High-precision measurements of the quantized Hall resistance: Experimental conditions for universality

Beat Jeckelmann and Blaise Jeanneret
Swiss Federal Office of Metrology, CH-3084 Wabern, Switzerland

Dave Inglis
Institute for National Measurement Standards, National Research Council, Ottawa, Canada K1A OR6
(Received 2 December 1996)

We report comprehensive high-precision direct and indirect comparisons of quantized Hall resistances in Si-metal-oxide-semiconductor field-effect transistor and GaAs/$Al_xGa_{1-x}As$ heterostructures. We find no evidence for a step, device, or material dependence of the quantized Hall resistance (QHR) at the level of 3.5 parts in 10^{10} provided the resistance of each contact used is sufficiently low. However, we observe deviations from ideal QHR values of up to 5 parts in 10^7 if the resistance of at least one of the voltage probes used in the measurement is in the kΩ range. The deviations in the QHR are always accompanied by nonzero values for longitudinal voltage on at least one side of the sample. We propose that deviations such as these are responsible for reports in the literature that purport to show deviant QHR values with zero dissipation. The magnitude of the QHR deviations varies inversely with both current and temperature, for temperatures below 1.2 K and currents below the device critical current. [S0163-1829(97)02019-5]

I. INTRODUCTION

Since the discovery of the integer quantum Hall effect (IQHE), there has been great interest in the question of the universality of the effect that is, whether the quantized values of resistance (QHR) depend in any way on the properties of the material in which the two-dimensional electron gas (2DEG) is established. The metrology community specifically is concerned with the possibility of device dependence of the QHR, since the IQHE has been used to realize a representation of the SI-unit ohm in national standards laboratories around the world since 1990. In the absence of a quantitative theoretical description of the IQHE at high currents and macroscopic device dimensions, this question has essentially been approached experimentally, several authors reporting the quantization to be exact for different materials, devices, and step numbers.[1–3]

In contrast with these findings there has been a number of reports of high-precision measurements that purport to show that the situation is not so straightforward, and that the QHR values are not identical from sample to sample.[4–7] Most of these reports concern measurements comparing metal-oxide-semiconductor field-effect transistors (MOSFET's), made at Gakushuin University on Sony manufactured wafers, indirectly or directly with GaAs heterostructure samples. These MOSFET's are reported to exhibit quantized resistance values of up to 4 parts in 10^7 higher than expected.

In this paper we present data from high-precision measurements of the four-terminal quantized Hall resistance in various Si-MOSFET and GaAs/$Al_xGa_{1-x}As$ heterostructure devices. The measurements were made on metrological-sized samples (a few mm along the side), configured as Hall bars, using currents of tens of microamperes. We unambiguously show that apparently anomalous values of Hall resistance can be related to the resistance of the contacts used to measure the potential differences.

We recently published a small subset of these data for the benefit of the electrical metrology community, where we were concerned with the precision and accuracy of electrical resistance standards at the highest level.[1,8] However, we feel it is important to present a fuller account here for the physics community in general. This is of particular importance since attempts are being made to develop theories[9] to explain the reported anomalies to the quantized Hall effect,[4–7] which in light of our data could very well be explained as a consequence of poor electrical contacts, incorrectly handled samples, or incomplete measurements.

The paper is laid out as follows. The QHE devices used in this work and the measurement techniques are described in Secs. II and III. In Sec. IV, we describe measurements on, and present data from, two types of MOSFET. In Sec. V, we present data from GaAs samples from various sources. The data concern the influence contact resistance can have on measured values of the QHR in these types of samples. Our findings regarding the influence of nonideal contacts on the QHR and the effects of current and temperature on this influence will finally be discussed in Sec. VI.

II. QHE SAMPLES

The relevant properties of all the devices used in this work are listed in Table I. The Hall bar structures have the classical geometry (see Fig. 1) with two large current contacts and three pairs of potential contacts. The first type of Si-MOSFET samples were grown at Southampton University. They are relatively large (5.1×0.5 mm^2 and 2.2×0.3 mm^2, respectively) with a 800-nm-thick gate oxide and their characteristics are described fully in Ref. 10. The second type were considerably smaller (0.6×0.1 mm^2), and were configured by Gakushuin University, Tokyo, on Sony Corporation wafers. Details of these devices can be found in

TABLE I. Device characteristics. l and w are the sample length and width, respectively, a is the distance between two adjacent voltage contacts, and b is the width of the channel connecting the voltage contacts to the main Hall bar. μ is the mobility and n the carrier concentration in the 2DEG in the dark at 4.2 K. $R_{c1,5}$ is the resistance of the current contacts measured on the $i=2$ plateau. For the MOSFET devices, μ is given for the gate voltage V_g; d is the thickness of the oxide layer.

Device	l (mm)	w (mm)	a (mm)	b (μm)	μ (T^{-1})	n (10^{15} m^{-2})	d (nm)	V_g (V)	$R_{c1,5}$ (m Ω)
Si-MOSFET									
Nott 92-01	5.1	0.52	2.00	50	0.84		800	49.6	20
Nott 92-03	2.2	0.33	0.70	20	0.84		800	49.6	20
SONY 72-17H53-NB1, NB3; 72-1 H53-1	0.6	0.10	0.20	10	1.20		230	14.4	300
GaAs									
EPF 240/3	2.4	0.40	0.60	50	13.3	3.70			10
EPF 277/1,2,5	5.0	1.00	1.20	200	42.0	4.80			8
EPF S80/3	2.4	0.20	0.60	50	45.0	4.20			50
EPF 285/1	5.7	0.80	1.50	50	61.2	4.00			250
LEP 92-01	2.2	0.40	0.50	25	28.7	5.17			7
USSR G1	5.8	1.00	1.20	500	49.8	3.35			
HCØ 130-92	5.9	0.50	1.50	50	130.0	3.00			1000

Ref. 7. We refer to the devices as Nottingham MOSFET's and Sony MOSFET's, respectively.

Of the heterostructures, those labeled EPF were made by the Swiss Federal Institute of Technology, in Lausanne. Similar structures have been described in Ref. 11. HCØ refers to a sample supplied by the Center for Microelectronics of the Technical University of Denmark, and LEP to a sample made by the Laboratoires d'Electronique Philips.[12] USSR G1 is a device from the Institute for Métrological Services in Moscow. The GaAs samples have a range of carrier concentrations from 3.0 to 4.8×10^{15} m^{-2}, with mobilities spanning a range from 13 to 130 T^{-1}. All the heterostructure devices were defined using photolithographic and wet-etching techniques and have conventional alloyed AuGeNi contacts.

III. MEASUREMENT APPARATUS AND PROCEDURES

The measurements were all carried out at the Swiss Federal Office of Metrology (OFMET). Hall resistances were compared either directly or via transfer resistors, using a cryogenic current comparator (CCC). Two separate cryostats were available, a dilution refrigerator and a top-loading ^{3}He system. The latter comprised a 14-T magnet and a top-loading ^{3}He insert, the QHE samples being measured in liquid ^{3}He at temperatures of 0.3–1.2 K. The dilution refrigerator also has a 14-T magnet. Here the samples were measured in the mixing chamber, at temperatures between 100 mK and 1 K. All QHR comparisons were made using the OFMET CCC bridge which is described in detail in Ref. 13. The comparator is of the binary self-checking type; no winding errors were detectable at the level of $\leqslant 1\times10^{-10}$. The sensitivity of the CCC is 4 μA turn/Φ_0 where Φ_0 $(=h/2e)$ is the flux quantum. The windings used for these comparisons have a maximum of 2065 and a minimum of 16 turns. The rms noise for a typical $R_H(4)$:$R_H(4)$ comparison is 6.2 nV/$\sqrt{\mathrm{Hz}}$ $[R_H(4)=h/4e^2]$.

