Unification of the metal-insulator transitions driven by the impurity concentration and by the magnetic field in arsenic-doped germanium

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We show that one can unify the scaling behavior of the conductivity in Ge:As in the vicinity of the metal-insulator transition driven either by the concentration of impurities N or by the magnetic field B by introducing a new scaling variable $U=[(N/N_c)-(B/B^*)-1]$, where both the critical impurity concentration N_c and the characteristic magnetic field B^* are constant. [S0163-1829(97)10004-2]

The metal-insulator transition (MIT) has been the subject of intensive theoretical and experimental investigation for many years.[1] According to the scaling theory for doped semiconductors,[2] the conductivity at zero temperature $\sigma(0)=\sigma(T\rightarrow 0)$, when plotted as a function of the impurity concentration N, is equal to zero on the insulating side of the MIT and remains finite on the metallic side, obeying a power law in the vicinity of the transition

$$\sigma(0)\propto[(N/N_c)-1]^{\mu} \tag{1}$$

where N_c is the critical-impurity concentration and μ is the critical-conductivity exponent. The theory[2] predicts $\mu=1$. We will refer to this transition as N-MIT. For barely metallic samples with $N>N_c$, the MIT will occur upon the application of a critical magnetic field B_c, because a strong magnetic field leads to the shrinkage of the electron wave function. Scaling behavior of the conductivity is also expected in the neighborhood of this magnetic-field-driven metal-insulator transition (B-MIT):

$$\sigma(0)\propto[1-(B/B_c)]^{\nu}, \tag{2}$$

where ν is the critical exponent.[1] Most theoretical work[1,3–6] predicts that the critical exponents should be equal to unity for both N-MIT and B-MIT, i.e., $\nu=\mu=1$.

B-MIT has been studied for different doped semiconductors. There are many references on this subject. Therefore, we limit ourselves to investigations made in typical semiconductors, such as Ge,[7–9] GaAs,[10–12] InSb,[11,13] and Si,[14–18] where the temperature dependencies of the conductivity $\sigma(T)$ have been measured at fixed magnetic fields. B-MIT in the form (2) was described in Refs. 7 and 10 and ν was found to be close to unity. Regarding the dependence of N_c on B, inconsistent results have been obtained: in Ref. 17 it was found that for Si:B, N_c does not change at $B=1$ T and increases slightly at $B=7.5$ T, whereas, in Si:P,[18] it was found that N_c is the same at $B=0$ and 8 T. For Ge:Sb, an increase of N_c was observed in Ref. 8 at $B=4$ T; in Ref. 9 it was found that $N_c(B)-N_c(0)\propto B^{1/2}$ in weak fields ($B<2$ T).

In this work, we present the results of an investigation of the MIT, driven by the impurity concentration and by the magnetic field in Ge:As. The N-MIT in this series of samples was studied in our previous paper,[19] where we found that $N_c=3.50\times10^{17}$ cm^{-3} and $\mu=1$. Here we present the results of an investigation of the B-MIT. We show that $\nu=1$, and the values of B_c increase linearly with $\Delta N\equiv N-N_c$:

$$B_c=B^*(\Delta N/N_c). \tag{3}$$

Finally, we show that N-MIT and B-MIT can be unified by introducing a new variable

$$U=[(N/N_c-1)(1-B/B_c)]. \tag{4}$$

Using Eq. (3), one can rewrite U in the form

$$U=[(N/N_c)-(B/B^*)-1]. \tag{5}$$

As a result, the unified MIT $\sigma(0)\propto U^{\mu}$, $\mu=1$ can be characterized by only two parameters, N_c and B^*, which are constant.

The samples of uncompensated Ge, metallurgically doped by As with a concentration of impurities close to the MIT, were cut from crystals grown by the Czochralski method. The temperature-dependent data were obtained by a four-probe method using a dilution refrigerator combined with a superconducting magnet for measurements down to 100 mK in magnetic fields up to 9 T. In the vicinity of the MIT, the concentration of impurities measured by the Hall effect may not be equal to the effective concentration N^* responsible for the low-temperature conductivity because of the sample inhomogeneity. Therefore, we calculated N^* directly from the low-temperature resistance data using the method and scale proposed in Ref. 20.

