Incompressible liquid states in asymmetric double-layer electron systems

D. S. Hirashima
Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki 305, Japan
(Received 14 August 1996)

The ground state and excited states of asymmetric double-layer electron systems in a strong magnetic field is studied by diagonalizing the Hamiltonian for systems with finite electron numbers. It is found that the ground state is well approximated by a Jastrow-type wave function at certain fillings, at a certain degree of asymmetry, and for intermediate values of the distance between the two layers. [S0163-1829(97)05603-8]

Under a strong magnetic field, a two-dimensional electron system forms an incompressible liquid state at certain fractional fillings, leading to the fractional quantum Hall effect[1–3] (FQHE) owing to the electron-electron interaction. When the magnetic field is strong enough, the electron spins are totally polarized, and the electrons have only the orbital degree of freedom. Recent technological progress has made it possible to fabricate double-layer electron systems with high mobility. In these systems, the distance d between the two layers is so small, typically $d \lesssim 100$ Å, that the interlayer Coulomb interaction plays an equally important role as the intralayer Coulomb interaction. Owing to the extra (layer-index) degree of freedom, to which we can assign a pseudospin variable, these systems exhibit various interesting phenomena that are not observed in single-layer systems. For example, the FQHE is observed not only at odd-denominator fillings but also at even-denominator fillings ($\nu = 1/2$ and 3/2).[4–7] Moreover, the symmetry in the pseudospin space and the consequences of its breaking down have recently been discussed,[8–10] in particular, in connection with the sharp change in the excitation gap with the tilted magnetic field at $\nu = 1$.[11]

So far, most studies (in particular, most of the theoretical ones) have been done for balanced or symmetric double-layer systems where the electron density in each layer is equal. By changing the gate voltage electron number in each layer is easily controlled. We can also expect that interesting phenomena will be observed in asymmetric double-layer systems. The purpose of this paper is to study the possibility of the occurrence of incompressible liquid states in asymmetric double-layer systems.

Double-layer electron systems in a magnetic field B are characterized by (1) the ratio d/ℓ_B of the distance d between the two layers to the magnetic length $\ell_B(=\sqrt{\hbar c/eB})$, (2) the filling factors ν, ν_L, and ν_R where ν_L and ν_R stand for the filling factor in each layer, and $\nu = \nu_L + \nu_R$, and (3) the tunneling amplitude $\Delta_{\rm SAS}$, which is defined as the energy difference between the symmetric and antisymmetric wave functions resulting from the electron tunneling between the layers. In this study we consider only the cases with $\Delta_{\rm SAS} = 0$. In actual double-layer systems, the finite thickness of the layers has some quantitative effects,[5] but we neglect it and assume that the layers are strictly two dimensional. Moreover we assume that the magnetic field is so strong that we can consider only the spin-polarized lowest Landau level.

In the absence of the tunneling, the electron number in each layer is conserved, and an N-electron wave function of a double-layer system is generally expressed as

$$\Psi(r_1, \ldots, r_N) = \mathcal{A}[\Psi(z_a, z_\alpha) s_1^{(L)} \cdots s_{N_L}^{(L)} s_1^{(R)} \cdots s_{N_R}^{(R)}]$$
$$(a=1, \ldots, N_L; \alpha = 1, \ldots, N_R) \quad (1)$$

where $\mathcal{A}$ stands for the antisymmetrizer, $z_j = x_j - iy_j$ [(x_j, y_j) is the two-dimensional coordinate of the jth electron], N_L (N_R) is the number of electrons in the "left" ("right") layer, $N = N_L + N_R$, and $s^{(L)}$ and $s^{(R)}$ stand for the pseudospin part of the wave function.

