Solvable model of a random spin-$\frac{1}{2}$ XY chain

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This paper presents exact calculations of thermodynamic quantities for the spin-$\frac{1}{2}$ isotropic XY chain with random Lorentzian intersite interaction and transverse field that depends linearly on the surrounding intersite interactions. [S0163-1829(97)06321-2]

I. INTRODUCTION

Starting from the seminal paper by Lieb, Schultz, and Mattis the study of the one-dimensional spin-$\frac{1}{2}$ XY models has attracted much interest. A lot of exact results concerning thermodynamics, spin correlations, and their dynamics were found over the last 35 years.[1–6] The analytical results obtained for random versions of such models are not so impressive. Some results dealing with magnetic properties for special cases of XY models with random intersite interactions were found by Smith,[7] Barouch and McCoy,[8] and Zaitsev[9] using an approach developed by Dyson.[10] A somewhat different approach was suggested by Nishimori,[11] who presented exact calculations of thermodynamic quantities for the isotropic XY model in a random Lorentzian transverse field. Nishimori's exact solution is based on the Jordan-Wigner transformation of the spin Hamiltonian to a tight-binding model of noninteracting spinless fermions with diagonal Lorentzian disorder. For the latter fermionic model the random-averaged one-particle Green functions (and hence the density of states that yields thermodynamics) were found exactly by Lloyd[12] with the help of contour integrals. Later on Nishimori's work was generalized for models with alternating bonds[13] and additional intersite Dzyaloshinskii-Moriya interaction.[14] The main results obtained in the papers on models with a Lorentzian transverse field concern the changes in the temperature dependences of entropy, specific heat, and static transverse linear susceptibility as well as the ground-state transverse magnetization as a function of averaged transverse field once the randomness is introduced.

The idea of the present paper is to study the thermodynamics of a random spin-$\frac{1}{2}$ XY chain exploiting an extended version of the Lloyd model with off-diagonal disorder.[15] In the late seventies in a series of papers this extended Lloyd model was applied to disordered electron systems by one of the authors.[16–20] Similarly to Ref. 11 we present exact calculations for various thermodynamic quantities. However, due to the additional off-diagonal disorder our results concerning the influence of disorder on thermodynamic functions differ to some extent from those obtained by Nishimori.[11]

The paper is organized as follows. In Sec. II we describe the Jordan-Wigner transformation from spins to noninteracting spinless fermions and the evaluation of the averaged one-fermion Green functions. In Sec. III the averaged Green functions are used to calculate the thermodynamic properties, namely, entropy, specific heat, transverse magnetization, and static transverse linear susceptibility. Some conclusions are given in Sec. IV. A short report of these results was presented in Ref. 21.

II. JORDAN-WIGNER TRANSFORMATION AND AVERAGED ONE-FERMION GREEN FUNCTIONS

We consider a linear XY chain of N spins $\frac{1}{2}$ in a transverse field with periodical boundary conditions. The Hamiltonian reads

$$H=\sum_{n=1}^{N}\Omega_n s_n^z+\sum_{n=1}^{N}J_n(s_n^x s_{n+1}^x+s_n^y s_{n+1}^y)$$

$$=\sum_{n=1}^{N}\Omega_n\left(s_n^+ s_n^- -\frac{1}{2}\right)$$

$$+\sum_{n=1}^{N}\frac{J_n}{2}(s_n^+ s_{n+1}^- + s_n^- s_{n+1}^+),\quad s_{n+N}^{\alpha}=s_n^{\alpha}, \tag{1}$$

where Ω_n is the transverse field at site n and J_n is the exchange interaction between the sites n and $n+1$. The J_n are taken to be independent random variables with a Lorentzian probability distribution

$$p(J_n)=\frac{1}{\pi}\frac{\Gamma}{(J_n-J_0)^2+\Gamma^2}. \tag{2}$$

Here J_0 is the mean value and Γ is the width of the distribution (strength of disorder). In order to treat the model (1), (2) in an exact manner we assume the following relation between the transverse field at each site Ω_n and the surrounding intersite interactions (see Refs. 15–20):

$$\Omega_n-\Omega_0=a\left(\frac{J_{n-1}-J_0}{2}+\frac{J_n-J_0}{2}\right),\quad a \text{ is real, } |a|\geqslant 1, \tag{3}$$

where Ω_0 is the averaged transverse field at site.

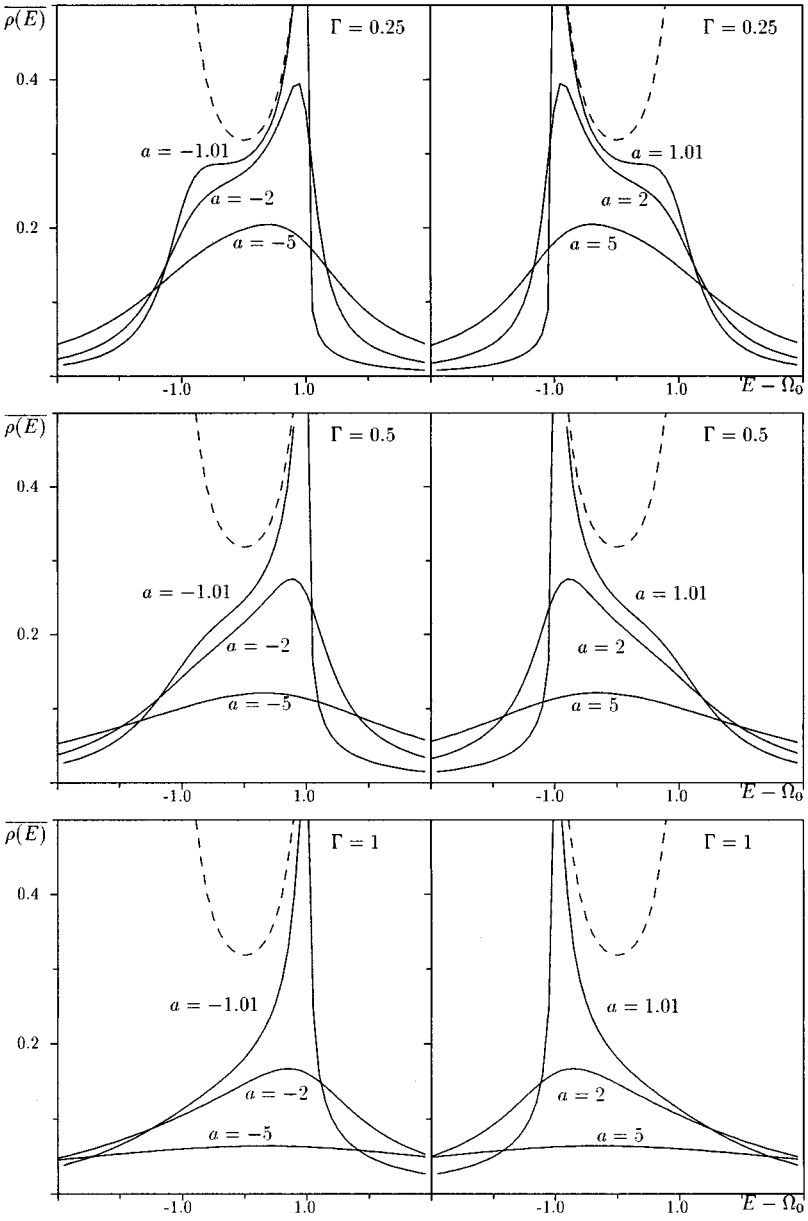

FIG. 1. The averaged density of states (13) $\overline{\rho(E)}$ vs $E-\Omega_0$.

