

Interactions and disorder in multichannel quantum wires

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Recent experiments have revealed that the temperature dependence of the conductance of quasiballistic quantum wires bears characteristic features of the Luttinger-liquid state. In this paper, the conductance of an N -channel quantum wire is calculated within the model of N coupled Luttinger liquids and under the assumption of weak disorder. It is shown that as the number of channels increases, a crossover from the Luttinger-liquid to the Fermi-liquid behavior occurs. This crossover manifests itself in the $1/N$ decrease of the scaling exponent of the temperature dependence. An exact expression for the scaling exponent for the case of N coupled Luttinger chains is obtained, and the large N limit is studied for the case of a quantum wire. The case of $N=2$ for electrons with spin is analyzed in detail, and a qualitative agreement with the experiment is achieved. [S0163-1829(97)00320-2]

I. INTRODUCTION

Although the conventional Landau theory for Fermi liquids^{1,2} has been very successful in understanding many condensed matter systems, its failure to describe quasi-one-dimensional (Q1D) systems, e.g., conducting polymers and organic conductors, has motivated the search for alternative models which can describe a non-Fermi liquid behavior. The most theoretically studied example of a non-Fermi liquid system is that of interacting electrons in one dimension (1D) known as the Tomonaga-Luttinger model.^{3,4} By now, the main properties of this model are theoretically well understood and form the concept of a "Luttinger liquid."⁵ Luttinger liquids are very different from their higher dimensional counterparts, Fermi liquids, in many respects, including the following: the absence of single-particle excitations at low energies, spin-charge separation, and the absence of a well-defined Fermi surface, even at zero temperatures.

Despite the vigorous theoretical activity in this field, there has been only limited experimental evidence for the existence of Luttinger liquids in the conventional Q1D systems, such as organic conductors. The genuine Luttinger-liquid behavior is easily masked in these systems by other effects, such as Peierls transitions and dimensionality crossovers resulting from the coupling among the conducting chains. However, recent advances in semiconductor technologies have made high-mobility quantum wires new and promising candidates for studying Luttinger-liquid effects in Q1D interacting electron systems. The most obvious advantages of the quantum wires are (i) the absence of unintentional dimensionality crossovers; (ii) a very low and controllable degree of disorder; and (iii) the absence of Peierls transitions. Indeed, the first evidence for Luttinger-liquid behavior has recently been obtained in transport measurements^{6,7} on GaAs quantum wires, where the temperature dependence of the

conductance of a weakly disordered single-channel⁶ and multichannel⁷ quantum wires was interpreted in terms of the Tomonaga-Luttinger model.⁸ Luttinger-liquid behavior has also been observed in transport experiments on fractional quantum Hall systems,^{9,10} where the edge states are believed to be in the chiral Luttinger-liquid state.¹¹

Luttinger liquids and Fermi liquids are two fixed-point regimes, which are well understood on their own. The crossover between these regimes, which is expected to occur when several Luttinger liquids are coupled together, is of significant interest from the general theoretical point of view and has been investigated by a number of authors.¹²⁻²¹ However, it is difficult to compare the results of various theoretical approaches with the experiment, because the parameters driving this crossover such as the number of Luttinger-liquid chains coupled together by interchain tunneling, electron-electron interactions or both, cannot be changed smoothly in a real sample. The semiconductor quantum wires again appear to be ideal candidates for studying the Luttinger-liquid to Fermi-liquid crossover, because the number of conducting channels is an adjustable parameter of the experimental setup. The indication for such a crossover in the conductance of GaAs quantum wires has recently been observed.⁷ Theoretically, Matveev and Glazman^{22,23} have used a multichannel model to calculate the tunneling rate into a clean wire.

In this paper, we study the conductance of a multichannel quantum wire in the presence of disorder. Our main goal is to follow the crossover from the single-channel case to the multi-channel, when the wire is expected to be in the Luttinger-liquid state and the Fermi-liquid state, respectively. The second motivation for this study comes from recent experiments,⁷ in which an indication of such a crossover has been observed. Our main result is that the temperature-dependent conductance of a weakly disordered Luttinger-liquid wire is reduced with an increasing number of occupied channels N , and disappears in the limit of an infinite number

of channels. We find that the scaling exponent of the temperature dependence behaves as $1/N$, for $N \gg 1$.

This paper is organized as follows. In Sec. II, we describe the model of a multichannel quantum wire in the presence of long-range disorder and short-range electron-electron interactions. In Sec. III, we present a general formalism for the calculation of the conductance and derive the expression for the exponent of the temperature scaling. This exponent is analyzed for various situations in Sec. IV. In Sec. IVA, the general result for spinless electrons is studied, and the comparison with the experimental results is made in Sec. IV B. Our conclusions are given in Sec. V.

II. FORMULATION OF THE MODEL

In this section, we outline the main assumptions and approximations used to calculate the temperature-dependent conductance of a weakly disordered Luttinger-liquid wire including the following: the geometry of the wire, the effects of disorder, the nature of electron-electron interactions, and the effect of electron reservoirs. We rely on the approaches developed by Glazman and Jonson²⁴ and by Matveev and Glazman.²³

A. Geometry

Consider a quantum wire of width d , *adiabatically* connected to the leads. For simplicity, the confinement in the transverse direction is modeled by a square well potential. The wave function of the n th mode of transverse quantization $\Psi_n(x, y)$ is expanded over the adiabatic basis of transverse wave functions $\xi_{m\perp}(y)$,

$$\Psi_n(x, y) = \sum_m \psi_{nm}(x) \xi_{m\perp}(y). \quad (1)$$

Limiting to the leading (zeroth) order in the adiabatic expansion, $\psi_{nm}(x)$ takes the form

$$\psi_{nm}^{(0)}(x) = \frac{1}{\sqrt{L}} e^{ik_n x} \delta_{nm}, \quad (2)$$

where k_n is the longitudinal wave vector of an electron with a total Fermi momentum $\hbar k_F$,

$$k_n = k_F \sqrt{1 - \left[\frac{n}{z} \right]^2}, \quad (3)$$

where $z = k_F d / \pi$. The number of occupied transverse channels in the wire is $N = [z]$. An *effective* Fermi velocity for channel n is defined as $v_F(n) = \hbar k_n / m^*$.