For direct comparisons, one QHE device is placed in each cryostat. For indirect comparisons, either cryostat could be used, the transfer resistors being one or both of a pair of well-characterized 100-Ω resistance standards (type: Tinsley 5685 A), maintained in an air bath, nominally at 30 °C.

During resistance measurements, the currents through both sides of the bridge are reversed periodically, to remove thermal effects. One data point is obtained typically by averaging for about 30 min leading to an uncertainty of less than 1 part in 10^9 (ppb) for a R_H measurement. For the determination of the longitudinal resistivity ρ_{xx}, the voltage drop along the sample side was measured directly using the bridge detector (nanovoltmeter type EM N11).

All resistance measurements presented in this work are referred to the QHR on step $i=2$, $R_H(2)$, observed in the GaAs device EPF 277/1. All our tests have shown EPF 277/1 to be an ideal device, with good contacts and very high critical current (around 600 μA in the middle of the $i=2$ plateau).

The quality of the contacts may be characterized by their

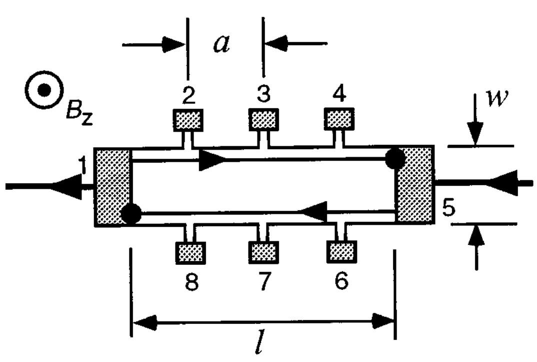

FIG. 1. Sample configuration showing the two current contacts (1,5) and the six voltage contacts (2,3,4 and 6,7,8). The direction of the current flow and the direction of the edge currents (electrons) in the device for "forward" magnetic induction are indicated. Values for l, w, and a are given in Table I.

resistance R_c which is measured in the QHE regime as follows. The voltage drop across the contact j to be characterized and the next contact situated at the same Hall potential is measured while passing a current through contact j and one of the current contacts. If the sample is well quantized, the longitudinal resistivity ρ_{xx} can be neglected and R_{cj} is obtained directly. When determining the resistance of current contacts, one needs to apply the same current as for usual R_H measurements. When characterizing the voltage contacts, the magnitude of the current is chosen as small as possible ($<0.5\ \mu$A), in order to minimize dissipation and/or breakdown of the quantization in the narrow contact arm.

IV. MEASUREMENTS ON MOSFET'S

A. Measurements on Nottingham MOSFET's

In the first series of measurements, Nottingham MOSFET's were compared with various GaAs heterostructures. We measured two of these devices, the measurements being made at 0.3 K after cooling the devices slowly with a positive emf of 10 V applied to the gate. Unlike the Sony samples described later in this section, these devices are usually electrically stable once cooled, and can be measured for several days with little change in their overall characteristics. To measure precision values of $R_H(4)$, the magnetic flux density is set just below 14 T and the gate voltage adjusted to the center of the plateau, typically to 49.6 V for these devices.

The Nottingham MOSFET's have a particularly thick (800 nm) oxide layer and the main features observed in their electrical properties are as follows.

They have low contact resistances, typically 7 Ω for the voltage contacts and less than 50 mΩ for the current contacts.

Their ρ_{XX} plateaus are flat within the measurement resolution across a considerable span of gate voltage V_g, typically 0.8 V. This is clearly seen in Fig. 2 which shows how the longitudinal voltage V_{xx} and the Hall resistance R_H vary across step $i=4$ as a function of gate voltage. This contrasts sharply with the Sony samples (see Sec. I B).

They carry relatively high current without dissipation. This is illustrated in Fig. 3 which clearly shows that there is no deviation in the Hall resistance of step $i=4$ for currents up to 60 μA.

We have carried out an extensive series of high-precision comparisons in which $R_H(4)$ of the two Nottingham MOSFET's in the middle of the plateau, in both field directions and different currents was compared to the QHR of our reference GaAs device. As a result we find for the difference in R_H between Si-MOSFET and GaAS,

$$\frac{R_H(4,\mathrm{Si})}{R_H(4,\mathrm{GaAs})}=1-(1.6\pm 2.3)\times 10^{-10}.$$

This result includes our data published in Ref. 1 and data measured since then. Our result confirms the findings of Hartland *et al.*[3] who showed that $R_H(2)$ observed in a GaAs heterostructure device is the same as $2\times R_H(4)$ in a Si-MOSFET within an uncertainty of $\pm 3.5\times 10^{-10}$.

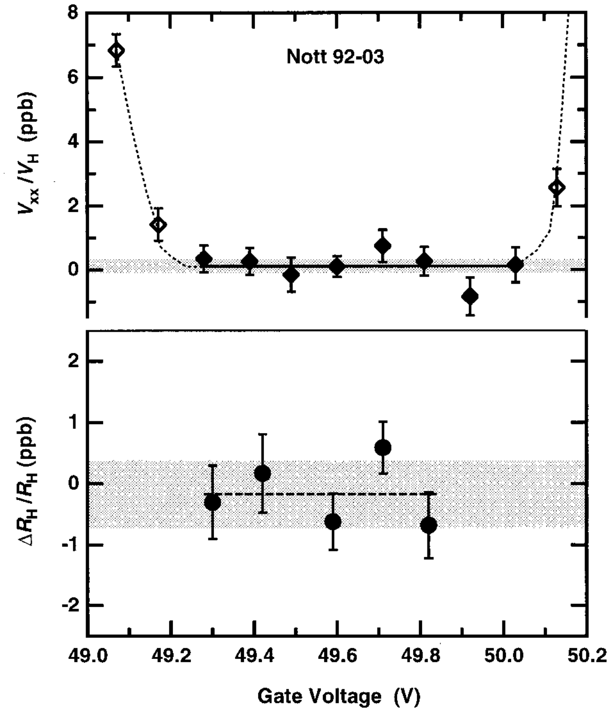

FIG. 2. Variation in ppb (parts in 10^9) with gate voltage of both the quantized Hall resistance $R_H(4)$ and the longitudinal voltage V_{xx} for Nottingham MOSFET Nott 92-03. The temperature was 0.3 K, the magnetic induction 13.8 T, and the measuring current 30 μA.

B. Measurements on Sony MOSFET's

We measured three of these devices. One of them (72-17H53-NB1) is equivalent to the sample used by Yoshihiro *et al.* where deviations from the QHR have been observed. Again we cooled the samples with $+10$ V applied to the gate. Because of the thinner gate oxide, these MOSFET's are measured on step $i=4$ at 14 T with an applied gate voltage of about 14 V. These devices are considerably smaller than the Nottingham MOSFET's and in addition they have higher contact resistances, typically around 18 Ω for each voltage contact and 0.3 Ω for the current contacts.

