Mode locking in the Gunn effect

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A model describing the Gunn effect is discussed. Numerical results show that the model exhibits the expected characteristics of the self-sustained oscillations. The model-locking characteristics under an externally applied microwave field are analyzed. As the frequency of the applied microwave field is changed, a devil's staircase of frequency-locked oscillations develops. [S0163-1829(97)05323-X]

The Gunn effect[1] has been extensively studied by experimental[2] and theoretical[3] techniques. It is well known that n-type GaAs and a number of other compound semiconductors can exhibit self-sustained current oscillations (the Gunn effect) in the microwave range when the applied drift field exceeds a characteristic threshold value. Because of inefficient energy relaxation, the electron gas heats up to temperatures well above that of the crystal lattice, and a transfer of carriers from the high-mobility conduction-band minimum to a set of low-mobility satellite valleys takes place. If this transition is fast enough, a bulk negative differential conductivity may arise. The spatially homogeneous electron distribution then becomes unstable, and propagating high-field domains are formed.

Since chaotic behavior in semiconductors was first observed by Aoki, Kobayashi, and Yamamoto[4] in n-type GaAs, the nonlinear and chaotic dynamics of semiconductors has now become of considerable interest both experimentally and theoretically. Because modern electronic devices associated with nonlinear carrier transport may encounter serious chaotic noise which prevents reliable device operation, it is important to know how a nonlinear electronic device responds to the external field, particularly, in the hot-electron regime.

A majority of the theoretical works are based on several assumptions[3,5] which give a picture of the Ridley-Watkins-Hilsum (RWH) mechanism.[6] We proposed four coupled-mode equations[7] for the Gunn effect. The four-coupled-mode equations are not good enough in describing the basic self-sustained oscillations. In this paper we employ an often-used drift velocity[8] with negative differential mobility to represent the RWH mechanism, and then propose a theoretical model composed of six-coupled-mode equations. This model gives the expected results that the self-sustained oscillating frequency decreases as the static electric field increases which is consistent with the experiments.[5] The mode-locking characteristics are also discussed with this model.

In one dimension, the Gunn effect is governed by the following equation for the electric field $E(x,t)$:[8,9]

$$\frac{\partial E(x,t)}{\partial t} = -e' n_0 v(E) - v(E)\frac{\partial E}{\partial x} + D\frac{\partial^2 E}{\partial x^2} + \frac{J(t)}{\varepsilon}, \quad (1)$$

where $e' = e/(4\pi\varepsilon)$, e is the electronic charge, ε the dielectric constant of the semiconductor (for GaAs, $\varepsilon = 12.5\varepsilon_0$, and ε_0 is the static dielectric constant.), n_0 the equilibrium electron density, and D the diffusion constant. Here $v(E)$ is the electron drift velocity, and $J(t)$ is the total current density. For explicit calculations it is advantageous to use an explicit form of $v(E)$to represent the RWH mechanism which has the form[8]

$$v(E) = \left(\mu_1 E + \frac{\mu_2 E^2}{E_c}\right) \Bigg/ \left(1 + \frac{E^2}{E_c^2}\right). \quad (2)$$

μ_1 is the electron mobility of the lower band, μ_2 the electron mobility of the upper band, and E_c the threshold field for the negative differential mobility.We Fourier analyze $E(x,t)$ in the form

$$E(x,t) = E_0 + \sum_{m\neq 0} E_m(t)\exp(imk_0x).$$

The summation goes over all positive and negative integers, and the fundamental wave number is defined by

$$k_0 = 2\pi/L.$$

L is the length of the sample. Expanding $v(E)$ into a power series of E, and then comparing the coefficients of the same exponential function in Eq. (1) yields the following set of equations:

$$\frac{\partial E_m}{\partial t} = (\alpha_m - i\beta_m)E_m - \sum_{s=2}^{\infty} \sum_{m_1+m_2+\cdots+m_s=m} \frac{1}{s!} A_{s,m_1} E_{m_1} E_{m_2} \cdots E_{m_s}. \quad (3)$$

The different terms on the right-hand side of Eq. (3) are defined as follows:

$$\alpha_m = -e' n_0 v_0^{(1)} - Dk_0^2 m^2,$$

$$\beta_m = mk_0 v(E_0),$$

$$A_{s,m_1} = e' n_0 v_0^{(s)} + im_1 s k_0 v_0^{(s-1)},$$

where the derivatives of $v(E)$ are defined by

$$v_0^{(s)} = d^s v(E)/dE^s|_{E=E_0}.$$

E_0 is the applied electric field.