The N-MIT for the series of Ge:As samples in zero magnetic field has been shown in Fig. 1 of Ref. 19. Figure 1 of the present paper shows the B-MIT for one of the sample of Ge:As with $N=4.60\times10^{17}$ cm^{-3}. One can see that for both kinds of transitions, in the vicinity of the MIT, the conductivity $\sigma(T)$ takes the form $\sigma(T)=a+bT^{1/3}$ and crosses the MIT at a critical concentration $N=N_c$ or at a critical magnetic field $B=B_c$ where $\sigma(T)=bT^{1/3}$. This result is in agree-

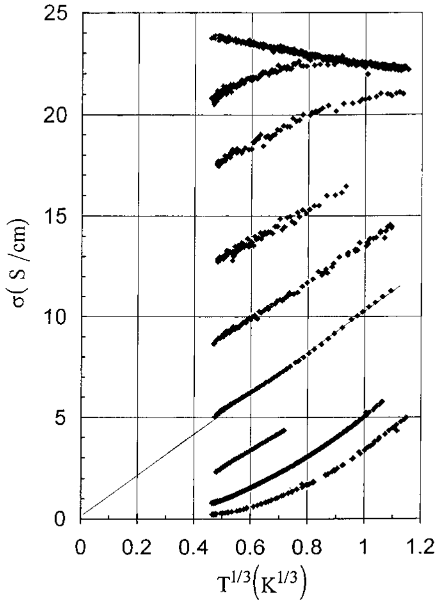

FIG. 1. Temperature dependence of the conductivity of one sample of Ge:As with $N=4.60\times10^{17}$ cm^{-3} in different magnetic fields B. Magnetic field from top to bottom (in T): 0, 1, 2, 3, 4, 5, 6, 7, and 8. The ''critical field'' $B_c=5$ T, the straight line corresponds to $\sigma(T)=bT^{1/3}$.

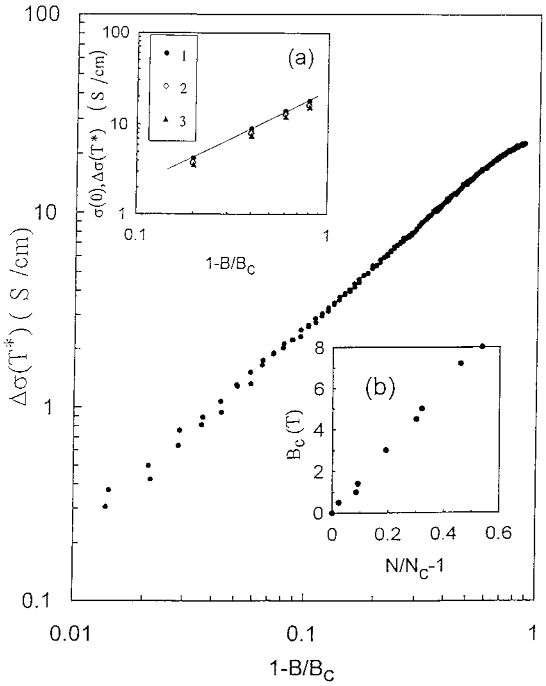

FIG. 2. Dependence of $\Delta\sigma(T^*)$ at $T^*=0.216$ K as a function of $[1-(B/B_c)]$ for one sample of Ge:As ($N=5.38\times10^{17}$ cm^{-3}, $B_c=8$ T). Inset (a) shows the scaling dependencies of $\sigma(0)$, (1) and $\Delta\sigma(T^*)$ measured at $T^*=0.1$ K (2) and at 0.216 K (3) for the sample of Ge:As ($N=4.60\times10^{17}$ cm^{-3}, $B_c=5$ T). The straight line corresponds to $\nu=1$. Inset (b) shows the dependence of B_c on ΔN for a series of samples of Ge:As.

ment with the Aronov-Altshuler model,[21] which predicts that at the critical point of the MIT the temperature dependence of conductivity must obey $\sigma(T)=bT^{1/3}$.