In every case it turns out to be considerably more difficult to make precision measurements on these devices than on the Nottingham MOSFET's. The current is limited to $\leqslant 10\ \mu$A for two reasons. Owing to the small device width, the critical

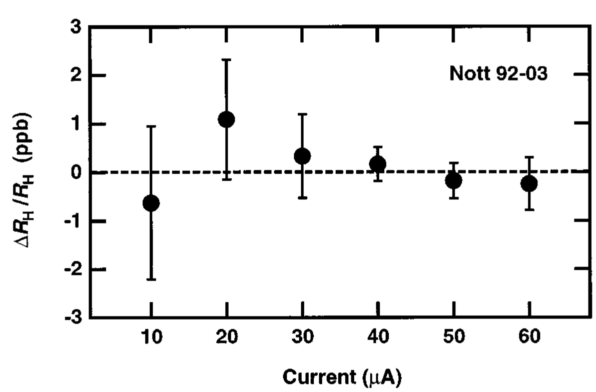

FIG. 3. Current independence of $R_H(4)$ for device Nott 92-03. The temperature is 0.3 K and the magnetic induction 13.8 T.

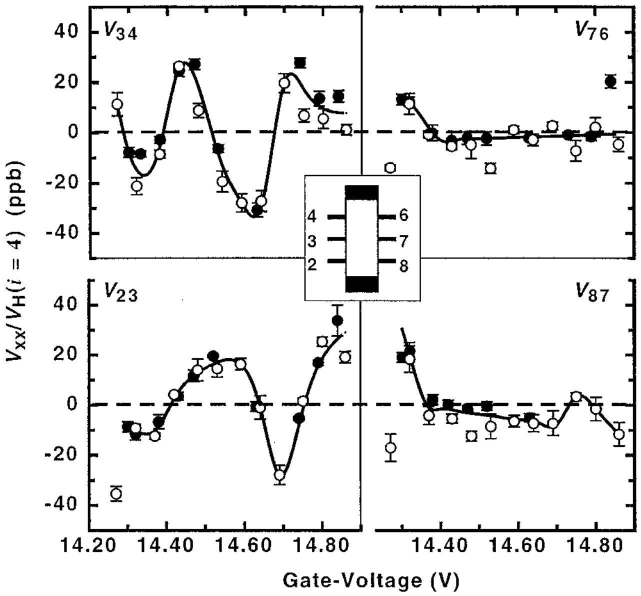

FIG. 4. Variation with gate voltage of the longitudinal voltage for all four potential contact pairs, plotted as a fraction of the Hall voltage for a Sony MOSFET (72-17H53-NB1). Open and solid circles refer to the measuring current of 6 and 10 μA, respectively. The inset shows the contact numbering convention used.

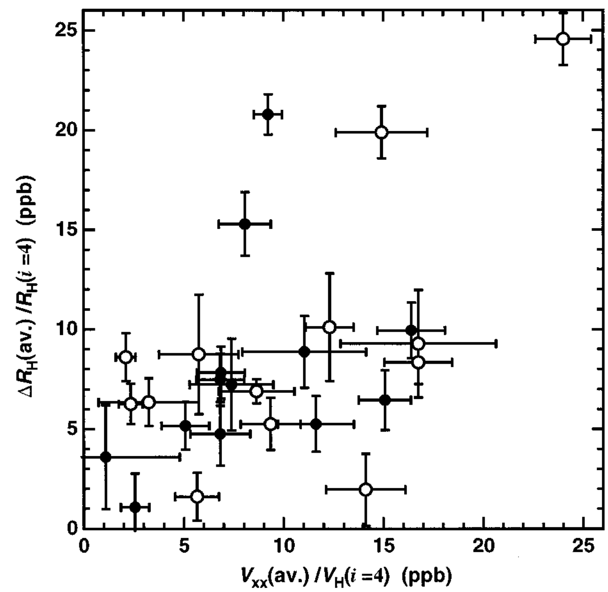

FIG. 5. Scatter plot of the mean values of deviations from the expected value of R_H for two adjacent contact pairs on step $i=4$, plotted against the mean values of V_{xx} for the same contacts. Open and solid circles indicate forward and reversed B-field directions. $V_{xx}(\mathrm{av})=|V_{23,15}+V_{87,15}|/2$, $\Delta R_H(\mathrm{av})=|R_{28,15}+R_{37,15}|/2 -R_H(4, \mathrm{GaAs,EPF277})$.

current (current at which dissipation in the bulk plays a role) is rather low. The plateau width is only about 0.4 V at 0.3 K, partly owing to the small gate voltage of 14.4 V. In our experimental setup, this plateau width is further reduced by the Hall voltage since the gate voltage is referenced to the device current contact. The carrier concentration is therefore given by the combination of gate and Hall voltage. The Hall voltage changes its sign on current reversal and we do not compensate for this change.

A further problem was caused by the tendency of the gate-voltage plateau to slowly drift, so that one had to apply an ever-increasing gate voltage to maintain the operation of the device. Presumably, this drift is a consequence of trapping off electrons from the 2DEG into localized sites, leading to the need to increase the gate voltage to maintain the carrier concentration at the same value. The drift rate varied slightly from sample to sample, and from cooldown to cooldown, but typically was around 10–20 mV per hour. When making a series of measurements across a plateau with a slowly drifting gate voltage we referred all the data to the plateau center voltage. It was found partway through the measurements that the drift in gate voltage could be stabilized by shining a short pulse from a light-emitting diode on the device. The data obtained after a pulse were identical with those obtained without a pulse but the increased stability made the measurement easier. This observation confirms the trapping hypothesis mentioned above.

The most significant feature of the Sony MOSFET measurements was the structure that we always measured in the gate plateaus, at 0.3 K. A typical example is shown in Fig. 4 where we show plots of V_{xx} as a function of gate voltage for all four contact pairs of device 72-17H53-NB1. The numbering sequence for the contacts is shown by the inset. Although the details of the structure vary from sample to sample, and in some instance from cooldown to cooldown, we always find oscillations in the gate voltage plateaus for these devices at 0.3 K. The structure varies along the sample length and differs for opposite sides. The voltage pattern thus differs from one contact pair to the next but the four sets do not appear to be correlated in any obvious way.

For any precision measurement on the plateau, a set of four voltage measurements around any rectangle defined by a set of four potential contacts yields two values for $R_H(4)$ and two values for R_{xx} (on opposite sides of the sample). In all cases, the sum of the voltages around this rectangle is zero within the experimental uncertainty. However, as a consequence of the structure, we find that the two values for R_H are in general not the same, even when *one* of the R_{xx} values is zero. As we have previously pointed out,[8] this has very important implications if incomplete sets of measurements are used to infer a value for the Hall resistance. We return to this point in the discussion. Finally, sweeping the magnetic field at a temperature of 0.3 K. while holding the gate voltage constant produces similar structure.

In Fig. 5, we show a scatter plot that summarizes the R_H versus R_{xx} data for the Sony MOSFET's where we have measured a set of four pairs of contacts in a rectangular configuration at a temperature of 0.3 K and a current of 10 μA. The plotted points are determined by taking the absolute values of the mean of the pairs. At this point, it is crucial to note that a deviation from the expected QHR is *always* related to a finite longitudinal resistance.