B. Disorder

In the absence of disorder, the conductance is quantized in units of e^2/h per spin orientation, where each plateau of quantization corresponds to a newly occupied channel.

The quasiballistic regime, where the wire length L is shorter than the (transport) mean free path ℓ is considered. This regime is realized in the experiments by Tarucha *et al.*,^{6,7} in which $\ell/L > 6$.

We assume also that the disorder potential in GaAs heterostructures varies slowly on the scale of the Fermi

wavelength,²⁵ i.e., $k_F \ell_c > 1$, where ℓ_c is the correlation length of the disorder potential.²⁶ Backscattering processes in which the longitudinal momentum of the electron in the i th channel is changed by $\hbar \delta k_{ij} = \hbar k_i + \hbar k_j$, $j = 1 \dots N$, give the contribution to the resistance. The probability of these processes P_{BS} for a long-range disorder potential depends exponentially on δk_{ij} : $P_{BS} \sim \exp(-2\delta k_{ij} \ell_c)$.²⁵ Therefore two different regimes may be distinguished.²⁴

In the first regime, P_{BS} is exponentially small for all occupied channels, except for the topmost one ($i = N$). In this channel, two situations can occur. (1) When the N th channel is just opened, the Fermi energy is equal to the threshold energy. The momentum carried by this channel is small and is strongly affected by impurity scattering. In this case, the scattering in channel N clearly gives the dominant contribution to the resistance. (2) As the Fermi energy is increased, the momentum increases, P_{BS} decreases and finally becomes exponentially small. However, because δk_{ij} is minimal for $i = j = N$, i.e., for the backscattering within channel N , this process dominates the resistance. Thus, regardless of the position of the Fermi energy to the threshold energy the largest contribution to the resistance is given by the backscattering in the topmost channel, and the contributions from the rest of the channels are negligible.

In the second regime, P_{BS} ceases to be exponentially small for some channel $N_c < N$, such that $k_{N_c} \ell_c \sim 1$. Then, all the channels with $N_c \leq n \leq N$ are subject to strong backscattering.

As was shown by Glazman and Jonson,²⁴ the first (second) regime is realized if $N < N^*$ ($N > N^*$), where $N^* = 8(k_F \ell_c)^2$. In a typical experimental situation, $N^* \approx 100$ ($N^* \approx 60$ in the experiment⁷). As the number of observed plateau is usually significantly smaller than N^* , it suffices to consider only the first regime and take into account the backscattering only in the topmost channel.

Although the disorder is smooth on the scale of the Fermi wavelength ($k_F \ell_c > 1$), it can be shown (cf. Appendix) that the effective potential describing the backscattering of left- and right-moving excitations can be represented by a δ -function form: The only information on the smooth variations of the original potential are hidden in the exponentially small prefactor of the δ function.

C. Electron-electron interactions

The same Luttinger-liquid model for a multichannel wire as that proposed by Matveev and Glazman²³ is employed. We specify the assumptions needed for this model and we begin with the case of spinless electrons. The Hamiltonian of interacting 2D spinless electrons is given by

$$H = H_0 + H_{\text{int}}, \quad (4)$$

where H_0 is the Hamiltonian of free electrons and

$$H_{\text{int}} = \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' U(\mathbf{r} - \mathbf{r}') \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}^\dagger(\mathbf{r}') \hat{\Psi}(\mathbf{r}') \hat{\Psi}(\mathbf{r}), \quad (5)$$

where $U(\mathbf{r} - \mathbf{r}')$ is a (repulsive) Coulomb interaction and $\hat{\Psi}(\mathbf{r})$ represents the fermionic field operator. Our first assumption is that the interaction term [Eq. (5)] can be re-

placed by the direct density-density interaction between electrons occupying different channels, i.e.,

$$H_{\text{int}} \Rightarrow \frac{1}{2} \sum_{ij} \int dx \int dx' \rho_i(x) \rho_j(x') U_{ij}(x-x'), \quad (6)$$

where ρ_k is the density operator of the k th channel and

$$U_{ij}(x-x') = \int dy' \int dy |\xi_i(y)|^2 |\xi_j(y')|^2 U(\mathbf{r}-\mathbf{r}'). \quad (7)$$

This assumption neglects the interchannel exchange interactions, which are usually considered to be less important than the direct ones due to the smaller values of the overlap integrals.

The Coulomb potential is assumed to be screened by the metallic gates forming the channel, and in the dc limit the actual form of the potential $U(\mathbf{r}-\mathbf{r}')$ can be replaced by the delta function: $U_0 \delta(\mathbf{r}-\mathbf{r}')$.²³ For a δ -function 2D potential, the effective 1D potential is also a δ function. Using the eigenfunctions of a square-well confinement potential for $\xi_{i\perp}$ in Eq. (7), the 1D coupling constant is channel independent and it is related to U_0 by

$$\hbar V_0 = \frac{U_0}{d}, \quad (8)$$

where the numerical coefficient has been absorbed into the redefinition of U_0 .

