Roughness scattering in a finite-length wire

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(Received 12 December 1996)

Semiclassical conductance of a finite two-dimensional wire is theoretically evaluated. The bulk scattering is neglected. Rigorous formulas for situations with completely diffuse and partially diffuse straight boundaries are given. For the short limit of wire length the conductance decreases according to $1/(1+ca)$, where a is an aspect ratio parameter (length divided by wire width), and c is a prefactor depending on the specularity parameter of the boundary. If the wire is long enough, conductance decreases with $c'\ln a/a$ using another constant c', which is consistent with the classical analysis of resistivity by boundary roughness scattering of an infinite wire. [S0163-1829(97)05123-0]

I. INTRODUCTION

Ballistic transport in one- and two-dimensional electron systems has been the subject of intensive research. Quantized conductance was observed first in point contacts formed by split gate,[1,2] and then in quantum wires formed from a high-mobility substrate by electron-beam lithography and wet chemical etching.[3–5] A necessary condition for observing quantized conductance is that the elastic mean free path l_e is much longer than the sample length L and the width W. In Hall-bar geometries, bend resistance[6,7] and several anomalies in low-field Hall resistance[8–10] are characteristics typical of ballistic electron transport, and, chaotic behavior occurs at low temperatures.[11,12] Some of these effects are not intrinsic to the quantum wire, but are instead due to junction characteristics. More recently, correction in the quantum conductance stemming from a one-dimensional electron many-body effect has been found, and was explained in terms of the Tomonaga-Luttinger liquid picture.[13] Ballistic long wire is crucial to see these many-body effects, because one-dimensional power-law behavior disappears for the temperatures lower than $\hbar v_F/L$, where v_F is the Fermi velocity. A ballistic transport system can therefore be said to be an ideal experimental setup for studying quantum mechanics and many-body effects.

Of course the more we obtain samples free from bulk scattering, the more the transport is controlled by the condition of the potential boundary. The effect of boundary roughness in a quantum wire transport was first reported by Thornton *et al.*, who found the effect in a low-field magnetoresistance peak.[14] Such a magnetosize effect has been investigated theoretically and explained by semiclassical[15] and quantum treatments.[16,17] Conductance fluctuation[18–20] and chaotic behavior[21,22] induced by the boundary scattering have also been investigated recently.

Another many-body effect was discovered by Molenkamp and de Jong[23] in the nonlinear transport characteristics of a quantum wire with cleverly situated thermometers. Excess current up to about 20 μA raises the electron temperature while keeping the lattice temperature almost constant. If the current is small enough, the electrons have negligible interaction with boundaries. The heating drastically decreases the electron-electron scattering time, which in turn increases the probability of electrons being scattered into the boundaries and increases the resistance. A successive increase of the electron temperature then makes the electrons scatter frequently with each other, which prevents the electrons from diffusing to the boundaries. This results in a decrease of the resistance.[24] They analyzed these effects quantitatively by solving Boltzmann's equation for an infinite wire.[25]

The transition from Sharvin ballistic resistance to diffusive Ohmic resistance is another interesting topic related to nearly ballistic transport in a wire, and semiclassical solutions for short-range bulk scatterers[26] and small-angle scatterers[27] have been obtained. In these analyses, boundary scattering was assumed to be specular. As yet there seems to have been no rigourous treatment of resistance (not resistivity) of boundary roughness scattering, although there have been several attempts to evaluate it using the billiard model.[28–31] A Landauer-type treatment is of course quite a powerful tool for obtaining the quantum-mechanical resistance,[17–19] but a rigorous semiclassical understanding of boundary roughness in a finite wire is quite important to single out the quantum-mechanical effects.

This paper therefore evaluates the boundary roughness scattering in a finite wire by using a semiclassical Boltzmann treatment without bulk scatterings. We find a formula for the conductance for general boundary conditions, and we find analytical characteristics of the conductance in the limiting parameters by a power-series expansion method. Section II describes the theoretical model and basic formulation in terms of Boltzmann's equation. Section III presents a closed formula for completely diffuse boundaries. Section IV extends the result to boundaries with general specularity parameters. Section V gives some numerical results of conductance and current distribution. Section VI discusses the results, and presents our conclusions. Appendixes A and B shows some details of the calculations.

