Low-temperature magnetoconductance transition to Mott's conductance

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The conductivity of an amorphous sample at low temperatures in a strong magnetic field is calculated. While Mott's and Ono's theories consider infinite samples, the proposed formalism treats finite ones. It turns out that this is a crucial difference. The model shows a transition between two conductivity behaviors: $\ln(\sigma)\approx T^{-1/3}$ for $T<\widetilde{T}$ and $\ln(\sigma)\approx -T^{-1/2}$ for $T>\widetilde{T}$ (the transition temperature $\widetilde{T}$ depends on the magnetic field and on the sample's size). The former one resembles the simple two-dimensional Mott conductivity behavior, while the latter resembles Ono's theory. [S0163-1829(97)00223-3]

In 1969 Mott presented his variable-range hopping (VRH) theory of an amorphous system.[1] It predicted that for a d-dimensional amorphous sample the electrical conductivity (σ) has the following temperature dependence: $\ln[\sigma(T)]\approx -T^{-1/(d+1)}$. This theory was found to be in good agreement with early experiments of three-dimensional[2] (3D) and two-dimensional[3] (2D) systems, where Mott's theory predicts

$$\sigma\approx\exp[-(T_M/T)^{1/3}]. \quad (1)$$

The first experiments of the quantum Hall effect stimulated the interest in the temperature dependence of the transverse conductivity (σ_{xx}). Since the magnetic field causes a Gaussian localization instead of an exponential one (i.e., the electron's wave function looks like $\varphi\approx e^{-(r/l)^2/2}$, l is the magnetic length), VRH theory predicts an exponential dependence $\exp[-(T_0/T)^{1/2}]$.

Ono took a percolation approach[6] in order to calculate the conductivity more rigorously. His theory predicts the following:[5]

$$\sigma\approx T^{-1}\exp[-(T_0/T)^{1/2}]. \quad (2)$$

The experimental results of Ref. 4 show excellent agreement with this prediction.

In this paper we present a rigorous calculation of the transverse conductivity in the resonant tunneling regime, i.e., in the regime where the coherence length is longer than the sample's size. It should be emphasized that Mott's and Ono's theories assume infinite sample. This paper shows that the finite sizes of the sample cause an important effect on the conductivity. It is found that there is a transition temperature $\widetilde{T}$ where the temperature dependence behaves like Eq. (2) when $T>\widetilde{T}$ and like Eq. (1) when $T<\widetilde{T}$.

Take a system where electrons are scattered over a large number of impurities (Fig. 1), which are uniformly placed in an opaque potential barrier in a strong magnetic field. The stationary-state Schrödinger equation can then be written as

$$[(\hat{\mathbf{p}}_y+\mathbf{A}_y)^2+\hat{\mathbf{p}}_x^2]\psi-(E-U)\psi=\sum_i D(|\mathbf{r}_i-\mathbf{r}|)\psi. \quad (3)$$

Hereafter, we use the units $h=2m=-e=c=1$ (Planck constant, the electron's mass and charge, and the velocity of light, respectively). $\hat{\mathbf{p}}_{x,y}$ are the momentum operators, E is the incoming electrons' energy, and U is the potential:

$$U\equiv\begin{cases} V & \text{for } -L<x<L \\ 0 & \text{otherwise,}\end{cases}$$

and the D's are the impurities' potentials, which are short-range ones (see Ref. 7). V is a positive potential, and thus the electrons with energy $0<E<V$ tunnel through the barrier. The exact form of the impurities' potentials (D) is unimportant as long as they are short-range ones, i.e., on atomic scales. Mathematically, they can be represented by the impurity D function (IDF, see Ref. 7), which is infinitely shallower than the two-dimensional δ function (2DDF), but unlike the 2DDF, it has an eigenvalue (say $\widetilde{E}_{i0}$). Because of its infinitely small dimensions, the short-range potential can be fully determined by this single parameter (the eigenvalue). Thus, each impurity creates a resonance level $E_i\equiv V-\widetilde{E}_{i0}$, to which the particles can tunnel. If the electrons' energy equals the resonance energy of the impurity, the impurity's presence will be felt in the conductance, since many electrons will tunnel through it. But if their energy does not match, the influence of the specific impurity will be negligible. In the following, the Landau gauge is chosen as $A_y=Bx$ (B is the magnetic field).