The interaction Hamiltonian [Eq. (6)] causes forward and backward scattering processes. In a multichannel case, the forward scattering is defined as the process in which none of the momenta of the electrons is reversed, although the momentum transfer, Q , may not be equal to zero as the electrons can be exchanged between the channels. Forward scattering includes processes with $Q \approx 0$ (for momentum transfer between electrons in the same channel and in different channels) and $Q \approx k_F(i) - k_F(j)$ (for momentum transfer between electrons in channel i and channel j). The density-density interaction in Eq. (6) conserves the total number of electrons in a given channel. Therefore, for temperatures low enough, i.e., $T \ll \min\{v_F(i), v_F(j)\} |k_F(i) - k_F(j)|$, the forward scattering with $Q \neq 0$ involves electron states only far away from the Fermi level and can thus be neglected. Apart from the renormalization of parameters, the backscattering processes produced by repulsive interactions do not change the low energy properties of the system,^{12,27,28} and therefore they are not included in the model.²⁹ Finally, the Umklapp processes are not included. Due to the low electron densities in semiconductor heterostructures a typical quantum wire is very far away from the half-filling condition.

Each 1D channel is described by a Luttinger-liquid model in which the electron density fluctuations of the i th channel are represented by a boson field $\phi_i(x, \tau)$ defined so that

$$\rho_i(x) - \bar{\rho}_i = \frac{1}{\sqrt{\pi}} \partial_x \phi_i, \quad (9)$$

where $\bar{\rho}_i$ is the average electron density in this channel. The (number) current flowing in the i th channel is

$$j = -i \partial_\tau \phi / \sqrt{\pi}. \quad (10)$$

The (Euclidean) action of the system of interacting electrons occupying N channels is given by²³

$$S_1 = \frac{\hbar}{2} \int_0^\beta d\tau \int dx \sum_{i=1}^N \left[\frac{1}{K_i v_i} (\partial_\tau \phi_i)^2 + \frac{v_i}{K_i} (\partial_x \phi_i)^2 \right], \quad (11a)$$

$$S_2 = \frac{\hbar}{2} \sum_{i \neq j}^N \frac{V_0}{\pi} \int_0^\beta d\tau \int dx \partial_x \phi_i \partial_x \phi_j. \quad (11b)$$

The action S_1 describes a set of N Luttinger liquids with parameters K_i, v_i which depend on the Fermi velocities $v_F(i)$ and the effective coupling constant V_0 . The action S_2 describes the forward part of the density-density interaction between the channels. The electron spin will be included in Sec. IV B.

D. Effect of reservoirs

Two characteristic features are predicted for the conductance of a single-channel Luttinger-liquid wire, g . First, in the absence of disorder, g is expected to be renormalized by the electron-electron interactions to the value of $g = K e^2/h$ per spin orientation,^{30,31} where $g = e^2/h$ for a noninteracting system, when $K = 1$. Second, in the presence of weak disorder, g had been shown to decrease with the temperature, revealing a tendency to interaction-enhanced Anderson localization.^{30,32,33} At temperatures lower than $T_L \equiv v_F/L$, this temperature dependence crosses over to a length dependence. However, as has recently been shown by a number of authors,^{34–37} the first prediction does not survive if the presence of the Fermi-liquid electron reservoirs attached to the wire is taken into account. Instead, the conductance remains at its noninteracting value $g = e^2/h$. This result was obtained in Refs. 34–37 in a model in which the Fermi-liquid reservoirs were imitated by switching off the interactions in the outer parts of the system, i.e., by putting $K = 1$ outside the wire.³⁸ On the other hand, the second prediction was shown to survive even in the presence of the reservoirs.^{39–41} Moreover, the scaling exponent of the leading term in the T dependence was found to be the same as in the absence of the reservoirs.^{39–42} This occurs because when $T \gg T_L$, i.e., when $L \gg L_T$, the density-density correlation function, whose $2k_F$ Fourier component determines the value of disorder-induced corrections to the conductance, decays inside the wire and is only minimally affected by the presence of the reservoirs.⁴³ Thus, in order to determine the temperature dependence of the conductance, the original model of a homogeneous Luttinger liquid^{30–33} may be employed and the presence of the reservoirs may be ignored. If the interactions are not strongly attractive,⁴² the error introduced by this simplification will be in an incorrect numerical prefactor of the T -dependence term, a nonuniversal quantity. Using this reasoning, we consider only the model of homogeneous coupled Luttinger liquids.

III. CONDUCTANCE OF AN N -CHANNEL WIRE

In this section a general scheme for the calculation of the corrections to the conductance of a quantum wire carrying N occupied channels due to the presence of weak disorder is presented. The current $I = ej$ is related to the electric field by

$$I(x, t) = \int_{-L/2}^{L/2} dx' \int \frac{d\omega}{2\pi} e^{-i\omega t} \sigma_{\omega}(x, x') E_{\omega}(x'), \quad (12)$$

where $E_{\omega}(x)$ is the temporal Fourier component of the electric field and $\sigma_{\omega}(x, x')$ is the nonlocal ac conductivity. To calculate $\sigma_{\omega}(x, x')$, we make use of the Kubo formula⁴⁴

$$\sigma_{\omega}(x, x') = \frac{ie^2}{2\pi\hbar\omega} \int_0^{\beta} d\tau \langle T_{\tau}^* j(x, \tau) j(x', 0) \rangle e^{i\bar{\omega}\tau} \Big|_{\bar{\omega} = -i\omega + \epsilon}, \quad (13)$$

where $ej(x, \tau)$ is the total current through the wire, T_{τ}^* is a time ordered product as defined in Ref. 44, and $\bar{\omega}$ is the Matsubara frequency.