II. MODEL AND BASIC EQUATION

We consider a quantum wire with length L and width W in a two-dimensional x-y plane as shown in Fig. 1 (x is along the wire). Ideal reservoirs are attached to both ends of the quantum wire. The two confining walls at $y=0$ and $y=W$, which are straight on a scale much larger than Fermi

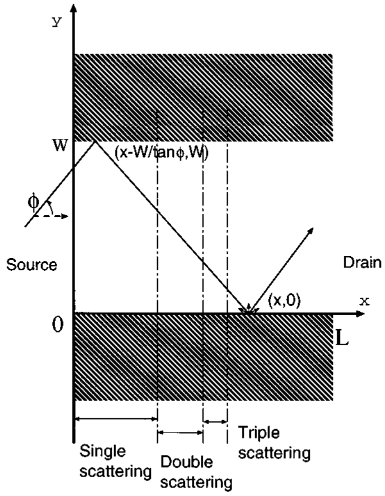

FIG. 1. Schematic diagram of the system considered. Shaded regions are where the infinitely high confinement potential defines the wire region with length L and width W.

wavelength, are diffusive. Consider steady-state Boltzmann's equations at zero temperature for the distribution function $f(r,k)$ of two-dimensional electrons with electric field $E(r)$ applied to the system, where r is a two-dimensional position and k is the electron momentum,

$$eE(r)\frac{\partial f(r,k)}{\hbar\partial k}+v\cdot\frac{\partial f(r,k)}{\partial r}=\left.\frac{\partial f(r,k)}{\partial t}\right|_{\rm coll}, \qquad (1)$$

where the first term shows the response to the external field, the second shows the effect due to electron drift motion, and v is the electron velocity. The right-hand side corresponds to the bulk scattering due to impurities or to other electrons. The distribution function is expanded around equilibrium Fermi-Dirac distribution function $f_0(\varepsilon)$ as follows:[26,32]

$$f(r,k)=f_0(\varepsilon)+\left(-\frac{\partial f_0}{\partial\varepsilon}\right)e[Vu(r,\phi)-\Phi(r)], \qquad (2)$$

where ε is the electron Fermi energy and $\Phi(r)$ is the electrostatic potential. The new function $u(r,\phi)$ corresponds to a deviation of the local chemical potential from ε when it is averaged in ϕ. Deep inside the reservoirs, the second term of Eq. (2) vanishes since the chemical potentials, $\varepsilon+eV$ in the left reservoir and ε in the right reservoir, cancel with $e\Phi(r)$ from the charge neutrality condition. We neglect the change of velocity and set $v=v_F(\cos\phi,\sin\phi)$ using the Fermi velocity v_F, where we measure the angle from the positive x axis. Then we can expand Boltzmann's equation to the lowest order of the field by using the identity $-\partial\Phi/\partial r=E(r)$:

$$v\cdot\frac{\partial u(r,\phi)}{\partial r}=\left.\frac{\partial u(r,\phi)}{\partial t}\right|_{\rm coll}. \qquad (3)$$

In the following discussions we completely neglect bulk scattering processes, and the right-hand side of Eq. (3) is zero. The bulk (electron-electron or other) scatterings *in the reservoirs* maintain the equilibrium Fermi-Dirac distribution. The electron current density including the spin freedom is

$$j(r)=2e\sum_k f(r,k)\cdot v$$
$$=e^2VDv_F\int_0^{2\pi}\frac{d\phi}{2\pi}u(r,\phi)(\cos\phi,\sin\phi), \qquad (4)$$

where $D=m/(\pi\hbar^2)$ is the two-dimensional density of states, and m is the electron effective mass. Therefore, Eq. (3), with no collision term, is nothing but a bulk current conservation relation: $\mathrm{div}j(r)=0$. The total current $I(x)$ across a line $x=X$ is given by integrating the x component of $j(r)$ from $y=0$ to $y=W$:

$$I(X)=VG_0\frac{\pi}{W}\int_0^W dy\int_0^{2\pi}\frac{d\phi}{2\pi}u(X,y,\phi)\cos\phi, \qquad (5)$$

where $G_0=(2e^2/h)(Wk_F/\pi)$ is the inverse of classical Sharvin's resistance with Fermi momentum k_F and Wk_F/π is the total number of subbands.[4,33] In the following we will show formulas for the dimensionless conductance $g(x)=I(x)/(VG_0)$, where the current conservation is expressed as $dg(x)/dx=0$.

To solve Boltzmann's equation, we should define boundary conditions at four regions. Since we are considering perfect reservoirs, we use the following idealized conditions at the boundaries to source $(x=0)$ and to drain $(x=L)$, respectively:

$$u(0,y,\phi)=1, \quad \phi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right], \qquad (6)$$

$$u(L,y,\phi)=0, \quad \phi\in\left[\frac{\pi}{2},\frac{3\pi}{2}\right]. \qquad (7)$$

