Magnetoconductance of Aharonov-Bohm rings with half-bound states

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The effects of half-bound states or zero-energy resonances on the electronic magnetotransport in Aharonov-Bohm rings are investigated. These states can be obtained by adding attractive impurity potentials within the ring. In the most symmetric case of attractive impurities located on the two crossings with the leads connecting the ring to the external reservoirs, the zero-energy resonance condition can be reduced to a simple analytical expression containing the magnetic field and the impurity parameters. If the resonance occurs at zero magnetic field, it is pushed into the discrete spectrum for finite fields, and the two bound states become degenerate for magnetic flux $\Phi = h/2e$ for arbitrary ring size. This system provides an ideal experimental setup for studying the particular properties of half-bound states, which correspond to the exceptional solutions of Levinson's theorem. It is also shown that the presence of a zero-energy resonance has dramatic consequences in the magnetic-field dependence of the conductance. [S0163-1829(97)01804-3]

I. INTRODUCTION

Mesoscopic systems provide the most elegant and instructive tool for the study of quantum mechanical phenomena related to the interference effects of one-particle wave functions. The theoretical as well as the experimental developments in this field have been supported by the enormous recent progress in device technology,[1] which in turn has been pushed forward by the progressive understanding of the underlying physics. Aharonov-Bohm rings and related structures are the most famous examples of such quantum interference devices and have been extensively studied during the past.[1–7]

The theoretical approach commonly used to treat the dc transport properties of mesoscopic systems is the Landauer theory, which provides a link between transport properties and scattering theory.[8] This approach is particularly suitable when elastic scattering is present but can also be extended to include the effects of dissipative inelastic scattering by introducing additional scattering side channels that connect the system to ''external reservoirs.''[9,10]

In this paper we investigate magnetoconductance and bound-state properties of the *ad hoc* Aharonov-Bohm ring (ABR) shown in Fig. 1. It contains two splitters, which provide the connections between the ring and two external reservoirs held at slightly different chemical potentials. Bound states are introduced by adding attractive potentials. We investigate the case of attractive potentials located directly at the splitters. The principal interest of this ABR configuration with impurities is that it can support a zero-energy resonance (ZER), depending on the distance between the crossings and on the penetrating magnetic flux.

The fundamental relationship between the low-energy limit of the phase of the transmission amplitude and the total number of bound states of a given potential is given by Levinson's theorem.[11,12] Here, ZER's or ''half-bound states'' correspond to the exceptional case with nonzero transmission probability $|\tau_r(q)|^2$ for $q \to 0$, $\tau_r(q)$ being the transmission amplitude of the ABR from the left to the right side channel and $q = \sqrt{2mE}/\hbar$ the free-electron wave vector. Despite the well-established importance of Levinson's theorem in quantum mechanics,[13] to our knowledge, the implications of a ZER on the transport properties of a real system have not yet been investigated. Semiconductor superlattices that show narrow resonances near the zero of energy have been recently investigated by Capasso and co-workers.[14] Even if these structures can support zero-energy resonances under well-defined conditions,[15] the practical realization of superlattices with *exactly* a ZER is not straightforward. In the following we show that the ABR is an ideal system to investigate the effects of a ZER on the transport properties, since the spectrum of the system depends strongly on the magnetic field, which thus can be used to *tune* resonances

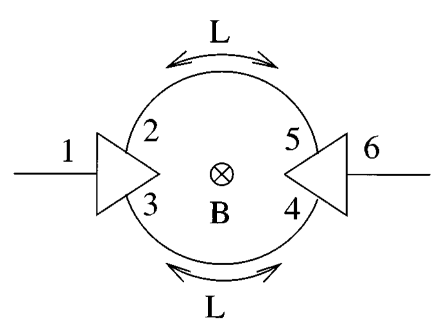

FIG. 1. The Aharonov-Bohm ring. The triangles indicate the splitters with attractive potential.

