## Magnetically induced suppression of phase breaking in ballistic mesoscopic billiards

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The phase-breaking time of electrons ( $\tau_{\phi}$ ) trapped in a ballistic quantum dot is determined using two independent analyses of its low-temperature conductance fluctuations. In the first approach the amplitude of the fluctuations is analyzed in terms of random-matrix theory, while the second estimate is obtained from a study of the correlation field. Values of  $\tau_{\phi}$  determined by these two techniques are found to differ by a factor of 6, and comparing with the results of previous experiments we suggest this discrepancy results from the random-matrix-theory-based analysis overestimating the phase-breaking rate. The correlation analysis is also found to be consistent with a sudden suppression of the phase-breaking rate, by more than an order of magnitude at high magnetic fields. [S0163-1829(97)09904-9]

Mesoscopic quantum dots have recently attracted much interest as an experimental probe of quantum chaos.1-13 Since bulk disorder is largely eliminated in such devices, electron scattering within them is thought to mirror the ballistic motion of classical particles in the same geometry. Distinct electric behavior is therefore expected for devices which generate either classically chaotic or regular scattering and, for such effects to be resolved in experiment, it is necessary for electrons to be coherently trapped for long enough to fully sample the confining geometry. While a quantitative measure of this criterion is provided by the phase coherent lifetime of the electrons  $(\tau_{\phi})$ , the processes which limit this time scale in ballistic dots remain poorly understood. The few experimental studies performed to date suggest that, at temperatures of the order of a degree Kelvin, the magnitude and temperature-dependent scaling of  $\tau_{\phi}$  are similar to that of disordered thin films.<sup>8,11</sup> At lower temperatures, however, an unexpected saturation in  $\tau_{\phi}$  is observed, reminiscent of the behavior previously reported in disordered<sup>14</sup> and quasiballistic<sup>15</sup> quantum wires.

In this paper, we determine the phase-breaking time of electrons trapped in a ballistic quantum dot, by two independent analyses of its conductance fluctuations.<sup>7–11</sup> In the first approach, the amplitude of fluctuation is analyzed in terms of random matrix theory,<sup>5</sup> while the second estimate is obtained from a study of the correlation field<sup>11</sup> ( $B_c$ ). Values of  $\tau_{\phi}$  determined by these two techniques are found to differ by a factor of 6 and, comparing with the results of previous experiment, we suggest this discrepancy results from the random-matrix-theory-based analysis overestimating the

phase-breaking rate. The correlation analysis is also found to be consistent with a sudden suppression of the phasebreaking rate at much higher fields, and we briefly consider the possible origin of this effect.

Split-gate dots are realized in GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterojunctions using standard techniques. Prior to gate deposition, optical lithography was used to define a Hall bar pattern in the wafers and subsequent low-temperature measurements revealed a typical electron carrier density of  $4.1 \times 10^{15}$  $m^{-2}$  and a mobility of 20  $m^2/V$  s. Here we focus on results from a multigate dot with stadiumlike geometry (Fig. 1 lower inset). The lithographic size of the dot was roughly 1  $\mu$ m, shorter than the calculated mean free path in the bulk wafer (2.2  $\mu$ m). After mounting on a header, the sample was clamped to a dilution refrigerator and audio-frequency magnetotransport measurements were made at a cryostat temperature of 10 mK. The four-probe configuration used includes a series contribution from the source and drain regions and, at low magnetic fields, the resistance of this was smaller than that of the dot. At higher fields, however, the configuration is only sensitive to edge-state transmission through the dot.<sup>16</sup> Care is taken to ensure good thermal contact to the samples, and a source-drain excitation of less than 3  $\mu$ V is employed for the current-biased measurements.

Figure 1 shows a typical magnetoresistance measurement, obtained with a common bias applied to all six gates of the dot. Prior to investigating the characteristics of the dot, the conductance of its two-point contacts was measured as a function of gate voltage. These calibrations enabled us to determine the number of spin-degenerate modes (N) occu-

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FIG. 1. Magnetoresistance of the dot for a bias of -0.6 V on its gates. Lower inset: scanning electron microscopy micrograph of the dot (spacer bar is 1  $\mu$ m). Upper inset: Conductance fluctuations, obtained from the data in the main figure by subtracting a low-frequency background (Ref. 9). Vertical axis: conductance (in units of  $e^2/h$ ), horizontal axis: magnetic field (T).

pied in the point contact leads, as the negative bias applied to them was varied. The voltage on the central split-gate pair was held at a constant -0.6 V throughout the course of the experiment, however. As can be seen from Fig. 1, reproducible fluctuations persist across the entire range of magnetic field. The fluctuations are thought to result from interference between electron partial waves, multiply scattered by the dot geometry, and in the absence of time-reversal symmetry, their root mean square amplitude is predicted to take the form<sup>5</sup>

$$\delta g = N / [(4N^2 - 1)^{1/2} + N_{\phi}], \qquad (1)$$

where  $\delta g$  is measured in units of  $e^2/h$ . Equation (1) is an interpolation formula derived from random-matrix theory, where  $N_{\phi}$  accounts for the effects of phase breaking and is the number of modes in an imaginary lead, assumed to connect the dot to a thermalizing reservoir (one method of introducing the phase-breaking processes).<sup>8</sup> For a chaotic billiard with cross-sectional area A,  $N_{\phi}$  can be related to a phase coherent lifetime via<sup>8</sup>

$$\tau_{\phi} = [h/N_{\phi}\Delta], \qquad (2)$$

where  $\Delta(=h^2/2\pi m^*A)$  is the average level spacing in the billiard, and  $m^*$  is the effective electronic mass.