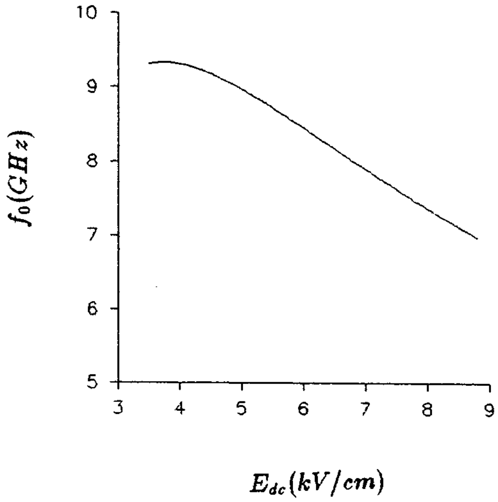

FIG. 1. Calculated variation of self-sustained oscillating frequency f_0 with electric field E_{dc}. This variation is very consistent with experiments (Ref. 5).

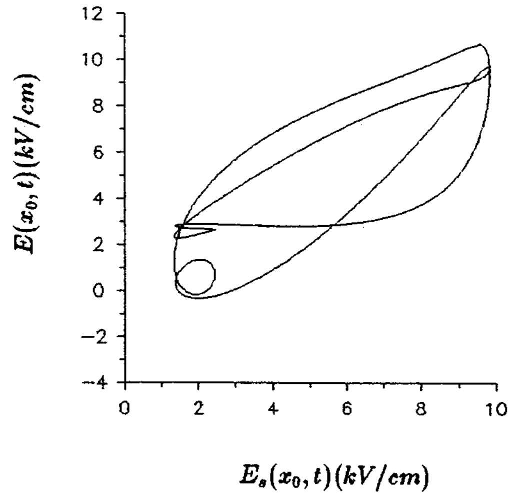

FIG. 2. A phase plot for the stationary 1:2 solution upon application of a microwave signal frequency f_{ac}=4.5 GHz and relative amplitude $K = E_{ac}/E_{dc} = 0.2$.

According to Nakamura,[9] we disregard all higher-order terms other than $s=3$, and omit the imaginary part of A_{s,m_1}. Decomposing complex variables E_m into real and imaginary parts as $E_m = x_{2m-1} + i x_{2m}$ for positive integers m, and using the relation $E_m = E^*_{-m}$ for negative integers m, we can reduce Eq. (3) to six coupled-mode equations,

$$\frac{dx_1}{dt} = \alpha_1 x_1 + \beta_1 x_2 - R_2(x_1x_3 + x_2x_4 + x_3x_5 + x_4x_6) - R_3 x_1 X_1 - R_3[(x_1^2 - x_2^2 + x_3^2 - x_4^2)x_5 + 2(x_1x_2 + x_3x_4)x_6]/2,$$

$$\frac{dx_2}{dt} = \alpha_1 x_2 - \beta_1 x_1 - R_2(x_4x_1 - x_2x_3 + x_6x_3 - x_4x_5) - R_3 x_2 X_1 - R_3[(x_1^2 - x_2^2 - x_3^2 + x_4^2)x_6 - 2(x_1x_2 - x_3x_4)x_5]/2,$$

$$\frac{dx_3}{dt} = \alpha_2 x_3 + \beta_2 x_4 - R_2[(x_1^2 - x_2^2)/2 + x_1x_5 + x_2x_6] - R_3 x_3 X_3 - R_3[(x_1x_3 + x_2x_4)x_5 + (x_4x_1 - x_2x_3)x_6], \tag{4}$$

$$\frac{dx_4}{dt} = \alpha_2 x_4 - \beta_2 x_3 - R_2(x_1x_2 + x_6x_1 - x_5x_2) - R_3 x_4 X_3 - R_3[(x_1x_3 + x_2x_4)x_6 - (x_4x_1 - x_2x_3)x_5],$$