In the presence of N channels, the total current is the sum of the currents carried by each channel. Upon bosonization, the expression for the conductivity takes the form

$$\sigma_{\omega}(x, x') = \frac{e^2}{2\pi\hbar} \frac{i\bar{\omega}^2}{\omega} G_{\bar{\omega}}(x, x') \Big|_{\bar{\omega} = -i\omega + \epsilon}, \quad (14)$$

where

$$G_{\bar{\omega}}(x, x') = \int_0^{\beta} d\tau \sum_{i,j=1}^N \langle T_{\tau}^* \phi_i(x, \tau) \phi_j(x', 0) \rangle e^{i\bar{\omega}\tau}, \quad (15)$$

and ϕ_i is defined in Eq. (9).

For a 2D disorder potential $W(\mathbf{r})$, the effective 1D potential $W_{ij}(x)$ is obtained by taking the matrix element of $W(\mathbf{r})$ between the transverse wave functions $\xi_{i\perp}$ and $\xi_{j\perp}$. The action representing the impurity-electron interaction takes the form

$$S_I = \frac{\hbar}{2} \sum_{i,j}^N \int dx \psi_i^*(x) W_{ij}(x) \psi_j(x), \quad (16)$$

where $\psi_k(x)$ is defined by Eq. (2). Although the impurity scattering includes processes where electrons can be transferred from channel i to channel j , as discussed in Sec. II B, only the backscattering in the last occupied channel N is important. In this case, $W_{ij}(x) = W(x) \delta_{iN} \delta_{jN}$.

The part of S_I describing the backscattering is

$$S_{IB} = \frac{\hbar}{\pi a} \int_0^{\beta} d\tau \int dx W_B(x) \cos[2k_F(N)x + 2\sqrt{\pi}\phi_N], \quad (17)$$

where a is the microscopic cutoff length and $W_B(x)$ is the effective backscattering potential (cf. Appendix).

We introduce the new fields $\chi(x, \tau)$ by rescaling the fields $\phi(x, \tau)$ as

$$\phi_i(x, \tau) = \sqrt{K_i v_i} \chi_i(x, \tau). \quad (18)$$

Using these fields, the action of the system without disorder takes the form

$$S_1 + S_2 = \frac{\hbar}{2} \int_0^{\beta} d\tau \int dx \sum_{i,j=1}^N [(\partial_{\tau} \chi_i)^2 + v_i^2 (\partial_x \chi_i)^2 + V_{ij} (\partial_x \chi_i) \times (\partial_x \chi_j)], \quad (19)$$

where $V_{ij} = (V_0/\pi) \sqrt{K_i v_i K_j v_j}$.

Due to the separability of the interaction term [Eq. (11b)], this action can be diagonalized exactly to give

$$S_1 + S_2 = \frac{\hbar}{2} \int_0^{\beta} d\tau \int dx \sum_{i=1}^N [(\partial_{\tau} \tilde{\chi}_i)^2 + w_i^2 (\partial_x \tilde{\chi}_i)^2], \quad (20)$$

where $\tilde{\chi}_m = \sum_{i=1}^N A_{mi} \chi_i$ and w_i are the eigenvalues of A_{mn} satisfying the following equation:

$$\frac{\pi}{V_0} = \sum_{j=1}^N \frac{K_j v_j}{[w_i^2 - v_j^2 + (V_0/\pi) K_j v_j]}. \quad (21)$$

The elements of the diagonalization matrix A are given by

$$A_{ij}^2 = \frac{K_i v_i}{[w_j^2 - v_i^2 + (V_0/\pi) K_i v_i]^2} \times \left[\sum_{l=1}^N \frac{K_l v_l}{[w_j^2 - v_l^2 + (V_0/\pi) K_l v_l]^2} \right]^{-1}. \quad (22)$$

The expression for the impurity action now takes the form

$$S_I = \frac{\hbar}{\pi a} \int_0^{\beta} d\tau \int dx W(x) \cos \left[2k_F(N)x + \sqrt{4\pi} \sum_{i=1}^N A_{Ni} \sqrt{K_i v_i} \tilde{\chi}_i \right]. \quad (23)$$

Because of the assumption of weak disorder, the conductance g can be obtained via the perturbation expansion in $W(x)$:

$$g = N \frac{e^2}{2\pi\hbar} + \delta g. \quad (24)$$

The leading term in Eq. (24) is taken to be unrenormalized by the interactions (cf. Sec. II B). δg is expressed in terms of the correlation function $\overline{W(x)W(0)}$. As is shown in the Appendix, this correlation function can be taken in the following form:

$$\overline{W_B(x_1)W_B(x_2)} = n_i u^2 \delta(x_1 - x_2), \quad (25)$$

where u is the effective impurity strength. The only information about the long-range nature of the disorder is now contained in the parameter u^2 , which is proportional to the backscattering probability P_{BS} of the original potential (cf. Sec. II B). Under this assumption, the leading (second order in W) contribution for the correction to the Green's function is given by

$$\begin{aligned} \delta G_{\bar{\omega}}(x, x') = & -\frac{2n_i u^2}{(\pi a^2)} \int_{-L/2}^{L/2} dx_1 [F_0^{(N)}(x_1) - F_{\bar{\omega}}^{(N)}(x_1)] \\ & \times \left[\sum_{i,j,l,m=1}^N c_{il} c_{jm} c_{Nl} c_{Nm} \tilde{G}_i(\bar{\omega}; x - x_1) \right. \\ & \left. \times \tilde{G}_m(\bar{\omega}; x_1 - x') \right], \end{aligned} \quad (26)$$

where

$$\begin{aligned} F_{\bar{\omega}}^{(N)}(x) = & \int_0^\beta d\tau e^{i\bar{\omega}\tau} \exp \left(-4\pi \sum_{j=1}^N c_{Nj}^2 [\tilde{G}_j(x, 0) \right. \\ & \left. - \tilde{G}_j(x, \tau)] \right). \end{aligned} \quad (27)$$

