successive collisions no longer applies. The crossover to effectively strong coupling occurs in strong enough quantizing fields even if $\tau_{ex}^{-1} \gg \tau^{-1}$ for $B=0$, since, for $\hbar \omega_c > T$ and for short-range scattering, $t_{coll} \propto B^{1/2}$ and $\tau^{-1} \propto B^{3/2}$. These arguments explain why in some cases the single-electron theory which ignores correlations in the nondegenerate electron system is in reasonable agreement with experiment,[11,12] whereas in other cases many-electron effects are dominating.[7,8,13,14,17,45]

ACKNOWLEDGMENTS

We are grateful to P. M. Platzman for valuable discussions.

APPENDIX: QUANTUM CORRECTIONS TO THE MANY-ELECTRON CLASSICAL RELAXATION RATE

In the range of weak to moderately strong magnetic fields where $T \gg eE_f \chi_T \gg \omega_c$ an electron is moving nearly classically. The characteristic duration of a collision with a short-range scatterer $t_{coll} = T^{-1}$ (44) is small compared to ω_c^{-1}. The change of the velocity of an electron due to the acceleration in the fluctuational field over the time t_{coll} is small compared to the thermal velocity $(T/m)^{1/2}$. Therefore in evaluating the increment $\hat{\mathbf{r}}_n(t) - \hat{\mathbf{r}}_n(0)$ of the electron coordinate in $\xi(\mathbf{q})$ (37) for $t \lesssim T^{-1}$ it suffices to allow for a few lowest-order terms in electron acceleration. These terms include the acceleration due to the Lorentz force and the fluctuational electric field [which have been taken into account in (28)] as well as the acceleration due to the electric field being time-dependent itself:

$$\hat{\mathbf{E}}_n(t) \approx \hat{\mathbf{E}}_n(0) + t \frac{1}{m} \sum_{n'} (\hat{\mathbf{p}}_{n'}(0) \nabla_{n'}) \hat{\mathbf{E}}_n(0) \tag{A1}$$

[the explicit form of the operator $\hat{\mathbf{E}}_n$ is given by Eq. (27) with $\mathbf{r}_{n'}$ replaced by $\hat{\mathbf{r}}_{n'}$; to the lowest approximation in the acceleration of the electrons we have set the velocity of an n'th electron equal to its value $\hat{\mathbf{p}}_{n'}(0)/m$ at $t=0$]. An estimate of $\nabla_n \mathbf{E}_n$ for a classical system follows from the estimate (3) of $\langle E_f^2 \rangle$ and from the relation (cf. Ref. 46)

$$e^2 \langle E_f^2 \rangle \equiv \langle (\nabla_n H_{ee})^2 \rangle = -eT \langle \nabla_n \mathbf{E}_n \rangle. \tag{A2}$$

Allowing for this estimate we see that, for the characteristic $p_n \sim (mT)^{1/2}$, $t \sim T^{-1}$, the time-dependent term in (A1) gives rise to a correction to the reduced electron displacement $\delta r_n / \chi_T$ of the order of $e^2 \langle E_f^2 \rangle \chi_T^2 / T^2$. This is the quantum correction we are looking for.

With account taken of (A1) the expressions (28), (29) are modified:

$$e^{i\mathbf{q}\hat{\mathbf{r}}_n(t)} \approx \exp[i\mathbf{q}\tilde{\mathbf{F}}(t, \hat{\mathbf{p}}_n(0))] e^{i\mathbf{q}\hat{\mathbf{r}}_n(0)} \times \exp\left[-i \frac{q^2}{2m} t \left(1 - \frac{1}{6} \omega_c^2 t^2 + \frac{e}{12m} (\nabla_n \mathbf{E}_n) t^2 \right) \right], \quad t \ll \omega_c^{-1}, (eE_f \chi_T)^{-1}, \tag{A3}$$

where

$$\tilde{\mathbf{F}}(t, \hat{\mathbf{p}}_n) = \frac{\hat{\mathbf{p}}_n}{m} t \left(1 - \frac{1}{6} \omega_c^2 t^2 \right) + e \frac{\hat{\mathbf{p}}_n \times \mathbf{B}}{2m^2 \omega_c} \omega_c t^2 + \frac{e}{2m} \hat{\mathbf{E}}_n t^2 + \frac{e}{6m^2} t^3 (\hat{\mathbf{p}}_n \nabla_n) \hat{\mathbf{E}}_n . \tag{A4}$$

