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Composite-fermion Hall conductance at $\nu=\frac{1}{2}$

S. A. Kivelson
Department of Physics, University of California at Berkeley, Berkeley, California 94720
and Department of Physics, University of California at Los Angeles, Los Angeles, California 90095

D-H. Lee, Y. Krotov, and J. Gan
Department of Physics, University of California at Berkeley, Berkeley, California 94720
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We show that in the limit of vanishing bare electron mass, and in the presence of particle-hole symmetric disorder (which can be of vanishing strength), the composite-fermion Hall conductance is constrained to be $-\frac{1}{2}e^2/h$. We discuss the implications of this result for the existence and nature of a composite Fermi liquid in the lowest Landau level. [S0163-1829(97)00824-2]

The observation of a seemingly metallic dc magnetotransport[1] and the subsequent discovery of an acoustic wave anomaly[2] near $\nu=1/2$, opened a new chapter in the studies of quantum Hall effects. (Here $\nu\equiv\phi_0\overline{\rho}/B$, where $\overline{\rho}$ is the mean electron density, $\phi_0=hc/e$, and B is the externally applied magnetic field.) A *very* intriguing idea, the composite-fermion theory, has been put forward to explain these phenomena.[3,4] In this theory, each electron is represented as a composite fermion[5] carrying two quanta of fictitious magnetic flux which pierce the physical plane in the direction opposite to that of the real magnetic flux. Formally, this transformation maps the problem of electrons in a strong magnetic field onto a system of ''composite fermions'' moving in the same external field while interacting with a fluctuating ''statistical'' gauge field governed by a Chern-Simons action.[6]

In the absence of disorder, the ground-state electron density is uniform. Thus at the *mean-field* level, the averaged statistical magnetic field, $\overline{b}=2\phi_0\overline{\rho}=|B|$, cancels the external one, and the composite fermions see no net field. When one tries to improve upon the mean-field theory (MFT) by including the fluctuations of the statistical magnetic field, one encounters divergences.[3] Attempts to sum these divergences have led to suggestive, but so far inconclusive results.[7–11] Despite this difficulty, it has been *conjectured* that the full effect of statistical-gauge-field fluctuations is to renormalize the parameters (perhaps in a singular way) of a zero field ''composite Fermi liquid.''

At this point it is useful to differentiate two concepts. The first is the composite-fermion approach, and the second is the composite Fermi-liquid theory. The former is simply an exact reformulation of the original problem, but the latter is a conjecture about the final solution. It is also worth pointing out that although the magnetic field is canceled out at the mean-field level in the composite-fermion approach, there is no symmetry reason to expect $\sigma_{xy}^f=0$ since the full composite-fermion action lacks time reversal symmetry. It is *our understanding*, however, that the composite Fermi-liquid conjecture requires that $\sigma_{xy}^f=0$.

In any case, it has been argued that the transport properties of the electrons near $\nu=1/2$ simply reflect the underlying Fermi-liquid (or, possibly, the marginal Fermi liquid) behavior of the composite fermions in *zero* magnetic field. This intriguing picture acquired further support when Fermi-surface-like features were observed in recent experiments.[12]

The principal purpose of the present paper is to reexamine the Fermi-liquid picture when there is a finite (but possibly arbitrarily small) amount of disorder. In the presence of disorder, the ground-state electron density is no longer uniform. In the regions of high electron density, the statistical magnetic flux overcompensates the external one, and in the low density region it undercompensates. Thus from the viewpoint of the composite fermions, the plane is divided into regions with net effective fields opposite to each other. Nominally, if the average field is zero, one would expect a vanishing Hall conductance for composite fermion (i.e., $\sigma_{xy}^f=0$). This naive expectation is *incorrect* because of the *correlation* between the composite-fermion density (which is equal to the electron density) and the effective magnetic field. Thus even if there are as many regions with the positive and negative net magnetic field, one expects the composite-fermion Hall conductance to be negative (i.e., $\sigma_{xy}^f<0$), since there are more fermions seeing the negative than the positive field. The existence of this correlation between the flux and charge also raises questions concerning the validity of models of composite-fermion transport in which this correlation is ignored.[4]

Here, we shall concentrate on a particular limit (the limit where the bare electron mass m_b is vanishingly small, and the disorder potential is particle-hole symmetric[13]), where we will show that at $\nu=1/2$ the *electron* Hall conductivity is

$$\sigma_{xy}=\frac{1}{2}\frac{e^2}{h}. \tag{1}$$

The *electron* resistivity tensor is related to the *composite-fermion* resistivity tensor by a ''connection formula'' (which will be discussed below),

$$\rho_{xx}=\rho_{xx}^f,$$

$$\rho_{yx}=2\frac{h}{e^2}+\rho^f_{yx}; \tag{2}$$

combining this expression with Eq. (1), we will show that, so long as $\sigma_{xx}\neq 0$, it follows that

$$\sigma^f_{xy}=-\frac{1}{2}\frac{e^2}{h}, \tag{3}$$

independent of the strength of the disorder or whether the temperature is zero or finite.

Disorder (or some other interaction which breaks Galilean invariance) is *essential* to establish the above constraint on σ^f_{xy}. Galilean invariance requires the *electron* resistivity tensor to be

$$\rho_{xx}=0,$$

$$\rho_{yx}=2\frac{h}{e^2}. \tag{4}$$

The above result combined with the connection formula Eq. (4) implies that

$$\rho^f_{xx}=\rho^f_{yx}=0. \tag{5}$$

In this latter case the composite-fermion resistivity tensor is noninvertable. Therefore, our conclusions concerning σ^f_{xy} apply in the limit that the disorder tends to zero, but may not apply in the absence of disorder.

The remainder of the paper is organized as follows: In Sec. I we derive Eq. (1). In Sec. II, we show how Eq. (3) follows from Eq. (1). In the remainder of the paper, we attempt to understand the implications of this result on the fundamental character of the physical state at $\nu=1/2$. Section III contains some discussion of the nature of the ground state in the presence of disorder in the $m_b\to 0$ limit. In Sec. IV, we examine the problem of computing σ^f_{xy} in the absence of disorder, but including the perturbative effects of fluctuations about the mean-field state. We find that, at least to the lowest order, the mean-field result $\sigma^f_{xy}=0$ is unchanged. We also discuss our reasons for believing that, even though the present results are derived in a way that depends critically on the existence of a disorder potential, they raise important questions concerning the nature of the ground state in the lowest Landau level (LLL) at $\nu=1/2$, even in the absence of disorder. Section V is a discussion of some other perturbative results in the absence of disorder; formally this section is a digression from the main thrust of the paper, except in that it sheds some light on the nature of the composite-Fermion ground state at $\nu=1/2$. Section VI contains a discussion of results, some speculations concerning their possible implications, and a discussion of their possible relevance to experiment. For the remainder of the paper, we adopt units in which $e/c=k_B=\hbar=1$.