In Fig. 6, we show how the structure in V_{xx} varies as a function of temperature. Increasing the temperature from 0.3 to 1.2 K gradually removes all signs of the structure, and the plateau becomes flat and zero valued by 1.2 K, albeit considerably narrower. Reducing the temperature reverses this change. The deviation of $R_H(4)$ in the middle of the plateau from the reference value as a function of temperature is plot-

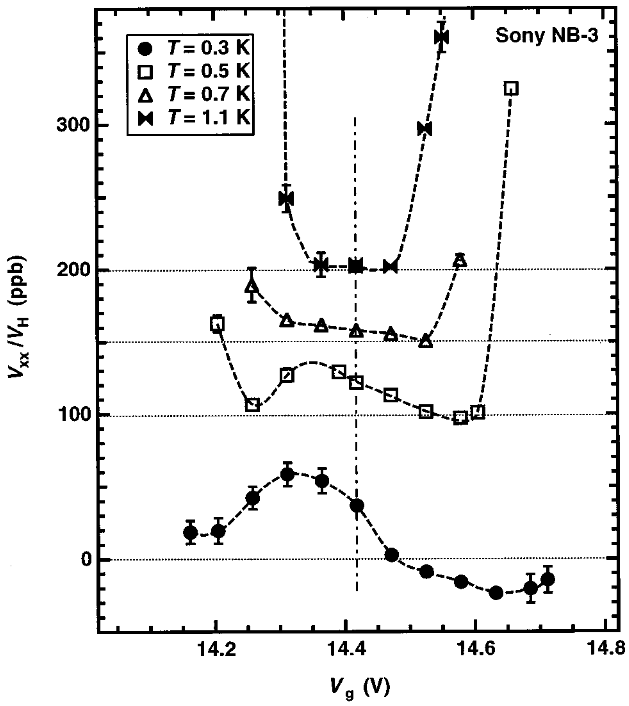

FIG. 6. Temperature variation of V_{xx} vs gate voltage, in the range 0.3–1.2 K for the Sony MOSFET device 72-17H53-NB3 on step $i=4$. The curves are offset in the y direction for clarity. The dotted lines indicate the respective zero positions.

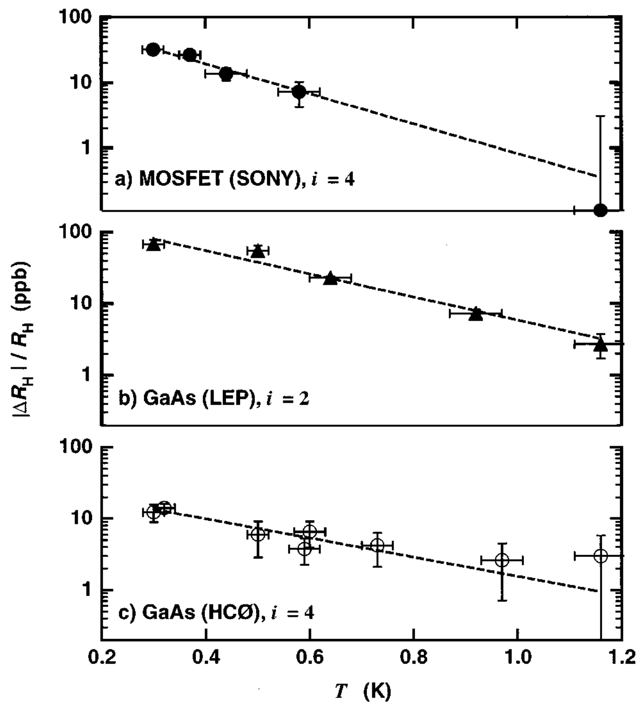

FIG. 7. Temperature variation of the QHR deviations. (a) Sony MOSFET device 72-17H53-NB3 at a fixed gate voltage of 14.4 V, magnetic induction of 13.8 T (equal to the center of $i=4$ plateau), and a current of 10 μA. (b) GaAs device LEP 92-01 at the center of the $i=2$ plateau. $I=30$ μA. One of the voltage contacts used for the measurement of the QHR had a resistance of 12.9 kΩ. (c) GaAs device HCØ 130-92 at the center of the $i=4$ plateau. $I=10$ μA. One of the voltage contacts used for the measurement of the QHR had a resistance of 1.4 kΩ. The data are fitted with exponential functions indicated by the dotted lines.

ted in Fig. 7(a). As can be seen, the deviation has an exponential temperature dependence and decreases from 50×10^{-9} at 0.3 K to a value of $\leqslant 3\times10^{-9}$ at 1.2 K.

V. MEASUREMENTS ON GaAs/Al$_x$Ga$_{1-x}$As HETEROSTRUCTURES

A. Devices with ''ideal'' contacts

We have made high-precision measurements of the QHR for the GaAs devices listed in Table I. The samples vary widely in their characteristics (mobility, geometry) and the aim was to check the consistency of the results at the highest level of accuracy presently available. For all the measurements in this category, the contact resistances were below 1 Ω. It was checked that the longitudinal voltages on both sides of the device were zero within the measurement resolution. In case of dissipation in the bulk, the current dependence of the dissipation was determined and the R_H values extrapolated to zero current.[14]

First, $R_H(2)$ and $R_H(4)$ of all the devices were compared to the QHR of the reference device (EPF 277/1). The results are shown in Fig. 8 as a function of device mobility. The weighted mean of the measured differences is $(-1.2\pm3.2)\times10^{-10}$ and $(1.0\pm2.8)\times10^{-10}$ for the $i=2$ and $i=4$ plateaus, respectively. Furthermore, the QHR of GaAs devices on steps $i=1,3,4,6,8$ were compared to $R_H(2)$ of the reference sample. The results are given in Fig. 9. Again, no deviations from the expected ratios were found within the measurement resolution and our experimental limit for a possible step-ratio dependence of the QHR can be summarized as

$$\frac{R_H(i=1,3,4,6,8)i}{R_H(2)}=2\times[1-(1.1\pm2.9)\times10^{-10}].$$

More details about these measurements and data evaluation procedure are given in Ref. 1.

B. Nonideal contacts

So far, there have been few reports in the literature of the role of the contacts to the 2DEG in relation with high-precision determinations of the QHR. However, it is well known[15] that in the adiabatic regime (i.e., small devices of high mobility, distance between contacts $\leqslant 100$ μm) and for small enough currents (linear regime), large deviations from the QHR are caused by imperfect voltage contacts. The effects can be explained in the framework of the Landauer-Büttiker formalism.[16] However, in the metrological application of the QHE where much higher currents are passed through a device of macroscopic dimensions, charge transport must be described by a nonlinear theory and the pure edge-state description is no longer appropriate. The effects caused by imperfect voltage contacts in a typical high-precision measurement are discussed next.

The quality of the contacts is characterized by their resistance R_c that is measured as described in Sec. III. The resistance of a good AuGeNi contact is usually below 1 Ω (see

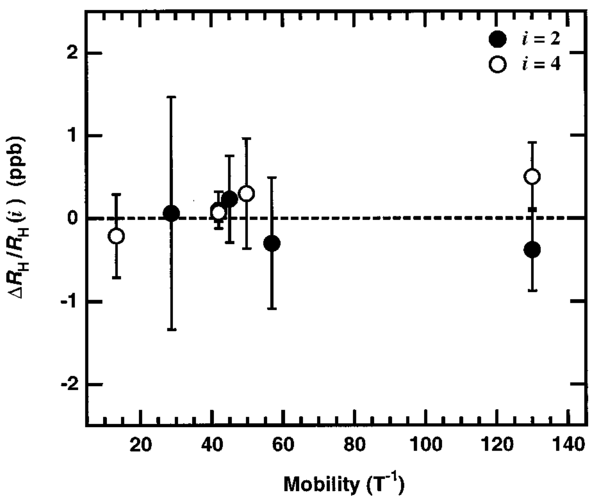

FIG. 8. Device independence of the QHR observed in various GaAs heterostructures. The differences between the QHR observed in various devices and the QHR of the reference device EPF 277/1 are plotted against the device mobility.