A striking feature of Fig. 1 is a clear reduction in highfrequency content as the magnetic field is increased above 0.5 T. Similar behavior in stublike dots was previously associated with the onset of discrete Landau quantization.<sup>10,11</sup> The basic idea is that, at low-magnetic fields where the cyclotron orbit size is larger than the dot dimensions, the characteristic area for interference is limited by the dot size. As the magnetic field is increased, however, the cyclotron orbit ultimately shrinks within the dot diameter and edge states begin to form. The relevant area for interference is then enclosed between the edge states and the size of this region



FIG. 2. Variation of  $\delta g$  with *N*. Solid line is a single parameter fit to Eq. (1) with  $N_{\phi} = 65$ .

shrinks with increasing magnetic field. Consequently, the correlation field<sup>2</sup> of the fluctuations is expected to take the form<sup>10,11</sup>

$$B_{c}(B) = [8\pi^{2}m^{*}B/hk_{F}^{2}\tau_{\phi}], \qquad (3)$$

where  $k_F$  is the Fermi wave vector, and *B* is the applied magnetic field.

In Fig. 2, we plot the evolution of  $\delta g$  with lead mode number. The value of N at each gate voltage is calculated by assuming  $N = [N_1 + N_2]/2$ , where  $N_1$  and  $N_2$  are determined from the conductance characteristics of the individual point contacts. This approximation is justified by our observation that the mode occupation in the two point contacts typically differed by no more than 20%. The significant reduction in  $\delta g$ , observed in Fig. 2 as N is decreased, is in qualitative agreement with the behavior expected from random-matrix theory.<sup>5</sup> The solid line through the data is a least-squares fit to the form of Eq. (1), obtained by taking the single fitting parameter  $N_{\phi} = 65$ . To convert this to a phase-breaking time, we note that an analysis of the gate depletion widths, determined from the conductance characteristics of the two point contacts, yields an effective dot area of 0.5  $\mu$ m.<sup>2</sup> Substituting the appropriate values in Eq. (2), we then obtain  $\tau_{\phi} = 5$  ps. We return to a discussion of this number below, after discussing the evaluation of  $au_{\phi}$  from the correlation function of these fluctuations.

A typical correlation analysis is shown in Fig. 3. As the magnetic field is increased beyond 0.4 T,  $B_c$  initially increases approximately linearly.<sup>10,11,17–20</sup> Equation (3) suggests that the linear increase is consistent with a magnetic-field-independent phase-coherent lifetime, and from the slope of this region we estimate  $\tau_{\phi}$ = 30 ps. As the magnetic field is further increased, however, the data deviate from their initial linear dependence and ultimately appear to saturate. Considering the form of Eq. (3), the saturation implies a sudden suppression of the phase-breaking rate by nearly an order of magnitude (Fig. 3 inset).

The results of our study are summarized in Fig. 4. For the correlation analysis, the values we plot are those determined from the straight-line segment at intermediate fields, and the amplitude analysis is performed below 0.4 T. While both



FIG. 3. Magnetic-field dependence of  $B_c$ , for the data of Fig. 1. Solid lines are drawn to guide the eye. Inset: Magnetic-field dependence of  $\tau_{\phi}$ . The horizontal line marks the magnetic-field-independent value of  $\tau_{\phi}$  found in an earlier study (Ref. 11).

techniques indicate that  $au_{\phi}$  is independent of lead opening, their quantitative values consistently differ by a factor of approximately 6. This discrepancy can partially be accounted for by the fact that Eq. (1) is derived for phase breaking at absolute zero. Nonetheless, previous experiments have shown that the electron gas in the devices typically cools to a base temperature of 50 mK.<sup>16</sup> Convolution over the derivative of the Fermi function then suggests that the zerotemperature variance of the fluctuations should be reduced by a factor of 2.<sup>5</sup> While this implies that we may have overestimated  $N_{\phi}$  by something like 40%, this is not sufficient to account for the observed discrepancy. While space limitations prevent an exhaustive study here, a similar discrepancy was also apparent in our earlier experiment which showed that with two modes occupied in the leads, the amplitude of fluctuation  $\delta g = 0.12^{.9-11}$  Substitution of these parameters into Eqs. (1) and (2) gives  $\tau_{\phi} = 30$  ps. The correlation analysis, on the other hand, gives<sup>11</sup>  $\tau_{\phi} = 0.2$  ns, again more than six times larger than the estimate from random-matrix