Next we transform the spin model to a fermionic model by Jordan-Wigner transformation: $c_1=s_1^-$, $c_1^+=s_1^+$, $c_j=P_{j-1}s_j^-$, $c_j^+=P_{j-1}s_j^+$, $j=2,\ldots,N$, $P_j\equiv\Pi_{n=1}^{j}(-2s_n^z)$. The resulting Hamiltonian reads

$$H=H^-+BP^+=H^+P^++H^-P^-,$$

$$H^{\pm}\equiv-\frac{1}{2}\sum_{n=1}^{N}\Omega_n+\sum_{n=1}^{N}\Omega_n c_n^+c_n+\sum_{n=1}^{N}\frac{J_n}{2}(c_n^+c_{n+1}-c_nc_{n+1}^+),$$

$$B\equiv-J_N(c_N^+c_1-c_Nc_1^+),$$

$$P^{\pm}\equiv\frac{1\pm P}{2},\quad P\equiv\prod_{n=1}^{N}(-2s_n^z) \tag{4}$$

with anticyclic boundary conditions for H^+ and cyclic boundary conditions for H^-. For the calculation of thermodynamic properties of model (1) one can omit the boundary term B,[22] i.e., it is sufficient to study the thermodynamics of spinless fermions described by the c-cyclic Hamiltonian $H=H^-$. This c-cyclic fermionic Hamiltonian corresponds to the one-dimensional version of Anderson's model with off-diagonal disorder considered by John and Schreiber.[15]

Following Ref. 15 one introduces the retarded and advanced temperature double-time Green functions

$$G_{nm}^{\mp}(t)\equiv\mp i\theta(\pm t)\langle\{c_n(t),c_m^+\}\rangle,$$

$$G_{nm}^{\mp}(t)=1/2\pi\int_{-\infty}^{\infty}d\omega e^{-i\omega t}G_{nm}^{\mp}(\omega\pm i\varepsilon),$$

$$\varepsilon\to+0.$$

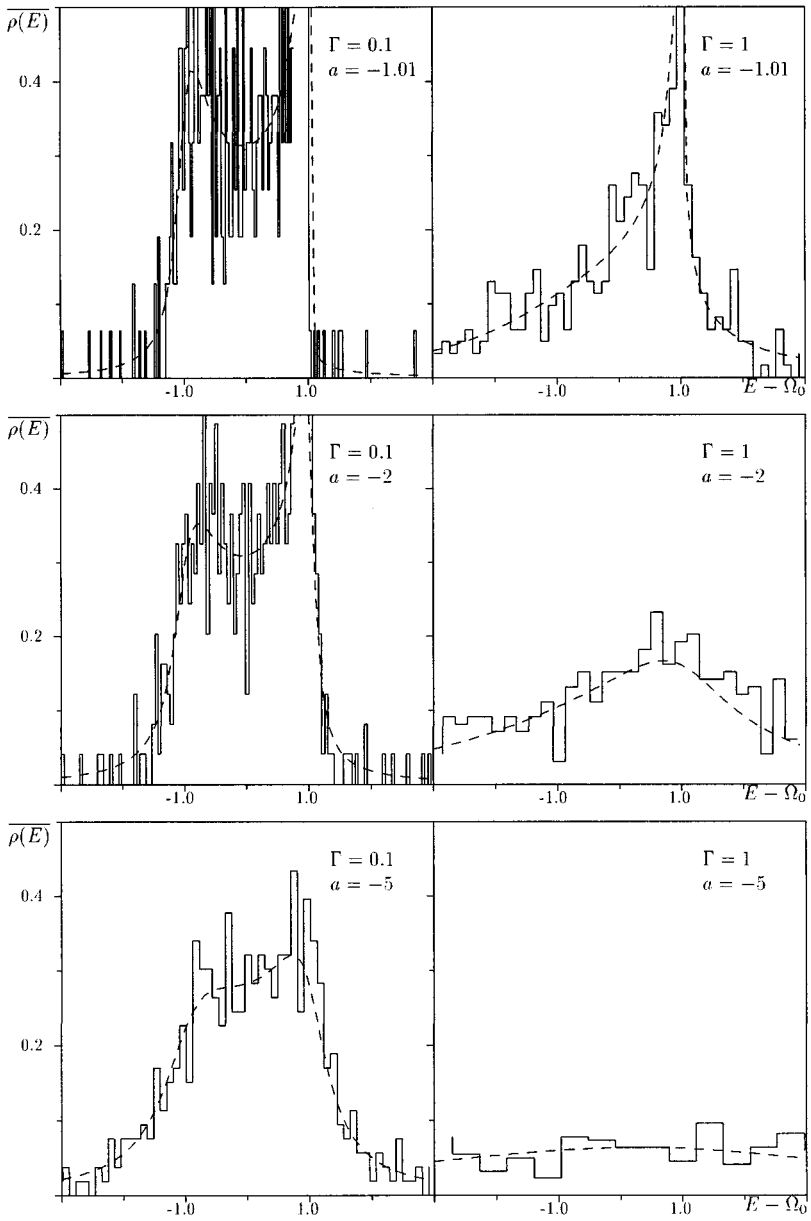

FIG. 2. The averaged density of states (13) $\overline{\rho(E)}$ (broken lines) and the density of states for certain realization of random intersite interactions obtained by exact finite-chain calculations (solid lines) vs $E-\Omega_0$.

For $G^{\mp}_{nm}(\omega \pm i\varepsilon)$ one finds the following set of equations:

$$(\omega \pm i\varepsilon - \Omega_n) G^{\mp}_{nm}(\omega \pm i\varepsilon) - \left[\frac{J_{n-1}}{2} G^{\mp}_{n-1,m}(\omega \pm i\varepsilon) + \frac{J_n}{2} G^{\mp}_{n+1,m}(\omega \pm i\varepsilon) \right] = \delta_{nm}. \tag{5}$$

Suppose that, $\ldots, J_n, \ldots,$ (and hence, $\ldots, \Omega_n, \ldots$) are complex variables. The singularities of $G^{\mp}_{nm}(\omega \pm i\varepsilon)$ are given by the zeros of $\det(\omega \pm i\varepsilon - H)$. $\det(\omega \pm i\varepsilon - H)$ is different from zero if the eigenvalues λ of $\mathrm{Im}(\omega \pm i\varepsilon - H)$ are either all positive or all negative. From the Gershgorin criterion[23,24] for the complex matrix $\mathrm{Im}(\omega \pm i\varepsilon - H)$ one gets that for any eigenvalue λ at least one of the conditions

$$\left| \mathrm{Im}(\omega \pm i\varepsilon - \Omega_n) - \lambda \right| \leqslant \frac{1}{2} \left| \mathrm{Im} J_{n-1} \right| + \frac{1}{2} \left| \mathrm{Im} J_n \right| \tag{6}$$

has to be fulfilled. Using Eq. (3) the inequalities (6) can be written as

$$\left| \mathrm{Im}(\omega \pm i\varepsilon) - \frac{a}{2} (\mathrm{Im} J_{n-1} + \mathrm{Im} J_n) - \lambda \right| \leqslant \frac{1}{2} |\mathrm{Im} J_{n-1}| + \frac{1}{2} |\mathrm{Im} J_n|, \quad |a| \geqslant 1. \tag{7}$$

Let us consider the retarded Green function $[\mathrm{Im}(\omega + i\varepsilon) > 0]$. Then according to Eq. (7) for $a \geqslant 1$ all λ must be positive if all $\mathrm{Im} J_n < 0$, whereas for $a \leqslant -1$ all λ must be positive if all $\mathrm{Im} J_n > 0$. Similarly, for the advanced Green function $[\mathrm{Im}(\omega - i\varepsilon) < 0]$ according to Eq. (7) for $a \geqslant 1$ all λ are negative if all $\mathrm{Im} J_n > 0$, and for $a \leqslant -1$ all λ are negative if all $\mathrm{Im} J_n < 0$.