In the last term in (A3), which will be evaluated to the lowest order of the perturbation theory, we replaced $(\mathbf{q}\nabla_n)(\mathbf{q}\mathbf{E}_n)$ by $(1/2) q^2 (\nabla_n \mathbf{E}_n)$. In the last term in (A4) we dropped the terms with $n' \neq n$ in (A1), because for a classical electron system the momenta of different electrons are uncorrelated.

The evaluation of the correlator $\xi(\mathbf{q})$ (37) comes to statistical averaging of $\exp(i\mathbf{q}\tilde{\mathbf{F}})$ and further integration over time. In doing averaging we have to retain terms $\sim \omega_c^2 t^2 \sim \omega_c^2 / T^2$ and $\sim e^2 \langle E_f^2 \rangle \chi_T^2 / T^2$ (clearly, there may be no corrections of first order in ω_c or in E_f). Therefore the commutator

$$[(\mathbf{q}\hat{\mathbf{p}}_n), (\mathbf{q}\hat{\mathbf{E}}_n)] \Rightarrow -\frac{1}{2} i q^2 (\nabla_n \mathbf{E}_n)$$

should be allowed for to the lowest order of the perturbation theory. Then one can write $\exp(i\mathbf{q}\tilde{\mathbf{F}})$ as a product of the exponential of the first two $\hat{\mathbf{p}}_n$-dependent terms in (A4) (multiplied by $i\mathbf{q}$) and of the exponential that contains the remaining $\hat{\mathbf{r}}_n$-dependent terms in $\tilde{\mathbf{F}}$, the above commutator, and the last term in (A4). When this is done one may perform configuration averaging of the terms that depend on the electron coordinates:

$$\left\langle \exp\left[i\frac{e}{2m}\mathbf{q}\hat{\mathbf{E}}_n t^2 - i\frac{e}{6m^2}t^3 q^2(\nabla_n \mathbf{E}_n) + i\frac{e}{6m^2}t^3(\hat{\mathbf{p}}_n\nabla_n)(\mathbf{q}\hat{\mathbf{E}}_n)\right]\right\rangle_{\rm conf} \approx \exp\left[-\frac{e^2\langle E_f^2\rangle}{m^2}\left(\frac{q^2t^4}{16} - i\frac{q^2t^3}{6T} + i\frac{\mathbf{q}\hat{\mathbf{p}}_n t^3}{12T} + \frac{\mathbf{q}\hat{\mathbf{p}}_n t^2}{8T^2}\right)\right]. \tag{A5}$$

Here we have allowed for the fact that, to zeroth order in $\hbar/T$ (the expansion parameter, Ref. 46), the configuration averaging $\langle \hat{K}(\{\mathbf{r}_{n'}\})\rangle_{\rm conf}$ comes to integration over the coordinates $\mathbf{r}_{n'}$ with the weight $\exp(-\beta H_{\rm ee})$. To first order in $\hbar/T$ one should add the configuration average of the commutator $(\hbar/2T)[\hat{K},H_0]$. In (A5) we used

$$\frac{1}{2}[(\mathbf{q}\hat{\mathbf{E}}_n), e^{-\beta H_0}] \approx -\frac{1}{2m} i\beta \sum_{n'} (\mathbf{p}_{n'}\nabla_{n'})(\mathbf{q}\mathbf{E}_n) e^{-\beta H_0} \Rightarrow -i\frac{1}{4mT}(\mathbf{q}\mathbf{p}_n)\langle(\nabla_n \mathbf{E}_n)\rangle$$

and also allowed for (A2).