I. THE HALL CONDUCTANCE AT $\nu=1/2$ IN THE LIMIT OF VANISHING BAND MASS

A. Intuitive discussion: Particle-hole symmetry in the lowest Landau level

In the limit of small m_b, or equivalently when the cyclotron frequency, $\omega_c=B/m_b$ is the largest energy in the problem, we expect that the low-lying eigenstates for $\nu\leq 1$ can be constructed out of states lying entirely in the lowest Landau-level plus perturbative effects of Landau-level mixing. It is easy to see[14] that even in the presence of electron-electron interactions and particle-hole symmetric disorder,[13] the Hamiltonian projected onto the lowest Landau level is particle-hole symmetric. This is roughly, but not quite, adequate for our present purposes. What we seek to investigate is the nature of this symmetry for the full problem, in the physically meaningful limit $m_b\to 0$; intuitively, this limit is related to Landau-level projection, but there are effects of Landau-level mixing which survive in this limit,[15] especially when the current operator is involved.

Nonetheless, we will start our discussion by assuming that particle-hole symmetry is an exact low-energy symmetry, and discuss its consequences. (In the following subsections we will demonstrate that, subject to some reasonable assumptions, the inferences we have made can be substantiated.)

Since the ground state at $\nu=1$ is unique, it can play the role of a reference vacuum equally well as the state with no electrons. What this means is that a system with electron concentration $\nu<1$ can be viewed, equivalently, as a system of holes with concentration $1-\nu$. The corresponding conductivity tensor as a function of filling factor can be expressed in either electron or hole language as

$$\sigma(\nu)=\sigma(1)+\sigma^h(1-\nu), \tag{6}$$

where $\sigma^h(1-\nu)$ is the hole conductivity tensor at hole concentration $1-\nu$ (electron concentration ν). Particle-hole symmetry, in turn, implies that

$$\sigma_{xx}(\nu)=\sigma^h_{xx}(\nu) \tag{7}$$

and

$$\sigma_{xy}(\nu)=-\sigma^h_{xy}(\nu). \tag{8}$$

From these equations, we can exactly relate the Hall conductivity at $\nu=1/2$ to the Hall conductivity at $\nu=1$:

$$\sigma_{xy}(\nu=1/2)=(1/2)\sigma_{xy}(\nu=1). \tag{9}$$

Since $\sigma_{xy}(\nu=1)=(e^2/2\pi)$, Eq. (9) implies Eq. (1).

Equation (9) is a strong result, and it applies not only to the dc conductivity, but to finite frequency, ω, finite wave number, $\vec{k}$, and finite temperature, T, to the extent that none of these are large enough to imply substantial Landau-level mixing and hence breaking of the particle-hole symmetry; indeed, we expect corrections due to finite temperature and finite frequency to vanish in the $\omega_c\to\infty$ limit, and finite k corrections to go like $(kl)^2$, where $l=\sqrt{B}$ is the magnetic length.

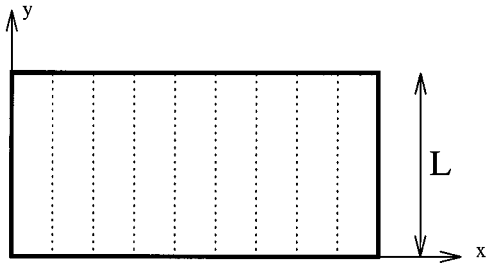

FIG. 1. The geometry for the quantum Hall system.

B. Particle-hole symmetry at zero temperature

In this subsection, we show that in the limit $m_b \rightarrow 0$ and in the presence of particle-hole symmetric disorder,[13] the $T=0$ and $|\vec{k}|=0$ electronic Hall conductivity is given by Eq. (1), if the following conditions are fulfilled. (i) As a function of m_b there are no nonanalytic contributions to the Hall conductivity which survive in the $m_b \rightarrow 0$ limit, and (ii) there is no spontaneous particle-hole symmetry breaking.

The general expression for the Hall conductivity is given by the Kubo formula

$$\sigma_{xy}(\omega)=\frac{A}{\omega}\int dt e^{i\omega t}\theta(t)\langle\langle g[U]|[J_x(t),J_y(0)]|g[U]\rangle\rangle_{\rm dis}, \tag{10}$$

where A is the total area, $\theta(t)$ is the Heaviside function, and J_α, the averaged current density, is given by

$$J_\alpha=\frac{e}{A}\int d^2r\frac{1}{2m_b}\left[\Psi^\dagger(r)\left(\frac{\partial_\alpha}{i}-A_\alpha\right)\Psi(r)+\text{H.c.}\right]. \tag{11}$$

In the following we shall consider $\omega \ll \omega_c$. In Eq. (10) $|g[U]\rangle$ is the ground state for a given external potential $U(r)$, and $\langle\ \rangle_{\rm dis}$ denotes the disorder average. [To simplify the notation, we shall henceforth leave implicit the dependence of eigenstates on $U(r)$.] Next we choose the eigenstates of the kinetic energy operator as a basis and expand $\Psi(r)=\Sigma_{nk}\psi_{nk}(r)a_{nk}$, where a_{nk} annihilates an electron in the state

$$\psi_{nk}=\frac{1}{(Ll)^{1/2}}e^{iky}\chi_n\left(\frac{x-kl^2}{l}\right). \tag{12}$$

Here we have chosen the gauge $\vec{A}=(0,Hx)$; L is the size of the system in the y direction (Fig. 1) and

$$\chi_n(x)=(2^n n!\,\pi^{1/2})^{-1/2}H_n(x)e^{-x^2/2}, \tag{13}$$

where $H_n(x)$ are the Hermit polynomials. After some trivial algebra we obtain

$$J_x=\frac{e}{iAm_b l}\sum_{n,k} X_n[a^\dagger_{nk}a_{n+1k}-a^\dagger_{n+1k}a_{nk}], \tag{14}$$

where $X_n=\int dx\chi_n(x)\partial_x\chi_{n+1}(x)$. Similarly,

$$J_y=-\frac{e}{Am_b l}\sum_{n,k} Y_n[a^\dagger_{nk}a_{n+1k}+a^\dagger_{n+1k}a_{nk}], \tag{15}$$

where $Y_n=\int dx x\chi_n(x)\chi_{n+1}(x)$. Note that $\vec{J}$ is purely off diagonal in the Landau-level index, but has nonzero matrix elements only between neighboring Landau levels. In particular, for our purposes, we need to know only the matrix elements, $X_0=Y_0=1/\sqrt{2}$.