For the orbital part $\Psi(z_a, z_\alpha)$, Halperin proposed the following wave function $\Psi_{mm'n}(z_a, z_\alpha)$:[12]

$$\Psi_{mm'n}(z_a, z_\alpha) = \prod_{a<b} (z_a - z_b)^m \prod_{\alpha<\beta} (z_\alpha - z_\beta)^{m'} \times \prod_{a,\alpha} (z_a - z_\alpha)^n \times \exp\left[-\left(\sum_a |z_a|^2 + \sum_\alpha |z_\alpha|^2\right) \Big/ 4\ell_B^2\right]$$
$$(a=1, \cdots, N_L; \alpha = 1, \ldots, N_R), \quad (2)$$

where m and m' are odd positive integers, and n is a non-negative integer. This wave function (with $m = m'$) was first proposed for the cases where the (real) spin degree of freedom has to be considered.[12] Later it was pointed out[4] that this wave function could be applied to double-layer systems. For the symmetric cases where $N_L = N_R$, $m = m'$, and the wave function corresponds to the filling factor $\nu = 2/(m+n)$. The FQHE observed in the double-layer systems at the filling factor $\nu = 1/2$ (Refs. 6 and 7) is believed to be owing to the incompressible state represented by the wave function Ψ_{331}.[4,5]

For asymmetric cases ($N_L \neq N_R$), m may be different from m'. The wave function $\Psi_{mm'n}$ corresponds to the filling factor

$$\nu = \frac{m + m' - 2n}{mm' - n^2}, \quad (3)$$

and the ratio of the electron number in each layer is given by

 © 1997 The American Physical Society

TABLE I. Possible examples of $\Psi_{mm'n}$ states.

$mm'n$	ν	ν_L	ν_R
351	$\frac{3}{7}$	$\frac{2}{7}$	$\frac{1}{7}$
371	$\frac{2}{5}$	$\frac{3}{10}$	$\frac{1}{10}$
571	$\frac{5}{17}$	$\frac{3}{17}$	$\frac{2}{17}$
352	$\frac{4}{11}$	$\frac{3}{11}$	$\frac{1}{11}$

$$\frac{N_L}{N_R}=\frac{\nu_L}{\nu_R}=\frac{m'-n}{m-n}. \tag{4}$$

From these relations we have restrictions on the values of m, m' and n: $n\neq m,m'$ when $m\neq m'$, and $(m'-n)/(m-n)>0$. (In real systems, the interlayer correlation is generally weaker than the intralayer correlation. It is therefore unlikely that n is larger than m or m'.) Several possible examples are shown in Tablel I. In this paper we investigate the possibility of incompressible states represented by these wave functions by diagonalizing the Hamiltonian of small electron-number systems.

In actual calculations we use the spherical geometry.[13,14] The two layers are represented by concentric spheres of radius R, and from the quantization condition the total flux $4\pi R^2 B$ piercing the spheres is an integral multiple of the unit flux hc/e. We then have

$$B=\frac{\hbar cS}{eR^2}, \tag{5}$$

where $2S$ is an integer. For the intralayer Coulomb interaction we take

$$V_{\mathrm{tra}}(z_1,z_2)=\frac{e^2}{\epsilon R}\frac{1}{\sqrt{(\Omega_1-\Omega_2)^2}}, \tag{6}$$

where ϵ is the dielectric constant, and Ω_i stands for the position of an electron on the sphere. For the interlayer Coulomb interaction, we consider

$$V_{\mathrm{ter}}(z_1,z_2)=\frac{e^2}{\epsilon}\frac{1}{\sqrt{R^2(\Omega_1-\Omega_2)^2+d^2}}. \tag{7}$$

From Eqs. (6) and (7), we calculate the pseudopotentials[13,14] $V_{\mathrm{tra}}^{(\ell)}$ and $V_{\mathrm{ter}}^{(\ell)}$ ($\ell=0,\ldots,2S$), and use them in the diagonalization. Note that $V_{\mathrm{tra}}^{(\ell)}$ stands for the potential energy between two electrons in the same layer having the relative angular momentum ℓ, and $V_{\mathrm{ter}}^{(\ell)}$ between an electron in one layer and an electron in the other layer having the relative angular momentum ℓ.