Table I). With one exception, all the contacts of the devices listed in Table I have R_c values in this range, provided they are cooled slowly from room temperature down to the working temperature of 0.3 K. The exception is the device EPF 285/1, where three of the voltage contacts have Ohmic resistances of several kilohms when measured on the $i=2$ or $i=4$ plateau. These values are reproducible on different cooldowns and probably originate from the resistance of the metallic contacts or the heavily doped region immediately below the metal pad, and not from the resistance of the 2DEG close to the contacts.

The resistance of the contact region can also be varied in a controlled way by using a gate placed over the probe, i.e., the narrow arm that links the contact pad to the main channel of the device in the usual Hall bar geometry. Applying a voltage to the gate partially depletes the 2DEG under the gate. In metrological applications of the QHE when standard ungated Hall bar devices are used, a similar local reduction

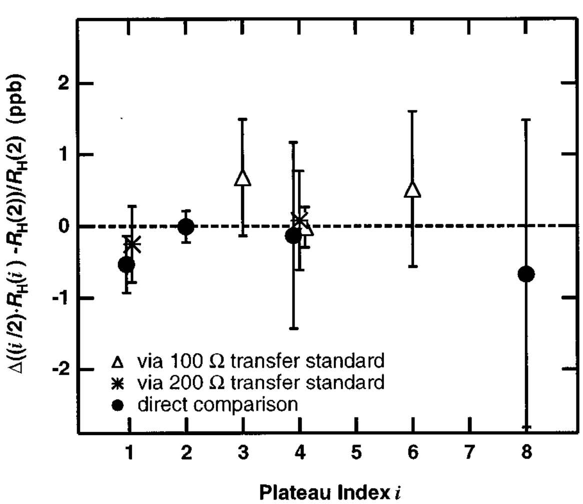

FIG. 9. Results for the step ratio measurements $R_H(i\neq 2)$: $R_H(i=2)$.

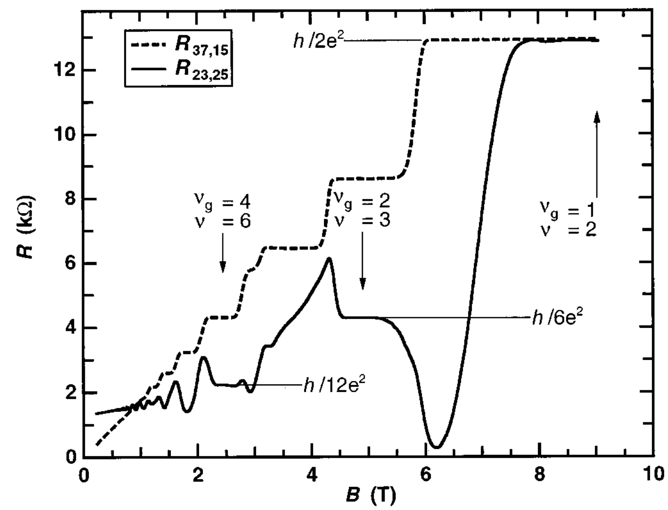

FIG. 10. Resistance measured across a partially depleted region in a voltage probe as a function of the magnetic flux density. Plateaus appear when the filling factors in the depleted region ν_g and the rest of device ν are integers.

of the carrier concentration in the narrow voltage probes can be caused accidentally by cooling a device too fast, by passing a current above the critical current through the potential probe or even by leaving the device in the cold for several days. For the measurements described here, we have intentionally generated high R_c values by applying negative voltage pulses to the chosen voltage contact. The resulting depletion of the carrier concentration in the probe is illustrated in Fig. 10 where the contact resistance is plotted as a function of the magnetic induction B. Plateaus appear in the regions where the filling factors of the Hall bar ν and the partially depleted voltage probe ν_g are integers (see, e.g., Ref. 17 for an explanation). The original contact properties are restored by cycling the device through room temperature or by illuminating the device at low temperature with infrared radiation.[18]

We have performed a series of QHR measurements[19] on devices where at least one of the voltage contacts had an increased resistance as described above. Special care was taken to evaluate the effects caused by the presence of a high R_c value in the measurement circuit. In general, the contact with the increased resistance was placed close to ground potential in order to avoid unwanted voltage drops produced by leakage currents. To test the consistency of the measurements, we regularly checked that the four voltages (two longitudinal voltages, V_{xx} and two Hall voltages, V_H) formed by two pairs of potential contacts summed to zero within the measurement uncertainties.

Figure 11 shows the data obtained using the device HCØ 130-92. The resistance of voltage contact 2 was 1.4 kΩ whereas all the other R_c values were below 1 Ω. The four-terminal resistance $R_{28,15}$ (current between contacts 1 and 5 and Hall voltage between 2 and 8) measured in the middle of the $i=4$ plateau, clearly differs from the quantized value $h/4e^2$. This deviation varies as the inverse of the current as illustrated by the line in Fig. 11(a). Moreover, it remains unchanged on field reversal. Figure 11(b) shows that the longitudinal voltage V_{xx} on the side of the device with good

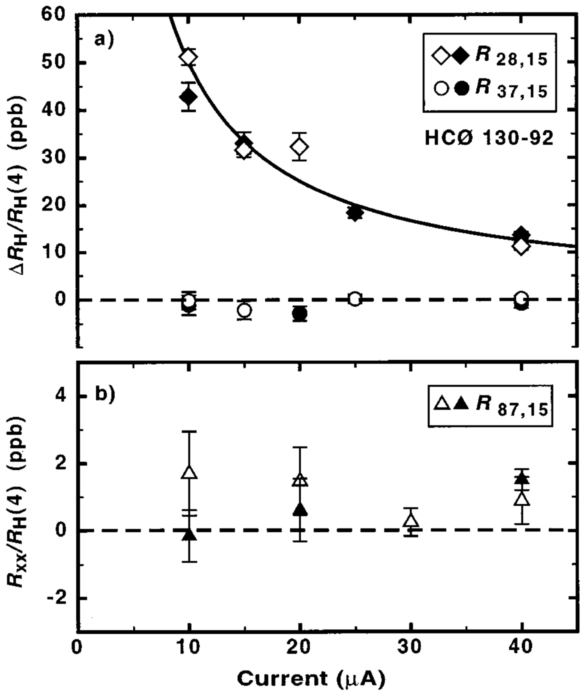

FIG. 11. Influence of high contact resistance on the QHR observed in the device HCØ 130-92. Voltage contact 2 has a resistance of 1.4 kΩ. Part (a) shows the deviations of $R_H(4)$ for two contact pairs as a function of the current I. The deviations $R_{28,15}$ follow the fitted $1/I$ dependence (solid line). Part (b) shows the longitudinal voltage measured on the side of the device with low contact resistances. Open and solid symbols indicate forward and reversed B field, respectively.

contacts is zero within the measurement resolution, except at $I=40\ \mu$A where dissipation in the bulk starts to play a role. At the same time, the QHR over the middle contact pair ($R_{37,15}$) was measured in both field directions and for three different currents between 20 and 40 μA. The difference to the reference QHR is found to be $(-1.8\pm5.0)\times10^{-10}$, showing that only the contact pair including the bad contact is affected.

The inverse current dependence of deviations of the Hall resistance caused by imperfect voltage contacts was also seen in other devices. Figure 12 shows the behavior of sample EPF 277/1 with one bad voltage contact. The deviations vary with $1/I$ above 15 μA but show a completely different behavior below (see Sec. VI).