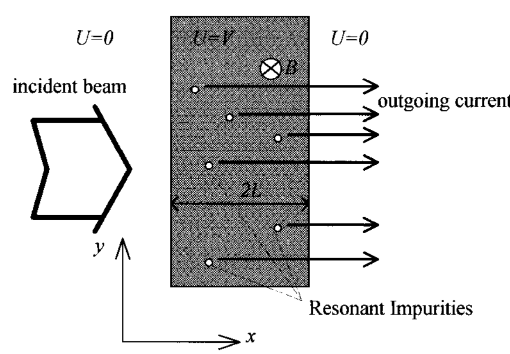

FIG. 1. The substantial contribution to the current through the barrier comes from the resonant impurities, i.e., from the impurities whose energy is equal to the incoming electrons' energy.

In the case of a very strong magnetic field ($BL^2 \gg 1$), the solution $\psi(\mathbf{r})$ can be written as a summation over the contributions from each of the impurities $\psi_i(\mathbf{r})$, i.e.,

$$\psi(\mathbf{r}) = \sum_i \psi_i(\mathbf{r},\mathbf{r}_i). \tag{4}$$

Notice that the strong magnetic field allows us to neglect the coherent multicenter scattering. Moreover, for a specific particle energy, only the resonance impurities give a considerable contribution to ψ, the rest can be ignored.

The absolute value of each one of these contributions at the end of the barrier (for a very strong magnetic field $B \gg \sqrt{V}/L$) is

$$|\psi_i(x=L)| \approx \frac{c e^{-B(L^2+x_i^2)/2} e^{-B(y-y_i)^2/4}}{(E-E_i)/\Delta + i e^{-B(L^2+x_i^2)/2+BL|x_i|}}, \tag{5}$$

where x_i, y_i, and E_i are the x and y locations of the ith impurity, and its resonance energy, respectively; c and Δ change slowly with the energy and the magnetic field. We assumed in Eq. (5) that the magnetic field is so strong, i.e., $B \gg \sqrt{V}/L$, that one can neglect the dependence of ψ on the barrier strength; that is why V is absent in Eq. (5) (actually, we can find it indirectly in $E_i \equiv V - \widetilde{E}_{i0}$). According to Ref. 8,

$$G_L = 4i \sum_\lambda \frac{\partial f}{\partial \varepsilon} \int dy \left(\frac{\partial \psi_\lambda^*}{\partial x} \psi_\lambda - \text{c.c.} \right), \tag{6}$$

where $f(\varepsilon_\lambda - \zeta)$ is the Fermi distribution function (ε_λ are the energy levels and ζ is the electrochemical potential), the summation over λ denotes summation over the quantum numbers (i.e., the energy levels E and the generalized momentum y component k), the integration over y is over the width of the sample, and the asterisk stands for a complex conjugate (c.c.).

In Eq. (6) only few impurities are at resonance and all the rest are out of resonance. Therefore, only these few make the major contribution to the conductivity (a resonant tunneling current). Now suppose that the energies of the impurities are uniformly distributed. Then, instead of integrating over the energy, one may sum over the contributions (5) from *all* the impurities.

For a very low temperature (T), one can also use the approximation

$$\partial f / \partial \varepsilon_\lambda \xrightarrow[E-\zeta \gg T]{} T^{-1} e^{-|E-\zeta|/T}. \tag{7}$$

Then by substituting Eqs. (5) and (7) in Eq. (6),

$$G_L(x=L) \approx T^{-1} \int dy\ g(y), \tag{8}$$

where

$$g(y) = \sum_i e^{-|E_i-\zeta|/T - 2BL|x_i| - B(y-y_i)^2/2}, \tag{8$'$}$$

where the sum was taken only for the resonance contribution; i.e., only resonance energies contribute to the sum. Hence, in (8′) we have substituted only the ψ_i with $E - E_i = 0$, and the summation was taken over all the impurities, which is equivalent to a summation over all the resonance energies (which is approximately equal to the integration over all the energies).