$\tilde{G}_j(x, \tau)$ is the propagator of the field $\tilde{\chi}_j$ defined as

$$\tilde{G}_j(x - x', \tau) = -\langle T_\tau \tilde{\chi}_j(x, \tau) \tilde{\chi}_j(x', 0) \rangle, \quad (28)$$

and $c_{ij} = \sqrt{K_i v_i} A_{ij}$. The function $F_{\bar{\omega}}^{(N)}$ is the Fourier transform of the $2k_F$ density-density correlation function for the channel N . Because the action $S_1 + S_2$ as written in Eq. (20) is quadratic in $\tilde{\chi}_j$, the propagator \tilde{G}_j and the function $F_{\bar{\omega}}^{(N)}$ can be calculated straightforwardly. Following the procedure of Ref. 39, we obtain for $L \gg L_T$ that the correction to the conductance is given by

$$\delta g = -\frac{e^2}{2\pi\hbar} C \frac{L}{\ell^*} \left(\frac{4\pi T}{\hbar\omega_F} \right)^{-\alpha_N}, \quad (29)$$

where

$$\alpha_N = 2 \left(1 - K_N v_N \sum_{j=1}^N \frac{A_{Nj}^2}{w_j} \right). \quad (30)$$

In Eq. (29), ℓ^* is an effective elastic mean free path defined by $1/\ell^* = n_i u^2 / a^2 \omega_F^2$, with ω_F being the (nonuniversal) ultraviolet energy cutoff, and C is a positive constant depending on K_i, v_i . Equation (29) is the most general result given in terms of the initial parameters of the model. In the next section various regimes in which the scaling exponent α can be calculated are discussed.

IV. SCALING EXPONENT

A. One and N channels for spinless electrons

The case of a wire with only one occupied spinless channel is the simplest one and the well-known value for the temperature exponent is easily recovered. In this case $N=1$, $A_{11}=1$, and from the eigenvalue equation (21) $w=v$. Thus, the temperature exponent reduces to $\alpha_1 = 2(1-K)$ in agreement with previous results.^{30,32}

To analyze the case of N occupied channels, it is necessary to solve the eigenvalue equation (21) which amounts to finding all zeros of an N degree polynomial. The solutions to this equation (w_i) have the meaning of the sound velocities of the new fields $\tilde{\chi}_i$, whereas the original fields ϕ_i propagate with velocities v_i . The main feature of these solutions is that all but one w_i lie between the values of two adjacent

v_i ($v_{i-1} < w_i < v_i$), whereas one w_j is bigger than the maximal v_i . This biggest velocity corresponds to the collective mode $\tilde{\chi}_j = \sum_i c_{ji} \phi_i$, where all the coefficients c_{ji} have the same sign. The linear combination for the rest of w_i include coefficients c_{il} with different relative signs.

In a model case where the N coupled channels are viewed as N equivalent coupled chains with disorder scattering only in one chain, the eigenvalue equation for $N \geq 2$ can be solved exactly. In this case $K_i = K$; $v_i = v$, $i=1, \dots, N$; Eq. (21) can be solved analytically and the matrix elements A_{ij} can be found explicitly. The expression for the temperature exponent is given by

$$\begin{aligned} \alpha_N = & 2 \left[1 - \frac{1}{N} \frac{K}{\sqrt{1 + [V_0 K (N-1)] / (\pi v)}} \right. \\ & \left. - \left(1 - \frac{1}{N} \right) \frac{K}{\sqrt{1 - (V_0 K) / (\pi v)}} \right]. \end{aligned} \quad (31)$$

In Eq. (31), the second term is the contribution coming from the collective mode, and the last one is the contribution from all the other modes. In the limit of large N , we obtain a finite value for α , for generic values of K , v , and V_0 . However, these parameters are not independent, but are related as

$$K = \left[1 + \frac{V_0}{\pi v_F} \right]^{-1/2}, \quad (32a)$$

$$v = v_F \left[1 + \frac{V_0}{\pi v_F} \right]^{1/2}. \quad (32b)$$

Substituting these equations into Eq. (31), we find

$$\alpha_N = \frac{2}{N} \left(1 - \left[1 + \frac{V_0 N}{\pi v_F} \right]^{-1/2} \right). \quad (33)$$

In the limit $N \rightarrow \infty$, the exponent vanishes rendering a temperature-independent conductance.