Now let us consider the correlation function

$$I([U];t)=\langle g|[J_x(t),J_y(0)]|g\rangle$$
$$=\sum_\alpha \{e^{-i(E_\alpha-E_g)t}\langle g|J_x(0)|\alpha\rangle\langle\alpha|J_y(0)|g\rangle-\text{c.c.}\}, \tag{16}$$

where $|\alpha\rangle$ are the true many-body eigenstates of the system in the presence of external disorder potential $U(r)$. To proceed, let us perform a canonical transformation

$$|\psi'\rangle=e^{iT}|\psi\rangle, \tag{17}$$

so that the transformed Hamiltonian

$$H'=e^{iT}He^{-iT}=H_{-1}+H_0+\cdots \tag{18}$$

has no matrix element connecting pure LLL states with those with a higher Landau-level component.[15,16] The Hermitian operator T can be constructed as a series in m_b [which actually is an expansion in powers of the ratio of the Landau-level mixing matrix element to ω_c (Ref. 17)], as follows:

$$T=\sum_{k=1}^{\infty}(m_b)^kT_k. \tag{19}$$

Thus in Eq. (18) $H_k=O(m_b^k)$. The transformed current operator has the expansion

$$\vec{J}'=e^{iT}\vec{J}e^{-iT}=\vec{J}_{-1}+\vec{J}_0+\cdots, \tag{20}$$

where $\vec{J}_{-1}=\vec{J}$.

After the transformation, the eigenstates separate into two groups: one group $\{|\alpha_l\rangle\}$ lies entirely in the LLL, and the other $\{|\alpha_h\rangle\}$ contains higher Landau-level components. (By assumption, $|g\rangle\in\{|\alpha_l\rangle\}$.) By construction, the lowest group $\{|\alpha_l\rangle\}$ are an eigenstates of the projected Hamiltonian $H_L=P_LH'P_L$, where P_L is the operator that projects onto the subspace spanned by states in the LLL. To $O(m_b^0)$

$$H_L=\mu\int d^2r\rho_L(r)+\frac{1}{2}\int d^2rd^2r'v(r-r'){:}\rho_L(r)\rho_L(r'){:}$$
$$+\int d^2rU(r)\rho_L(r). \tag{21}$$

Here $\rho_L(r)=\psi_L^+(r)\psi_L(r)$, with $\psi_L(r)\equiv\Sigma_k\psi_{0k}(r)a_{0k}$, and U and v are the disorder and interaction potential, respectively.

Consider first the contribution to Eq. (16) due to inter-Landau-level excitations:

$$I_1([U];t)=\sum_{\alpha_h}\{e^{-i(E_{\alpha_h}-E_g)t}\langle g|J_x'(0)|\alpha_h\rangle$$
$$\times\langle\alpha_h|J_y'(0)|g\rangle-\text{c.c.}\}. \tag{22}$$

Since $|g\rangle$ lies entirely in the lowest Landau level,

$$\langle\alpha_h|J'_x|g\rangle=\frac{eX_0}{iAm_bl}\langle\alpha_h|K^+|g\rangle+O(m_b^0),$$

$$\langle\alpha_h|J'_y|g\rangle=-\frac{eY_0}{Am_bl}\langle\alpha_h|K^+|g\rangle+O(m_b^0), \tag{23}$$

where

$$K^+\equiv\sum_k a^\dagger_{1k}a_{0k}. \tag{24}$$

Thus the corresponding contribution to σ_{xy} is

$$\sigma^{(1)}_{xy}([U];\omega)=\frac{A}{\omega}\int dt\,\theta(t)e^{i\omega t}I_1(t)$$
$$=\frac{e^2}{A(m_bl)^2}\sum_{\alpha_h}\frac{\langle g|K|\alpha_h\rangle\langle\alpha_h|K^+|g\rangle}{(E_{\alpha_h}-E_g)^2-\omega^2}+O(m_b). \tag{25}$$

Let us write

$$E_\alpha-E_g\equiv\omega_c+\Delta_\alpha; \tag{26}$$

to the lowest order in m_b, we can approximate Δ_α by 0 in Eq. (25). Thus, the leading order contribution to $\sigma^{(1)}_{xy}$ in the $m_b\to 0$ limit is

$$\sigma^{(1)}_{xy}=\frac{e^2}{AB}\langle g|\sum_k a^+_{0k}a_{0k}|g\rangle, \tag{27}$$

where we have used the fact that $(m_bl\omega_c)^2=B$.

Next we look at the contribution to σ_{xy} due to intra-Landau-levels excitations:

$$\sigma^{(2)}_{xy}([U];\omega)=\frac{A}{\omega}\int dt\,\theta(t)e^{i\omega t}I_2(t), \tag{28}$$

where

$$I_2([U];t)=\sum_{\alpha_l}\{\langle g|J'_x(t)|\alpha_l\rangle\langle\alpha_l|J'_y(0)|g\rangle-\langle g|J'_y(0)|\alpha_l\rangle \times\langle\alpha_l|J'_x(t)|g\rangle\}. \tag{29}$$

To $\mathcal{O}(m_b^0)$, we can replace $\vec{J}'$ in Eq. (29) by

$$\vec{J}''\equiv P_L(\vec{J}_{-1}+\vec{J}_0)P_L=P_L(\vec{J}_0)P_L. \tag{30}$$

Thus,

$$I_2[U]=\langle g|[J''_x(t),J''_y(0)]|g\rangle+O(m_b), \tag{31}$$

and $J''_\alpha=(1/A)\int d^2r\,j''_\alpha(r)$, where[16]

$$j''_\alpha(r)=P_Lj_\alpha(r)\frac{1}{\hbar\omega_c\hat{N}/2-\hat{H}_K}(1-P_L)VP_L+P_LV(1-P_L)\frac{1}{\hbar\omega_c\hat{N}/2-\hat{H}_K}j_\alpha(r)P_L. \tag{32}$$