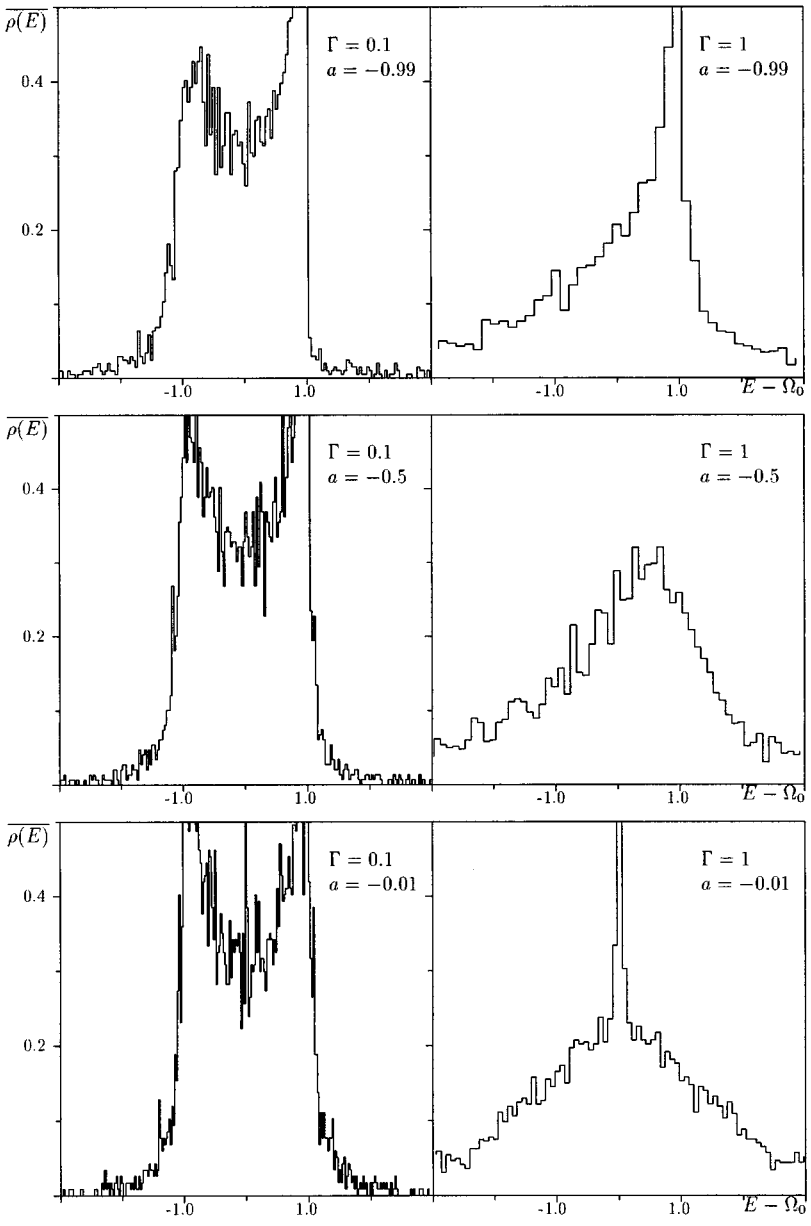

FIG. 3. The density of states $\rho(E)$ averaged over 10 realizations for $|a|<1$ obtained by exact finite-chain calculations vs $E-\Omega_0$.

Consequently, for $a \geqslant 1$ $(a \leqslant -1)$ the retarded Green function $G^{-}_{nm}(\omega+i\varepsilon)$ cannot have a pole in the lower (upper) half-planes of complex variables J_n, whereas the advanced Green function $G^{+}_{nm}(\omega-i\varepsilon)$ cannot have a pole in the upper (lower) half-planes of complex variables J_n for $a \geqslant 1$ $(a \leqslant -1)$. Using these properties one can perform the averaging of Eqs. (5), defined by

$$\overline{(\ldots)} \equiv \prod_{n=1}^{N} \int_{-\infty}^{\infty} dJ_n \frac{1}{\pi} \frac{\Gamma}{(J_n-J_0)^2+\Gamma^2}(\cdots) = \prod_{n=1}^{N} \int_{-\infty}^{\infty} dJ_n \frac{1}{\pi} \frac{\Gamma}{(J_n-J_0+i\Gamma)(J_n-J_0-i\Gamma)}(\cdots), \tag{8}$$

by means of contour integrals. For the averaging of a function $F(\ldots,\Omega_n,J_n,\ldots)$ that has no poles in lower half-planes J_n, one can close the contours of integration in Eq. (8) in these half-planes. One obtains

$$\overline{F(\ldots,\Omega_n,J_n,\ldots)} = F(\ldots,\Omega_0-ia\Gamma,J_0-i\Gamma,\ldots). \tag{9}$$

Similarly, for the function without poles in upper half-planes J_n one gets by contour integration

$$\overline{F(\ldots,\Omega_n,J_n,\ldots)} = F(\ldots,\Omega_0+ia\Gamma,J_0+i\Gamma,\ldots). \tag{10}$$

Then the averaged equations for Green functions (5) due to Eqs. (9), (10) read

$$[\omega \pm i\varepsilon - (\Omega_0 \mp i|a|\Gamma)]\overline{G^{\mp}_{nm}(\omega \pm i\epsilon)} - \frac{J_0 \mp i\,\mathrm{sgn}(a)\Gamma}{2} \times [\overline{G^{\mp}_{n-1,m}(\omega \pm i\varepsilon)} + \overline{G^{\mp}_{n+1,m}(\omega \pm i\varepsilon)}] = \delta_{nm}. \tag{11}$$

Equations (11) possess translational symmetry and there-

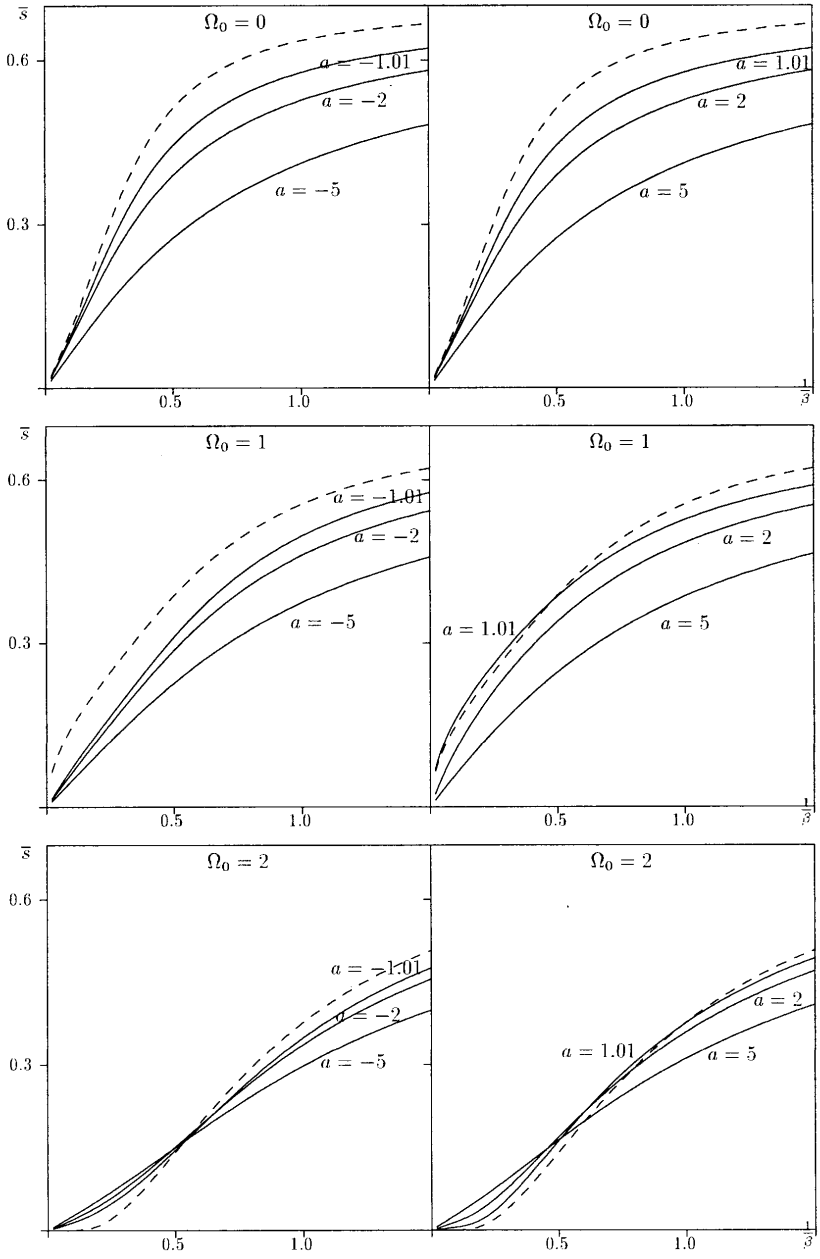

FIG. 4. The entropy $\overline{s}$ (14) vs temperature $1/\beta$, $\Gamma = 0.25$.