At the confinement boundaries we impose the following general conditions using specularity parameter $p(\phi)$ (Refs. 34 and 35) depending on the incident angle ϕ:

$$u(x,0,\phi)=p(\phi)u(x,0,-\phi)+\int_\pi^{2\pi}\frac{d\theta}{2}[1-p(\theta)]|\sin\theta|u(x,0,\theta),$$
$$\phi\in[0,\pi] \qquad (8)$$

$$u(x,W,\phi)=p(\phi)u(x,W,-\phi)+\int_0^{\pi}\frac{d\theta}{2}[1-p(\theta)]\sin\theta u(x,W,\theta),$$
$$\phi\in[\pi,2\pi], \qquad (9)$$

where the specularity parameter must have the symmetry $p(\phi)=p(\pi-\phi)=p(-\phi)$ from physical considerations. The completely diffuse scattering boundary corresponds to $p=0$, and the specular boundary corresponds to $p=1$. The factor $\sin\theta$ corresponds to the effective solid angle of a unit segment of the boundary for an incident flux with incident angle θ. The current conservation is fulfilled with these boundary conditions, as shown in Appendix A. In other words, the current density at the boundary normal to the boundary is shown to vanish with the above boundary conditions. de Jong and Molenkamp[25] used another boundary condition, where $|\sin\theta|/2$ is replaced with $1/\pi$ in the above equations, which does not conserve current. However, in the case of an infinite-length wire, which they considered, the second terms in Eqs. (8) and (9) disappear because of the symmetry.[25,34,35] In our finite-wire calculation the second terms, which is set to $\omega(x)$ in the following sections, remain finite and play an important role.

III. COMPLETELY DIFFUSE BOUNDARIES

This section shows a conductance of a wire with completely diffuse boundaries ($p=0$). Here we define a function $\omega(x)$ which is the uniformly scattered flux into the wire at a point $r=(x,0)$ of a bottom boundary. This function is actually the second term of the right-hand side of Eq. (8) with $p(\phi)=0$. We notice the auxiliary relation $u(x,y,\phi)=u(x,W-y,-\phi)$ which is due to the symmetry. Using the method of the characteristic equation, we can modify the angular integral in Eq. (8) into line integrals at the left source [the source term $\omega_0(x)$] and at the top boundary [integral of $u(x,W,-\phi)=u(x,0,\phi)=\omega(x)$ with $\phi\in[\pi,2\pi]$]. The integral equation to be solved is

$$\omega(X)=\int_0^L dx\ G(X-x)\omega(x)+\omega_0(X), \tag{10}$$

with

$$G(X)=\frac{W^2}{2(X^2+W^2)^{3/2}}, \tag{11}$$

$$\omega_0(X)=\frac{1}{2}-\frac{X}{2\sqrt{X^2+W^2}}, \tag{12}$$

which is an inhomogeneous Fredholm equation of the second kind. In the following we move the origin of x to the wire center and normalize the length with W—that is, $Z=[X-(L/2)]/W$—and define an aspect ratio parameter $a=L/(2W)$. Then by defining a new function $\Omega(z)=\omega[(z+a)W]-\frac{1}{2}$ in $z\in[-a,a]$, and by transforming Eq. (10), we find that the function $\Omega(z)$ satisfies the following equation:

$$\Omega(Z)=\frac{1}{2}\int_{-a}^{a}dz\frac{\Omega(z)}{[(Z-z)^2+1]^{3/2}}+\frac{1}{4}\left[\frac{a-Z}{\sqrt{1+(a-Z)^2}}-\frac{a+Z}{\sqrt{1+(a+Z)^2}}\right]. \tag{13}$$

Therefore, $\Omega(z)$ is an odd function of z. We evaluate the approximated solution by the power series expansion around $z=0$, $\Omega(z)=a_1z+a_3z^3/3!+a_5z^5/5!\ \ldots$. After a lengthy but straightforward calculation, we found analytical forms of the approximated solution up to fifth order of z. The coefficients have limiting values for very small a (short wire), $(a_1,a_3,a_5)\to(-\frac{1}{2}+\frac{3}{4}a^2,\frac{3}{2}-\frac{45}{4}a^2,-\frac{45}{2}+\frac{1575}{4}a^2)+O(a^3)$; and for large a (long wire), $(a_1,a_3,a_5)\to[-\frac{3}{7}/a+\frac{11}{14}/(a\ln a),-\frac{2}{7}/(a^3\ln a),-\frac{6}{7}/(a^5\ln a)]$. The conductance defined in Eq. (5) can be similarly evaluated by integrating all fluxes from the source, top boundary, and bottom boundary as

$$g(Z)=\int_{-a}^{a}dz\left[\ \mathrm{sgn}(Z-z)-\frac{Z-z}{\sqrt{(Z-z)^2+1}}\right]\Omega(z)+\tfrac{1}{2}[\sqrt{1+(Z+a)^2}+\sqrt{1+(Z-a)^2}]-(Z+a), \tag{14}$$