Here $\hat{N}$ and $\hat{H}_K$ are the particle number operator and kinetic energy operator, respectively, and V is the sum of the potential (disorder) and the two-body interaction part of the Hamiltonian. The time dependent operator $J''_x(t)$ is related to $J''_x(0)$ via

$$J''_x(t)=e^{itH_L}J''_x(0)e^{-itH_L}. \tag{33}$$

Given Eq. (32) we perform the integration over space[18] and obtain

$$J''_x=\frac{e}{A}\int d^2r\left\{\rho_L(r)\partial_yU(r)+\int d^2r_1\rho_L(r)\rho_L(r_1)\partial_yv(r-r_1)+\int d^2r_1\rho_L(r_1)\rho_L(r)\partial_yv(r_1-r)\right\} \tag{34}$$

and

$$J''_y=-\frac{e}{A}\int d^2r\left\{\rho_L(r)\partial_xU(r)-\int d^2r_1\rho_L(r)\rho_L(r_1)\partial_xv\times(r-r_1)-\int d^2r_1\rho_L(r_1)\rho_L(r)\partial_xv(r_1-r)\right\}. \tag{35}$$

At $\nu=1/2$, and when $\int d^2rU(r)=0$, the value of μ is such that $H_L[U]\to H_L[-U]$ under the *LLL p-h* transformation,

$$\psi_L(r)\to\psi^+_L(r). \tag{36}$$

Equation (36) amounts to the change

$$a_{0k}\to a^\dagger_{0k}, \tag{37}$$

and complex conjugation of the basis wave function. Under this transformation (since complex conjugation is equivalent to the transformation $y\to -y$),

$$J''_x(U)\to -J''_x(-U),$$

$$J''_y(U)\to J''_y(-U). \tag{38}$$

If

$$|g_p[U]\rangle\to|g_P[-U]\rangle, \tag{39}$$

(i.e., if there is no spontaneous particle-hole symmetry breaking), then (since $\Sigma_k[1]=AB/\phi_0=AB/2\pi$)

$$\sigma^{(1)}_{xy}[U]=\frac{e^2}{2\pi}-\sigma^{(1)}_{xy}[-U],$$

$$\sigma^{(2)}_{xy}[U]=-\sigma^{(2)}_{xy}[-U]. \tag{40}$$

Therefore

$$\sigma_{xy}\equiv\langle\sigma_{xy}[U]\rangle_{\rm dis}=\int D[U]P[U](\sigma_{xy}^{(1)}[U]+\sigma_{xy}^{(2)}[U])$$
$$=\frac{e^2}{2\pi}-\int D[U]P[U]\{\sigma_{xy}^{(1)}[-U]+\sigma_{xy}^{(2)}[-U]\}$$
$$=\frac{e^2}{2\pi}-\int D[U]P[-U]\sigma_{xy}[U]$$
$$=\frac{e^2}{2\pi}-\int D[U]P[U]\sigma_{xy}[U]$$
$$=\frac{e^2}{2\pi}-\langle\sigma_{xy}[U]\rangle_{\rm dis}\equiv\frac{e^2}{2\pi}-\sigma_{xy}\,. \tag{41}$$

In the above equation we have used the fact that the disorder is particle-hole symmetric, i.e.,

$$P[U]=P[-U]. \tag{42}$$

After restoring $\hbar$ Eq. (41) is equivalent to Eq. (1).

C. Particle-hole symmetry at nonzero temperature

The derivation presented above can be generalized to a finite temperature. In that case $\langle g|[J_x(t),J_y(0)]|g\rangle$ in Eqs. (10) and (16) is replaced by the thermal average, i.e.,

$$\langle g|[J_x(t),J_y(0)]|g\rangle\rightarrow\frac{{\rm Tr}\{e^{-\beta H}[J_x(t),J_y(0)]\}}{{\rm Tr}\{e^{-\beta H}\}}. \tag{43}$$

By making the same assumptions as in the above, we see that in the $m_b\rightarrow 0$ limit, we can evaluate the trace over states in the LLL. Thus Eq. (22) is replace by

$$I_1[U]=\frac{1}{Z_l}\sum_{\alpha_l}e^{-\beta E_{\alpha_l}}\sum_{\alpha_h}\{e^{-i(E_{\alpha_h}-E_{\alpha_l})t}\langle\alpha_l|J_x'(0)|\alpha_h\rangle\times\langle\alpha_h|J_y'(0)|\alpha_l\rangle-{\rm c.c.}\}, \tag{44}$$

where

$$Z_l\equiv\sum_{\alpha_l}e^{-\beta E_{\alpha_l}}. \tag{45}$$

To the lowest order in m_b, we again replace $\vec{J}'$ by $\vec{J}$ in Eq. (44). Again, as we did in Eq. (26), we make the replacement, valid to $O(m_b^0)$, $(E_{\alpha_h}-E_{\alpha_l})\rightarrow\omega_c$. Then

$$\sigma_{xy}^{(1)}=\frac{e^2}{AB}\frac{1}{Z_l}\sum_{\alpha_l}e^{-\beta E_{\alpha_l}}\langle\alpha_l|\sum_k a_{0k}^+a_{0k}|\alpha_l\rangle. \tag{46}$$

Finally, Eq. (31) is replaced by

$$I_2[U]=\frac{1}{Z_l}{\rm Tr}'\{e^{-\beta H_L}[J_x''(t),J_y''(0)]\}. \tag{47}$$

Here ${\rm Tr}'\{\}\equiv\Sigma_{\alpha_l}\langle\alpha_l|\ldots|\alpha_l\rangle$ denotes the partial trace over the LLL eigenstates only. At a finite temperature the condition of no particle-hole symmetry breaking is generalized to the statement that we may use Eq. (43) without including in H an infinitesimal symmetry-breaking field.

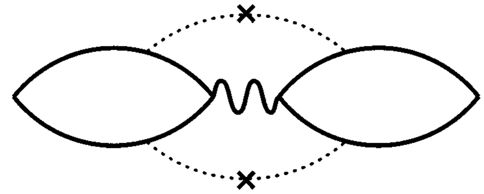

FIG. 2. An example where an impurity averaged irreducible bubble diagram does not appear after averaging the irreducible bubble diagrams for specific disorders.