In Fig. 1, we show the overlap between Ψ_{351} and the calculated ground-state wave function for $N=5$ ($N_L=3$, $N_R=2$, and $2S=8$) (Ref. 15) as a function of d/ℓ_B. [To map the wave function defined in a two-dimensional plane onto the sphere we use stereographic mapping,[14] and for $N=5$ we obtain the wave function Ψ_{351} by directly expanding the Jastrow factors in Eq. (2).] The overlap is close to unity around $d=2\ell_B$. We also show the results for Ψ_{371} and Ψ_{352} for $N=6$ ($N_L=4$, $N_R=2$, and $2S=11$ for Ψ_{371} and $2S=13$ for Ψ_{352}).[15] We again find that the ground-state wave functions are well approximated by $\Psi_{mm'n}$ for intermediate values of d.

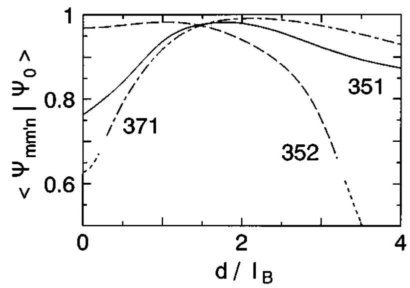

FIG. 1. The overlap between the calculated ground-state wave functions and the $\Psi_{mm'n}$ state ($N=5$ for Ψ_{351}, and $N=6$ for Ψ_{352} and Ψ_{371}). For Ψ_{352} (Ψ_{371}), the ground state is not in the $L=0$ sector for $d>3.2\ell_B$ ($d<0.3\ell_B$).

The size of the systems that we have investigated is so small that the finite-size effects are inevitable. To obtain more reliable results we proceed to $N=8$ ($N_L=5$, $N_R=3$, and $2S=15$) (Ref. 15) for Ψ_{351}. In this case the dimension of the Hamiltonian is so large that we use the Lanczos method to diagonalize it. Furthermore, to obtain the wave function Ψ_{351} numerically, we use the following model pseudopotentials: $V_{\mathrm{tra}}^{(\ell)}=0$ for $\ell\geqslant 3$ in the left layer, $V_{\mathrm{tra}}^{(\ell)}=0$ for $\ell\geqslant 5$ in the right layer, and $V_{\mathrm{ter}}^{(\ell)}=0$ for $\ell\geqslant 1$. Actually, the nonzero components of the pseudopotentials can be quite arbitrary, and we use the values calculated from the Coulomb interaction, Eqs. (6) and (7). For this set of pseudopotentials we find that the ground state (in the $L=0$ sector where L is the total angular momentum) is nondegenerate and its energy is zero and therefore it is Ψ_{351}. The overlap is found to be rather size dependent, but it is still close to unity around $d=2\ell_B$ (see Fig. 2). Moreover, the overlaps between Ψ_{351} and excited states (in the $L=0$ sector) take strong minima around $d=2\ell_B$ where the overlap with the ground state is the largest. These results strongly suggest that the ground state at $d\simeq 2\ell_B$ is very well approximated by Ψ_{351}.

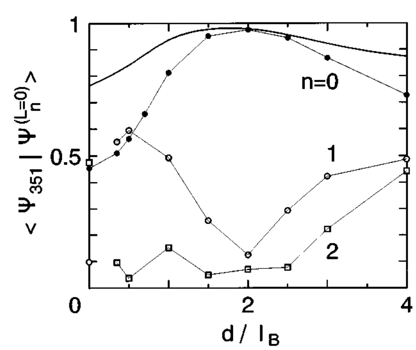

FIG. 2. The overlap between the Ψ_{351} state and the calculated three lowest-energy states in the $L=0$ sector for $N=8$. There is level crossing between the first excited state (open circles) and the second excited state (squares) between $d=0$ and $d=0.35\ell_B$. For comparison, the overlap between the Ψ_{351} state and the ground state for $N=5$ is also shown (solid curve).