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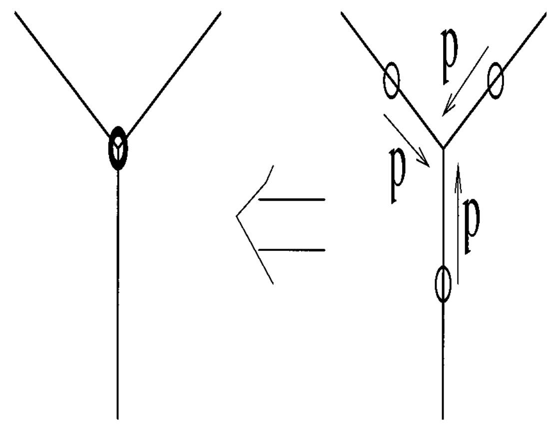

FIG. 2. Construction of a symmetric splitter with on-site attractive potential: Three attractive δ potentials are put a distance d away from the crossing (left). The scattering matrix of this system is calculated and then the limit of $d\to 0$ is performed (right).

and bound states. We further show that the conductance of the ABR is dramatically changed in the presence of a ZER. Hence, the ABR provides a convenient and simple experimental system to study the exceptional situation of Levinson's theorem for the ''half-bound state'' case.

In the following, we first construct a symmetric splitter with an attractive on-site potential. Then, the scattering matrix of the most symmetric ABR containing two such splitters is calculated using the theoretical approach of Ref. 16. A method is proposed to calculate the bound states of a general mesoscopic multichannel system from its scattering properties. We derive an analytic expression for the ZER condition for our specific ABR. Finally, the conductance of the ABR is calculated at different Fermi energies.

II. THE ATTRACTIVE SPLITTER

The attractive symmetric splitter is constructed following the procedure indicated in Fig. 2, where three attractive δ potentials are placed on the leads at a distance d from the crossing point. The scattering matrix S_δ for a single δ potential can be obtained by solving the Schrödinger equation of a potential sink of depth F and thickness a, and calculating the transmission amplitude in the limit $a\to 0$ keeping $\gamma = Fa$ constant. We thus obtain

$$S_\delta = \begin{pmatrix} \rho & \tau \\ \tau & \rho \end{pmatrix} \tag{1}$$

with

$$\tau = \frac{q}{q - i\gamma}, \quad \rho = \tau - 1, \tag{2}$$

where τ and ρ are the transmission and reflection amplitudes. The scattering matrix of the composed system (left-hand side of Fig. 2) is then obtained by solving the respective multiple-scattering problem, as explained in Ref. 16. Here, the central splitter is described by a real symmetric scattering matrix (see, e.g., Ref. 17). The desired scattering matrix $\widetilde{S}$ of the attractive local splitter (right-hand side of Fig. 2) is then obtained after performing the limit $d\to 0$. This yields

$$\widetilde{S} = \begin{pmatrix} r & t & t \\ t & r & t \\ t & t & r \end{pmatrix}, \tag{3}$$

with

$$r = \frac{6i\gamma - q}{-6i\gamma + 3q}, \quad t = 1 + r. \tag{4}$$

The bound states of the so-obtained attractive splitter can be found looking for the poles of the transmission amplitude in the complex plane. According to $\widetilde{S}$, the splitter has a single bound state at an energy corresponding to $q_b = 2i\gamma$.

III. THE AHARONOV-BOHM RING

We now consider the case of an ABR containing two equal attractive splitters, each described by $\widetilde{S}$ (see Fig. 1). The labeling of the scattering channels is indicated in Fig. 1. A wave traveling, e.g., from 2 to 5 acquires a spatial phase factor $\kappa = \exp(iqL)$. The influence of the magnetic field is described by magnetic factors β. For convenience, we choose a gauge for which β is nonzero only along the connection $2-5$;[16] in this case we have $\beta = \exp(2\pi i\alpha)$, $\alpha = \Phi/\Phi_0$ being the ratio between the magnetic flux Φ through the cross section of the ring and the flux quantum $\Phi_0 = h/e$. According to Ref. 16, the scattering matrix of the composed ABR as depicted in Fig. 1, is given by $S^c = [(K^{-1} - T)^{-1}]_{i,j}$, where K is a 6×6 block diagonal matrix with the matrices $\widetilde{S}$ on its diagonals, $i,j = \{1,6\}$ are the indices of the open channels, and T is a matrix containing the phase factors, which are acquired by waves traveling between the connections. With the above gauge we obtain $T_{25} = \kappa\beta$, $T_{52} = \kappa/\beta$, $T_{34} = T_{43} = \kappa$, $T_{ij} = 0$ else.