A summary of all the QHR measurements with high R_c values is given in Fig. 13 where we show a scatter plot of the deviations in R_H and the corresponding values of R_c for both of step $i=2$ and $i=4$, for all the devices tested. The measurements were carried out at 0.3 K and current values between 20 and 50 μA. To allow a better comparison of the results, the deviations were scaled to 20 μA by the $1/I$ relationship presented above. The principal characteristics of these deviations can be summarized as follows.

The deviations may have either sign, positive or negative, and no preference for one or the other could be established.

There is no simple relationship between values of R_c and the deviation from R_H. The deviations tend to increase when R_c becomes larger, but cases have been observed where no

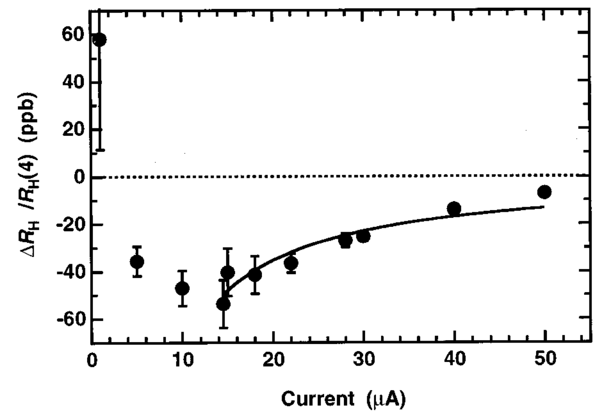

FIG. 12. Deviation of $R_{28,15}(i=4)$ from the QHR due to a high resistance of contact 2 ($R_{c2}=6.4$ kΩ) in device EPF 277/1 as a function of current. The fit for the data points above 15 μA has a $1/I$ dependence.

deviation was caused by a bad contact with a value equal to the QHR in the bulk of the device.

In general the deviations are not symmetric on field reversal.

In the case of a deviation, the longitudinal resistance on the side of the device with the bad contact is not zero, although the longitudinal resistance on the other side remains unaffected.

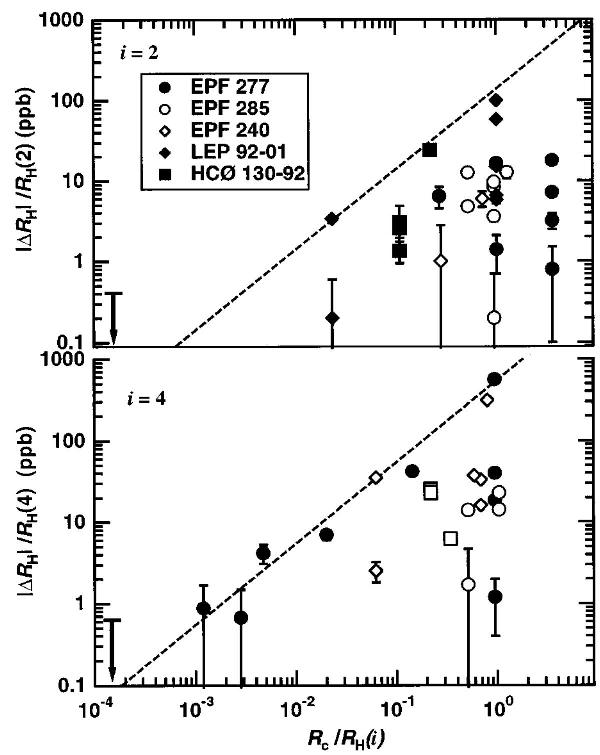

FIG. 13. Variation in the deviation of the Hall resistance from the value expected for (a) step $i=2$ and (b) $i=4$. The measurements were a taken for both B-field directions, at 0.3 K. If one assumes a relation of the type $\Delta R_H(i)_{\max}=kR_c$ then the upper limits of possible deviation would be given by the dotted lines. The experimental upper limit for deviations in the case of good contacts is shown by the small arrow in the bottom left corner of each plot.

These effects due to high-contact resistances are often metastable. Deviations can vary in time (over days) and can even change their sign.

In all cases where differences caused by high R_c values were seen, the control measurements made on the same device using a good contact pair never gave a measurable difference from the reference QHR. The experimental standard deviations deduced from all these control measurements amount to 5×10^{-10} for both $i=4$ and $i=2$. These values are reported as upper limits in Fig. 13 for $R_c\leqslant1\ \Omega$ (see arrows). We have analyzed our data under the assumption that the maximum deviation caused by a voltage contact is proportional to its relative resistance δ, where $\delta=R_c/R_H$ (straight lines in Fig. 13). This coarse approximation shows (1) the deviation from the correct value is well below the actual measurement uncertainty of a few ppb if the R_c values are below 1 Ω (which is usually the case for a good metrological sample) and (2) the deviations are about four times larger for the $i=4$ than for the $i=2$ plateau.

The temperature dependence of the contact effect was also measured. The deviations from exact quantization of the four-terminal Hall resistance due to a bad voltage contact are shown as a function of temperature for two different GaAs devices and plateaus in Figs. 7(b) and 7(c). The deviations exhibit a strong temperature dependence whereas the contact resistances themselves do not change appreciably in the same temperature range. This result will further be discussed in Sec. VI B.

VI. DISCUSSION

A. Previously reported anomalies of the quantized Hall resistance

Under the experimental conditions described here, the Nottingham devices are apparently ideal metrological samples. As shown in Fig. 3 they carry sufficient current without dissipation to allow CCC measurements to be made with 50 μA in the primary circuit of the bridge—the usual situation for comparison with 100-Ω resistance standards, for example. And it is clear from Fig. 2 that the plateau is wide and flat enough (±0.3 V on step $i=4$ at 50 μA) to allow current reversal at this current level.

Particularly important is the fact that we never see any sign of structure in either the R_H or V_{xx} data when plotted as a function of gate voltage, even at the lowest measurement temperatures used (0.3 K). This suggests that the structure in the data that we always observe at 0.3 K and similar temperatures with the Sony MOSFET's is in some way a device-dependent effect.

Perhaps the most obvious, and apparently trivial, differences between the Sony devices and the Nottingham MOSFET's are, firstly, that they are much smaller than the Nottingham devices, and, secondly, that they have considerably higher contact resistances. In fact, we believe that the combination of these two features leads to the observed plateau structure, and to the apparent anomalies in the Hall resistance. We outline our arguments for this connection in the next subsection. First we would like to discuss our data in terms of the so-called anomalous quantized Hall effect which has been previously reported by other workers who measured Sony devices.[4–7] Yoshihiro *et al.*[7] claim to see deviations in the Hall resistance of up to 1×10^{-7} in these devices *despite the absence of dissipation* within the experimental resolution. Heinonen and Johnson[9] present a model in which such deviations can be caused by short-range elastic scatterers located at the edges.

It is clear from Fig. 4 that the value of V_{xx} which one measures for device 72-17H53-NB1 depends critically on the gate voltage, on the side of the device measured, and indeed on the pair of contacts chosen. For the data shown in Fig. 4, one could measure a value of $V_{xx}=0$ for any combination of the voltage contacts on the right-hand side of the device (as drawn) and come to the conclusion that, since there is no dissipation, a simultaneous deviation in the Hall resistance implies an anomalous quantized Hall resistance. At a gate voltage of around 14.6 V, it is even possible to simultaneously measure zero along both sides of the device, and nevertheless have measurable values of V_{xx} along each of the top and bottom halves of the device. This again leads to an apparent deviation from the correct value if one measures R_H across the middle terminals 3–7.