Averaging over electron momenta comes to integration over $\mathbf{p}_n$ with the Maxwell distribution as a weighting factor. As is well known[46] the lowest-order quantum corrections can be reduced to renormalization of temperature

$$T \Rightarrow T^*, \quad T^* = T\left[1 + \frac{e^2\langle E_f^2\rangle}{24mT^3} + \frac{\omega_c^2}{12T^2}\right]. \tag{A6}$$

The evaluation of the corresponding Gaussian integral over $\mathbf{p}_n$ is straightforward, and the result is

$$\langle \exp[i\mathbf{q}\hat{\mathbf{r}}_n(t)]\exp[-i\mathbf{q}\hat{\mathbf{r}}_n(0)]\rangle \approx \exp\left[-\frac{q^2}{2mT}\left(t^2T^2 + itT - \frac{1}{4}\mathcal{F}(t)\right)\right], \tag{A7}$$

$$\mathcal{F}(t) = \left(\frac{\omega_c^2}{T^2} + \frac{e^2\langle E_f^2\rangle}{2mT^3}\right)\left(\frac{1}{3}t^4T^4 + \frac{2}{3}it^3T^3 - \frac{1}{3}t^2T^2\right).$$

The value of the function $\xi(\mathbf{q})$ is given by the integral of the expression (A7) over time. In doing integration the function $\mathcal{F}(t)$ should be considered as a perturbation. The resulting expression for $\xi(\mathbf{q})$ is of the form (45), and the parameter $\mathcal{F}$ in (45) is given by the value of $\mathcal{F}(t)$ for $t=-i/2T$ [where the exponent in (A7) has a saddle point, to zeroth order in $\mathcal{F}(t)$].

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*Also at Department of Physics, MIT, Cambridge, MA 02139.

[1] M.W. Cole and M.H. Cohen, Phys. Rev. Lett. **23**, 1238 (1969).

[2] (a) T. Ando, A.B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982); (b) *2D Electron Systems on Helium and Other Substrates*, edited by E.Y. Andrei (Kluwer Academic, New York, 1997); (c) for an introductory review, see A.J. Dahm and W.F. Vinen, Phys. Today **40**, 43 (1987).

[3] C.C. Grimes and G. Adams, Phys. Rev. Lett. **42**, 795 (1979).

[4] D.S. Fisher, B.I. Halperin, and P.M. Platzman, Phys. Rev. Lett. **42**, 798 (1979).

[5] G. Deville, J. Low Temp. Phys. **72**, 135 (1988).

[6] M.A. Stan and A.J. Dahm, Phys. Rev. B **40**, 8995 (1989).

[7] Y. Iye, J. Low Temp. Phys. **40**, 441 (1980).

[8] V.S. Edel'man, JETP **50**, 338 (1980); L. Wilen and R. Giannetta, Phys. Rev. Lett. **60**, 231 (1988); Surf. Sci. **196**, 24 (1988).

[9] R. Mehrotra, C.J. Guo, Y.Z. Ruan, D.B. Mast, and A.J. Dahm, Phys. Rev. B **29**, 5239 (1984).

[10] V.A. Buntar', Yu.Z. Kovdrya, V.N. Grigoriev, Yu.P. Monarkha, and S.S. Sokolov, Sov. J. Low Temp. Phys **13**, 451 (1987); V.A. Buntar', V.N. Grigoriev, O.I. Kirichek, Yu.Z. Kovdrya, Yu.P. Monarkha, and S.S. Sokolov, J. Low Temp. Phys. **79**, 323 (1990).

[11] R.W. van der Heijden, M.C.M. van de Sanden, J.H.G. Surewaard, A.T.A.M. de Waele, H.M. Gijsman, and F.M. Peeters, Europhys. Lett. **6**, 75 (1988).

