Current-temperature phase diagram of layered superconductors

Stephen W. Pierson*
Department of Physics, Worcester Polytechnic Institute (WPI), Worcester, Massachusetts 01609-2280
(Received 4 November 1996)

The behavior of clean layered superconductors in the presence of a finite electric current and in zero-magnetic field behavior is addressed. The structure of the current temperature phase diagram and the properties of each of the four regions will be explained. We will discuss the expected current-voltage and resistance characteristics of each region as well as the effects of finite-size and weak disorder on the phase diagram. In addition, the reason for which a weakly non-Ohmic region exists above the transition temperature will be explained. [S0163-1829(97)06721-0]

I. INTRODUCTION

The understanding of a current's effect on the behavior of vortices in clean layered superconductors is important on its own merit and is essential for the interpretation of measurements which use an electric current as a probe. This issue has been addressed by many authors. Glazman and Koshelev[1] found that in contrast to quasi-two-dimensional superconducting films, there is a nonzero critical current because a minimum current is needed to overcome the attraction due to the Josephson coupling between the layers and to drag apart vortex pairs. Jensen and Minnhagen[2] have generalized the two-dimensional current voltage (I-V) relation to the layered case by considering the effect of the Josephson coupling on the intralayer vortex attraction. They find that the voltage goes from being exponentially small to finite with the following nonlinear dependence on current:

$$V=k(T)I[I-I_c^1(T)]^{\alpha(T)}, \tag{1}$$

where $I_c^1(T)$ represents the temperature-dependent threshold current needed to overcome the Josephson attraction and $k(T)$ and $\alpha(T)$ are generally taken to be independent of current. The relevance of this equation to such layered materials as the high-temperature superconductors (HTSC's) has been established by several experimental groups.[3–5]

A rigorous self-consistent study of the effects of a current on the critical behavior of vortices in a clean layered system was conducted by the author using a renormalization group analysis which culminated in the I-T phase diagram.[6] As will be described below, the I-T space was found to be divided into four regions separated by three characteristic currents which included a second-order phase transition line. This too has been shown to describe well the behavior of the copper oxides.[7–9] Martynovich and Artemov[10] have also studied this system in zero and finite current and also find a second-order phase transition, but only at larger currents. Langevin simulations have also been used to study the effect of a driving current on an anisotropic Josephson-junction array.[11] Grønbech-Jensen, Domínguez, and Bishop confirmed the structure of the I-T phase diagram[6] finding that the phase transition occurs at a temperature higher than the temperature marking the onset of resistance and that the slopes of the two characteristic currents that they studied [$I_c^1(T)$ and $I_c(T)$] were linear. While the theoretical and simulational work seems sufficient for describing clean, layered systems, there is now evidence that finite-size effects can produce a linear current dependence in the V-I relations at small current which give way to a nonlinear behavior at larger currents.[8,12–14] This has also been seen in two-dimensional (2D) systems.[15,16] These and other properties of layered superconductors in the presence of a current need to be addressed.

In this paper, we explain in more detail and expand upon the results of Ref. 6. In particular the details of how the I-T phase diagram was derived (Sec. II B), the nature of the actual phase transition (Sec. II C), and the properties of each region (Sec. III) are explained more thoroughly. The origin of a weak non-Ohmic contribution above the transition temperature will be discussed in Secs. III C and III D as will the effects of free vortices created due to finite-size effects (Sec. IV). Finally, the effects of weak disorder as calculated via the replica technique will be reported (Sec. V).

II. MODEL AND DERIVATION OF THE I-T PHASE DIAGRAM

A. Model and recursion relations

We begin with a description of our model which has been described in detail elsewhere[17,18] and so will be kept brief here. Vortices in layered systems are modeled as a ''gas'' of charges in a stack of weakly coupled layers interacting via two-body interactions which approximate those of vortices in layered superconductors. Interactions between vortices in the same layer and between vortices in neighboring layers are included but interactions between vortices separated by more than one layer are neglected. The effect of the current **J** is to exert a constant force on vortices in a direction perpendicular to **J** but in opposite directions for oppositely charged vortices. Knowing the interaction potentials and the effect of the current, one can write down the partition function and perform a real-space renormalization group (RG) study on it in the fashion of Kosterlitz.[19] The results are

$$dx/d\epsilon=2y^2[1-(1/16)\lambda+J^2], \tag{2}$$

$$dy/d\epsilon=2y[x+(1/2)\lambda\ln\lambda]/(1+x)+Jy, \tag{3}$$

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$$d\lambda/d\epsilon = 2\lambda[1-4y^2(1+J^2)/(1+x)], \tag{4}$$

$$dJ/d\epsilon = J. \tag{5}$$

λ is the ratio of the interlayer coupling to the intralayer coupling $p^2, \epsilon = \ln(l/\xi_0)$ (where l is the relevant length scale), $x = 4/(\beta p^2) - 1$, and ξ_0 is the zero-temperature correlation length. $y = \exp(-\beta E_c)/l^2$ is the fugacity, where E_c is the ''core energy.'' Note that the current affects not only the interaction strength parameters x and λ but also the fugacity directly. This is because the current lowers the core energy of vortices and therefore directly affects the fugacity.