When $T = 0$, the main contribution to the conductivity comes from the impurities, whose energy is equal to the Fermi energy and their position is at the center of the barrier [as Eq. (5) suggests]. However, for $T > 0$ most of the contributions to G_L come from other impurities. Because of the uniform distribution of the impurities' resonance energy, it obeys the relation $E - \zeta \approx R^{-2}$, where $R^2 \equiv x_i^2 + (y_i - y)^2$. That is, inside a circle with a radius R, which is centered at the center of the barrier ($x_i = 0, y_i - y = 0$), the number of impurities with a resonance energy E is proportional to R^2.

Next, since the summation is taken over all the impurities, and since they are distributed randomly over the barrier, we can replace the index i with two indexes: one for the x coordinate and the other for the y coordinate. This means that now every impurity will be identified by two indexes (its coordinates) instead of one (in both cases we sum over all the impurities so the order of summation is not important). We can also assume that since the number of impurities is huge, the fluctuations of g as a function of y are very small, and thus $g(y) \approx$ const. So, it is enough to calculate $g(y)$ at a specific position y.

Thus, by replacing the summation in (8′) by the summation over the impurities' components $\eta \equiv x_i$ and $\xi \equiv y_i - y$, it follows that

$$G_L \approx \sum_{\eta,\xi} e^{-f(\eta,\xi)}, \tag{9}$$

where

$$f(\eta,\xi) \equiv \frac{1}{(\eta^2+\xi^2)NT} + 2BL|\eta| + B\xi^2/2 \tag{10}$$

and N is the density of states at the Fermi energy.

In general, $f(\eta,\xi)$ has four minima of two kinds. However, when

$$T > T_c \equiv \frac{1/8}{BNL^4} \tag{11}$$

(in ordinary physical dimensions $k_B T_c = \hbar c / 8eNBL^4$) only two minima exist at $\eta = 0$, $\xi = \pm(2/BNT)^{1/4}$, and thus the conductivity can be evaluated:

$$G_L \approx T^{-1} e^{-(2B/NT)^{1/2}}. \tag{12}$$

Here $T_0 = 2B/N$, and in ordinary physical dimensions $k_B T_c = 2Be/N\hbar c$. This expression resembles Ono's result. When

$$T < T_c \tag{13}$$

two additional minima emerge at $\xi = 0$, $\eta = \pm(BLNT)^{-1/3}$ (see Fig. 2), however, the contribution from these points is negligible as long as

$$T > \widetilde{T} \equiv \frac{(2/9)^3}{BL^4 N} \tag{14}$$

(in ordinary physical dimensions $k_B \widetilde{T} \approx 0.01 \hbar c / eNBL^4 = 0.09 k_B T_c$) since in the temperature range $T_c > T > \widetilde{T}$ the

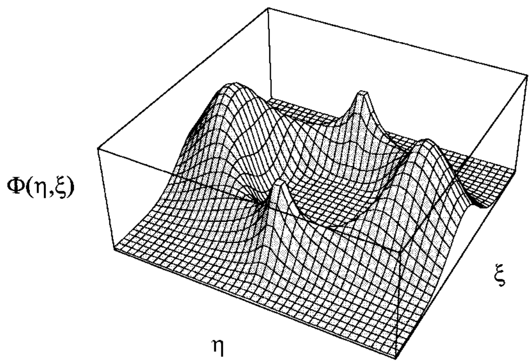

FIG. 2. The surface graph of the function $\Phi(\eta,\xi)=e^{-f(\eta,\xi)}$. The maxima at $\eta=0$ determine the $\sigma\approx T^{-1}e^{-(T_0/T)^{1/2}}$ behavior, while the maxima at $\xi=0$ determine the $\sigma\approx T^{-1}e^{-(T_M/T)^{1/3}}$ behavior.

two dominant minima remain at $\eta=0$, $\xi=\pm(2/BNT)^{1/4}$. But when

$$T<\tilde{T}, \tag{15}$$

the two dominant minima are at $\xi=0$, $\eta=\pm(BLNT)^{-1/3}$, and then

$$G_L\approx T^{-1}e^{-3(B^2L^2/NT)^{1/3}}. \tag{16}$$

This last result exhibits the properties of a 2D Mott conductivity in the absence of a magnetic field. Here $T_M=27(BL)^2/N$, and in ordinary physical dimensions $k_BT_M=27(eBL)^2/N(\hbar c)^2$.