fore they can be solved in a standard way. The resulting averaged Green functions read

$$\overline{G^{\mp}_{nm}(\omega)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\kappa \quad e^{i(n-m)\kappa} \times \frac{1}{\omega - [\Omega_0 \mp i|a|\Gamma + [J_0 \mp i\,\mathrm{sgn}(a)\Gamma]\cos\kappa]} = \frac{[(\sqrt{x^2-y^2}-x)/y]^{|n-m|}}{\sqrt{x^2-y^2}} \tag{12}$$

with $x \equiv \omega - \Omega_0 \pm i|a|\Gamma$, $y \equiv J_0 \mp i\,\mathrm{sgn}(a)\Gamma$.

III. ENTROPY, SPECIFIC HEAT, TRANSVERSE MAGNETIZATION, AND STATIC TRANSVERSE LINEAR SUSCEPTIBILITY

The obtained averaged Green functions (12) allow us to study the thermodynamics of the spin model (1)–(3). For this we diagonalize the bilinear in Fermi operators form H^- (4) by the canonical transformation $\eta_k = \Sigma_{n=1}^N g_{kn} c_n$, $\Lambda_k g_{kn} = \Sigma_{i=1}^N g_{ki} A_{in}$, $A_{ij} = \Omega_i \delta_{ij} + \frac{1}{2} J_i \delta_{j,i+1} + \frac{1}{2} J_{i-1} \delta_{j,i-1}$, $\Sigma_{i=1}^N g_{ki} g_{pi} = \delta_{kp}$, $\Sigma_{p=1}^N g_{pi} g_{pj} = \delta_{ij}$ with the result $\Sigma_{p=1}^N \Lambda_p (\eta_p^+ \eta_p - \frac{1}{2})$. The thermodynamics for certain realizations of random intersite interactions is then determined by the spectrum of elementary excitations Λ_p or its density $\rho(E) \equiv (1/N)\Sigma_{p=1}^N \delta(E - \Lambda_p)$. For example, the Helmholtz free energy per site is given by $f = (1/N)(-(1/\beta)\ln\Pi_{p=1}^N \{\exp[-(\beta\Lambda_p/2)] + \exp(\beta\Lambda_p/2)\}) = -(1/\beta)\int dE \rho(E) \ln[2\cosh(\beta E/2)]$. The result of averaging over realizations of random variables is given by

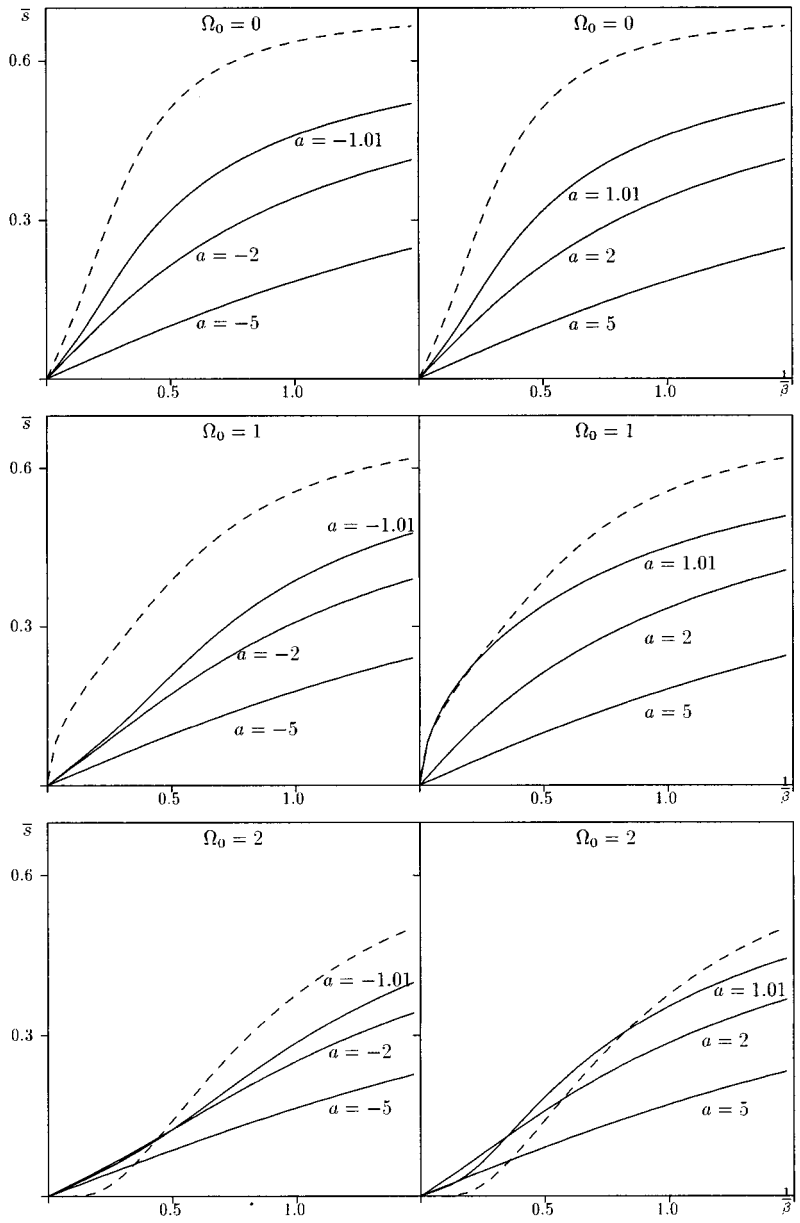

FIG. 5. The entropy $\overline{s}$ (14) vs temperature $1/\beta$, $\Gamma = 1$.

$\overline{f} = -(1/\beta)\int dE\,\overline{\rho(E)}\ln[2\cosh(\beta E/2)]$. The required averaged density of states is then calculated by the averaged one-particle Green functions (12)

$$\overline{\rho(E)} = -\frac{1}{\pi}\mathrm{Im}\overline{G^{-}_{nn}(E)} = \frac{1}{\pi}\mathrm{Im}\overline{G^{+}_{nn}(E)}$$

$$= \mp\frac{1}{\pi}\mathrm{Im}\frac{1}{\sqrt{(E-\Omega_0 \pm i|a|\Gamma)^2 - [J_0 \mp i\,\mathrm{sgn}(a)\Gamma]^2}}$$

$$= \frac{1}{\pi}\sqrt{\frac{\sqrt{A^2+B^2}-A}{2(A^2+B^2)}},$$

$$A \equiv (E-\Omega_0)^2 + (1-|a|^2)\Gamma^2 - J_0^2,$$

$$B \equiv 2\Gamma[|a|(E-\Omega_0) + \mathrm{sgn}(a)J_0]. \qquad (13)$$

Really, $\rho(E) = (1/N)\Sigma_{p=1}^{N}[-(1/\pi)\mathrm{Im}\Gamma^{-}_{pp}(E+i\varepsilon)] = (1/N)\Sigma_{p=1}^{N}[(1/\pi)\mathrm{Im}\Gamma^{+}_{pp}(E-i\varepsilon)]$, where $\Gamma^{\mp}_{pq}(t) \equiv \mp i\,\theta(\pm t)\langle\{\eta_p(t), \eta_q^{+}\}\rangle = \Sigma_{i=1}^{N}\Sigma_{j=1}^{N} g_{pi}g_{qj}G^{\mp}_{ij}(t)$, and therefore $\rho(E) = (1/N)\Sigma_{j=1}^{N}[-(1/\pi)\mathrm{Im}G^{-}_{jj}(E+i\varepsilon)] = (1/N)\Sigma_{j=1}^{N}[(1/\pi)\mathrm{Im}G^{+}_{jj}(E-i\varepsilon)]$; the averaging yields the first two equalities in the left-hand side of Eq. (13).