II. IMPLICATIONS FOR THE COMPOSITE-FERMION CONDUCTIVITY

An important ingredient of the composite-fermion approach is the relation between the electron and composite-fermion correlation functions. It is the nature of the mapping that the density of composite fermions equals that of the electrons, but the relation between current operators is more complicated. To compute the electron current-current correlation function, we need to string together the composite-fermion ‘‘irreducible bubbles’’[19] using the Chern-Simons bare gauge propagator. As shown in Refs. 3, 19, and 6[b], this results in the relation Eq. (2) between the resistivity tensor of electrons, $\rho_{\alpha\beta}$, and that of the composite fermions, $\rho^f_{\alpha\beta}$. Physically, this expresses the fact that associated with the composite-fermion current, there is a statistical flux current, which produces a corresponding electromagnetic field (EMF) proportional to the statistical flux carried by each composite fermion times the electrical current. When $\rho^f_{xx}\neq 0$, the resistivity tensor can be inverted with the consequence that Eq. (2) is equivalent to

$$\sigma^f_{xy}=\frac{e^2}{2\pi}\,\frac{\frac{e^2}{2\pi}\sigma_{xy}-2(\sigma_{xx}^2+\sigma_{xy}^2)}{4\sigma_{xx}^2+\left(\frac{e^2}{2\pi}-2\sigma_{xy}\right)^2},$$
$$\sigma^f_{xx}=\frac{e^2}{2\pi}\,\frac{\frac{e^2}{2\pi}\sigma_{xx}}{4\sigma_{xx}^2+\left(\frac{e^2}{2\pi}-2\sigma_{xy}\right)^2}. \tag{48}$$

In the above σ_{xx} and σ_{xy} are the *impurity averaged* conductivity tensor of the electrons. $\sigma^f_{xx},\sigma^f_{xy}$ are the conductivity deduced from the *impurity averaged* bubble diagrams that are irreducible with respect to cutting a statistical-gauge propagator. Here we stress that the latter is *not* necessarily equal to first taking the statistical-gauge-propagator-irreducible bubble in fixed disorder, and then averaging over the disorder realization. For example, the diagram shown in Fig. 2 belongs to the former, while not the latter. By substituting $\sigma_{xy}=e^2/4\pi$ into Eq. (48), we obtain

$$\sigma^f_{xx}=\left(\frac{e^2}{4\pi}\right)^2\frac{1}{\sigma_{xx}},$$
$$\sigma^f_{xy}=-\frac{e^2}{4\pi}. \tag{49}$$

The above is valid so long as $\rho_{xx}\neq 0$ and when the particle-hole symmetry is maintained, so it applies with or without electron-electron interactions, for finite or infinite systems, and at zero or nonzero temperature.

III. WHAT IS THE CORRECT STATE IN THE LIMIT OF ZERO BAND MASS IN THE PRESENCE OF DISORDER?

Now the remaining question is ''what is the correct state in the limit of $m_b\rightarrow 0$ when there is a nonzero amount of particle-hole symmetric disorder?'' For that purpose let us consider the composite-boson representation where the electrons are viewed as composite bosons carrying one quantum of fictitious magnetic flux each (i.e., the $\theta=1$ boson Chern-Simons theory). Here we recall that in this representation, the Bose superfluid phase corresponds to the $\nu=1$ quantum Hall liquid, and the Bose insulator (or the vortex superfluid) phase corresponds to the electron insulator. In between we can have a particular situation where the bosons and vortices are in the same state. The latter is marked by the so-called[19] ''self-duality condition'' where

$$(\rho_{xx}^b)^2+(\rho_{yx}^b)^2=\frac{1}{(\sigma_{xx}^b)^2+(\sigma_{xy}^b)^2}=\left(\frac{2\pi}{e^2}\right)^2. \tag{50}$$

To translate this condition into a statement concerning the electronic response, we use the connection formula between the electron and composite boson resistivity tensor,[19]

$$\rho_{xx}=\rho_{xx}^b,$$

$$\rho_{yx}=\frac{2\pi}{e^2}+\rho_{yx}^b. \tag{51}$$

With this identity, it is easy to see that Eq. (50) is equivalent to Eq. (1). Thus the particle-hole symmetric condition $\sigma_{xy}=(e^2/4\pi)$ is equivalent to the statement of self-duality.[20]

One example of the self-duality of the $\theta=1$ boson Chern-Simons theory is the critical point of the $\nu=0$ to $\nu=1$ plateau transition.[19] For the latter, it was argued that

$$\sigma_{xy}^b=0,$$

$$\sigma_{xx}^b=\frac{e^2}{2\pi}. \tag{52}$$

We note that Eq. (52) constitutes a special solution to Eq. (50). Values of the conductivity consistent with Eq. (52) were found for both particle-hole symmetric and nonsymmetric disorder in numerical studies of noninteracting electrons at this transition.[21] Recently, experiments have been performed which dramatically support the notion that there is a universal resistivity tensor at the critical point, with measured values in all cases consistent with the conjectured values of the composite-boson conductivities [Eq. (52)].[22,23]

The plateau transition [Eq. (52)], being a critical point, obviously is infrared unstable with respect to a single perturbation [which turns out to be $\sigma_{xy}-(e^2/4\pi)$]. The fact that it is experimentally observable, implies that given the constraint that $\sigma_{xy}=e^2/4\pi$, it is infrared stable. There are infinitely many other possible solutions to Eq. (50),[24] with all of them consistent with $\sigma_{xy}=(e^2/h)$; the question is whether any of them corresponds to an infrared stable fixed point in the presence of disorder (which, again, can be vanishingly small). If the answer is no, then even in the limit of vanishing disorder, the ground state of the system at $\nu=1/2$ is asymptotically equivalent to the critical state at the $0\rightarrow 1$ plateau transition. If the answer is yes, much new physics remains to be explored.

IV. PERTURBATIVE RESULTS FOR COMPOSITE-FERMION HALL CONDUCTIVITY IN THE ABSENCE OF DISORDER

In this part of our paper we address the case where there is no disorder. As we stressed earlier, in that case the fact that $\sigma_{xy}=(e^2/4\pi)$ does not uniquely determine the value of σ_{xy}^f. For example, so long as $\sigma_{xx}^f=\infty$, σ_{xy}^f can have any finite value. In particular, $\sigma_{xx}^f=\infty$ and $\sigma_{xy}^f=0$ is a perfectly legitimate solution.