B. The correlation length and the *I*-*T* phase diagram

The *I*-*T* phase diagram is derived by studying the correlation length $\xi(T)$ which is related to the value of l at which one terminates the integration of the recursion relations. Because the calculation leading to Eqs. (2)–(5) assumed small vortex density (y), current, and interlayer coupling, the integration of the recursion relations must be terminated when any of these quantities becomes large. The value of l at the cutoff is called $l_{\max}$ and because only one length (the correlation length) dominates the critical behavior, the following identification follows: $l_{\max} = \xi$. The temperature dependence of ξ is found by considering the first integral of the 2D recursion relations $y^2 + 2\ln(1+x) - 2x = c = y_i^2 + 2\ln(1+x_i) - 2x_i$, where the subscript i denotes the initial or ''bare'' value of that parameter. c is a constant of integration and it can be shown that $c \propto (T - T_{KT})/T_{KT}$ (Ref. 19) where T_{KT} is the 2D transition temperature. Because we have assumed the interlayer coupling to be very weak in our model, it is expected that c should be linear in temperature in the immediate vicinity of T_{KT} for our system.

Plotting $\ln(l_{\max}/\xi_0)$ versus c for various currents gives an indication of the temperature dependence of $\xi(T)$ and the plots are qualitatively similar to that of Fig. 2 in Ref. 18. One finds that there is a peak in $\xi(T)$ which occurs at the transition temperature. As one increases the current, the peak shifts to the left corresponding to a decrease in the transition temperature. This process allows one to determine $T_c(I)$ [or equivalently $I_c(T)$] which decreases linearly with current. $I_c(T)$ is plotted in Fig. 1 as calculated for $x_i = 0.5$ and $\lambda_i = 10^{-6}$. (y_i and J_i were varied to sweep through *I*-*T* space.)

Two more characteristic currents can be derived by comparing the correlation length for the layered, finite-current case (referred to as the full correlation length) to the 2*D* finite-current correlation length and to the layered *zero-current* correlation length. In Fig. 1 of Ref. 6, the three correlation lengths are plotted. As expected, the 2D and full correlation lengths coincide at larger temperatures signifying the 2D behavior of the layered system for temperatures larger than $T_c(I)$. The small temperature range over which these two correlation lengths separate is taken to be the 3D/2D crossover region and marks another characteristic temperature $T_c^2(I)$ [or equivalently $I_c^2(T)$] which is also found to vary linearly with current. (See Fig. 1.)

The third characteristic current comes from comparing the full correlation length with the layered, zero-current correlation length. Where these two correlation lengths begin to separate is the temperature at which the current starts to affect the system and identifies $T_c^1(I)$ [or $I_c^1(T)$]. Like $I_c^2(T)$, it is a crossover current and not a phase transition. By varying the current, it is found that $I_c^1(T)$ is linear at larger currents and is roughly parallel to $I_c(T)$ but crosses over to join $I_c(T)$ at small currents. In other words, $I_c^1(T_c) = I_c(T_c)$ as illustrated in Fig. 1. As we shall discuss below, what happens in finite current at two different temperatures, occurs at one temperature in zero current.

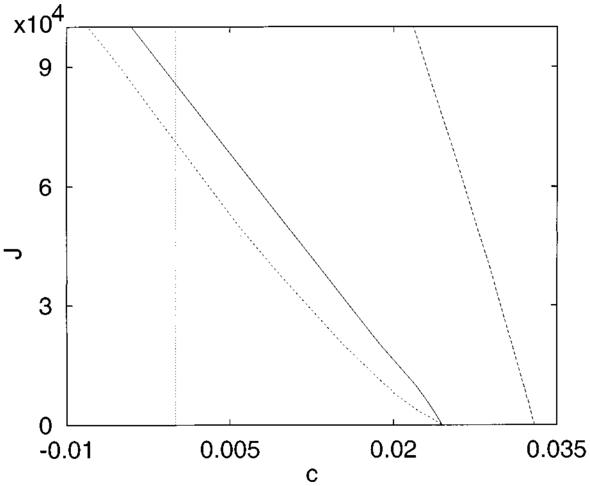

FIG. 1. The *I*-*T* phase space with the three characteristic currents derived by studying the correlation lengths as explained in the text. The x-axis label c is proportional to $T/T_{KT} - 1$.