where the function $\mathrm{sgn}(z)$ gives 1 for $z>0$ and -1 for $z<0$. We can confirm the current conservation by directly differentiating $g(Z)$ in Eq. (14) with respect to Z, and by using Eq. (13). The conductance at $Z=0$, which corresponds to $X=L/2$ (the wire's center), is most accurate since the integral kernel of Eq. (14) has peaks near $z=Z$, and Ω is expanded around $z=0$. We evaluate $g(0)$ at two limits: If $a\to0$, then $g\to1-a+a^2+O(a^3)$, and if $a\to\infty$ then $g\to(3/7a)\ln2a+(2/7a)$. The leading term of the latter limit approaches $(1/2a)\ln 2a$ if we take higher-order expansions of $\Omega(z)$, whereas the limit for $a\to0$ approaches $1/(1+a)$. This logarithmic dependence of conductance on length is reminiscent of the ''catastrophic'' disappearance of the resistivity,[34,36] which has been known to occur in an infinite wire, where the resistivity ρ decreases logarithmically and disappears when the bulk mean free path l_e approaches infinity, i.e., $\rho_0/\rho\sim\frac{3}{4}\kappa\ln(1/\kappa)$, where $\rho_0\propto1/l_e$ is the resistivity of bulk and $\kappa=W/l_e$. Notice that the resistance remains finite even if there is no bulk scattering.

IV. PARTIALLY DIFFUSE BOUNDARIES

The result of Sec. III can be extended to the case in which the boundaries are partially diffuse. Here we use a closed formula to give a conductance, and show some of the analytical approximated results.

Since there is no bulk scattering, the incident flux at $(X,0)$ with angle ϕ in Eq. (8) originates from the source $(0,y)$ or from the upper boundary $(x-W/\tan\phi,W)$ (see the ray trace in Fig. 1). Defining $X_n(\phi)=X-nW/\tan\phi$ and using the auxiliary relation of u, we have

$$\begin{aligned}u(X,0,\phi)&=\omega(X)+p(\phi)u(X_1(\phi),W,-\phi)\\&=\omega(X)+p(\phi)u(X_1(\phi),0,\phi)\\&=\omega(X)+p(\phi)[\omega(X_1(\phi))+p(\phi)u(X_2(\phi),0,\phi)],\end{aligned}$$

where now $\omega(x)$ contains the factor $1-p$. We can extend the iteration until $X_n(\phi)<0$ or $X_n(\phi)>L$, where the integer n depends on X and ϕ. Using the boundary condition of u at $x=0$ and L, we can reduce the above equation to

$$u(X,0,\phi)=\omega(X_0(\phi))+p(\phi)\omega(X_1(\phi))+\cdots+p^{n-1}(\phi)\omega(X_{n-1}(\phi))+Q, \tag{15}$$

where $Q=p^n(\phi)$ for $X_n(\phi)<0$ or $0<\phi<\pi/2$, and $Q=0$ for $X_n(\phi)>L$ or $\pi/2<\phi<\pi$. Therefore, if we know $\omega(X)$ for all X we can solve the problem. In a way similar to that in Sec. III, we can evaluate $\omega(X)$ by integrating at the source and at the top boundary by defining incident angle $\phi_{X-x}=\arctan[W/(X-x)]$:

$$\omega(X)=\int_0^W dy\ K(X,y)\left[1-p\left(\frac{y}{X}\right)\right]+\int_0^L dx\ G(X-x)\times\left[1-p\left(\frac{W}{X-x}\right)\right]u(x,0,\phi_{X-x}), \quad (16)$$

where we redefined $p(\phi)$ as $p(\tan\phi)$ and $K(X,y)=Xy/2/(X^2+y^2)^{3/2}$. The first term on the right-hand side is the flux from the source, and the second integral is the flux from the top boundary. We have an equivalent equation to Eq. (10), when we replace $G(X)$ with

$$G_p(X)=\sum_{n=0}^{\infty}\frac{W_n^2}{2(X^2+W_n^2)^{3/2}}\left[1-p\left(\frac{W_n}{X}\right)\right]p^n\left(\frac{W_n}{X}\right), \quad (17)$$

and $\omega(X)$ with

$$\omega_{0p}(X)=\int_0^W dy\ K(X,y)\left[1-p\left(\frac{y}{X}\right)\right]+\int_0^X dx\ G(X-x)\left[1-p\left(\frac{W}{X-x}\right)\right]-\int_0^X dx\ G_p(X-x)\left[1-p\left(\frac{W_n}{X-x}\right)\right], \quad (18)$$

where W_n is $n\times W$. A detail derivation is given in Appendix B. Of course these functions reduce to $G(X)$ and ω_0 if $p=0$.

The current is evaluated similarly, and the result is

$$g(X)=\frac{2}{W}\int_0^W dy\int_0^L dx\sum_{n=1}^{\infty}K(X-x,ny)q^n\omega(x)+\frac{2}{W}\int_0^W dy\int_0^X dx\left[K(X-x,y)-\sum_{n=1}^{\infty}K(X-x,ny)\times q^n(1-q)\right]+\frac{\sqrt{X^2+W^2}-X}{W}, \quad (19)$$

where we defined $q=p[ny/(X-x)]$ for simplicity.