$$\frac{dx_5}{dt} = \alpha_3 x_5 + \beta_3 x_6 - R_2(x_1x_3 - x_2x_4) - R_3 x_5 X_5 - R_3\left[x_1\left(\frac{x_1^2 - x_2^2}{3} + x_3^2 - x_4^2\right) - 2x_2\left(\frac{x_1x_2}{3} - x_3x_4\right)\right]\Big/ 2,$$

$$\frac{dx_6}{dt} = \alpha_3 x_6 - \beta_3 x_5 - R_2(x_4x_1 + x_2x_3) - R_3 x_6 X_5 - R_3\left[x_2\left(\frac{x_1^2 - x_2^2}{3} - x_3^2 + x_4^2\right) + 2x_1\left(\frac{x_1x_2}{3} + x_3x_4\right)\right]\Big/ 2,$$

where

$$R_2 \equiv \mathrm{Re} A_{2,m_1} = e' n_0 v_0^{(2)},$$

$$R_3 \equiv \mathrm{Re} A_{3,m_1} = e' n_0 v_0^{(3)},$$

$$X_1 = (x_1^2 + x_2^2)/2 + x_3^2 + x_4^2 + x_5^2 + x_6^2,$$

$$X_3 = x_1^2 + x_2^2 + (x_3^2 + x_4^2)/2 + x_5^2 + x_6^2,$$

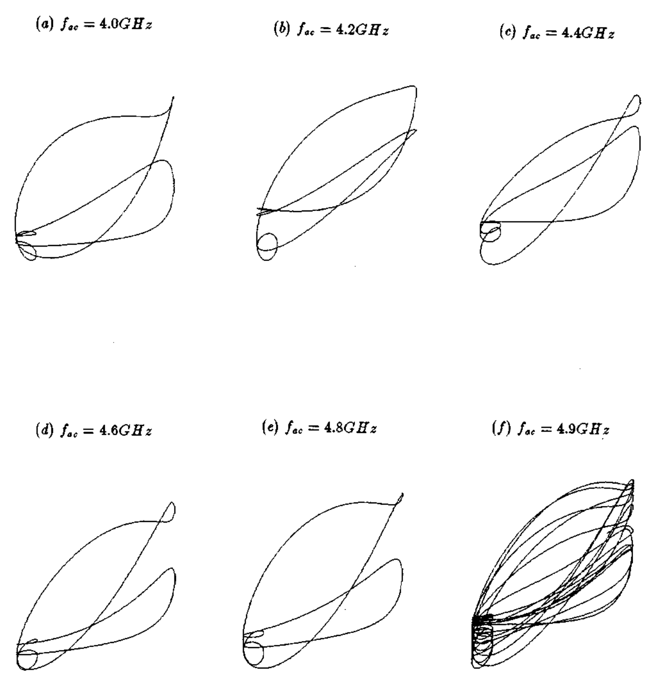

FIG. 3. Series of phase plots obtained by increasing the microwave frequency from 4.0 to 4.9 GHz. The Gunn mode maintains a 1:2 frequency-locked solution up to microwave frequencies of approximately 4.8 GHz. At f_{dc}=4.9 GHz, a quasiperiodic solution (or a periodic solution with a period longer than 2 ns) is observed.

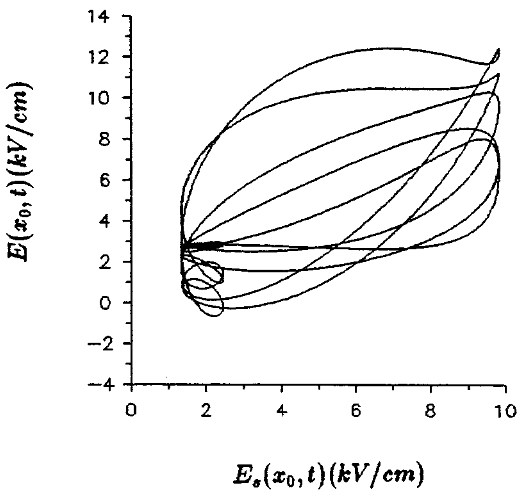

FIG. 4. The phase-space trajectory of the stationary 1:5 frequency-locked solution existing for $f_{\rm ac}=1.8$ GHz and $K=0.2$.

$$X_5=x_1^2+x_2^2+x_3^2+x_4^2+(x_5^2+x_6^2)/2.$$

Steady-state solutions $x_{i,0}$ of Eqs. (4) were numerically calculated by setting $dx_{i,0}/dt=0$. There are only zero solutions ($x_{i,0}=0$). Performing a linear stability analysis, we observe that $x_{i,0}$ becomes unstable for $\alpha_m>0$. This can be the case if $v_0^{(1)}$ is negative, i.e., if $E_0>E_c$. The unstable case is the Gunn oscillations.