While $I_c(T)$ can be determined to some precision, the other two characteristic currents cannot be since they are crossover currents. One can determine them in different ways which yield results which are in qualitative agreement with each other. One can simply take the temperature at which the difference between $l_{\max}(J,\lambda)$ and, say, $l_{\max}(J=0,\lambda)$ is a certain value [or at which this difference is a certain percentage of $l_{\max}(J,\lambda)$] to be $T_c^1(I)$. While the values are arbitrary, it is found that the temperature dependencies of $I_c^1(T)$ and $I_c^2(T)$ do not depend on the chosen values nor on whether one chooses the absolute difference or the relative difference. What does depend on these factors are the slopes. For this reason, it should be emphasized that Fig. 1 is only a rendition of the phase diagram. The slopes of these lines as well as the temperature and current scales will vary not only from the less anisotropic materials to the more anisotropic material but also from sample to sample.[8] The fact that the temperature scale and the current scale cannot be determined in our RG analysis contributes to this ambiguity.[18]

The temperature dependence of $I_c^1(T)$ has also been considered by Jensen and Minnhagen[2] who found it to be linear at all currents[20] unlike the small-current behavior found in Ref. 6. Using mean-field theory, they found that $dI_c^1/dT \propto 1/\gamma$, where $\gamma = \xi_{ab}/\xi_c$ is the anisotropy factor. (The relationship between this quantity and our parameter for the strength of the interlayer coupling is $\gamma \propto 1/\sqrt{\lambda}$.) This issue could not be addressed in our approach for the following reason. When the anisotropy is large, $T_c^1 \ll T_c$ and our temperature scale, which is linear in $t = T/T_{KT} - 1$, is only valid for small t. For similar reasons, the dependence of $I_c^1(T)$ on

γ could not be determined. For $I_c(T)$, no systematic dependence on γ could be identified.

C. The phase transition at $I_c(T)$

As mentioned above, a second-order phase transition of this system occurs at $I_c(T)$. It is here that the nonanalyticity in the correlation length occurs and above which $y(l)$ goes to infinity for large l and below which it goes to zero. In terms of vortices, we believe that at this current, free vortex lines can be spontaneously created as opposed to being created by a thermal unbinding of vortex loops. This means that the total vorticity of the system need not be zero but will fluctuate around zero. This leads to the meaning of $I_c^1(T)$ which we see as a natural extension of the definition of $I_c(T)$. As we have stated before, at this current the system starts to be affected by the current which we have taken to mean that it is now large enough for vortex loops to be thermally unbound into vortex lines. In other words, there are two mechanisms for creating vortex lines, one is by an unbinding of vortex loops and the other is by spontaneous creation. The latter is possible because the energy of the vortex line becomes finite due to screening. In zero current these two mechanisms become possible at the same temperature whereas in finite current, they occur at different temperatures. This is why $I_c(T)$ and $I_c^1(T)$ start out at the same point in zero current but become separate lines at finite current. As mentioned above, Martynovich and Artemov[10] have studied this system in zero and finite current. Without a current, they predict a first-order phase transition which turns into a second-order phase transition at a finite current. No evidence for this has been presented, however, and in fact there is evidence for a second-order phase transition at zero field.[21]

Grønbech-Jensen, Domínguez, and Bishop[11] have arrived at conclusions similar to ours based on Langevin simulations of anisotropic 3D, current-driven Josephson-junction arrays. There they calculate two quantities, the voltage and the helicity modulus. They find that the helicity modulus goes to zero at a temperature above which the voltage becomes finite. They conclude that the phase transition occurs at a temperature higher than that at which vortex loops start to thermally unbind in agreement with our results. They also find that $I_c^1(T)$ and $I_c(T)$ are linear in temperature.

To summarize this section, Fig. 1, where $I_c^1(T)$, $I_c(T)$ and $I_c^2(T)$ are plotted, represents the ‘‘raw’’ results of our analysis. $I_c^1(T)$ is the value of the current at which the system starts to be affected by the current, $I_c(T)$ is the current at which the phase transition occurs, and $I_c^2(T)$ is the current at which the 3D/2D crossover occurs. These results were obtained for a system composed strictly of vortex pancakes and neglect the structure and behavior of the underlying superfluid. Amplitude fluctuations of the superfluid become significant near the mean-field transition temperature T_{c0} and the superfluid is next to nonexistent making vortices a minor detail. Minnhagen[22] has taken this into account and we will see an example of its effect when we consider Eq. (6). [The position of $T_{c0}(I)$ relative to $T_c^2(I)$ was not determined in our analysis.] In the next section we will relate our results to layered superconductors such as the high-temperature superconductors. In particular we will describe the expected electric transport properties that are heavily influenced by vortices in each region for these materials.