In order determine the critical index ν, we plot $\sigma(0)=\sigma(T\to0)$ as a function of the scaling variable $[1-(B/B_c)]$. This dependence is shown in inset (a) to Fig. 2. The data of $\sigma(0)$ were obtained by extrapolating the straight lines $\sigma(T)\propto T^{1/3}$ to $T=0$. To avoid the need to extrapolate to $T=0$, which is the main source of inaccuracy (especially due to the lack of straight lines), we consider the conductivity of any sample in the ''critical regime,'' i.e., at $B=B_c$, as a ground level of conductivity, and instead of $\sigma(0)$, we plot the exact measured quantities $\Delta\sigma(T^*)=\sigma_B(T^*)-\sigma_{B_c}(T^*)$ at temperatures T^*, where the law $\sigma(T)\propto T^{1/3}$ is observed. The data for $\Delta\sigma(T^*)$ at $T^*=0.1$ and 0.216 K are also shown in inset (a) to Fig. 2. One can see that the obtained value of $\nu=1$ does not depend on this replacement. This is due to the fact that in the immediate vicinity of the MIT, all the curves for $\sigma(T)$ are almost parallel (see Fig. 1) and therefore differences in the temperature corrections to the conductivity at fixed T are much smaller than those caused by changing the scaling variable B. This gives us reason to plot the values of $\Delta\sigma(T^*)$, obtained by sweeping magnetic field at fixed temperature. The result of this plot is presented in Fig. 2. One sees that $\nu=1$, in accordance with the theoretical prediction.

Inset (b) shows that B_c increases linearly as a function of N: $B_c=B^*(\Delta N/N_c)$, with $B^*=15$ T for Ge:As. This allows us to propose the unification of both N-MIT and B-MIT by introducing a new scaling variable $U=[(N/N_c-1)(1-B/B_c)]$. Taking Eq. (3) into account, one can rewrite U for uncompensated samples in the form given by Eq. (5), $U=[(N/N_c)-(B/B^*)-1]$. Knowledge of B^* and N_c allows us to calculate U for any sample with $N>N_c$ and $B<B_c$ and plot $\sigma(0)$ or $\Delta\sigma(T^*)$ as a function of U. We normalize the values of $\sigma(0)$ and plot the dimensionless ra-

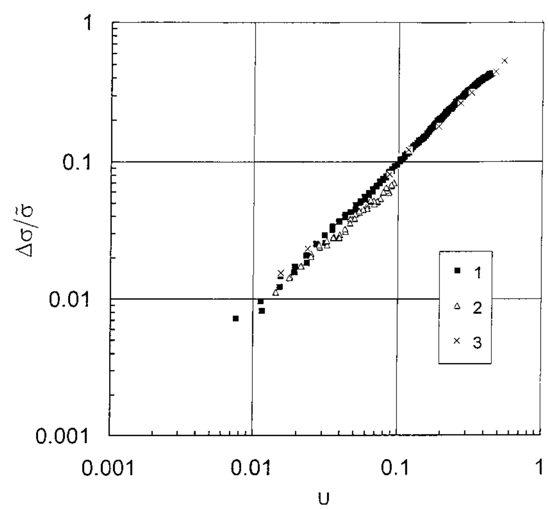

FIG. 3. Normalized scaling conductivity $\Delta\sigma/\tilde{\sigma}$ for Ge:As vs new universal variable $U=[(N/N_c)-(B/B^*)-1]$. 1, 2, B-MIT for two samples of Ge:As with $N=5.38$ and 4.17×10^{17} cm^{-3} ($N_c=3.50\times10^{17}$ cm^{-3}, $B^*=15$ T); 3, N-MIT for series of Ge:As at $B=0$.

tio $\sigma(0)/\widetilde{\sigma}$ or $\Delta\sigma/\widetilde{\sigma}$ where $\widetilde{\sigma}=C_0(E^2/\hbar)N_c^{1/3}$ is the Mott minimum metallic conductivity. The result of this procedure is shown on Fig. 3. (The adjustable numerical coefficient was chosen for Ge to be $C_0=0.32$, so for Ge:As $\widetilde{\sigma}\cong 55$ S/cm.) One can see from Fig. 3 that all the data exhibit universal linear scaling behavior: $\Delta\sigma/\widetilde{\sigma}=U^{\mu}$, $\mu=1$. This plot includes also the data for N-MIT at $B=0$.[19] It means that in doped Ge:As the N-MIT and the B-MIT can be unified by introducing a new scaling variable U.

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