[12] P.W. Adams and M.A. Paalanen, Phys. Rev. B **37**, 3805 (1988); A.O. Stone, P. Fozooni, M.J. Lea, and M. Abdul-Gader, J. Phys. Condens. Matter **1**, 2743 (1989); P. Scheuzger, J. Neuenschwander, and P. Wyder, Physica B **165&166**, 845 (1990); **194-196**, 1231 (1994); P.J.M. Peters, P. Scheuzger, M.J. Lea, Yu.P. Monarkha, P.K.H. Sommerfeld, and R.W. van der Heijden, Phys. Rev. B **50**, 11 570 (1994).

[13] M.I. Dykman, M.J. Lea, P. Fozooni, and J. Frost, Phys. Rev. Lett. **70**, 3975 (1993); Physica B **197**, 340 (1994).

[14] M.J. Lea, P. Fozooni, P.J. Richardson, and A. Blackburn, Phys. Rev. Lett. **73**, 1142 (1994).

[15] E.Y. Andrei, S. Yücel, and L. Menna, Phys. Rev. Lett. **67**, 3704 (1991); G.F. Saville, J.M. Goodkind, and P.M. Platzman, *ibid.* **70**, 1517 (1993); L. Menna, S. Yücel, and E.Y. Andrei, *ibid.* **70**, 2154 (1993); E.Y. Andrei, Physica B **197**, 335 (1994) and references therein.

[16] K. Shirahama and K. Kono, Phys. Rev. Lett. **74**, 781 (1995).

[17] K. Kono and K. Shirahama, J. Low Temp. Phys. **104**, 237 (1996).

[18] A. Kristensen, K. Djerfi, P. Fozooni, M.J. Lea, P.J. Richardson, A. Santrich-Badal, A. Blackburn, and R.W. van der Heijden, Phys. Rev. Lett. **77**, 1350 (1996).

[19] See, e.g., I.V. Kukushkin, V.I. Fal'ko, R.J. Haug, K. von Klitzing, K. Eberl, and K. Tötemayer, Phys. Rev. Lett. **72**, 3594 (1994); A.A. Shashkin, V.T. Dolgopolov, G.V. Kravchenko, M. Wendel, R. Schuster, J.P. Kotthaus, R.J. Haug, K. von Klitzing, K. Ploog, H. Nickel, and W. Schlapp, *ibid.* **73**, 3141 (1994) and references therein.

[20] C.C. Grimes and G. Adams, Phys. Rev. Lett. **36**, 145 (1976); C.C. Grimes, Surf. Sci. **73**, 379 (1978).

[21] D.B. Mast, A.J. Dahm, and A.L. Fetter, Phys. Rev. Lett. **54**, 1706 (1985); D.C. Glattli, E. Andrei, G. Deville, J. Pointrenaud, and

F.I.B. Williams, *ibid.* **54**, 1710 (1985); P.J.M. Peters, M.J. Lea, A.M.L. Janssen, A.O. Stone, W.P.N.M. Jacobs, P. Fozooni, and R.W. van der Heijden, *ibid.* **67**, 2199 (1991); O.I. Kirichek, P.K.H. Sommerfeld, Yu.P. Monarkha, P.J.M. Peters, Yu.Z. Kovdrya, P.P. Steijaert, R.W. van der Heijden, and A.T.A.M. de Waele, *ibid.* **74**, 1190 (1995).

[22] A.L. Fetter, Phys. Rev. B **32**, 7676 (1985); **33**, 3717 (1986); **33**, 5221 (1986); S.S. Nazin and V.B. Shikin, Sov. Phys. JETP **67**, 288 (1988); V.A. Volkov and S.A. Mikhailov, *ibid.* **67**, 1639 (1988); I.L. Aleiner and L.I. Glazman, Phys. Rev. Lett. **72**, 2935 (1994).

[23] R.S. Crandall and R. Williams, Phys. Lett. **34A**, 404 (1971); A.V. Chaplik, Sov. Phys. JETP **35**, 395 (1972).