Knowing the averaged Helmholtz free energy we can calculate the entropy and specific heat by the formulas

$$\overline{s} = \beta^2\frac{\partial\overline{f}}{\partial\beta} = \int dE\,\overline{\rho(E)}\left[\ln\left(2\cosh\frac{\beta E}{2}\right) - \frac{\beta E}{2}\tanh\frac{\beta E}{2}\right], \qquad (14)$$

$$\overline{c} = -\beta\frac{\partial\overline{s}}{\partial\beta} = \int dE\,\overline{\rho(E)}\left(\frac{\beta E/2}{\cosh(\beta E/2)}\right)^2. \qquad (15)$$

Due to the magic property of (13) $(\partial/\partial\Omega_0)\overline{\rho(E)} = -$

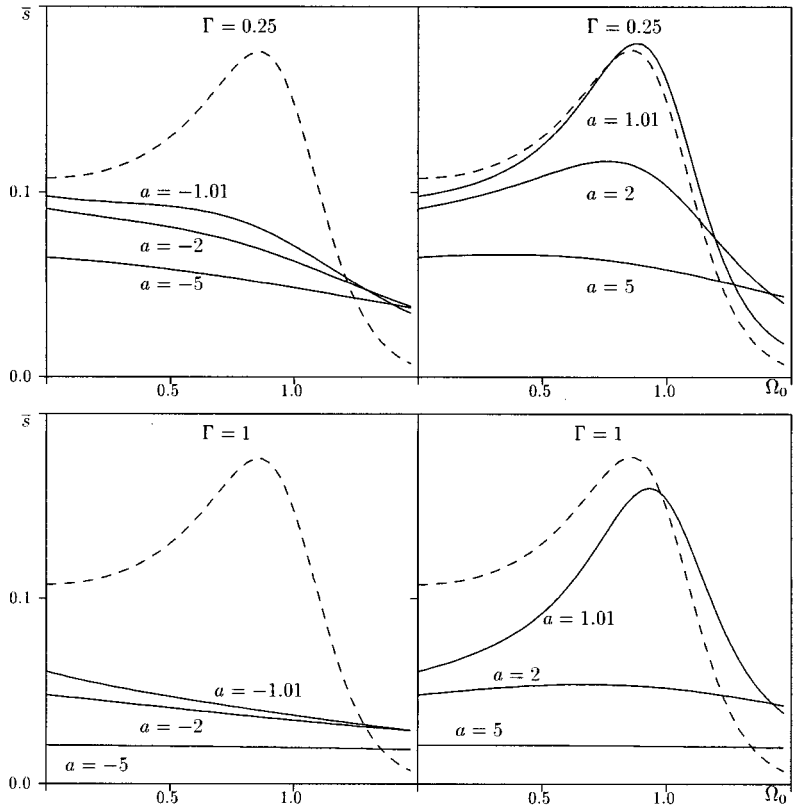

FIG. 6. The entropy $\overline{s}$ (14) vs transverse field Ω_0, $1/\beta=0.1$.

$(\partial/\partial E)\overline{\rho(E)}$ one can express transverse magnetization and static transverse linear susceptibility through the density of states

$$\overline{m_z}\equiv\overline{\left\langle \frac{1}{N}\sum_{n=1}^{N} s_n^z \right\rangle}=\frac{\partial\overline{f}}{\partial\Omega_0}=-\frac{1}{2}\int dE\,\overline{\rho(E)}\tanh\frac{\beta E}{2}, \tag{16}$$

$$\overline{\chi_{zz}}=\frac{\partial\overline{m_z}}{\partial\Omega_0}=-\beta\int dE\,\overline{\rho(E)}\frac{1}{[2\cosh(\beta E/2)]^2}. \tag{17}$$

Let us discuss the obtained results.

Note at first, that in the absence of randomness Eq. (13) reduces to the well-known result

$$\overline{\rho(E)}=\mp\frac{1}{\pi}\mathrm{Im}\frac{1}{\sqrt{(E-\Omega_0\pm i\varepsilon)^2-J_0^2}}$$

$$=\begin{cases}\dfrac{1}{\pi}\dfrac{1}{\sqrt{J_0^2-(E-\Omega_0)^2}} & \text{if} \quad |E-\Omega_0|\leqslant|J_0|,\\ 0, \quad \text{otherwise}\end{cases} \tag{18}$$

as anticipated.

The considered model (1)–(3) essentially differs from the one with diagonal disorder treated by Nishimori[11] ($J_n=J$, Ω_n are independent random variables with Lorentzian distributions). However, Nishimori's model can be obtained as a certain limit of the present model, namely, $\Gamma\to0$, $|a|\Gamma=\mathrm{const}=\Gamma_N$. The density of states (13) in contrast to the case of diagonal disorder is not symmetric with respect to the change $E-\Omega_0\to-(E-\Omega_0)$. However, it remains the same after the replacement $E-\Omega_0\to-(E-\Omega_0)$, $a\to-a$, or $E-\Omega_0\to-(E-\Omega_0)$, $J_0\to-J_0$, since the simultaneous change of signs of J_0 and a in Eq. (13) does not affect $\overline{\rho(E)}$. Without loss of generality we choose $\Omega_0, J_0>0$ throughout the rest of the paper. It is also convenient, although by no means essential, to put hereafter $J_0=1$. The above-mentioned symmetry of the density of states can be seen in Figs. 1–3, where the averaged density of states (13) (Fig. 1), the averaged density of states (13) in comparison with histograms $\rho(E)$ calculated for a certain realization of random intersite interactions using exact finite-chain calculations[25] (Fig. 2), and the histograms $\overline{\rho(E)}$ obtained by the latter approach for $|a|<1$ (Fig. 3) are displayed. The density of states for the nonrandom case (18) is depicted in Fig. 1 by dashed lines. For large $|a|$ the edges of the zone are completely smeared out with increasing strength of disorder Γ; for $|a|\approx1$ the increase of disorder results in a smearing out of mainly one edge of the zone. A further decrease of $|a|$ up to 0 leads to a recovering of the symmetry with respect to $E-\Omega_0\to-(E-\Omega_0)$ and to transforming of $\overline{\rho(E)}$ into $\delta(E-\Omega_0)$. Some consequences induced by this dependence of $\overline{\rho(E)}$ on Γ and a will be seen in the behavior of thermodynamic quantities.