In the following we shall compute σ_{xy}^f perturbatively. The starting point of our subsequent analysis is the composite-fermion Euclidean Lagrangian:

$$L[\overline{\psi},\psi,a]=\int d^2x\,\overline{\psi}(\partial_0+ieA_0-ia_0)\psi-\frac{1}{2m_b}\int d^2x\,\overline{\psi}(\vec{\nabla}+i\vec{A}-i\vec{a})^2\psi+L_a[a], \tag{53}$$

where

$$L_a=\frac{1}{8\pi^2\theta^2}\int d^2x d^2x'[b(x,t)-\overline{b}]v(x-x')[b(x',t)-\overline{b}]+\frac{i}{4\pi\theta}\int d^2x\,\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda. \tag{54}$$

$\overline{\psi}$ and ψ are the Grassmann fields associated with the composite fermions; A_μ and a_μ are the external and statistical-gauge fields, respectively; $b=\vec{\nabla}\times\vec{a}$; $v(x-x')$ is the bare interaction between electrons; $\overline{b}\equiv 2\pi\theta\overline{\rho}$ is the averaged statistical magnetic field. Moreover, we have made use of the Chern-Simons constraint that $b(x,t)=2\pi\theta\rho(x,t)$. By rescaling space, time, and the fermion fields, so that $x\rightarrow k_Fx, t\rightarrow tk_F^2/m_b$, and $\psi,\overline{\psi}\rightarrow k_F^{-1}\psi,k_F^{-1}\overline{\psi}$ (here $k_F\equiv\sqrt{\rho/\pi}$) one can easily prove that in Eqs. (53) and (54) the only dimensionless parameters are θ, $\alpha\equiv\hbar\omega_c/E_c$, and $\overline{\rho}\theta/2\pi B$, where $E_c=e^2/2\epsilon\sqrt{\rho/\pi}$ is the typical strength of the Coulomb interaction. Here, we will consider only the case in which the magnetic field satisfies the commensurability condition $2\pi\overline{\rho}\theta/B=1$, so that at the mean-field level the net effective magnetic field seen by the composite fermions is zero.

Within the class of models described by Eqs. (53) and (54), the problem of physical interest corresponds to $\theta=2$, while the problem is simple in the limit $\theta\rightarrow 0$ with α fixed. In that limit, the composite fermions are the bare electrons, and the MFT is exact.[7] The question we are trying to address is ''what are the fluctuation corrections to this mean-field picture?'' In carrying out the calculations, we choose to work in Coulomb gauge, in which the gauge-field propagator,

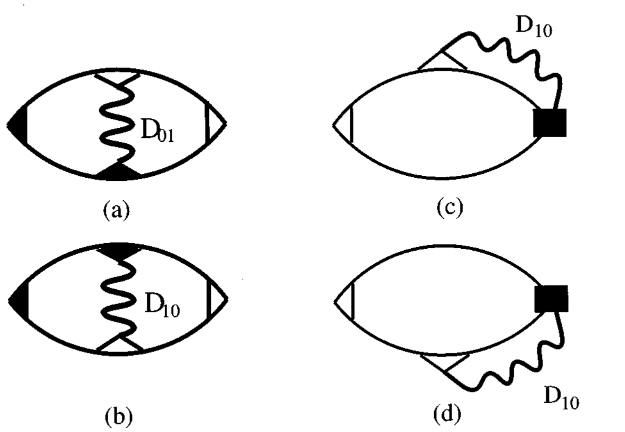

FIG. 3. Feynman diagrams for σ^f_{xy}. For $q\to 0,\omega\to 0$, while $\omega \gg q$, each individual graphs in (a)–(d) vanishes. For $q\to 0,\omega\to 0$, while $\omega \ll q$, the diagrams (a), (b) cancel diagrams (c), (d). The diagrams corresponding to self-energy insertions vanish due to symmetry.

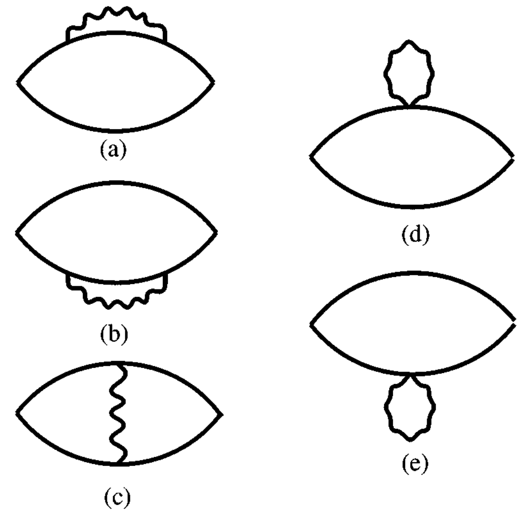

FIG. 4. Feynman diagrams for $\Pi(q_0,\vec{q})$. For logitudinal gauge fluctuations, diagrams (d) and (e) are absent.

D_{ij}, is a 2×2 matrix, with $j=0,1$ representing the time and space components, respectively.

We have calculated σ^f_{xy} perturbatively by evaluating the Feynman diagrams shown in Fig. 3. In that figure the wavy line represents the mixed gauge propagator D_{01} and D_{10}. The open triangle, solid triangle, and square represent the density, current, and the diamagnetic vertices, respectively. To the lowest order in θ and α we can use the bare gauge propagator, $D_{00}(q_0,\vec{q})=V(\vec{q})$, $D_{11}=0$, and $D_{10}(q_0,\vec{q}) = D_{01}(q_0,\vec{q}) = i2\pi\theta/|q|$, where $V(q)$ is the Fourier transform of $v(\vec{r})$. In this case, since D_{01} does not depend on frequency, the integration can be easily done. Let ω and $\vec{q}$ be the external frequency and momentum, respectively. We have looked at two limits: (i) $|\vec{q}|\to 0$ first and $\omega\to 0$ second (this is the canonical limit for defining the conductivities), and (ii) $\omega\to 0$ first and $|\vec{q}|\to 0$ second. In case (i) all the individual graphs shown in Figs. 1(a)–1(d) are zero. In case (ii) the contributions to σ^f_{xy} from Figs. 1(a), 1(b), and 1(c), 1(d) are $\pm\theta e^2/16\pi$, respectively; thus the net result is again zero. (We note that for this case we found that the characteristic momentum carried by the gauge line is of order k_F.) Since $|\vec{q}|\neq 0$ breaks Galilean invariance, we regard the value of σ^f_{xy} in case (ii) as a more stringent test of whether the time reversal symmetry of composite fermions is restored.

Therefore to this order we obtain $\sigma^f_{xy}=0$. This result is *consistent* with the notion of a composite Fermi liquid in zero magnetic field. *If* this were true to all orders, i.e., $\sigma^f_{xy}=0$ in the absence of disorder, we would be left with the following situation: In the limit of $m_b\to 0$, the composite-fermion Hall conductance in the presence of particle-hole symmetric disorder of vanishing strength would differ by $-e^2/4\pi$ from its value in the absence of disorder. The singular behavior in the limit of zero disorder is, of course, not unprecedented; For noninteracting electrons in two dimensions, the zero-temperature conductivity is infinite in the absence of disorder, and 0 (due to localization) for arbitrarily small disorder. However, this behavior is due to a subtle, infrared instability of the Fermi-liquid fixed point in two dimensions, and can be circumvented by considering a finite size system, or a system at finite temperature, in which case the zero disorder and vanishing disorder results coincide. The situation for the composite fermions is, we believe, fundamentally different. This is because the fact that particle-hole symmetry implies $\sigma_{xy}=(e^2/4\pi)$ does not rely on either the zero temperature or the thermodynamic limits; the fact that $\sigma^f_{xy}\neq 0$ is not a delicate infrared phenomenon. For this reason, *we believe* (without proof) that the results in the presence of weak disorder are pertinent to understanding the properties of the system in the absence of disorder.