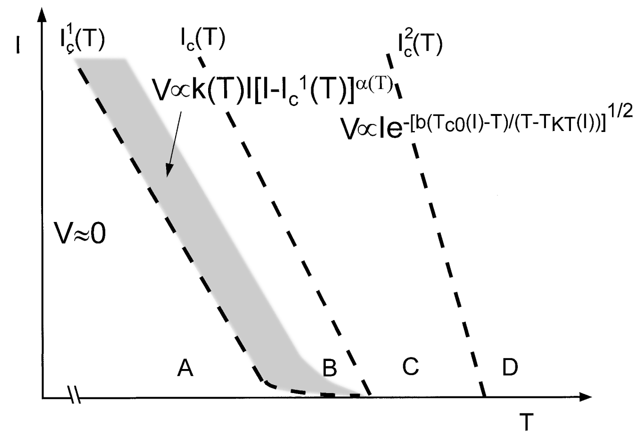

FIG. 2. The expected current-voltage characteristics of the I-T phase diagram. In region A, V is exponentially small but not 0 because, as discussed in Sec. III A, vortex loops can blow out in a direction perpendicular to the layers. In region B, Eq. (1) holds at small currents (shaded area) but will break down at larger currents. Region D has a weak non-Ohmic contribution [Eq. (6)] which enters through the renormalization of the transition temperature.

III. I-V AND $R(T)$ CHARACTERISTICS OF THE I-T PHASE DIAGRAM

In this section, the regions of the I-T phase diagram will be discussed and the expected electrical transport properties [e.g., I-V curves and resistance $R(T)$ measurements] of each region will be explained. (See Fig. 2.) Flux transformer measurements and the detection of $I_c^2(T)$ are also discussed.

A. Region A

This region is defined by where the full correlation length and the layered, zero-current correlation length are the same. The coincidence of these two quantities is interpreted to mean that in this region the current does not have a significant effect on the system. In the original paper reporting the I-T phase diagram,[6] the current-voltage characteristic was taken to be $V(I)=0$. In reality, it should read $V(I)\simeq 0$ since vortex loops do have a exponentially small chance of being unbound in the presence of a current[1,23] because the current also exerts a force on the segments of the loop which lie between and parallel to the layers. In a weakly coupled layered system this can be neglected and $I_c^1(T)$ signifies a blowout of the loops in a direction predominantly parallel to the planes. As one goes to the more isotropic systems, this quantity loses its meaning since the loop can blow out in both the parallel and perpendicular directions simultaneously. In their simulations, Grønbech-Jensen, Domínguez, and Bishop[11] do find a small number of blowouts at currents less than $I_c^1(T)$. It should be noted that in the two-dimensional limit, this region does not exist because $I_c^1(T;\gamma=\infty)=0$.

B. Region B

In this region, the full correlation length and the layered, zero-current correlation length differ from one another signifying that the current has a significant effect on the critical behavior. This implies that just above $I_c^1(T)$, vortex loops are being thermally unbound in the direction parallel to the layers and the I-V relation [Eq. (1)] of Jensen and Minnhagen[2] should hold. But this equation should not be expected to hold in all of region B for a number of reasons. First of all, the equation is based upon the formula for rare escapes over a barrier.[24] This breaks down severely as one increases the current and the energy of the most energetic pairs approaches that of the barrier height. Secondly, it is based upon vortex loop unbinding in the parallel direction (i.e., 2D unbinding). Close to the transition temperature, the system is three dimensional in its behavior and loop blowouts in the perpendicular direction become significant. These effects may be represented to first order by introducing a current dependence into the parameters of Eq. (1). In other words, to be more strict one should write Eq. (1) as $V=k(T)I[I-\mathcal{I}(T,I)]^{\alpha(T,I)}$ with $\mathcal{I}[T,I\sim I_c^1(T)]\simeq I_c^1(T)$. At larger currents, $\mathcal{I}(T,I)$ goes to zero.[25] The behavior of $\alpha(T,I)$ is less clear. Our renormalization group analysis showed that $\alpha(T,I)$ should be independent of current over a substantial part of region B at fixed temperature but this analysis does not incorporate either of the effects mentioned above. What is certain is that $\alpha(T,I)$ goes to zero at $I_c(T)$. The approximate region in which Eq. (1) is expected to hold is represented in Fig. 2 by the shaded region.