A. Bound States

The bound states of the ABR are given by the poles of the transmission amplitude $\tau_r = S^c_{61}$. Using the substitution $q \to ik$ together with the convention $k > 0$, we obtain (see also Ref. 16) after some algebraic simplifications

$$\tau_r = \frac{4(1+\beta)k^2\kappa(1-\kappa^2)}{9(k-2\gamma)^2 - 2\kappa^2(k-6\gamma)^2 + \kappa^4(k+6\gamma)^2 - 8\kappa^2k^2\cos(2\pi\alpha)}, \tag{5}$$

and hence the poles of τ_r are given by the zeros of the function

$$f(k;\alpha,\gamma) = 9(k-2\gamma)^2 - 2\kappa^2(k-6\gamma)^2 + \kappa^4(k+6\gamma)^2 - 8\kappa^2k^2\cos(2\pi\alpha), \tag{6}$$

where $\kappa^2 = \exp(-2kL)$. For infinite distance L, we have $\kappa = 0$, and Eq. (6) has a twofold zero at $k = 2\gamma$, as it should be.

Equivalently, the bound states of a composed system may also be determined from the solutions of the equations

$$[K^{-1}(k) - \lambda(k)T(k)]v(k) = 0. \tag{7}$$

Following the arguments of Ref. 16, it is easily seen that the bound eigenstates are given by the condition $\lambda(k) = 1$. One has simply to recognize that the operator $K(k)T(k)$ just corresponds to one scattering cycle; i.e., it transforms the outgoing waves in each channel into outgoing waves for the next scattering cycle. The eigenstate then corresponds to the situation where this operation leaves the vector $v(k)$ invariant. It is interesting to note that Eq. (7) gives also a direct physical meaning to the solutions for fixed real λ with $0 \leqslant \lambda < 1$. In fact, these solutions correspond to eigenstates in a system for which the matrix elements of $T(k)$, for given $k = \bar{k}$, are reduced by a common factor λ, which may be written as $\lambda = e^{-|\bar{k}|l}$. Therefore, they describe the eigenstates for $k = \bar{k}$ in an expanded system, in which all distances in the system are increased by l. For $\lambda = 0$ and finite $\bar{k}$, d is infinite and we recover the solutions of the isolated scatterers. The generalized eigenvalue problem Eq. (7), apart from giving a further physical information, is evidently easier to handle when the geometry of the system becomes complicated and the number of interconnected channels increases.

B. Zero-energy resonances

For the ABR considered here, the ZER condition can be obtained analytically from the $k \to 0$ limit of Eq. (5). Taylor expansion of Eq. (6) yields

$$f(k\to 0;\alpha,\gamma) = 4k^2\{(2-6\gamma L)^2 - 2[1+\cos(2\pi\alpha)]\}. \tag{8}$$

Therefore, the condition for a ZER reads

$$(2-6\gamma L)^2 - 2[1+\cos(2\pi\alpha)] = 0. \tag{9}$$

For given length L and (positively counted) attractive potential γ, a ZER can thus only be obtained by varying the magnetic flux if

$$\gamma L \leqslant \tfrac{2}{3}. \tag{10}$$

Obviously, for large distances L this condition cannot be satisfied; in this case, the two bound states are progressively decoupled with increasing L and become independent of the magnetic field.

If we choose $\gamma L = 2/3$, the system has exactly one bound state and a ZER for $\alpha = 0$. This situation is shown in Fig. 3 (solid line), where the "eigenvalues" λ are shown for imaginary qL, whereas for real qL (corresponding to positive energy) we have plotted the transmission probability $|\tau_r|^2$. For $\alpha \neq 0$ the ZER becomes a bound state that approaches the second one with increasing α, whereas the transmission probability decreases dramatically in particular for small electron energies. For $\alpha = 1/2$, the two bound states are degenerate and the ABR becomes nontransparent for all incoming particle energies. It is interesting to note that, for the latter choice of parameters, the two bound states are degenerate for all λ values $(0 \leqslant \lambda < 1)$; i.e., they remain degenerate for an arbitrary expansion of the system (see Fig. 3, dashed-dotted line). This result can be easily checked by evaluating Eq. (6) for $\alpha = 1/2$. In this particular case the magnetic flux acts as an infinite distance (to be added to L) between the two bound states, whereas at the same time, the open channels $\{1,6\}$ are completely decoupled by quantum interference. We note that this situation is specific for magnetic-field coupling, and cannot be realized by variation of the distance L.