In order to understand the temperature dependence of the conductance for quantum wires, however, we need to work with a set of $2N$ different parameters K_i, v_i . These parameters are related to the Fermi velocity of the channel by $K_i v_i = v_F(i)$, with $v_F(i)$ defined in Sec. II A. Making use of relations similar to those given by Eqs. (32a), (32b),

$$K_i = \left[1 + \frac{V_0}{\pi v_F(i)} \right]^{-1/2}, \quad (34a)$$

$$v_i = v_F(i) \left[1 + \frac{V_0}{\pi v_F(i)} \right]^{1/2}, \quad (34b)$$

the eigenvalue equation reduces to

$$\frac{\pi}{v_0} = \sum_{i=1}^N \frac{s_i}{\tilde{w}_j^2 - s_i^2}, \quad (35)$$

where $v_0 = V_0 / v_F$, $s_j = \sqrt{1 - (j/z)^2}$, and $\tilde{w}_j = w_j / v_F$. Under the conditions $N \geq 1$, $V_0 / v_F \ll 1$, and $V_0 N \rightarrow \text{const}$,

$$\alpha_N = \frac{2}{N} \frac{V_0 N}{\pi v_F} \left[1 - O\left(\frac{1}{N}\right) \right]. \quad (36)$$

Equation (36) contains the combination $V_0 N/v_F$. Using Eq. (8) in the limits $N \rightarrow \infty$ and $V_0 \rightarrow 0$, this combination approaches the constant value $U_0 m/\pi \hbar^2$, which is the dimensionless coupling constant of the original 2D problem,¹⁸ common in Fermi-liquid theory.¹ Thus, in both cases of N occupied channels and N chains, the dependence of the temperature exponent with the number of channels (or chains) is $1/N$ as shown in Eqs. (33), (36). This result is in agreement with the dependence found by Matveev and Glazman²³ for the exponent of the tunneling conductance, after redefining the value of their one-dimensional interaction potential in such a way to get a finite two-dimensional coupling constant. In the same limit, the velocity of the collective mode $\tilde{\chi}_j$ approaches the zero-sound velocity of a two-dimensional Fermi liquid.^{1,18}

As is seen from Eqs. (33) and (36), in the limit $N \rightarrow \infty$ the temperature exponent vanishes, and the correction to the conductance, δg , becomes temperature independent. This corresponds to the conductance of a Fermi liquid at low temperatures in the presence of weak disorder. In the Born approximation, the lowest order of perturbation theory in impurity scattering, the temperature-dependent weak-localization corrections are not observed. Thus, as the number of occupied channels (or the number of chains) increases, the crossover between a Luttinger liquid and a 2D Fermi liquid is observed.

B. $N=2$ (electrons with a spin)

Finally, in order to compare theoretical with the experimental results,⁷ the temperature exponent for a two-channel wire for the case of electrons with a spin is calculated. To include the spin, a boson field for each spin orientation $\phi_{\sigma,i}$ ($\sigma = \uparrow, \downarrow$) is introduced, and the charge and spin fields are defined as follows:

$$\phi_{c,i} = \frac{\phi_{\uparrow,i} + \phi_{\downarrow,i}}{\sqrt{2}}, \quad (37a)$$

$$\phi_{s,i} = \frac{\phi_{\uparrow,i} - \phi_{\downarrow,i}}{\sqrt{2}}. \quad (37b)$$

The action in the absence of disorder is

$$\begin{aligned} \tilde{S} = \frac{\hbar}{2} \int_0^\beta d\tau \int dx \left\{ \sum_{\mu=c,s} \sum_{j=1}^2 \left[\frac{1}{K_{\mu j} v_{\mu j}} (\partial_\tau \phi_{\mu j})^2 \right. \right. \\ \left. \left. + \frac{v_{\mu j}}{K_{\mu j}} (\partial_x \phi_{\mu j})^2 \right] + (V_0/\pi) \partial_x \phi_{1c} \partial_x \phi_{2c} \right\}, \quad (38) \end{aligned}$$

where the parameters $K_{\mu j}, v_{\mu j}$ correspond to the charge and spin Luttinger-liquid parameters²⁸ for both channels. The backscattering part of the interaction has not been included in Eq. (38) according to the discussion in Sec. II C. The part of the impurity action describing the backscattering in the topmost ($i=2$) channel has the form

$$\begin{aligned} \tilde{S}_{IB} = \frac{2\hbar}{\pi a} \int_0^\beta d\tau \int dx W(x) \cos[2k_F(2)x \\ + \sqrt{4\pi} \phi_{2c}(x, \tau)] \cos[\sqrt{4\pi} \phi_{2s}(x, \tau)]. \quad (39) \end{aligned}$$

Following the same procedure as in the spinless case, the temperature exponent of the conductance is

$$\tilde{\alpha}_2 = 1 - K_{2c} v_{2c} \sum_{i=1}^2 \frac{A_{2ic}^2}{w_{ic}}, \quad (40)$$

where A_{lmc} is the matrix used to diagonalize the charge part of the action and w_{ic} are the corresponding eigenvalues. Here, the SU(2) symmetry requirement of $K_{2s}=1$ has already been satisfied. In complete analogy with the spinless case, the relation between K_{ci}, v_{ci} and the Fermi velocity of a given channel and the equality between the intra- and inter-channel interactions are used to solve the eigenvalue equation. For completeness, explicit analytic expressions for the eigenvalues and matrix elements are presented,

$$\begin{aligned} s_{1,2}^2 = \frac{x_1^2 + x_2^2}{2} + \frac{g}{2}(x_1 + x_2) \\ \pm \frac{(x_1^2 - x_2^2)}{2} \sqrt{1 + \frac{2g}{x_1 + x_2} + \frac{g^2}{(x_1 - x_2)^2}}, \quad (41a) \end{aligned}$$

$$A_{2jc}^2 = \frac{1}{\left[1 + \frac{x_1 (s_j^2 - x_2^2)}{x_2 (s_j^2 - x_1^2)} \right]^2}, \quad (41b)$$