C. Region C

Region C corresponds to temperatures above $T_c(I)$ where the full correlation length differs from the 2D finite-current correlation length. Because one is above the phase transition free vortex lines can be spontaneously created which one typically considers to mean that the region is Ohmic. However, as we will show here and in Sec. III D, a non-Ohmic contribution enters through the renormalization of the critical temperature due to the current. There are actually non-Ohmic contributions from two sources. The first source are, of course, blowouts of smaller vortex loops which still exist above $I_c(T)$. The other piece will enter through the term describing the spontaneously created free vortex lines which is ''largely'' Ohmic. The resistance is proportional to $1/\xi(T)^2$ but a current dependence enters into this through the current-dependent transition temperature $T_c(I)$. Because our RG analysis does not accurately describe the 3D region, the functional dependence of the resistance in region C cannot be determined. The situation is better in region D.

A curiosity of this region is that its width increases with current implying that the 3D region becomes larger even though the current tends to decouple the layers. We believe that this effect is best looked at by saying that the current has a stronger effect on $I_c(T)$ than $I_c^2(T)$. It should also be noted that because the full correlation length differs from the 2D finite-current correlation length in this region, the behavior should be of a 3D nature. At the same time, however, the interlayer coupling at large length scales is renormalized to zero[26–28] making 2D signatures conceivable.

D. Region D

In this region, the full correlation length and the 2D finite-current correlation length overlap meaning that the behavior of the layered system is 2D-like. One can therefore write down the resistance formula for this region based on Minnhagen's 2D formula[22] $R(T)=\exp\{-[b(T_{c0}(I)-T)/(T-T_{KT}(I))]^{1/2}\}$, where T_{c0} is the mean-field transition temperature, T_{KT} is the 2D transition temperature, and b is a constant. To use this for region D in our the layered system, we must insert the current dependences for the two transition temperatures:

$$R(T)=\exp\{-[b(T_{c0}(I)-T)/(T-T_{KT}(I))]^{1/2}\}. \qquad (6)$$

This equation is important because it shows how a weak current dependence can enter into the resistance formula making this region weakly non-Ohmic. Equation (6) has been checked for $YBa_2Cu_3O_{7-\delta}$ (YBCO) data and it is found to work well[29] for weak currents ($\leqslant 10^{-3}$ Å) using a constant T_{c0}. It should be stressed that $T_{KT}(I)$ is not the same as $T_c(I)$. $T_{KT}(I)$ is the temperature at which it appears that the correlation length in the 2D region would diverge, and $T_c(I)$ is the actual temperature at which it diverges once the 3D effects take over. As noted above in Sec. II C one should keep in mind that the underlying superfluid is also being destroyed in this vicinity and so one must also consider normal-state effects which can wash out the effects of vortices.

E. $I_c^2(T)$ and flux transformer geometries

Here we will show how our results can be used to explain measurements made in the flux transformer geometry[30] and explain how one could measure $I_c^2(T)$ which is not easily determined from the primarily in-plane Eqs. (1) and (6). In the flux transformer geometry arrangement,[30] a current is injected in the top of a sample while a secondary voltage $V_s(T)$ is measured on the bottom. Typically, the current is held constant while the temperature is varied so that horizontal slices of the I-T phase diagram can be studied. Such a measurement has been carried out on $Bi_2Sr_2CaCu_2O_{8-y}$ (BSCCO) samples in zero-field by Wan *et al.*,[31] where it was found that near the transition temperature a peak appeared in $V_s(T)$ for various current values. On the low-temperature side of the peak, $V_s(T)$ rises rather abruptly from a value 0 in contrast to the high-temperature side where V_s descends to 0 but starts to increase before reaching it.