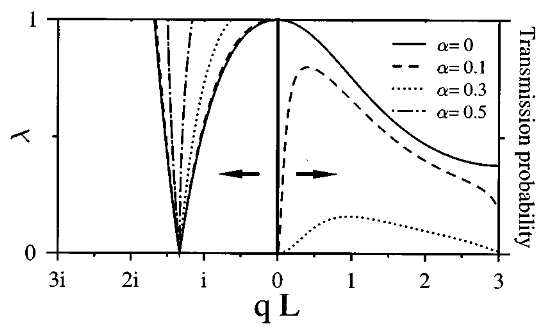

FIG. 3. Spectral properties of an ABR with $\gamma L = 2/3$ (see text) for different magnetic fluxes α. For pure imaginary qL, corresponding to negative energies, the "generalized eigenvalues" λ (as described in the text) are shown. For real qL corresponding to positive energies, the transmission probability is plotted. For $\alpha = 0.5$, the transmission probability vanishes and the bound state $(\lambda = 1)$ is twofold degenerate.

In practice it will be difficult to realize a device for which the condition Eq. (9) is satisfied at zero magnetic field. This is, however, not necessary. If for a given distance L we have a bound state and *no* ZER, we can modulate the magnetic flux until the ZER condition Eq. (9) is fulfilled, provided that the attractive potential is sufficiently strong such that the condition Eq. (10) be satisfied. In other words, the presently discussed ABR provides a convenient system for the study of the particular ZER situation.

The magnetotransport properties of the ABR discussed here are quite interesting. In order to see this, we calculate the conductance of the ABR between the open channels 1 and 6 (see Fig. 1). According to the Landauer theory, the conductance of the ABR is given by $G_{1,6} = (2e^2/h)|S^c_{1,6}|^2$. The α dependence of G is plotted in Fig. 4 for different Fermi wave vectors q. The ABR parameters are the same as in Fig. 3. For small q, i.e., for small Fermi energies, the conductance decays rapidly when increasing the magnetic flux. The ABR thus provides a magnetic switch in this case. It should be noted, however, that switching is only obtained for Fermi energies near the bottom of the conduction band. States at these energies are sensitive to potential fluctuations caused by, e.g., defects. Thus, the electronic transport could be suppressed by localization effects. Therefore, the experimental observation of the switching will be generally diffi-

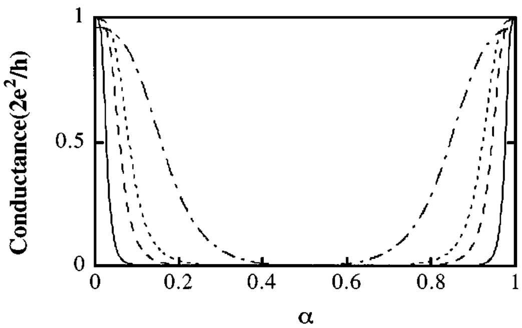

FIG. 4. Conductance of the ABR as a function of the magnetic flux α for different Fermi wave vectors $qL=0.01$ (solid line), 0.05 (dashed line), 0.1 (dotted line), and 0.5 (dashed-dotted line). The ABR parameters are the same as in Fig. 3.

cult. On the other hand, at variance with three- or two-dimensional systems the importance of long-range potential fluctuations is considerably reduced by screening effects at small energies in one dimension, which is the case considered here.

IV. CONCLUSIONS

In conclusion, we have investigated the magnetotransport and the bound-state properties of an ABR with on-site attractive potentials on each splitter. This system has the interesting property that it can support a half-bound state or ZER. Thus, it provides an ideal basis to study the properties of these particular states, which are known to be the exceptional solutions of Levinson's theorem. The necessary condition for the existence of the ZER has been given analytically. Furthermore, the presence of a ZER has been shown to have dramatic consequences on the magnetotransport of the ABR. We note that the presented results do not critically depend on any particular choice for the spatial location of splitters and scattering potentials; qualitatively the same results are found for different geometrical arrangements of splitters and attractive potentials.

ACKNOWLEDGMENTS

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