V. DIGRESSION: OTHER PERTURBATIVE RESULTS

In this section we report some other perturbative results that we have obtained. These results do not directly address the question of σ^f_{xy} in the pure system, but do shed some light on other properties of this system.

We shall concentrate on the density-density and current-current correlation functions. The effects of the bare D_{00} are identical to those of a static Coulomb interaction. As is customary in this case, a random phase approximation (RPA) resummation is performed to screen D_{00} and D_{11}. If one uses the renormalized D_{00} and D_{11} to compute the one-loop corrections to the composite-fermion self-energy, $\Sigma(q_0,\vec{q})$, the contribution from longitudinal fluctuations, i.e., those which involve D_{00}, diverges logaritmically with the size of the system for fixed q_0 and $\vec{q}$.[3] The contribution from transverse fluctuations, i.e., those involving D_{11}, are regular in the system size, but contribute a logarithmically diverging correction to the effective mass.[3]

In the following we shall prove that to the same level of approximation in the density-density and current-current correlation functions (see Fig. 4), the *divergent* contribution to the self-energy from longitudinal gauge-field fluctuations is exactly canceled by the corresponding vertex correction for *all* $\vec{q}$ and ω. However, a logarithmic singularity from transverse gauge fluctuations stays in these response functions, but *only* at wave vector $q=2k_F$.[10,11]

To begin with, let us recall the origin of the divergent contribution to the composite-fermion self-energy.[3] Since the divergence originates from the high-energy, small momentum region ($\omega \gg q$), we can use the following expression for the RPA screened D_{00}:

$$D_{00}(q)=\frac{2\pi\theta}{q^2}\frac{q_0^2\omega_c}{q_0^2+\omega_c^2}. \quad (55)$$

Substituting this expression into the formula for the lowest order self-energy correction, we obtain

$$\Sigma(k)=-\int\frac{d^3q}{(2\pi)^3}G_0(k-q)D_{00}(q). \quad (56)$$

From Eq. (56) a singular term can be extracted:

$$\Sigma(k)=\frac{\omega_c\theta}{2}\ln(q_{\max}L)\frac{\varepsilon_k-ik_0}{\omega_c+\mathrm{sgn}(\varepsilon_k)(\varepsilon_k-k_0)}. \quad (57)$$

Now let us consider the analogous contribution to the density-density correlation function, $\langle\rho\rho\rangle$. To the lowest order $\langle\rho\rho\rangle$ is given by the sum of three diagrams Figs. 4(a), 4(b), 4(c). The corresponding analytic expressions are

$$\Pi_1=-\int\frac{d^3p}{(2\pi)^3}G_0(p)G_0^2(p+q)\Sigma(p+q), \quad (58)$$

$$\Pi_2=-\int\frac{d^3p}{(2\pi)^3}G_0^2(p)G_0(p+q)\Sigma(p), \quad (59)$$

$$\Pi_3=-\int\frac{d^3p}{(2\pi)^3}G_0(p)G_0(p+q)\Gamma(p,p+q), \quad (60)$$

where the dressed vertex Γ is given by

$$\Gamma(p,p+q)=-\int\frac{d^3k}{(2\pi)^3}G_0(p+q+k)G_0(p+k)D_{00}(k). \quad (61)$$

It is straightforward to show that the following relation holds between the divergent contributions to Σ and Γ:

$$\Gamma(p,p+q)=\frac{\Sigma(p)-\Sigma(p+q)}{iq_0-\varepsilon_{p+q}+\varepsilon_p}. \quad (62)$$

Substituting this expression into Eqs. (58)–(60), and using the identity

$$G_0(p)G_0(p+q)=\frac{G_0(p)-G_0(p+q)}{iq_0-\varepsilon_{p+q}+\varepsilon_p}, \quad (63)$$

one can show that

$$\Pi=\sum_{i=1}^{3}\Pi_i=-\int\frac{d^3p}{(2\pi)^3}\frac{G_0^2(p)\Sigma(p)-G_0^2(p+q)\Sigma(p+q)}{iq_0-\varepsilon_{p+q}+\varepsilon_p}. \quad (64)$$

The above expression vanishes after integration over p_0 due to the analytic structure of the integrand, that is because the poles of $G_0(p)$ and $\Sigma(p)$ are on the same side of the real axis.

Now we turn to the current-current correlation function $\Pi^{\alpha\beta}=\langle j_\alpha j_\beta\rangle$. To get $\Pi_1^{\alpha\beta},\Pi_2^{\alpha\beta},\Pi_3^{\alpha\beta}$ associated with the diagrams in Figs. 4(a), 4(b), 4(c), we need to insert current vertices $p_\alpha(p+q)_\beta$, $p_\alpha(p+q)_\beta$ into Eqs. (58), (59), and $(p+k)_\alpha(p+q)_\beta=p_\alpha(p+q)_\beta+k_\alpha(p+q)_\beta$ into Eq. (61). Since the $k_\alpha(p+q)_\beta$ term in the last expression does not produce any divergence, it can be neglected. Therefore to obtain the singular contributions to $\Pi_1^{\alpha\beta},\Pi_2^{\alpha\beta},\Pi_3^{\alpha\beta}$, all we need to do is to multiply the integrands in Eqs. (58)–(60) by the factor $p_\alpha(p+q)_\beta$. Since the last operation does not affect the pole structure; the proof goes through as before and the singular contribution again vanishes.

Now, we turn to the contributions to these correlation functions from the transverse gauge-field fluctuations, where the singular contributions from the self-energy and vertex corrections do not cancel for $|\vec{q}|=2k_F$. (They do cancel at other $|\vec{q}|$.[10,11,25]) The graphs used in that calculation are summarized in Fig. 4. The result for the $2k_F$ density-density correlation function is given by

$$\Delta\Pi(\omega,2k_F)/\Delta\Pi_0(\omega,2k_F)=1+\theta\alpha C_1\ln\left[\frac{E_F}{|\omega|}\right]. \quad (65)$$

Here $\Pi(\omega,q)\equiv\int d^2x dt e^{i(\omega t-\vec{q}\cdot\vec{x})}\langle T[\rho(x,t)\rho(0,0)]\rangle$ with Π_0 being the density-density correlation function of free electrons, $\Delta\Pi(\omega,q)=\Pi(\omega,q)-\Pi(0,q)$, and $C_1=-(1/\pi)$ C1$=-(1/\pi)[\frac{1}{4}+\ln(\alpha\theta/2)]$. The above result shows that small α, hence *strong* Landau-level mixing, tends to stabilize the composite-fermion mean-field theory against divergent corrections arising from the transverse gauge-field fluctuations.