It is basically important for a model to be able to give the correct results of the self-sustained oscillations. The numerical results of Eq. (4) for self-sustained oscillating frequency f_0 vs static electric field $E_{\rm dc}$ (dc bias) are shown in Fig. 1, which are well consistent with experiments.[5] In the calculations, we take the parameters of the typical sample GaAs, $\mu_1=5000$ cm^2/V s, $\mu_2=320$ cm^2/V s, $E_c=3.5$ kV/cm, $L=10$ μm, and $n_0=10^{15}$ cm^{-3}.

In the presence of a microwave field, i.e., under an external electric field including both dc and ac bias,

$$E_0=E_{\rm dc}+E_{\rm ac}\sin(2\pi f_{\rm ac}t),$$

we observed a large variety of different modes of behavior, depending upon the amplitude and frequency of the microwave signal. As an example, Figs. 2 and 3 present the response of the Gunn diode to the application of a microwave signal of various frequencies $f_{\rm ac}$ for $E_{\rm dc}=4.9548$ kV/cm and relative amplitude $K=E_{\rm ac}/E_{\rm dc}=0.2$. Figure 2 shows the stationary phase-space trajectory obtained by plotting simultaneous values of the electric field $E(x_0,t)$ at $x_0=0.8$ L and the self-sustained oscillating electric field $E_s(x_0,t)$ at the same position x_0 over many oscillation periods. The domain mode entrains into a 1:2 frequency-locked solution with the applied microwave field.

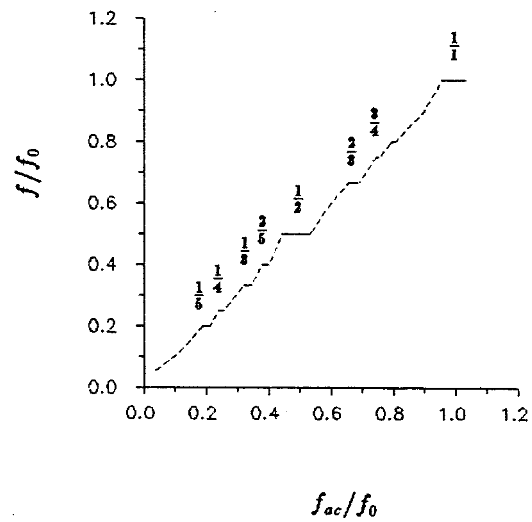

FIG. 5. The devil's staircase of frequency-locked oscillations for $E_{\rm dc}=4.9548$ kV/cm and $K=0.2$. f is the current oscillating frequency. f_0 is the self-sustained oscillating frequency corresponding to $E_{\rm dc}$, and $f_{\rm ac}$ is the ac bias frequency.

As the frequency of the microwave signal is increased, the 1:2 frequency-locked solution is maintained up to about 4.9 GHz. At this frequency, the oscillations becomes quasiperiodic. This is illustrated by the series of phase plots in Fig. 3.

For higher microwave frequencies, intervals exist in which the domain mode entrains into 1:3, 1:4, 1:5, etc., frequency-locked solutions. As an example, Fig. 4 shows the 1:5 frequency-locked solution that exists for $f_{\rm ac}=1.8$ GHz and $K=0.2$. In between the main frequency-locked solutions, more complex frequency-locked solutions are observed. Altogether, the interaction between the internally generated domain mode, or self-sustained oscillating mode, and the external microwave signal give rise to a devil's staircase of frequency-locked solutions, interspersed with quasiperiodic behavior. As the frequency of the applied microwave signal is changed, the devil's staircase of frequency-locked oscillations is illustrated in Fig. 5.

In conclusion, we discussed a model system for the Gunn effect which is composed of six coupled-mode equations. The numerical results of the model exhibit the expected characteristics of the self-sustained oscillations. The mode-locking characteristics of system (4) is also calculated. As the frequency of the applied microwave signal is changed, a devil's staircase of frequency-locked oscillations develops.[10]

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