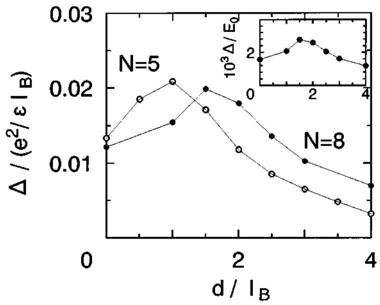

FIG. 3. The energy Δ required to create a quasiparticle-quasihole pair at a long distance as a function of d/ℓ_B. The inset shows Δ normalized by the ground-state energy E_0 for $N=8$.

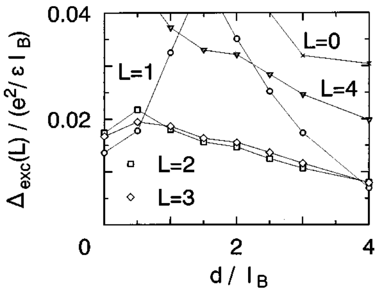

FIG. 4. The excitation energy $\Delta_{\rm exc}(L)$ $[L=0$ (crosses), 1 (circles), 2 (squares), 3 (diamonds), and 4 (triangles)] as a function of d/ℓ_B.

We then estimate the energy Δ required to create a quasiparticle-quasihole pair at a long distance. The energy Δ is considered to be observed as an activation energy in transport experiments. By changing the total flux by unity, $2S\rightarrow 2S-1$ (or $2S+1$), we can create two quasiparticles (or quasiholes) in the system. By denoting the ground-state energy of the system with two quasiparticles (quasiholes) by $E_0^{(\rm q.p.)}$ ($E_0^{(\rm q.h.)}$), I estimate Δ by

$$\Delta=(E_0^{(\rm q.p.)}+E_0^{(\rm q.h.)}-2E_0)/2, \tag{8}$$

where E_0 is the ground-state energy without quasiparticles (holes). This estimate is not very accurate quantitatively because of finite-size effects,[16] but it gives a good qualitative measure.[5] In Fig. 3, the energy Δ calculated for $N=5$ and $N=8$ is plotted as a function of d/ℓ_B. The finite-size effect is obvious, but it is seen that Δ takes a maximun where the overlap between the ground-state wave function and the Ψ_{351} state is large; this observation is also a case for the occurrence of an incompressible state.

The ground-state energy E_0 and the energies of the excited states themselves depend on d/ℓ_B because the interlayer Coulomb interaction, Eq. (7), depends on d/ℓ_B. We can get rid of this trivial dependence on d/ℓ_B by dividing Δ by the ground-state energy E_0. The result is shown in the inset of Fig. 3 (the contribution from the neutralizing background charge is not included in E_0), and it is more clearly seen that Δ takes a maximum value at $d\simeq 2\ell_B$.

In Fig. 4 are plotted the exciton energy $\Delta_{\rm exc}(L)$, which is the excitation energy of the lowest-energy state (with angular momentum L) of the system with the flux unchanged. For $d\simeq 2\ell_B$, the lowest excited state has the angular momentum $L=2$, and a state with $L=3$ is quite degenerate with it. (In the parameter region I have studied, the total angular momentum L of the lowest excited state is $L=1$, 2, or 3.) The dispersion curve is then similar to the magnetoroton dispersion[17] of the incompressible state in a single-layer system.