where $x_i = \sqrt{1 - (i/z)^2}$, $s_i = w_{ic}/v_F$, and $g = V_0/(\pi v_F)$. In the one-channel case with a spin,

$$\tilde{\alpha}_1 = 1 - K_{c1}. \quad (42)$$

In order to compare the one- and two-channel cases, the exponents $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are plotted as a function of K_{c1} as an effective measure of the interaction strength. As is shown in Fig. 1, the value of the exponent for two channels is smaller than for one channel. This is in qualitative agreement with the experimental observations,⁷ where the temperature dependence of the conducting wire was found to be weaker than in the one-channel case. The arrow indicates the value of $K_{c1} \approx 0.7$ obtained from the analysis of the data on the conductance of a single-channel wire.⁶ For this value of K_{c1} , $\tilde{\alpha}_2$ is smaller than $\tilde{\alpha}_1$ by only about 30%. Thus, although the Fermi-liquid-like behavior sets in for $N \gg 1$, a Luttinger-liquid behavior is still well-pronounced for a few-channel wire.

V. CONCLUSIONS

The correction to the conductance of an N -channel quantum wire arising from weak impurity scattering is calculated, assuming that the wire can be modeled by a set of N coupled Luttinger liquids. The general expression for the scaling exponent of the temperature-dependent conductance of a quantum wire is obtained, and it is shown that this exponent behaves as $1/N$ for $N \gg 1$. That is, as the number of channels increases, the temperature dependence diminishes and vanishes in the limit $N \rightarrow \infty$. This temperature-independent conductance is characteristic for a Fermi-liquid system. In this way, the results presented support the idea of a continuous crossover between the Luttinger-liquid and the Fermi-liquid pictures, as the number of channels (or chains) is increased,

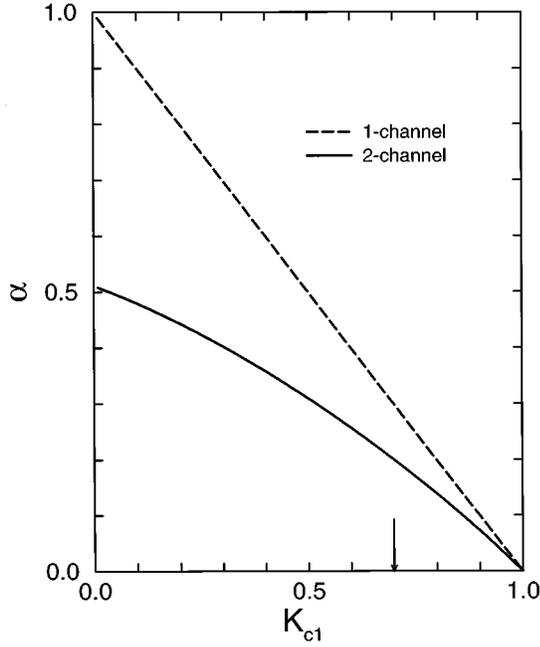


FIG. 1. Scaling exponents of the temperature-dependent correction to the conductance as a function of K_{c1} . Solid line: $N=2$; dashed line: $N=1$. For $N=2$, $z=2.5$ (middle of the plateau). The arrow points at $K_{c1}=0.7$, the experimental value for $N=1$ (Ref. 6).

as has been suggested by several authors.^{12,18} We have also learned of recent work by Kawabata and Brandes,⁴⁵ who have also found a $1/N$ scaling for the temperature exponent by using a different method.

Finally, the temperature exponent in the two-channel case is calculated including the electron spin. The result agrees qualitatively with the experimental observations.

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APPENDIX A

In this appendix, the bosonized expression for the impurity backscattering Hamiltonian for the one-channel case is derived. This calculation is necessary because, as will be shown below, the effective backscattering potential for the right and left movers is different from the original one. It is shown that even if the original potential is long-ranged, the effective backscattering potential takes the local (δ -function) form.

For a one-channel quantum wire, the Hamiltonian describing the electron-impurity interaction is

$$H = \int dx \psi^\dagger(x) W(x) \psi(x) \quad (\text{A1})$$

or, in momentum space,

$$H = \int_{-\infty}^{\infty} \frac{dq}{2\pi} \frac{dQ}{2\pi} \psi^\dagger(q+Q) \psi(q) W(Q), \quad (\text{A2})$$

where $W(x)$ is the disorder potential, whose correlation function $\overline{W(x)W(0)}$ is assumed to be known. The operators of right- and left-moving electrons are introduced and the contributions to the integral over Q in Eq. (A2) from the forward ($|Q| \approx 0$) and backward ($|Q| \approx 2k_F$) scattering processes are separated. Going back to the real-space representation, the backscattering part of Hamiltonian (A1) takes the form

$$H_B = \int \frac{dx}{\pi a} \cos(2k_F x + \sqrt{4\pi}\phi) W_B(x), \quad (\text{A3})$$

where $W_B(x)$ is the effective backscattering potential

$$W_B(x) = 2\text{Re} \int_{2k_F-\Lambda}^{2k_F+\Lambda} \frac{dQ}{2\pi} W(Q) e^{iQx}, \quad (\text{A4})$$

and $\Lambda = 1/a \ll k_F$ is a *hard* momentum cutoff. Note that $W_B(x)$ is not equal to the original $W(x)$ because the integration over Q in Eq. (A4) is taken over narrow regions near $2k_F$ only. Representing $W(x)$ by a sum of single impurity potentials $u(x)$, the correlation function $\overline{W_B(x)W_B(0)}$ takes the form

$$\overline{W_B(x)W_B(0)} = 2n_i \int_{2k_F-\Lambda}^{2k_F+\Lambda} \frac{dQ}{2\pi} |u(Q)|^2 \cos Qx. \quad (\text{A5})$$