The above equations are general for any angular dependence of the specularity parameter p. Here we evaluate the analytical limiting value of the conductance for a specularity parameter p which is *independent* of angle. This time $\omega(x)$ is not a sum of $\frac{1}{2}$, and an odd function of $x-L/2$ for a plausible reason: if $p\to 1$, then $\omega(x)=0$ for all z. But we again expand ω as $\omega(x)=c_0+c_1(x-L/2)+c_2(x-L/2)^2/2!+c_3(x-L/2)^3/3!+\cdots$, and find that the even coefficients and odd coefficients are independent and only the odd coefficients contribute to the current at wire center $x=L/2$. The result for small $a=L/(2W)$ is

$$g\to 1-(1-p)\zeta_1(p)a+\frac{1-p}{2}[\zeta_2(p)+(1-p)\zeta_1(p)^2]a^2, \quad (20)$$

where we defined power series functions of p, $\zeta_m(p)=\Sigma_{n=1}^{\infty}p^{n-1}/n^m$. Here $(1-p)\zeta_1(p)=-(1-p)/p\ln(1-p)$ is a monotonically decreasing function of p, and $\zeta_2(p)=(1/p)\int_0^p dt(1/t)\ln[1/(1-t)]$. The approximated limit for large a is

$$g\to\frac{3}{7}\frac{1-p}{a}\sum_{n=1}^{\infty}p^{n-1}n\ln\frac{2a}{n}, \quad (21)$$

where again $\ln a/a$ characteristics are found, and the prefactor approaches $\frac{1}{2}$ with higher-order approximation. The result for an infinite wire was $\rho_0/\rho\to\frac{3}{4}(1-p)\kappa\ln(1/\kappa)$.[34]

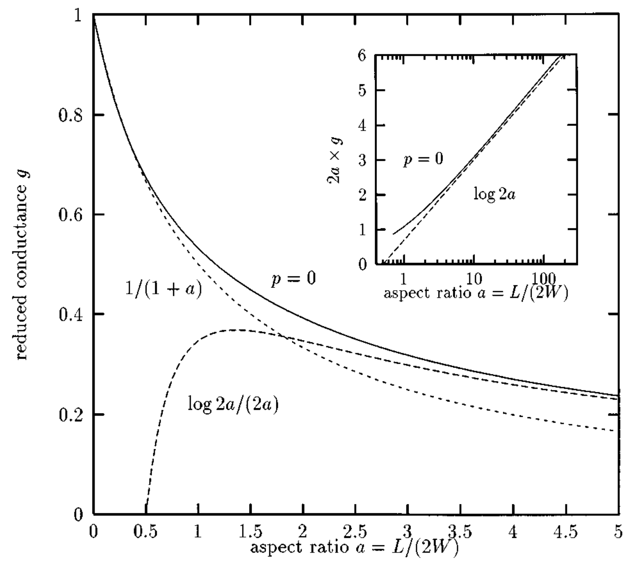

FIG. 2. Aspect ratio a dependence of the conductance g for completely diffuse boundary. Dashed curve and dotted curves are for $a\to 0$ and $a\to\infty$ limits, respectively. The inset shows a vs $2ag$ for larger range of a, and the dashed line is the limiting approximations $\ln 2a$ for $a\to\infty$ limit.

V. NUMERICAL RESULTS

The integral equation (10) can also be evaluated by a numerical method. First discretize the x length L into N points, and transform the integral equation into a $N\times N$ matrix equation. By matrix inversion or by an iteration method, we can solve ω in an N-dimensional vector form. We compared the result obtained by power-series expansion with that obtained by the numerical method with $N=1000$ (typically), and found that they coincide within 5% even for the largest aspect ratio $a=30$. This numerical method has an advantage in that we can consider any angular dependence of $p(\phi)$.

First let us show the conductance g depending on the aspect ratio a for the completely diffuse boundaries in Fig. 2. The inset shows the plot of $\ln 2a$ vs $2a\times g$ for a wider range of a in order to demonstrate the $\ln 2a/(2a)$ dependence. There have been several empirical selections of the specularity parameter. We take two models: one with angularly independent p (model A) and other in which p depends on the angle as $\exp[-(\alpha\sin\phi)^2]$ (model B).[25,37] In model B the flux incident at the glancing angles ($\theta\to 0,\pi$) is always specularly scattered. Figure 3 shows the conductance for several specularity parameters. Two sequences of results for models A and B are shown. Comparing these two sets of results, we find that the conductance for model A has a shorter tail for large a than does the conductance for model B. This can be un-