The results of numerical calculations of thermodynamic quantities are presented in Figs. 4–13, namely, the temperature dependences of the entropy (14) (Figs. 4,5), the specific heat (15) (Figs. 7,8), the transverse magnetization (16) (Fig. 11) and the static transverse linear susceptibility (17) (Fig. 13), the dependence on averaged transverse field at low tem-

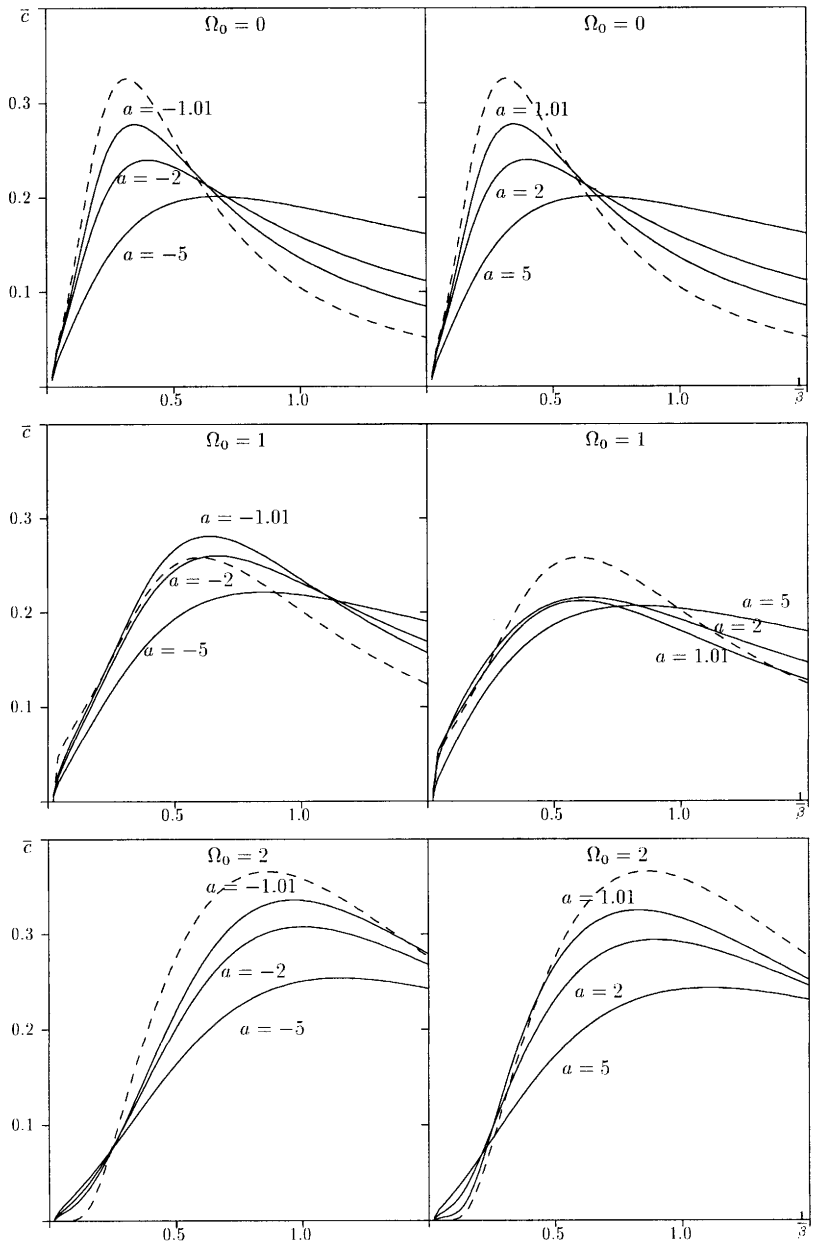

FIG. 7. The specific heat $\overline{c}$ (15) vs temperature $1/\beta$, $\Gamma = 0.25$.

peratures of the entropy (Fig. 6), the specific heat (Fig. 9), the transverse magnetization (Fig. 10), and the static transverse linear susceptibility (Fig. 12); the curves that correspond to the nonrandom case are depicted in Figs. 4–13 by dashed lines.

The influence of randomness on thermodynamics is mainly rather typical. It leads to (i) a weak deformation of the entropy versus temperature curve with a decrease of the entropy at high temperatures (Figs. 4,5), (ii) a broadening and decreasing of the peak in the dependence of the specific heat versus temperature (Figs. 7,8), (iii) a smearing out of the cast in the $\overline{m_z}$ versus Ω_0 curve at $T=0$ for $\Omega_0 = J_0$ and a nonsaturated transverse magnetization at any finite transverse field (Fig. 10), (iv) a decreasing and disappearing of the singularity (accompanying the saturation of $\overline{m_z}$ at $T=0$ for $\Omega_0 = J_0$) in the curve $\overline{\chi_{zz}}$ versus Ω_0 at $T=0$ (Fig. 12), and (v) a suppressing of static transverse linear susceptibility versus temperature curve (Fig. 13).

However, as can be seen in Figs. 4–13, the influence of disorder, especially for small a, essentially depends on the sign of a. Particularly interesting is the case of strong asymmetry in the density of states $\overline{\rho(E)}$ when $|a| \approx 1$. From a mathematical point of view the dependence of the computed quantities on temperature and averaged transverse field and the well-pronounced difference between the cases $a \approx -1$ and $a \approx 1$ can be understood keeping in mind that these quantities according to Eqs. (14)–(17) are integrals over E of products of $\overline{\rho(E)}$ (shown in Fig. 1) with functions with evident dependence on E at different β. It is interesting to note that for some Hamiltonian parameters and temperatures even very large randomness (controlled by Γ) almost does not affect the thermodynamic quantities. This can be nicely seen in Figs. 4–13.

It is worth emphasizing that the asymmetry of $\overline{\rho(E)}$ leads to the appearance of nonzero averaged transverse magnetiza-

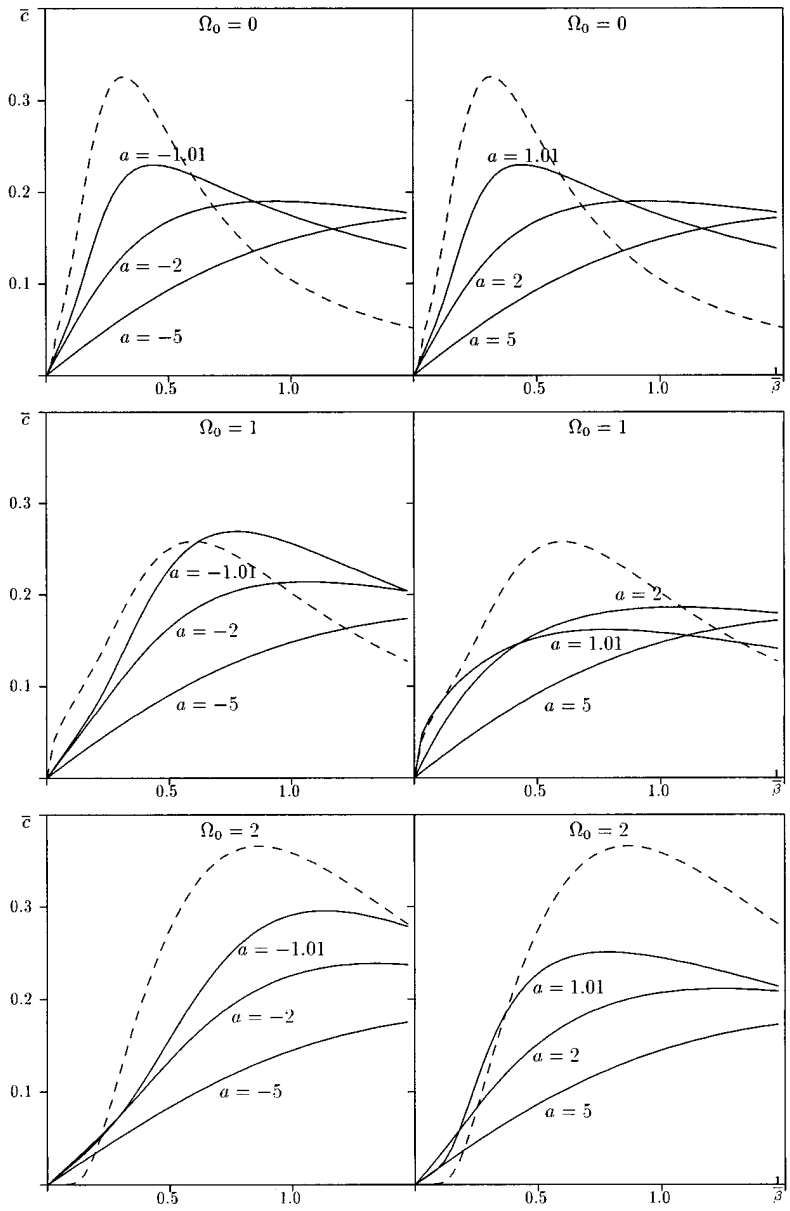

FIG. 8. The specific heat $\overline{c}$ (15) vs temperature $1/\beta$, $\Gamma = 1$.