To summarize, the perturbative results for the pure system are consistent with the existence of time reversal symmetry in the long wavelength low-energy properties of the composite fermions. The only sign of non-Fermi-liquid behavior is Eq. (65). Whether this singularity signals a true asymptotic state that lacks time reversal symmetry, cannot be determined on the basis of our results. However, along these lines, we would like to point out a *possible* implication of Eq. (65), i.e., that at finite Landau-level mixing, there exists a crossover temperature scale, T_{cr}, above which all divergent corrections to composite Fermi-liquid behavior are numerically insignificant.[10] If this interpretation is correct, then Eq. (65) suggests that this temperature is exponentially small in the limit of a large amount of Landau-level mixing, so there would exist a broad temperature range, $T_{cr}\ll T\ll\omega_c$, in which Fermi-liquid behavior would be observable.

Finally, the fact that the divergent self-energy correction from D_{00} is canceled by the vertex correction for *all* external momenta leads us to the following tentative conclusion: single composite-fermion excitations (and by extenuation, probably any excitation with net ''statistical charge'') are *not* part of the physical spectrum. Instead, the physical excitations are *statistical charge-neutral* particle-hole excitations.[25] Of course, we have proven the consistency of this viewpoint only to the lowest order in perturbation theory, so at this point we can only conjecture that it remains valid more generally.

VI. FINAL DISCUSSION

In this section we restore $e,c,\hbar$, and k_B. For real systems there *is* appreciable Landau-level mixing $(\alpha \sim 1)$. Thus an important question is ''what does the LLL and particle-hole symmetry constraint have to do with reality?'' (Here we should stress that although for real systems, it is not clear that $\sigma_{xy}^f = -\frac{1}{2}e^2/h$, but the general observation that disorder destroys flux cancellation and hence makes $\sigma_{xy}^f \neq 0$ should still be generically true.)

One way to address this question, is to examine it in the light of some recent experimental results of Wong and Jiang.[26] In that study[26] Wong and Jiang have attempted to map out the nature of the global, zero-temperature phase diagram in the density-filling factor plane in the neighborhood of $\nu = 1/2$ using gated GaAs heterojunctions with mobilities in the range $\mu \leq 2 \times 10^6$ cm^2/V s. (Since the mobility is a monotone increasing function, it is useful to think of varying the density as varying the degree of disorder.) Wong and Jiang have identified a line, which can be unambiguously associated with the $0 \to 1$ plateau transition, on which the full conductivity tensor (or resistivity tensor) is apparently temperature independent; moreover, everywhere on this line, $\sigma_{xx} \approx \sigma_{xy} \approx (1/2)e^2/h$, consistent with theoretical expectations. This line lies at $\nu \approx 1$ in the low mobility (high disorder) limit, and approaches $\nu = 1/2$ as the disorder is decreased. In the highest mobility samples, however, the boundary of the $\nu = 1$ phase can no longer be clearly identified, possibly due to finite temperature effects. (The lowest temperature in this experiment is 50 mK.) A similar line has been identified at $\nu < 1/2$ corresponding to the $0 \to 1/3$ plateau transition, on which $\sigma_{xx} \approx (1/10)e^2/h$ and $\sigma_{xy} \approx (3/10)e^2/h$, independent of temperature and density. In addition to these familiar phase boundaries, two other characteristic behaviors have been observed, which can be used to map out lines in the phase diagram of, as yet, undetermined meaning. One such line is more-or-less parallel to the density axis at $\nu \approx 1/2$, and occurs only at relatively high mobilities: On this line, $\sigma_{xy} = (1/2)e^2/h$, independent of temperature and density, while σ_{xx} varies with density, and is still temperature dependent, even at the lowest temperatures. At the low density end of this line, σ_{xx} approaches $(1/2)h/e^2$ and becomes ever more weakly temperature dependent, i.e., this line is apparently the continuation of the $0 \to 1$ phase boundary. However, for high density samples, the magnitude of σ_{xx} is about $(0.08)e^2/h$ at the lowest temperatures, where it is still quite noticeably temperature dependent. This result suggests that $\sigma_{xy} = e^2/2h$, i.e., particle-hole symmetry, is more robust than the universal dissipative transport. [This is also consistent with recent experiments on the nonlinear transport near quantum Hall transitions[20] which reveal that a form of self-duality (which for the $1 \to 0$ transition is the same as particle-hole symmetry) is observed over a much wider range of filling factors than the critical behavior, itself.] It remains to be seen whether upon further cooling σ_{xx} rises to the universal value (as would be expected if this is indeed the continuation of the $0 \to 1$ phase boundary). Finally, another line is observed on which $\rho_{xy} \approx 2h/e^2$, and is approximately temperature independent. (These two lines necessarily converge in the high mobility limit, as $\rho_{xx} \to 0$.) Along this line $\sigma_{xy}^f = 0$.

In a recent preprint[27] Simon, Stern, and Halperin have pointed out a difficulty in the mean field, and what they call the (M)RPA, approximations for the composite Fermi-liquid theory. They consider the limit of $m_b \to 0$, and an inhomogeneous external magnetic field $B(r) = B_{1/2} + \delta B(r)$. From the electron point of view, due to the zero-point kinetic energy $\hbar \omega_c(r)$, the region of $\delta B(r) < 0$ will be populated by electrons while that of $\delta B(r) > 0$ will not. From the composite-fermion point of view the same physics *is* reflected in energy associated with the zero-point composite-fermion density fluctuation. Instead, in the spirit of Landau theory, Simon, Stern, and Halperin suggested modifying the composite Fermi-liquid theory by attaching a magnetic moment, of a determined strength, to each composite fermion. After taking into account the magnetization current associated with this moment, they arrived at a new approximation—the M^2 RPA. While it seems to us unlikely that this same correction will simultaneously correct the value of σ_{xy}^f in the presence of disorder,[28] it is possible that a similar in spirit modification of the basic constituents of the composite Fermi-liquid theory might exist that would accomplish this task.

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