To avoid spurious oscillations introduced by a hard cutoff procedure, an actual calculation should be performed by using the soft cutoff procedure. Therefore, the following change is made:

$$\int_{2k_F-\Lambda}^{2k_F+\Lambda} \frac{dQ}{2\pi} \dots \Rightarrow \int_{-\infty}^{\infty} \frac{dQ}{2\pi} e^{-(Q-2k_F)^2/2\Lambda^2} \dots \quad (\text{A6})$$

$u(Q)$ is chosen to be

$$u(Q) = u_0 e^{-|Q|/c}. \quad (\text{A7})$$

This form captures correctly the exponential dependence of $u(Q)$ on Q for a realistic disorder potential in GaAs heterostructures.²⁵ Performing the integration in Eq. (A5), one obtains

$$\overline{W_B(x)W_B(0)} = n_i (u_0 e^{-2k_F/c})^2 [g_{1\Lambda}(x) \cos(2k_F x) + g_{2\Lambda}(x) \sin(2k_F x)], \quad (\text{A8})$$

where

$$g_{1,2\Lambda}(x) = \Lambda \sqrt{\frac{2}{\pi}} e^{2\Lambda^2/c^2} e^{-\Lambda^2 x^2/2} \times \begin{cases} \cos(2\Lambda^2/c x) \\ \sin(2\Lambda^2/c x) \end{cases}. \quad (\text{A9})$$

The correction to the bosonic propagator is given by

$$\begin{aligned} \delta G_{\bar{\omega}}(x-x') = & -\frac{2}{\pi a^2} \left(\frac{2\pi}{\omega_F \beta} \right)^{2K_1} \int_{-L/2}^{L/2} dx_1 dx_2 \\ & \times \overline{W_B(x_1-x_2) W_B(0) \cos 2k_F(x_1-x_2)} \\ & \times [G_{\bar{\omega}}^0(x-x_1) G_{\bar{\omega}}^0(x_1-x') F_0(x_1-x_2) \\ & - G_{\bar{\omega}}^0(x-x_1) G_{\bar{\omega}}^0(x_2-x') F_{\bar{\omega}}(x_1-x_2)], \end{aligned} \quad (\text{A10a})$$

$$F_{\bar{\omega}}(x) = \int_0^\beta d\tau \frac{e^{i\bar{\omega}\tau}}{[(\sinh \pi x/L_T)^2 + (\sin \pi \tau/\beta)^2]^{K_1}}, \quad (\text{A10b})$$

where $G_{\bar{\omega}}^0(x)$ is the propagator in the absence of disorder. The product $\overline{W_B(x) W_B(0) \cos 2k_F x}$ can be separated as

$$\begin{aligned} \overline{W_B(x) W_B(0) \cos 2k_F x} \propto & g_{1\Lambda}(x) + \cos(4k_F x) g_{1\Lambda}(x) \\ & + \sin(4k_F x) g_{2\Lambda}(x). \end{aligned} \quad (\text{A11})$$

An estimate of the length scales of the various functions entering the integrals over $x_{1,2}$ in Eqs. (A10a),(A10b) can be done. First of all, the product $\overline{W_B(x) W_B(0) \cos 2k_F x}$ contains a component oscillating on the scale $\approx 1/4k_F$, whereas the functions $g_{1,2\Lambda}(x)$ in this product oscillate on the scale $\approx 1/(2\Lambda^2/\ell_c)$ and decay rapidly on the scale $\approx 1/\Lambda$. The density-density correlation function (A10b) decays on the

scale $\approx L_T$. The propagator $G_{\bar{\omega}}^0(x)$ decays on the scale $\approx v_F/\bar{\omega}$. After the analytic continuation ($i\bar{\omega} \rightarrow \omega + i0$) is performed and the dc limit is taken, this scale becomes infinite. By the meaning of the cutoff procedure, $L_T, \ell_c \gg 1/\Lambda$. Also, when comparing the scales of $1/4k_F$ and $1/2\Lambda^2/\ell_c^2$, one has to recognize that the limit $k_F/\Lambda \rightarrow \infty$ is to be taken before the limit $\Lambda \rightarrow \infty$. Thus it is possible to establish the following hierarchy of scales:

$$\frac{1}{4k_F} \ll \frac{1}{\Lambda} \left(\frac{1}{2\Lambda/\ell_c} \right) \ll \frac{1}{\Lambda} \ll L_T. \quad (\text{A12})$$

This shows that the $4k_F$ -oscillating terms in Eq. (A11) can be neglected as these oscillations are the most rapid ones, whereas the function $F_{\bar{\omega}}(x)$ varies slowly compared to $g_{1\Lambda}(x)$. As can be easily checked, $g_{1\Lambda}(x)|_{\Lambda \rightarrow \infty} \rightarrow \delta(x)$. The effective correlation function takes the form

$$\overline{W_B(x) W_B(0)} \rightarrow n_i (u_0 e^{-2k_F \ell_c})^2 \delta(x), \quad (\text{A13})$$

which is the same as for a sum of δ -function impurities with the exception that the strength of each impurity is renormalized. This renormalization is the only effect of the actual form of the impurity potential on the effective backscattering potential. For a δ -function original potential ($\ell_c=0$), the renormalization is absent. For a long-ranged potential ($k_F \ell_c > 1$), such as the present in GaAs heterostructures, the backscattering potential is exponentially weak.

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