We have demonstrated that in asymmetric double-layer systems the ground state is well approximated by the generalized Halperin's wave function $\Psi_{mm'n}$ at certain fillings, for a certain degree of imbalance and for intermediate values of d. To discuss the observability of the FQHE effect caused by these states we have to consider the following factors: one is the competition against the fomation of the Wigner crystal. As the filling factor decreases, a two-dimensional electron system tends to form the Wigner crystal, although the critical filling factor (or the phase diagram) is not yet known exactly[2,18–21] (see below). In asymmetric double-layer systems the filling in one of the layers can become rather small even when the total filling is not so small. For Ψ_{351} the filling factor in one of the layers is $\frac{1}{7}$, and in the other $\Psi_{mm'n}$ states the filling factors in one of the layers are even smaller (Table I). In addition, in double-layer systems, the critical filling factor is expected to get larger than in single-layer systems.[22,23] In a single-layer system, although a plateau in the Hall resistivity ρ_{xy} has not been observed at $\nu=\frac{1}{7}$, anomalies in ρ_{xx} and in ρ_{xy} at $\nu=\frac{1}{7}$ were clearly observed.[24] In magneto-optical experiments, moreover, anomalies were clearly observed at $\nu=\frac{1}{7}$ and at an even smaller filling $\nu=\frac{1}{9}$.[25] From these observations it is believed that an incompressible liquid state persists at these particular fillings $\nu=1/m$ (Ref. 25) although it is difficult to observe it in magnetotransport experiments due to the localization induced by disorder. It is then very possible that in double-layer systems the incompressible liquid state also persists at certain fillings and at a certain degree of imbalance, e.g., at $\nu_L=\frac{2}{7}$ and $\nu_R=\frac{1}{7}$.

The other problem is a finite tunneling amplitude present in actual double-layer systems. It is clear that in the limit of large tunneling, the $\Psi_{mm'n}$ state is unlikely to be the ground-state. For example, at $\nu=\frac{3}{7}$, the ground state will also be an incompressible liquid state in the limit of large tunneling, but it will be the same one-component state as the one realized at $\nu=\frac{3}{7}$ in a single-layer system. To reach a quantitative result on the occurrence of the $\Psi_{mm'n}$ states in the presence of finite tunneling, one needs extensive calculations in which a finite tunneling amplitude and asymmetry of the layers are explicitly considered. At present it is fair to say that the $\Psi_{mm'n}$ states are most likely observed in the parameter region (d/ℓ_B and $\Delta_{\rm SAS}$) where the Ψ_{331} state is observed, because those states share common features.

In summary, I have investigated the possibility of the occurrence of incompressible liquid states in asymmetric double-layer electron systems. It is found that at certain total fillings, at a certain degree of asymmetry and for intermediate values of the distance between the layers, the ground state is well approximated by the generalized Halperin's wave function $\Psi_{mm'n}$, and is very likely to be an incompressible liquid state.

I would like to thank Song He for useful correspondence. I also thank the Yukawa Institute of Physics for hospitality, where a part of this work was carried out. Numerical calculations were done on FACOM-VPP 500 at the Supercomputer Center, ISSP, University of Tokyo, and on FACOM-VPP 500 at the Information Processing Center, University of Tsukuba.

[1] D.C. Tsui, H.L. Störmer, and A.C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).

[2] R.B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

[3] *The Quantum Hall Effect*, edited by R.E. Prange and S.M. Girvin (Springer, New York, 1990).

[4] D. Yoshioka, A.H. MacDonald, and S.M. Girvin, Phys. Rev. B **39** 1932 (1989).

[5] S. He, X.C. Xie, S. Das Sarma, and F.C. Zhang, Phys. Rev. B **43**, 9339 (1991); S. He, S. Das Sarma, and X.C. Xie, *ibid.* **47**, 4394 (1993).

[6] Y.W. Suen, L.W. Engel, M.B. Santos, M. Shayegan, and D.C. Tsui, Phys. Rev. Lett. **68**, 1379 (1992); Y.W. Suen, M.B. Santos, and M. Shayegan, Phys. Rev. Lett. **69**, 3551 (1992); Y.W. Suen, H.C. Manoharan, X. Ying, M.B. Santos, and M. Shayegan, Phys. Rev. Lett. **72**, 3405 (1994).