The behavior of the $V_s(T)$ peak can be explained in terms of the I-T phase diagram. The onset of the secondary voltage on the low-temperature side is $T_c^1(I)$, where the loops starts to blow out due to the current. Because there is finite interlayer coupling in this region, the movement of free vortices in the upper layers is translated through to the bottom of the sample.[32] The peak gets larger as one goes deeper into region B because more loops are blowing out. However, as one goes into region C, this effect is offset by the gradual layer decoupling and so $V_s(T)$ begins to decrease and in principle will decrease to a value zero at $T_c^2(I)$. If one could neglect amplitude fluctuations, $V_s(T)$ would decrease to zero at $T_c^2(I)$ but this was not the case in Ref. 31. We have determined $I_c^1(T)$ from the secondary voltage data of Ref. 31 using an onset voltage of 0.01μV as our threshold. It is plotted in Fig. 3 along with its error bars which were determined from the temperature resolution used in the measure-

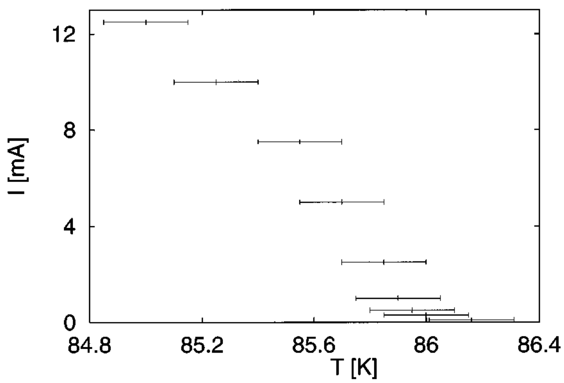

FIG. 3. $I_c^1(I)$ determined from $V_s(T)$ as measured in the flux transformer measurements of Ref. 31. As discussed in the text, within the error bars this data is consistent with our results that $T_c^1(I)$ be linear in current.

ment of $V_s(T)$. Within the error bars it is possible to say that the data is linear in T as found by others using in-plane I-V measurements[3–5,8] and also predicted in Ref. 6 although it is not as convincing as the other data.

Even though $T_c^2(I)$ could not be determined from the data of Ref. 31 this method remains the best hope for studying $I_c^2(T)$. It may be possible to find a sample in which the layers become decoupled before the underlying superfluid breaks down and a close inspection of the Wan *et al.* data suggests that $T_c^2(I)$ might be determined at small currents. It should also be noted that the their data do manifest the behavior predicted by our results and discussed in Sec. III C: as the current is increased, $T_c^2(I)-T_c^1(I)$ increases.

IV. FINITE-SIZE EFFECTS

In this section, we consider finite-size effects which make the energy of a single vortex finite and therefore make it possible to have free vortices in zero-current at temperatures below the critical temperature. The presence of these free vortices destroys the Kosterlitz-Thouless-Berezinskii (KTB) transition[33,34] turning it into a crossover. This is because the free vortices screen the interactions more strongly than the vortex pairs do , which in turn makes vortex pair unbinding possible at lower temperatures than originally discussed by Kosterlitz and Thouless.[33] Nevertheless, it is still possible to see KTB behavior as we explain below.

There are two ways in which finite-size effects enter. In a chargeless superfluid, the bare energy of a 2D, single vortex is $\ln L$, where L is the size of the system. In charged superfluids, the energy of a 2D, single vortex goes as $\ln\lambda_L^2/d$, where λ_L is the London penetration depth and d is the thickness of the film. Usually, however, L and λ_L are large in typical samples and thus do not smear the transition in a significant way. Beasley, Mooij, and Orlando[35] along with others[36] pointed this out in the late 1970s. In the presence of a current, the principle consequence of a finite L or λ_L is to contribute an Ohmic term to the otherwise purely nonlinear (below T_c) I-V characteristics which can dominate at low currents and can wash out the phase transition.

In the HTSC's there is evidence that λ_L is small and therefore can significantly affect the critical behavior in this system. Indeed, linear I-V characteristics at small currents have been observed by Paracchini and Romano[12,13] on YBCO, by Matsuo *et al.* on layered conventional superconductors,[14] in the author's own analysis[7,8] of I-V measurements from various groups on YBCO and BSCCO materials, and by Repaci and co-workers in ultrathin YBCuO films[15,16] who attributed the small current Ohmic behavior to finite-size effects. Repaci and co-workers[15,16] go on to report that the low-sensitivity data are consistent with a KTB transition. Here, we will address finite-size effects on the critical behavior and the I-T phase diagram. While their effect may ultimately destroy the phase transition, our position[37] is that KTB behavior can still be observed and that the structure of the I-T phase diagram is reflected in electronic transport measurements. (The effect of a finite L on 2D Josephson-junction arrays was recently considered theoretically and in simulations by Simkin and Kosterlitz.[38])