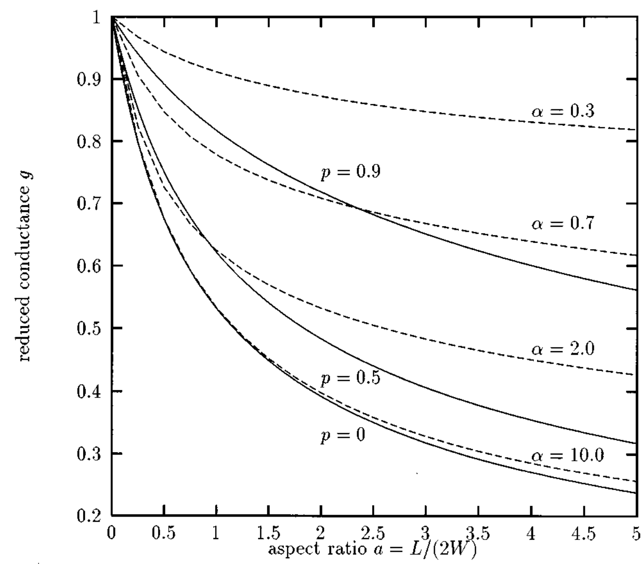

FIG. 3. Conductance g versus aspect ratio for several specularity parameters. See the text for the definition of α.

derstood because the dominant current component in wires with a large aspect ratio is flux with a smaller incident angle, which rarely suffers backscatterings in model B. Akera and Ando also evaluated the effect of roughness scatterings on the electron level population by the semiclassical Boltzmann equation. They found that most of the current is carried by the electrons in lower subbands for zero magnetic field,[10] although quantitative comparison with our result is not possible.

We also evaluated the angular distribution of current emitted from the wire into the right reservoir (drain) by calculating current at $x=L$. The current with a larger angle component suffers diffuse scattering more frequently. Therefore, the emission angular profile has a narrow peak at the angle parallel to x. This effect is enhanced for the angularly dependent p (model B) for the same reason as given in the previous paragraph. Several angular distributions at $x=L$, $\rho(\phi)$ defined by $\int_0^W dy u(L,y,\phi)/W$ are shown in Fig. 4. The current distribution in the drain measured at a distance much larger than W is given by $\rho(\phi)\cos\phi$ if we can neglect bulk scatterings. This diffuse-scattering-mediated *collimation* effect was analyzed by the billiard model, and was used to explain why the bend resistance peak is larger than the expected value, which is evaluated with assuming uniform injection into the cross geometry.[29–31] The collimation effect is larger for longer and more diffuse wires. Furthermore, an M-shaped peak is found in the distribution derived using model B. The flat region near $\phi=0$ is the direct injection from the source. The side peaks are the sum of the current injected at the glancing angle, which is scattered once almost specularly, and the diffusively scattered current after injected almost normal to the boundaries.[31]

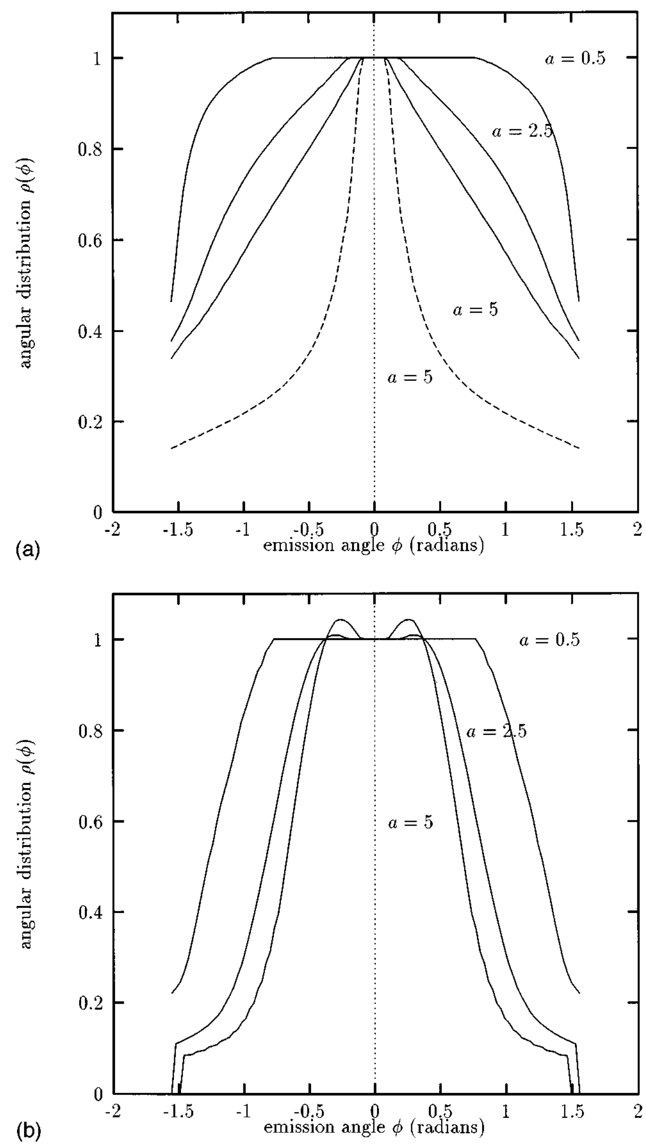

FIG. 4. Angular distribution at the exit of wires with several aspect ratios assuming the constant specularity parameter $p=0.9$ (solid lines) and $p=0.5$ (dashed line) (upper figure). The lower figure is the angular distributions for angularly depending specularity parameter with $\alpha=0.7$.