At the transition curve, i.e., $T=T_c(B)$, the conductivity behaves like (see Fig. 3)

$$G_L\approx T^{-1}e^{-\nu/(NTL^2)}, \tag{17}$$

where $\nu=4/27$. The exponent in Eq. (17) looks the same with ordinary physical dimensions.

In order to measure this transition the transition temperature $\tilde{T}$ should be within the measurable range. Thus, because we need at least $BL^2>10$, and a sample length $L<1\ \mu$m, the magnetic field should be $B\approx1$ T, and in order to get $\tilde{T}>0.1$ K the density of state should be $N<10^7\ \text{meV}^{-1}\ \text{cm}^{-2}$ (it can be achieved between the Landau levels, where the measurement is taken).

Even though the sample's length should be of mesoscopical size, its width should be much larger, and can be even macroscopic. This comes directly from the demand that the temperature should be larger than the interresonance energy (otherwise, there will not be a monotonic dependence on the Fermi energy), i.e., from

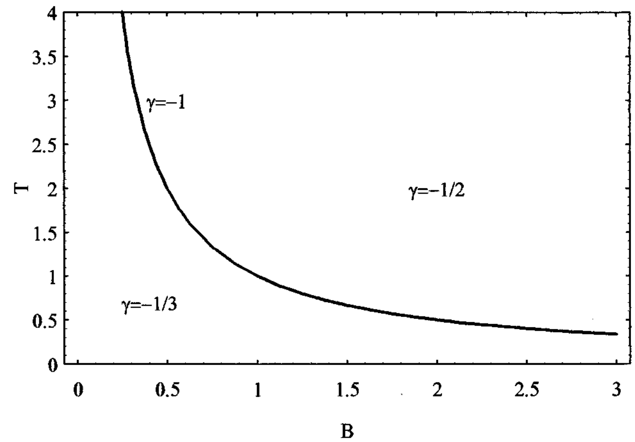

FIG. 3. The conductivity behavior $\ln\sigma\approx-T^{\gamma}$ divides the B-T (magnetic-field–temperature) diagram into two parts: above the transition curve (high T and B) with $\gamma=-1/2$ and below the transition curve (low T and B) with $\gamma=-1/3$. At the transition curve $\gamma=-1$.

$$\tilde{T}>\Delta E\approx1/LaN, \tag{18}$$

where a is the sample's width. It comes out directly that

$$a/L>10^3. \tag{19}$$

So, the width should be $a>1$ mm.

To summarize, a model based on resonant tunneling was presented in order to calculate the conductivity at low temperature of an amorphous sample in the presence of a strong magnetic field. Unlike in Mott's and Ono's theories, the conductivity was calculated for a finite-sized sample, and it was found to have a crucial impact on the conductivity. The model demonstrates a temperature transition: for $T>\tilde{T}$ the conductivity behaves like Ono's theory,

$$\sigma_h\approx T^{-1}\exp[-(T_0/T)^{1/2}],$$

but when $T<\tilde{T}$ it looks like the usual 2D Mott conductivity

$$\sigma_l\approx T^{-1}\exp[-(T_M/T)^{1/3}].$$

The transition temperature $\tilde{T}$ depends on the magnetic field B. Thus, the model also shows that when moving on the transition curve $T=\tilde{T}(B)$,

$$\sigma_t\approx T^{-1}\exp[-T_t/T]$$

(the subscripts h, l, and t stand for high, low, and transition temperature, respectively).

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