As mentioned above, finite-size effects introduce an Ohmic term into the I-V characteristics which are typically purely nonlinear below $T_c(I)$ which can be seen through the following simplified derivation. The kinetic equation for the formation of free vortices in terms of the density of free vortices n_F is

$$dn_F/dt=\Gamma_J(T,J)+\Gamma_{\rm FS}(T)-k_1n_F^2, \tag{7}$$

where $\Gamma_J(T,J)$ is the rate at which free vortices are being created by pair unbinding due to thermal activation over the barrier made possible by the current, $\Gamma_{\rm FS}(T)$ is the rate at which free vortices are created due to finite-size effects, and k_1 is a constant. It is a standard derivation[2] to show that

$$\Gamma_J(T,J)\propto[I-I_c^1(T)]^{2\alpha_B(T)}, \tag{8}$$

which corresponds to the rate at which bound vortex pairs are thermally activated over a barrier and $\alpha_B(T)$ corresponds to a bare exponent that must be renormalized. The rate at which free vortices are spontaneously created is proportional to a Boltzman factor:

$$\Gamma_{\rm FS}(T)\propto\exp[-E_{\rm FV}/k_BT], \tag{9}$$

where $E_{\rm FV}$ is the energy of a free vortex. As pointed out above, Eq. (8) is valid for rare escapes over a barrier and also for when the vortex loop blowouts are in a direction parallel to the planes. In that limit, one can take $\Gamma_{\rm FS}(T)$ to be independent of current. It is Eq. (7) that one uses to derive Eq. (1) in the limit of $E_{\rm FV}=\infty$.

Proceeding with Eq. (7), one assumes a steady state solution and finds

$$n_F\propto\sqrt{\Gamma_J(T,J)+\Gamma_{\rm FS}(T)}. \tag{10}$$

This is readily put in the form of a current-voltage relation since $R\propto n_F$ and $V=IR$:

$$V=I\sqrt{m_1(T)+k(T)^2[I-I_c^1(T)]^{2\alpha(T)}}. \tag{11}$$

At low currents, the first term will dominate and the I-V curves will be linear but at larger currents it is possible for the second term to dominate and for the I-V curves to become nonlinear. Therefore, by taking into account the first term, it is still possible to determine $I_c^1(T)$ and $\alpha(T)$. The

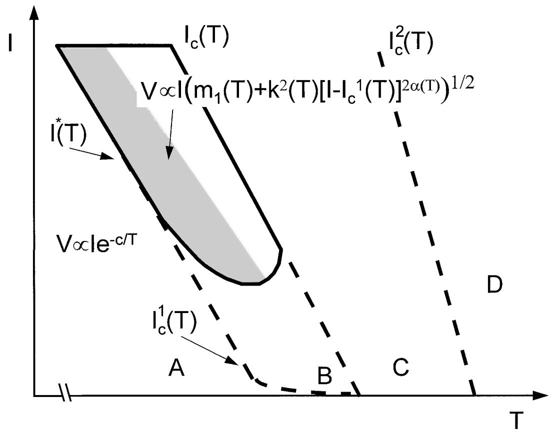

FIG. 4. The approximate modifications to the I-T phase diagram due to finite-size effects. Note that the original phase lines are shown here as dashed lines. Within the solid lines is where one would expect the effect of vortex-pair unbinding. The shaded region is where one would expect Eq. (11). Outside of the solid lines, Ohmic behavior is expected although it could be exponentially small at lower temperatures and currents. Slightly non-Ohmic behavior may persist to the right on the non-Ohmic region as a result of transition temperature renormalization.

current $I^*(T)$ at which the second term dominates introduces a fourth characteristic current into the I-T phase diagram which becomes quite different in the presence of finite-size effects as we discuss below. If the temperature dependence of $m_1(T)$ and $k(T)$ were known, it would be possible to determine the relationship between $I^*(T)$ and $\alpha(T)$. As it is we have,

$$I^*(T)=I_c^1(T)+[m_1(T)/k(T)^2]^{1/[2\alpha(T)]} \qquad (12)$$

which tells us that $I^*(T)$ lies above $I_c^1(T)$ and that their difference increases with temperature. [In this last statement we have made use of some empirical understanding of $m_1(T/k(T)$.] Based on this we can modify the I-T phase diagram to include finite-size effects as we illustrate in Fig. 4. Within the solid lines is where one would expect the nonlinear behavior due to vortex pair unbinding. The lower solid line is $I^*(T)$ and the upper is $I_c(T)$. We have not included the effect of the renormalization of $I_c(T)$ due to the free vortices here. To a first approximation, their effect will be to shift this quantity to lower temperatures and perhaps to smear it. The renormalization of the quantities in Eq. (11) should also be accounted for but here we do not expect any drastic changes. Figure 4 is similar to Fig. 8 of Ref. 13 but the differences are important. Their $I_0(T)$ is defined through a phenomenological equation different than Eq. (1) and is therefore distinct from $I_c^1(T)$ and $I^*(T)$. Furthermore, the Ohmic region and non-Ohmic regions have different shapes in the two figures.