[24] A.-M. Tremblay and V. Ambegaokar, Phys. Rev. B **20**, 2190 (1979).

[25] (a) M.I. Dykman, Solid State Commun. **35**, 753 (1980); (b) J. Phys. C **15**, 7397 (1982); (c) JETP **55**, 766 (1982).

[26] A.G. Eguiluz, A.A. Maradudin, and R.J. Elliott, Phys. Rev. B **24**, 197 (1981).

[27] A.J. Dahm and R. Mehrotra, J. Low Temp. Phys. **50**, 201 (1983); R. Mehrotra and A.J. Dahm, *ibid.* **50**, 235 (1983).

[28] M. Saitoh, J. Phys. Soc. Jpn. **55**, 1311 (1986); **56**, 706 (1987).

[29] E.M. Conwell, *High Field Transport in Semiconductors* (Academic Press, New York, 1967); K. Seeger, *Semiconductor Physics. An Introduction*, 5th ed. (Springer, Berlin, 1991).

[30] R. Mehrotra, J. Low. Temp. Phys. **68**, 161 (1987); **79**, 311 (1990).

[31] (a) M.I. Dykman and L.S. Khazan, JETP **50**, 747 (1979); (b) M.I. Dykman, Sov. J. Low Temp. Phys. **6**, 268 (1980).

[32] N.A. Krall and A.W. Trivelpiece, *Principles of Plasma Physics* (McGraw-Hill, New York, 1973).

[33] (a) P. Fozooni, P.J. Richardson, M.J. Lea, M.I. Dykman, C. Fang-Yen, and A. Blackburn, J. Phys. Condens. Matter **8**, L215 (1996); (b) C. Fang-Yen, M.I. Dykman, and M.J. Lea, following paper, Phys. Rev. B **55**, 16 272 (1997).

[34] L. Bonsall and A.A. Maradudin, Phys. Rev. B **15**, 1959 (1977).

[35] E.M. Baskin, L.N. Magarill, and M.V. Entin, JETP **48**, 365 (1978).

[36] R. Kubo, S.J. Miyake, and N. Hashitsume, Solid State Phys. **17**, 269 (1965).

[37] W. Kohn, Phys. Rev. **123**, 1242 (1961).

[38] J.P. Hansen, D. Levesque, and J.J. Weis, Phys. Rev. Lett. **43**, 979 (1979); R.K. Kalia, P. Vashishta, S.W. de Leeuw, and A. Rahman, J. Phys. C **14**, L991 (1981).

[39] For systems with equidistant or nearly equidistant energy levels the shape of the spectrum is not given just by the superposition of the lines that correspond to individual transitions, with the widths calculated using the results for two-level systems. This is called the harmonic oscillator ''paradox'' [V. Weisskopf and E. Wigner, Z. Phys. B **65**, 18 (1930)]. The theory of the spectra of such systems is reviewed by M.I. Dykman and M.A. Krivoglaz, in *Soviet Physics Reviews*, edited by I.M. Khalatnikov (Harwood, New York, 1984), Vol. 5, p. 265.

[40] L.D. Landau and E.M. Lifshitz, *Mechanics* (Pergamon, London, 1976).

[41] Yu.M. Vil'k and Yu.P. Monarkha, Sov. J. Low Temp. Phys. **15**, 131 (1989).

[42] P.W. Anderson, J. Phys. Soc. Jpn. **9**, 316 (1954).

[43] See A. Isihara, Solid State Phys. **42**, 271 (1989).

[44] M. Saitoh, J. Phys. Soc. Jpn. **42**, 201 (1977).

[45] M.J. Lea *et al.*, this issue, Phys. Rev. B **55**, 16 280 (1997).

[46] L.D. Landau and E.M. Lifshitz, *Statistical Physics*, 3rd ed., Pt. 1 (Pergamon, New York, 1980).