tion $\overline{m_z}$ at zero averaged transverse field Ω_0. As can be seen from Eq. (16) $\overline{m_z} = 0$ at $T = 0$, $\Omega_0 = 0$ if $\int_{-\infty}^{0} dE\overline{\rho(E)} = \int_{0}^{\infty} dE\overline{\rho(E)}$. This is evidently true for symmetric density of states $\overline{\rho(E)}$ (as in the case considered by Nishimori) but is not obvious in the considered case (13). The difference between the integrals $\int_{-\infty}^{0} dE\overline{\rho(E)}$ and $\int_{0}^{\infty} dE\overline{\rho(E)}$ can be clearly demonstrated by numerical finite-chain calculations as a difference between the numbers of negative and positive eigenvalues Λ_p of the $N \times N$ matrix $||A_{ij}||$ denoted by $\mathcal{N}_-$ and $\mathcal{N}_+$, respectively. Examples for a certain realization of the random model (1)–(3) are given in Tables I and II. The transverse magnetization for a certain realization at $T = 0$ is given by $m_z = (\mathcal{N}_- - \mathcal{N}_+)/2N$ and one finds a good agreement of these direct numerical finite-chain calculations for $-\overline{m_z}$ with the results depicted in Fig. 10 (e.g., for $\Gamma = 0.25$ $-\overline{m_z} \approx 0.051$, 0.038, 0.011 if $a = -1.01$, -2, -5, respectively, for $\Gamma = 1$ $-\overline{m_z} \approx 0.095$, 0.030, 0.003 if $a = -1.01$, -2, -5, respectively).

IV. CONCLUSIONS

In conclusion, we present exact calculations of the thermodynamics of the spin-$\frac{1}{2}$ isotropic XY chain with random Lorentzian intersite interaction and a transverse field that depends linearly on the surrounding intersite interactions (1)–(3). The derived exact expressions for the averaged density of states (13) and thermodynamic quantities (14)–(17) may serve as a testing ground for approximate methods of spin systems with off-diagonal disorder that usually involve an unclear error. Aside from this they are interesting in their own right, since experimentally accessible systems are always affected by randomness, and an understanding of disorder effects even within such a simple model can help in comparing experimental observations and theoretical predictions.

Unfortunately, the obtained results do not permit us to calculate exactly the averaged spin correlation functions since such calculation requires the knowledge of averaged many-particle fermion Green functions. Spin correlations

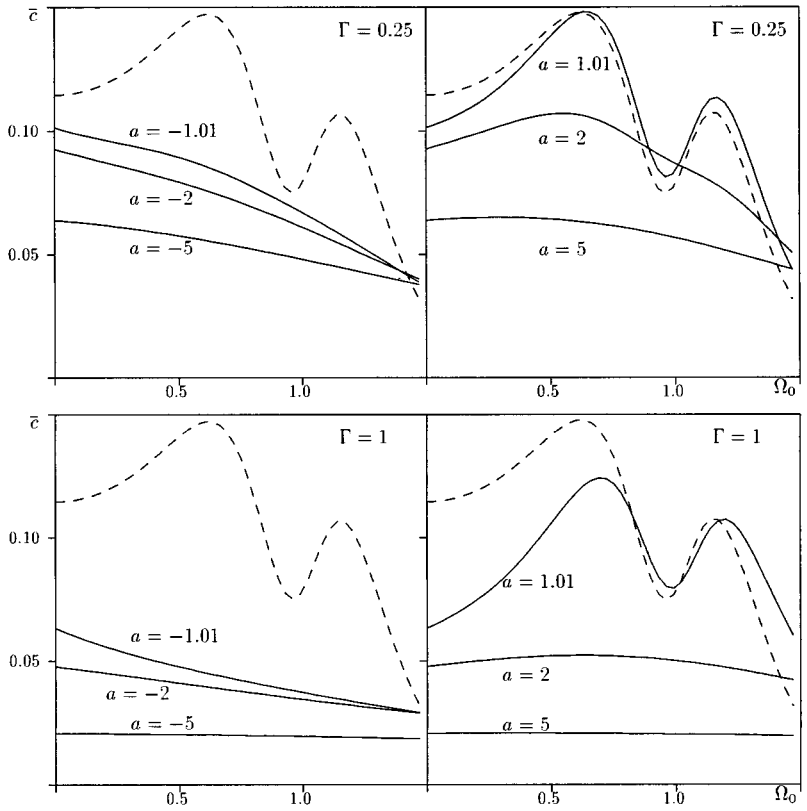

FIG. 9. The specific heat $\overline{c}$ (15) vs transverse field Ω_0, $1/\beta = 0.1$.

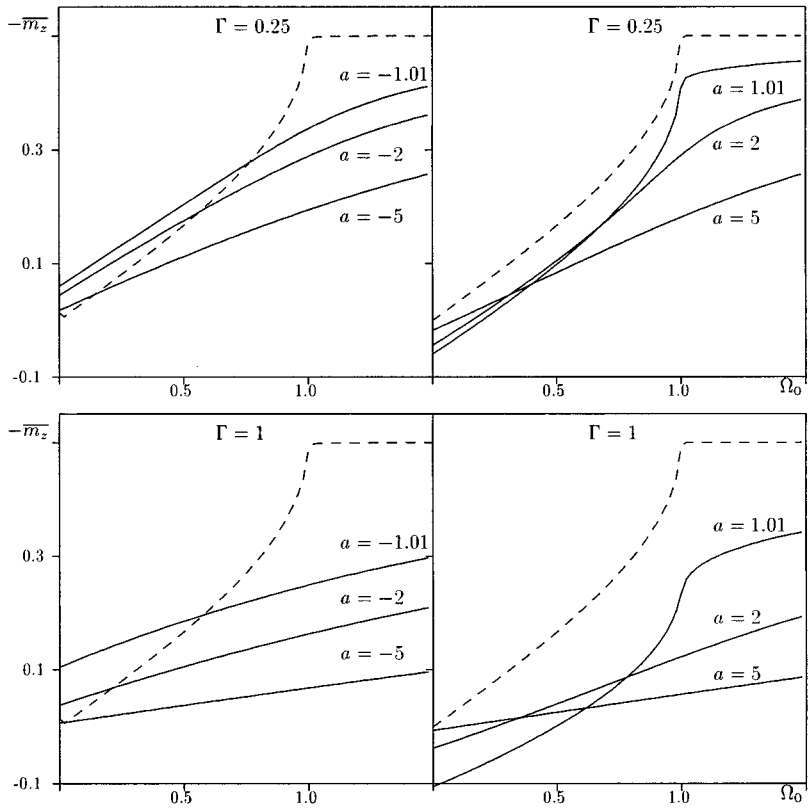

FIG. 10. The transverse magnetization $-\overline{m_z}$ (16) vs transverse field Ω_0 at low temperature ($1/\beta = 0.001$).