V. EFFECTS OF DISORDER

In this section, the effect of a quenched, random one-body potential $V_d(R)$ on the system will be considered using the replica technique.[39] The primary effect of this type of disorder is to pin a vortex, but various types of disorder on 2D systems have been considered in the past. Rubinstein, Shraiman, and Nelson[40] and Korshunov[41] have studied the effects of a random configuration of quenched vortex pairs. José[42] has investigated the 2D ferromagnetic planar model where the coupling is taken to be a random variable while Fischer[43] has studied a system where the disorder is introduced via the fugacity. The effect of a quenched, random one-body potential on a one-component vortex gas (i.e., no antivortices) has been considered by Menon and Dasgupta[44] and we will follow their notation here. We will present the calculation for the layered case which is also applicable to the 2D case.

The essence of the replica technique is to average the disorder over n replicas of the system which is done via the identity $[\ln Z]=\lim_{n\to 0}\int dV_d P(V_d)(Z^n-1)/n$, where Z is the partition function for the layered vortex gas and $[\cdots]$ represents an overage over the probability distribution of the disorder. $P(V_d)$ is a Gaussian distribution with zero mean value and short ranged spatial correlations $[V_d(\mathbf{r})V_d(\mathbf{r}')]\simeq\Delta\,\delta(|\mathbf{r}-\mathbf{r}'|)$, where Δ represents the strength of the disorder. The effect of this averaging is to introduce an interaction between vortices in different replicas which includes the standard logarithmic interaction in addition to a short-ranged disorder-induced interaction which is proportional to $-\beta\Delta\,\delta(|\mathbf{r}-\mathbf{r}'|)$ as described in more detail in Ref. 44. After this averaging, the model is similar to that of Fischer[43] except the additional disorder-induced interaction is long ranged there.

We have done a renormalization group analysis on the new partition function which consists of two steps, a course-graining and a rescaling. While the integrations for the course graining step cannot be done exactly, one can show that the resulting terms have no logarithmic or linear dependencies and so will not renormalize the interaction strength parameters p^2 and λ. To do this integral, one approximates the δ function by the short-ranged expression[44] $\exp[-r^2/\xi_0^2]$. The rescaling steps alone result in the following recursion relation for the disorder strength: $d\Delta/d\epsilon=-\Delta$ which is immediately seen by considering $-\beta\Delta\,\delta(R)$ with the substitution $R'=R/(1+d\tau/\tau)$. Since it is unlikely that coarse-graining effects will contribute to making the disorder more important, this parameter is irrelevant.

Because the disorder does not affect the recursion relations of the system parameters p^2,λ, and y and the disorder parameter Δ is irrelevant, our results show that a weak, random, one-body potential has no effect on the critical behavior of the system in agreement with the results of Ref. 43. This is expected since even with one vortex in a pair pinned, the pair can still unbind because of the ''freedom'' of the other vortex.

VI. DISCUSSION AND SUMMARY

Numerous aspects of the current-temperature phase diagram have been considered here including its derivation, the current-voltage characteristics of each region, properties of the phase lines, and the use of the flux transformer geometry to study $I_c^2(T)$. We have also considered finite-size effects and derived a current-voltage equation for this case. Finally, the effect of a weak, random, one-body potential was found to be weak. An interesting consequence of the current dependence of the critical temperature is to make the system slightly non-Ohmic above T_c. Within these topics, three new equations have been presented which could be tested experimentally. Equation (6) incorporates the current dependence into the critical temperatures in the resistance formula making the system slightly non-Ohmic above the transition temperature. Eq. (11) is the current-voltage relationship for systems where finite-size effects are important. Associated with this equation is Eq. (12) which marks the crossover current from finite-effect-induced Ohmic behavior to a pair-unbinding-induced non-Ohmic behavior. There remain many interesting aspects of the I-T phase diagram to consider including the dependencies of the characteristic currents $I_c^1(T)$,$I_c(T)$, and $I_c^2(T)$ on the anisotropy factor γ.

ACKNOWLEDGMENTS

The author gratefully acknowledges D. Domínguez, P. Minnhagen, L. Miu, C. Paracchini, L. Romano, and S. Shenoy, for useful conversations. I also thank S. Hebboul *et al.* for providing their $V_s(T)$ data, M. Friesen for his contribution in deriving Eq. (3), and T. M. Katona for assistance with Fig. 3. This work was supported by the Office of Naval Research and early stages of it were carried out at the Naval Research Laboratory.

*Electronic address: pierson@wpi.edu

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