In the data summarized in Fig. 5 we have only included measurements made with two adjacent pairs of contacts, allowing four measurements to be made (two of V_{xx} and two of R_H) around the rectangle defined by the contacts. In these circumstances, we find that deviations of R_H from the expected value always and only occur when V_{xx} for at least one of the sides of the rectangle is nonzero. Similarly, we find that if both values of V_{xx} are zero around the rectangle, for instance, at about 14.4 V for the data shown in Fig. 4, then both values of R_H are as expected.

Apparently, one has to take the greatest care with these devices in the choice of contacts used and the measurement procedure followed if one is to avoid spurious errors that can lead to the conclusion that there can be a deviation from the QHR despite a vanishing longitudinal resistance. In fact the appropriate procedure for making correct measurements of the quantized Hall resistance at the highest precision, as is necessary in national standards laboratories where the Hall resistance is used to realize a representation of the ohm,[20] recommends that wherever possible the measurement be made in the way we have described here. Failure to follow the full procedure may lead to incorrect values. The full measurement procedure is fairly onerous, requiring many measurements, and on a routine basis many workers actually omit part of the process, with no consequences. However, if one finds anomalies in the quantized value without following the full procedure, it is vital the measurement be rechecked using the full procedure outlined in the technical guidelines before anomalous QHR values are claimed.

There are two final points we would like to cover with respect to the data published in Refs. 4–7. In the reports of the measurements carried out at NIST, there are references to the effect on the Hall resistance of electrical disturbances in the laboratory. Further, Kawaji reported[5] that on occasion a GaAs sample also showed small deviations from the expected value for the QHR. We described in Sec. III how applying voltage pulses to the potential contacts of a device can induce small deviations in the Hall resistance. We suggest that the thunderstorm effects reported in the NIST measurements are merely less-controlled versions of the same experiment. And it is possible that Kawaji's deviant GaAs sample also suffered from a bad voltage contact.

We have studied in the literature all the reports[4–7] of deviant Hall quantization in metrological-sized devices. We have come to the conclusion that none of the reports describes a measurement procedure that is absolutely foolproof in that it excludes the possibility of any of the effects we have observed from being present in the measured samples. We propose therefore, that none of these claims of deviations from the quantized Hall resistance with at the same time vanishing longitudinal resistance can be upheld at present.

B. High contact resistances and erroneous Hall resistance values

In this section, we show that the fact of having nonideal voltage contacts in the measurement of the QHR accounts for all the measured deviations which we have seen to date, in both Si-MOSFETs and GaAs samples. The features we need to account for are the following. (i) The magnitude of the contact effects is not readily related to basic electrical characteristics of the devices such as carrier concentration or mobility. (ii) The effect can be reduced by increasing either the device temperature or the current. (iii) We observe similar effects and dependencies in MOSFET's that are small and have relatively high-contact resistances, but not in large MOSFET's that have low-contact resistances. (iv) In small MOSFET's the effect varies with both gate voltage and magnetic induction.

In our GaAs devices, bad contacts were deliberately made by a partial depletion of the electron gas in the channel connecting a voltage contact to the main Hall bar. In one of our samples, high R_c values are due to a improper metallization of the contact pads.

In the case of the Sony MOSFET's, the gate does not seem to extend sufficiently over the connection to the contact region to ensure that the carrier concentration is uniform in all regions of the probe. This variation in concentration is affected by changing the gate voltage and by sweeping the magnetic induction with a stable gate voltage. Changing the gate voltage directly modifies the carrier concentration of the main channel, whereas sweeping the B field changes the density of states available to the carriers.

Komiyama and co-workers[21] have developed a model based on the Büttiker formalism of contacts that allows an estimate of the upper limits for the deviations of the four-terminal Hall resistance as a function of the contact resistances. The calculation is based on the assumption that, at a given current and temperature, a nonequilibrium distribution of edge states created by an nonideal current contact extends over macroscopic distances and is only selectively detected by a voltage probe with a finite resistance. Expressions are given for the case of an uniform 2DEG where the nonuniform population of the edge states is produced by the current contact only and for the case where this unequal population is produced by the nonuniform 2DEG. The authors claim their model should qualitatively remain valid even in the regime when most of the current flows through the bulk of the device, which we assume to be our experimental situation.

We have tested the predictions for the uniform 2DEG. In this case the observed deviation should decrease by a factor of $\delta = R_c/R_H$ for every contact situated between the current contact and the voltage contact used in the measurement. We do not see this behavior. The observed deviations are of the same order for the case with one bad voltage contact next to a current contact and the case where the bad contact lies between two good voltage contacts.

In the case of a nonuniform 2DEG transmission of electrons from a contact to a nonadjacent contact along a path through the interior of the sample may be possible. A general treatment of all the possibilities is not practical so that only a simple case is treated in Ref. 21. This case is in general not compatible with our experimental situation, therefore, a comparison of this model with our data was not attempted.

A quantitative agreement between our results and the predictions of the edge-state theory of contacts cannot be expected. The Büttiker formalism is strictly valid only when the chemical potential difference between the edges is smaller than the Landau-level spacing. None of our measurements was done in this domain. An extension of the Büttiker model is needed to describe the QHE in the high-current regime more precisely. In a paper by van Son, de Vries, and Klapwijk,[22] independent edge and bulk-current components coupled at the contacts are assumed. The model yields a mechanism for the onset of nonlinear behavior and a proper extension of it may lead to a better quantitative understanding of our experimental results. Nevertheless, we see a qualitative agreement with Büttiker's edge-state model. Even good current contacts with resistances in the milliohm range are not ideal and consequently they do not populate all outgoing edge states equally. A bad voltage contact selectively detects the incoming edge states and, if the equilibrium between channels is not achieved on the way from the current contact, a deviation from exact quantization is measured. These deviations become larger as the resistance of the voltage contact increases. It is remarkable that we see this effect for our experimental situation where tens of microamps are passed through a device of macroscopic dimensions. This means that even in this regime, scattering between channels is strongly suppressed.

The deviation ΔR_H from exact quantization is related to the equilibration length l through the relation[23,24]

$$\frac{\Delta R_H}{R_H} = \alpha e^{-L/l},$$

where L is the boundary length between the selective contact and the adjacent one and α is a parameter characterizing the contact properties. The parameter α is closely related to the contact resistance R_c. As mentioned earlier, R_c does not change with temperature in the temperature range considered here. As in Ref. 23, a T independence of α is assumed. Because α is not known in our case, no numerical value for the equilibration length l can be extracted from our data. However, the measured temperature dependence of ΔR_H yields the temperature dependence of l.[23] As shown in Fig. 7, the data fit well to a function of the form $\Delta R_H / R_H = k_0 e^{-k_1 T}$, where T is the device temperature and k_0 and k_1 are the fitting parameters. This means that in the temperature range considered, the inverse equilibration length $1/l$

varies linearly with temperature. The slope is given by k_1/L which is (a) $(26\pm4)\times10^{-3}$ μm^{-1} K^{-1} at $i=4$ in MOSFET device 72-17H53-NB3, (b) $(7.4\pm0.6)\times10^{-3}$ μm^{-1} K^{-1} at $i=2$ in GaAs device LEP 92-01, and (c) $(2.0\pm0.5)\times10^{-3}$ μm^{-1} K^{-1} at $i=4$ in GaAs device HCØ 130-92.