FIG. 4. Variation of  $\tau_{\phi}$  with *N*. Open symbols: values from Eqs. (1) and (2). Solid symbols: values from Eq. (3). Solid lines are drawn to guide the eye.

theory. Since the device considered in this earlier paper was fabricated on a different wafer to that of the dot studied here, the disagreement between analyses appears to be a consistent effect. More specifically, since our correlation analysis has been shown to give good agreement with the results of another study,<sup>8</sup> it would seem that the random-matrix-theory-based analysis overestimates the phase-breaking rate. As for why this is the case, we note that Eqs. (1) and (2) result from the addition of an imaginary, thermalizing lead to the dot.<sup>8</sup> Estimating the width of this lead from the Fermi wavelength and  $N_{\phi}$ , we obtain a value significantly larger than the size of the dot itself. In this regime of strong phase breaking, it is far from obvious that the "imaginary-lead" approach will be valid.

Even considering our upper estimate, the phase-breaking time deduced here is an order of magnitude smaller than the experimental values previously reported,<sup>8,11</sup> consistent with the fact that the amplitude of fluctuation is also an order of magnitude smaller [Fig. 2, see also Eqs. (1) and (2)]. The obvious interpretation is to associate the reduced coherence with the factor of 2 lower mobility of the wafer employed here. This conclusion contradicts the behavior known from bulk, disordered films, however, in which the phase-breaking time is directly proportional to the mobility.<sup>21</sup> Furthermore, for a temperature of 50 mK this bulk model yields  $\tau_{\phi} = 0.6$ ns, 20 times larger than our estimate. A recent theory for diffusive dots<sup>22</sup> does even worse, yielding a value nearly two orders of magnitude larger than experiment. Poor agreement with these theories is also apparent in other studies, 8,11 which failed to exhibit the inverse-square dependence of the phasebreaking time on temperature, recently predicted for quantum dots.<sup>22</sup> Furthermore,  $\tau_{\phi}$  was found to saturate at temperatures well below a degree Kelvin, suggestive of a crossover to a new regime of transport where the broadening of the discrete dot levels is smaller than their average spacing.<sup>8,11</sup> In support of this conclusion, we note that numerical simulations have shown the persistence of transmission resonances in open dots, which are correlated to the known bound states of the corresponding closed system.<sup>23</sup> Since the theories mentioned above are not expected to be valid in this regime,<sup>22</sup> there is, therefore, a clear need for a model of phase breaking in ballistic dots which properly takes this discrete quantization into account.

Finally, we consider the origin of the magnetic suppression of the phase-breaking rate, implied by Fig. 3.<sup>24,25</sup> At the temperatures considered here, the main source of phase randomization is thought to be electron-electron scattering. According to Fermi's golden rule, the rate at which this scattering occurs is proportional to the weighted density of final states. As the magnetic field is increased, such that the cyclotron orbit fits within the dot diameter, the energy spectrum of the dot should condense into a series of Landau levels. Since the overlap between these decreases with magnetic field, it might therefore be expected that  $au_{\phi}$  should increase. Given these ideas, we would then conclude that the suppression of phase breaking apparent in Fig. 3 simply results from a crossover from bulklike scattering of electrons in the dot to adiabatic edge-state transport in a small number of Landau levels. The problem with this interpretation, however, is that an earlier study showed  $\tau_{\phi}$  to remain unchanged to much higher fields than those considered here.<sup>11</sup> As a

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possible explanation of this, we note that other independent reports have suggested that  $\tau_{\phi}$  may be limited to an order  $10^{-10}-10^{-9}$  s in GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As material.<sup>8,11,14,15</sup> In dots fabricated from high-mobility wafers,  $\tau_{\phi}$  is usually already of the same order as this "universal" value at zero magnetic field.<sup>11</sup> Here, however,  $\tau_{\phi}$  only approaches this level at the highest fields. Unfortunately, the insensitivity of  $B_c$  in this regime prevents us from establishing if  $\tau_{\phi}$  then saturates, or continues to increase.

An alternative interpretation is suggested by our observation that the increase in  $\tau_{\phi}$  first onsets at around 1 T, where the cyclotron radius  $r_c$  is of order 0.1  $\mu$ m. Since this is comparable to the width of the point contacts, one possibility is that phase breaking is suppressed once the cyclotron orbits pass through the dot leads without suffering extensive scattering. In this regime, we might then expect that the phase-

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breaking time should show a transition to a value characteristic of the bulk wafer (in this case an increase).<sup>21</sup> Given this interpretation, however, we would expect the crossover to the bulk  $\tau_{\phi}$  to occur at progressively higher fields as N is decreased. Such behavior was not apparent in the experiment, however.

In conclusion, we have determined the phase-breaking time of ballistic electrons in a split-gate quantum dot, by two independent analyses of its low-temperature conductance fluctuations. Values of  $\tau_{\phi}$  determined by the two techniques consistently differ by a factor of 6, and we suggest that this discrepancy results from the random-matrix-theory-based analysis overestimating the phase-breaking rate. The correlation analysis of the fluctuations was found to be consistent with a sudden suppression in the phase-breaking rate, by more than an order of magnitude in a strong magnetic field.

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