[7] J. P. Eisenstein, G.S. Boebinger, L.N. Pfeiffer, K.W. West, and S. He, Phys. Rev. Lett. **68**, 1383 (1992).

[8] X.G. Wen and A. Zee, Phys. Rev. Lett. **69**, 1811 (1992); Phys. Rev. B **47**, 2265 (1993).

[9] Z.F. Ezawa and A. Iwasaki, Int. J. Mod. Phys. B **19**, 3205 (1992); Phys. Rev. B **47**, 7295 (1993).

[10] K. Yang, K. Moon, L. Zheng, A.H. MacDonald, S.M. Girvin, D. Yoshioka, and S.C. Zhang, Phys. Rev. Lett. **72**, 732 (1994); K. Moon, H. Mori, K. Yang, S.M. Girvin, A.H. MacDonald, L. Zheng, D. Yoshioka, and S.C. Zhang, Phys. Rev. B **51**, 5138 (1995).

[11] S.Q. Murphy, J.P. Eisenstein, G.S. Boebinger, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **72**, 728 (1994).

[12] B.I. Halperin, Helv. Phys. Acta. **56**, 775 (1983).

[13] F.D.M. Haldane, Phys. Rev. Lett. **51**, 605 (1983); in *The Quantum Hall Effect* (Ref. 3), Chap. 8; F.D.M. Haldane and E.H. Rezayi, Phys. Rev. Lett. **54**, 237 (1984).

[14] G. Fano, F. Ortolani, and E. Colombo, Phys. Rev. B **34**, 2670 (1986).

[15] Equations (3) and (4) are applicable to infinite-size systems. For finite systems, we instead have the relations $2S=mN_L+nN_R-m=nN_L+m'N_R-m'$, and $(m-n)N_L-(m'-n)N_R=m-m'$.

[16] Since the quasiparticles (or quasiholes) are dilute, effects of the interaction between them are assumed to be negligible. Furthermore, the change in the neutralizing charge accompanying the creation of quasiparticles (holes), suggested in Ref. 13, is not taken into account. This change gives a correction $\delta\Delta$ for Δ. If the correction $\delta\Delta$ were included, it would dominate Δ; $\delta\Delta/(e^2/\epsilon\ell_B)\simeq 0.045$ and 0.033 for $N=5$ and 8, respectively.

[17] S.M. Girvin, A.H. MacDonald, and P.M. Platzman, Phys. Rev. Lett. **54**, 581 (1985); Phys. Rev. B **33**, 2481 (1986).

[18] P.K. Lam and S.M. Girvin, Phys. Rev. B **30**, 473 (1984).

[19] D. Levesque, J.J. Weis, and A.H. MacDonald, Phys. Rev. B **30**, 1056 (1984).

[20] X. Zhu and S.G. Louie, Phys. Rev. Lett. **70**, 335 (1993).

[21] R. Price, P.M. Platzman, and S. He, Phys. Rev. Lett. **70**, 339 (1993).

[22] L. Świerkowski, D. Neilson, and J. Szymański, Phys. Rev. Lett. **67**, 240 (1991).

[23] H.C. Manoharan, Y.W. Suen, M.B. Santos, and M. Shayegan, Phys. Rev. Lett. **77**, 1813 (1996).

[24] V.J. Goldman, M. Shayegan, and D.C. Tsui, Phys. Rev. Lett. **61**, 881 (1988); J. Wakabayashi, A. Fukano, S. Kawaji, K. Hirakawa, H. Sakaki, Y. Koike, and T. Fukase, J. Phys. Soc. Jpn. **57**, 3678 (1988).

[25] H. Buhmann, W. Joss, K. von Klitzing, I.V. Kukushkin, G. Martinez, A.S. Plaut, K. Ploog, and V.B. Timofeev, Phys. Rev. Lett. **65**, 1056 (1990); **66**, 926 (1991).