In agreement with our data, Alphenaar *et al.*[25] also find a linear temperature variation of $1/l$ in the range from 0.5 to 4 K in a device of mobility $\mu=36$ T^{-1}. The slope is 2.5 $\times10^{-3}$ μm K^{-1}, very close to our value for case (c). On the other hand, Komiyama *et al.*[24] see an exponential temperature dependence of $1/l$ in the temperature range from 1 to 12 K in a device of mobility $\mu=130$ T^{-1}. The temperature variation around 1 K has the same order of magnitude as ours for the two GaAs devices [cases (c) and (b), respectively]. Our temperature range is too small to clearly establish the T dependence of $1/l$. In fact, the data in Fig. 7(c) can as well be fitted to a function of the form $\Delta R_H/R_H=k_0T^{k_2}$ with $k_2=(-1.6\pm0.4)$. This may point to a nonlinear temperature dependence of $1/l$ if a wider temperature range is considered.

For device HCØ 130-92, $i=4$, the current dependence of ΔR_H was also measured (see Fig. 11). A $1/I$ dependence of the deviations from exact quantization was found. In combination with the $1/T^{1.6\pm0.4}$ dependence mentioned above, we therefore find for the current dependent effective temperature in the 2DEG: $T_e\propto I^{0.6\pm0.1}$. This current scaling perfectly agrees with the results obtained by others from the measurements of the width of the Shubnikov–de Haas oscillations[26] or from the slope of the R_{xy} versus B curve.[27] However, our measurements were performed on a plateau whereas the data presented in Refs. 26 and 27 were obtained in between the plateaus. Theoretical arguments for this scaling behavior are, e.g., given in Ref. 28.

VII. SUMMARY

We have shown that the QHR is independent of host material, device, and step number to within 3.5 parts in 10^{10} if each of the Ohmic contacts to the 2DEG used in the measurement has a resistance $\leqslant 1\ \Omega$. This is, to our knowledge, the most extensive study reported to test the device independence of the QHR at high precision. Our results do not prove the relation $R_H(i)=h/ie^2$. The correctness of this equation can only be shown in a comparison of R_H with an independent realization of h/e^2. But our demonstration of the universality of $R_H(i)$ at high precision adds considerable weight to the supposition that the equality is exact.

We have found that large deviations of up to 5 parts in 10^7 from the quantized Hall resistance can be caused by nonideal contacts. The measuring current in our experiments is in the range 10–50 μA corresponding to a chemical potential difference between the sample boundaries much larger than the Landau-level spacing. The pure edge-state formalism of contacts[16,21] is no longer valid in this domain and we have found no satisfactory quantitative explanation for our results. The QHR deviations caused by nonideal contacts decrease with increasing temperature and current. They vary as $e^{-\alpha T}$ for temperatures T in the range 0.3–1.2 K and as $1/I$ for a range of current smaller than the device critical current.

We have shown that the so-called anomalous behavior of quantized Hall plateaus reported in the literature by various authors can probably be ascribed to bad contacts. In light of our data to our knowledge, none of the claims made to date of deviant QHR values despite vanishing longitudinal resistance can be upheld at present.

ACKNOWLEDGMENTS

The authors are very grateful to H. J. Bühlmann, B. Gallagher, M. Ilegems, A. Kristensen, V. Pudalov, and K. Yoshihiro for supplying the samples. They would like to thank U. Feller and W. Schwitz of OFMET and B. Wood of NRC for their continuous support of this work and H. Bärtschi and W. Fasel for their skillful technical assistance.

[1] B. Jeckelmann, A. D. Inglis, and B. Jeanneret, IEEE Trans. Instrum. Meas. **IM-44**, 269 (1995).
[2] F. Delahaye and D. Bournaud, IEEE Trans. Instrum. Meas. **IM-40**, 237 (1991).
[3] A. Hartland K. Jones, J. M. Williams, B. L. Gallagher, and T. Galloway, Phys. Rev. Lett. **66**, 969 (1991).
[4] S. Kawaji *et al.*, IEEE Trans. Instrum. Meas. **IM-38**, 270 (1989).
[5] S. Kawaji, Physica B **164**, 50 (1990).
[6] C. T. van Degrift, K. Yoshihiro, M. E. Cage, D. Yu, K. Segawa, J. Kinoshita, and T. Endo, Surf. Sci. **263**, 116 (1992).
[7] K. Yoshihiro, C. T. van Degrift, M. E. Cage, and D. Yu, Phys. Rev. B **45**, 14 204 (1992).
[8] B. Jeckelmann, A. D. Inglis and B. Jeanneret, Metrologia **33**, 499 (1996).
[9] O. Heinonen, Phys. Rev. B **46**, 1901 (1992); O. Heinonen and M. D. Johnson, *ibid*. **49**, 11 230 (1994).
[10] T. Galloway, Ph.D. thesis, Nottingham University, 1990.
[11] B. Jeckelmann, W. Schwitz, H. J. Bühlmann, R. Houdré, M. Ilegems, D. Jucknischke, and M. A. Py, IEEE Trans. Instrum. Meas. **IM-40**, 231 (1991).
[12] F. Piquemal *et al.*, IEEE Trans. Instrum. Meas. **IM-42**, 264 (1993).
[13] B. Jeckelmann, W. Fasel, and B. Jeanneret, IEEE Trans. Instrum. Meas. **IM-44**, 265 (1995).
[14] M. E. Cage, B. F. Field, R. F. Dziuba, S. M. Girvin, A. C. Gossard, and D. C. Tsui, Phys. Rev. B **30**, 2286 (1984).
[15] S. Komiyama, H. Hirai, S. Sasa, and S. Hiyamizu, Phys. Rev. B **40**, 12 566 (1989); S. Komiyama, H. Hirai, S. Sasa, and T. Fujii, Surf. Sci. **229**, 224 (1990).
[16] M. Büttiker, Phys. Rev. B **38**, 9375 (1988).
[17] R. J. Haug, Semicond. Sci. Technol. **8**, 131 (1993).
[18] B. Jeanneret, B. Jeckelmann, H. J. Bühlmann, and M. Ilegems, IEEE Trans. Instrum. Meas. (to be published).
[19] B. Jeckelmann, and B. Jeanneret, IEEE Trans. Instrum. Meas. (to be published).
[20] F. Delahaye, Metrologia **26**, 63 (1989).

[21] H. Hirai and S. Komiyama, J. Appl. Phys. **68**, 655 (1990); S. Komiyama and H. Hirai, Phys. Rev. B **40**, 7767 (1989).
[22] P. C. van Son, F. W. de Vries, and T. M. Klapwijk, Phys. Rev. B **43**, 6764 (1991).
[23] S. Komiyama, H. Hirai, M. Ohsawa, H. Matsuda, S. Sasa, and T. Fujii, Phys. Rev. B **45**, 11 085 (1992).
[24] S. Komiyama H. Hirai, M. Ohsawa, H. Matsuda, S. Sasa, and T. Fujii, in *Proceedings of the 20th International Conference on the Physics of Semiconductors, Thessaloniki, 1990*, edited by E. M. Anastassakis and J. D. Joannopoulos (World Scientific, Singapore, 1990), p. 1150.
[25] B. W. Alphenaar, P. L. McEuen, R. G. Wheeler, and R. N. Sacks, Physica B **175**, 235 (1991).
[26] H. Scherer, L. Schweitzer, F. J. Ahlers, L. Bliek, R. Lösch, and W. Schlapp, Semicond. Sci. Technol. **10**, 959 (1995).
[27] H. P. Wei, L. W. Engel, and D. C. Tsui, Phys. Rev. B **50**, 14 609 (1994).
[28] D. G. Polyakov and B. I. Shklovskii, Phys. Rev. Lett. **70**, 3796 (1993).