VI. DISCUSSION

The most direct application of the present formulation is to analyze the transfer resistance[7,28] in a very high-mobility sample, where the resistance peak at zero-magnetic field is assumed in the first approximation as ideal bend resistance corrected with backscatterings by the diffuse boundaries in the intermediate channel of length L. Takagaki *et al.*[7] evaluated this backscattering in multilead geometry defined with Ar-ion milling in GaAs-$Al_xGa_{1-x}As$ heterostructures where l_e is $2\sim3$ μm. They fitted the experimental results in an exponential form $g\sim\exp[-L/L_B(W)]$ for $L<1$ μm (quasiballistic region), and found $L_B=5W$ for $W<0.1$ μ m. Comparing this form with Eq. (20) and equating $L/L_B(W)=(1-p)\zeta_1(p)a$, we found $p=0.8$. Sakamoto *et al.*[28] tried to demonstrate the result with a simplified billiard model. Their result is qualitatively similar to ours. As noted in Sec. V, diffuse scattering modifies the angular distribution of the current in the channel near the voltage probe (at the exit of wire or right reservoir), and then changes the probability of direct injection. Moreover, the initial distribution near the current probe (left reservoir), which has been assumed to be a uniform injection from a perfect reservoir in our analysis, is probably distorted quite a bit. But these imperfections can be corrected with minor modifications of the theory, and the present result serves as the first approximation.

The angular distribution of the current has recently been measured directly by using small superconducting quantum interference devices.[38,39] The results for rather wider ballistic quantum wires, where more than 30 subbands are occupied, show large collimation. This collimation is too large to be explained by the rounding of the opening (Horn effect), or the barrier collimation effect which arises from the reduction of carrier concentration in the constriction.[40] Part of this collimation effect might originate from the diffuse scattering, although other possibilities cannot be ruled out yet.[27]

Finally, several extensions of the present work seem possible. The first is to integrate the magnetic field effect, which was first treated phenomenologically by Thornton *et al.*[14] in order to explain the low-field magnetoresistance peak. The second extension is to include the effect of bulk scatterings. We can assume the effect of bulk scatterings and diffuse boundary scatterings independently in qualitative discussions. However, there seem to be two exceptions. One is in the approach to the limit of a long wire, where the effect of bulk scatterings become more and more important while the effect of diffuse scattering decreases logarithmically. The other is the case of electron-electron scatterings. As demonstrated by Molenkamp and deJong[23] and Gurzhi, Kalimenko, and Kopeliovich,[41] electron-electron scatterings and diffuse boundary scatterings are closely related with each other, and electron transport is affected by these two scattering processes in total. It seems it would be quite interesting to investigate these effects in a wire of finite aspect ratio. Finally, the quantum correction to the present result is an important subject. For example, the localization effect due to the bulk scattering is corrected by the properties of the boundary scattering.[42] In numerical studies, there is no long-tailed conductance versus length in quantum transport.[16,18,19] Moreover, the diffuse boundary scattering itself can cause localization and fluctuation phenomena that will be quite different from those due to bulk scatterings.[20,18,19]

In conclusion, we have investigated the semiclassical conductance of a finite straight two-dimensional wire theoretically, while neglecting the effect of bulk scattering. We presented conductance formulas for general diffuse boundary specularity parameters. The conductance depends on a (length divided by wire width) according to $1/(1+a)$ if the wire is short. We found that the conductance of wires that are long enough decreases with $\ln a/a$, which is consistent with the classical analysis of resistivity by boundary roughness scattering of an infinite wire. Collimated angular distribution at the exit of a wire is also demonstrated.

ACKNOWLEDGMENT

The authors wish to thank to Y. Takagaki for very informative discussions.