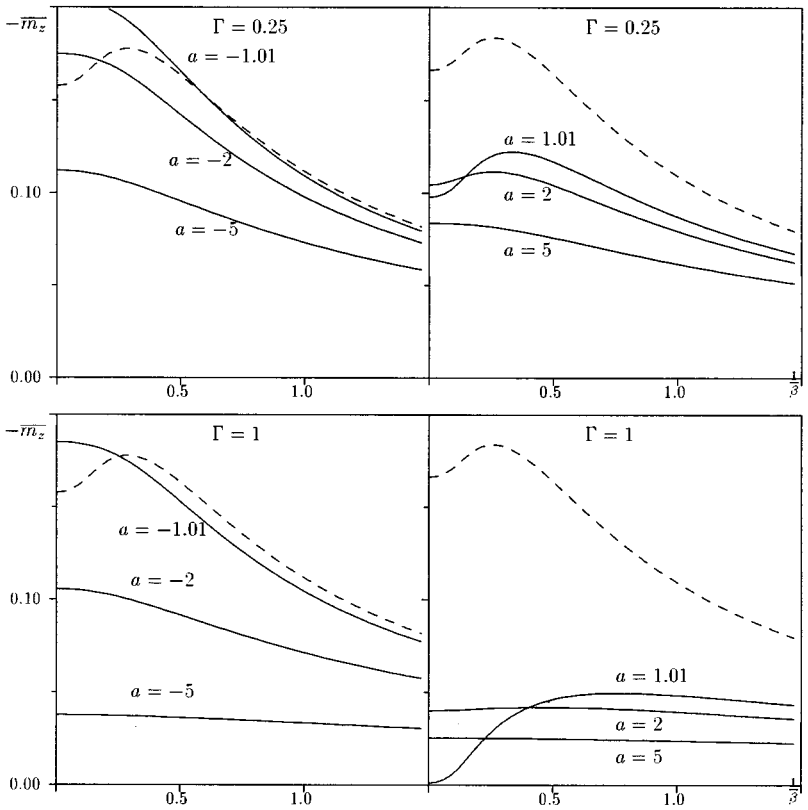

FIG. 11. The transverse magnetization $-\overline{m_z}$ (16) vs temperature $1/\beta$ at $\Omega_0 = 0.5$.

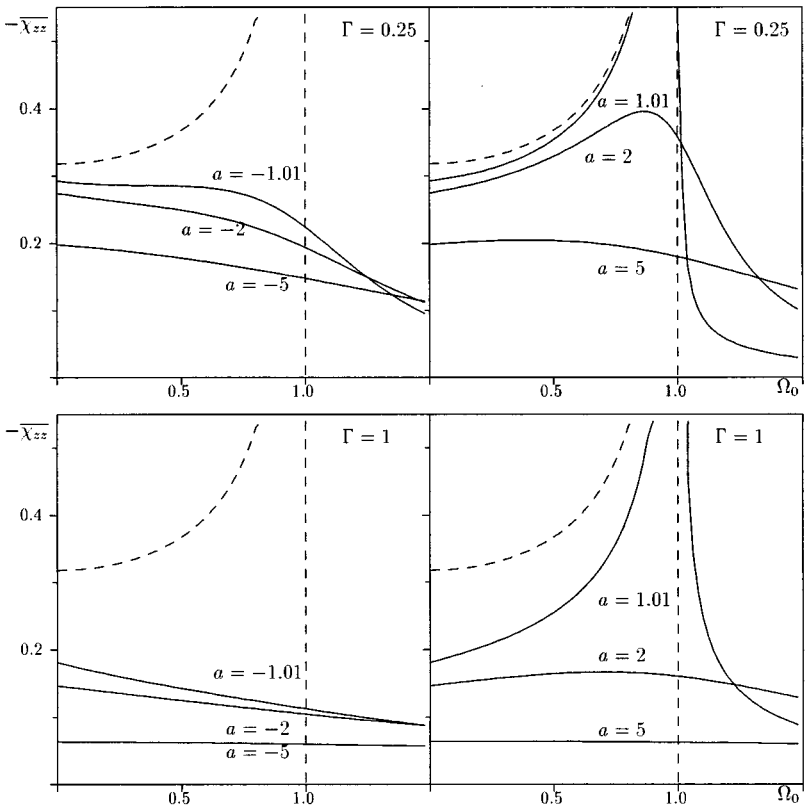

FIG. 12. The static transverse linear susceptibility $-\overline{\chi_{zz}}$ (17) vs transverse field Ω_0 at low temperature ($1/\beta = 0.001$).

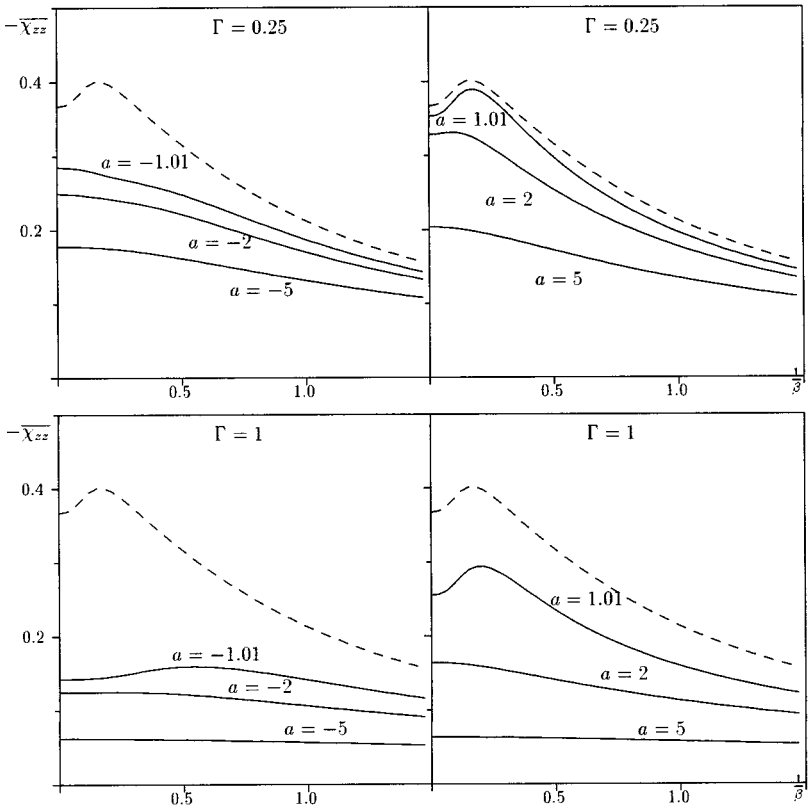

FIG. 13. The static transverse linear susceptibility $-\overline{\chi_{zz}}$ (17) vs temperature $1/\beta$ at $\Omega_0=0.5$.

TABLE I. The numbers of negative and positive Λ_p, $\mathcal{N}_-$ and $\mathcal{N}_+$, for three realizations of random intersite interactions in a finite model (1)–(3) ($N=1000$) with $\Omega_0=0$, $J_0=1$, $\Gamma=0.25$.

$(1/N)\Sigma_{j=1}^{N}J_j$	$a=-5$		$a=-2$		$a=-1.01$		$a=1.01$		$a=2$		$a=5$	
	$\mathcal{N}_-$	$\mathcal{N}_+$	$\mathcal{N}_-$	$\mathcal{N}_+$	$\mathcal{N}_-$	$\mathcal{N}_+$	$\mathcal{N}_-$	$\mathcal{N}_+$	$\mathcal{N}_-$	$\mathcal{N}_+$	$\mathcal{N}_-$	$\mathcal{N}_+$
0.997	491	509	458	542	454	546	546	454	542	458	509	491
0.984	490	510	464	536	442	558	558	442	536	464	510	490
1.008	487	513	463	537	452	548	548	452	537	463	513	487

TABLE II. The numbers of negative and positive Λ_p, $\mathcal{N}_-$ and $\mathcal{N}_+$, for three realizations of random intersite interactions in a finite model (1)–(3) ($N=1000$) with $\Omega_0=0$, $J_0=1$, $\Gamma=1$.

$(1/N)\Sigma_{j=1}^{N}J_j$	$a=-5$		$a=-2$		$a=-1.01$		$a=1.01$		$a=2$		$a=5$	
	$\mathcal{N}_-$	$\mathcal{N}_+$	$\mathcal{N}_-$	$\mathcal{N}_+$	$\mathcal{N}_-$	$\mathcal{N}_+$	$\mathcal{N}_-$	$\mathcal{N}_+$	$\mathcal{N}_-$	$\mathcal{N}_+$	$\mathcal{N}_-$	$\mathcal{N}_+$
1.009	495	505	470	530	402	598	598	402	530	470	505	495
0.986	503	497	471	529	408	592	592	408	529	471	497	503
1.034	494	506	469	531	406	594	594	406	531	469	506	494

and their dynamics may be examined using the exact finite-chain calculations developed in Refs. 26 and 27.

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