APPENDIX A: PROOF OF CURRENT CONSERVATION

Here we examine current conservation under the boundary condition specified in the text, Eqs. (8) and (9). The change of the current at $x=X$ is proportional to

$$\frac{\partial g(X)}{\partial X}\propto\int_0^W dy\int_0^{2\pi}d\phi\frac{\partial u(X,y,\phi)}{\partial X}\cos\phi \tag{A1}$$

$$=-\int_0^W dy\int_0^{2\pi}d\phi\frac{\partial u(X,y,\phi)}{\partial y}\sin\phi \tag{A2}$$

$$=\int_0^{2\pi}d\phi[u(X,0,\phi)-u(X,W,\phi)]\sin\phi, \tag{A3}$$

where we used the local current conservation condition or the Boltzmann equation Eq. (3). Then, using Eq. (8),

$$\int_0^{2\pi}d\phi\ u(X,0,\phi)\sin\phi=\int_0^{\pi}d\phi\ u(X,0,-\phi)p(\phi)\sin\phi+\int_{\pi}^{2\pi}d\phi\ u(X,0,\phi)p(\phi)\sin\phi \tag{A4}$$

$$=\int_0^{\pi}d\phi\ u(X,0,-\phi)\times[p(\phi)-p(-\phi)]\sin\phi \tag{A5}$$

$$=0, \tag{A6}$$

where we used the symmetry $p(\phi)=p(-\phi)$. Similarly, the term $\int_0^{2\pi}d\phi u(X,W,\phi)\sin\phi$ is also zero.

APPENDIX B: INTEGRAL EQUATION FOR PARTIALLY SPECULAR BOUNDARY

This appendix derives the kernel $G_p(X)$ and source term $\omega_{0p}(X)$ in the integral equation of $\omega(x)$ for a partially diffuse boundary. The second term of Eq. (16) is divided into two integrals of regions $x<X$ and $x>X$. Let us first consider $x<X$. For $0<x<X/2$, the specular part of the flux comes from the source after one reflection. For $X/2<x<2X/3$, the specular part of the flux comes from the source after two reflections, and vice versa. Noting that $x_1(\phi_{X-x})=x-(X-x)=2x-X$, and $x_n(\phi_{X-x})=x-n(X-x)=(n+1)x-nX$, we can rewrite the first part of the integrals,

$$\int_0^{X/2}dx\ G(X-x)(1-p)[\omega(x)+p]\times\int_{X/2}^{2X/3}dx\ G(X-x)(1-p)\times[\omega(x)+p\omega(2x-X)+p^2]+\cdots, \tag{B1}$$

where p is understood to be $p[W/(X-x)]$. This equation is reordered into

$$\int_0^{X}dx\ G(X-x)(1-p)\omega(x)\int_{X/2}^{X}dx\ G(X-x)(1-p)\times p\omega(2x-X)\int_{2X/3}^{X}dx\ G(X-x)(1-p)p^2\omega(3x-2X)+\cdots+D(X), \tag{B2}$$

where $D(X)$ is a sum of terms which is independent of $\omega(X)$. A similar argument is also applied to the second part of integral ($x>X$):

$$\int_X^L dx\, G(X-x)(1-p)\omega(x)\int_X^{(L+X)/2} dx\, G(X-x)(1-p)p\omega(2x-X)\int_X^{(L+2X)/2} dx\, G(X-x)(1-p)p^2\omega(3x-2X)+\cdots. \tag{B3}$$

Therefore after combining Eqs. (B2) and (B3), we obtain

$$\sum_{n=0}^{\infty}\int_{[nX/(n+1)]}^{(L+nX)/(n+1)} dx\; G(X-x)(1-p)p^n\omega[(n+1)x-nX]+D(X)$$
$$=\tfrac{1}{2}\sum_{n=1}^{\infty}\int_0^L ds\frac{W_n^2}{[(X-s)^2+W_n^2]^{3/2}}\left[1-p\left(\frac{W_n}{X-s}\right)\right]p^n\left(\frac{W_n}{X-s}\right)\omega(s)+D(X), \tag{B4}$$

where we defined $W_n=nW$, and changed the variable x to $s=(n+1)x-nX$. The constant term $D(X)$ is also summed in a closed form as

$$D(X)=\int_0^{X/2}dx\; G(X-x)(1-p)p+\int_{X/2}^{2X/3}dx\; G(X-x)(1-p)p[1-(1-p)]$$
$$+\int_{2X/3}^{3X/4}dx\; G(X-x)(1-p)p[1-(1-p)-(1-p)p]+\cdots$$
$$=\int_0^X dx\; G(X-x)[(1-p)-(1-p)^2]-\int_{X/2}^X dx\; G(X-x)(1-p)^2p-\int_{2X/3}^X dx\; G(X-x)(1-p)^2p^2-\cdots$$
$$=\int_0^X dx\; G(X-x)(1-p)-\sum_n\int_0^X dx\frac{W_n^2}{2[(X-x)^2+W_n^2]^{3/2}}\left[1-p\left(\frac{W_n}{X-x}\right)\right]^2p^n\left(\frac{W_n}{X-x}\right). \tag{B5}$$

Therefore, Eqs. (B4) and (B5) give the kernel and source of the integral equation for $\omega(x)$ when the boundary is partially diffuse.

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