Energy-loss rates of heavy and light charged particles in a two-dimensional electron gas

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Energy-loss rates for charged external particles moving in a two-dimensional electron gas of zero temperature are calculated. The transition probability per unit time for excitations in the host system is determined by using weak coupling, i.e., linear-response theory. Contributions arising from electron-hole and collective excitations to the total loss rate are separated at the levels of the random-phase approximation and a generalized mean-field approach for the linear-response function. Substantial phase-space reduction in the excitations of the fermion system is demonstrated for light external particles and interpreted as the effect of recoil. The results obtained for heavy external particles are compared with recent strong-scattering predictions based on the kinetic theory of the energy-loss process. A kinematical effect that influences the high-velocity form of the loss rate for different masses of the projectiles is pointed out. [S0163-1829(97)03620-5]

I. INTRODUCTION

The two-dimensional quantum electron gas (2DEG) is attracting a steady interest in modern physics. Many aspects of this model have been studied. An important problem in the characterization of interactions between a 2DEG at zero temperature and an in-moving external particle is the calculation of energy-loss rates. In a response to an external particle of mass M and velocity $\mathbf{v}$, the system absorbs energy ω and momentum $\mathbf{q}$, with a corresponding reduction in the initial energy E and momentum $\mathbf{K}=M\mathbf{v}$ of the projectile. For fixed values of the kinematical variables $(\mathbf{K},M,E)$, the energy-loss rate is constrained by conservation laws and regulated by statistical behaviors of the stationary host system in its equilibrium.

The energy-loss rate $(-dE/dt)$ is related to the stopping power (S) of a 2DEG for *heavy* external particles in a simple way: $S=(-dE/dt)v^{-1}$. The stopping power is the energy lost by a heavy projectile per unit length of its classical trajectory. The reasonable assumption of essentially constant velocity ($\mathbf{v}=$const) for the case $M \gg m$ (where m is the electron mass) allows simplifications in the inherent kinematics of the loss rate. Two main approaches have been developed.[1–4]

In the linear, dielectric treatment[1,2] one calculates the induced charge and potential. The retarding force (S) is interpreted as the gradient of this induced potential at the classical location of the heavy projectile. This gradient is proportional to the imaginary part of the response function $\chi(\mathbf{q},\omega)$ of the 2DEG with $\omega=\mathbf{q}\cdot\mathbf{v}$ in the large mass limit $(M \gg m)$.

In the kinetic (scattering) treatment[3,4] the interpretation is different. In the frame of reference of the heavy projectile $(M \gg m)$ the independent electrons are scattered by a fixed potential, which represents the effect of the particle on the system. The average momentum transfer suffered by the scattering electrons is the source of the stopping power. The kinetic treatment is, by its construction, a nonperturbative one. On the other hand, knowledge of the effective scattering potential is crucial for this treatment.

In this paper we investigate the role of inherent kinematics in the energy-loss process by using the golden-rule expression for the transition probabilities for excitations in the host system. In this treatment, $\omega=\mathbf{q}\cdot\mathbf{v}-q^2/2M$, where the term $-q^2/2M$ accounts for the effect of recoil at a given $\mathbf{v}$.

The paper is organized as follows. In Sec. II, the theoretical framework is outlined. The results obtained for a heavy particle $(M \gg m)$ are presented in Sec. III A, and compared with recent strong-scattering predictions.[3,4] Section III B deals with the light- $(M=m)$ particle case. Section IV is devoted to the summary and comments. We use atomic units $(e^2=\hbar=m=1)$ throughout this work.

II. FORMALISM

The projectile scatters from a given momentum state $\mathbf{K}$ into a momentum state $\mathbf{K}'=\mathbf{K}-\mathbf{q}$. Its energy E is lowered by $\omega=\mathbf{q}\cdot\mathbf{v}-q^2/2M$, where $\mathbf{v}=\mathbf{K}/M$. At zero temperature of the 2DEG the energy change is constrained by $\omega \geq 0$. The transition rate (W) for excitations in the host system is determined using the golden rule expression[5]

$$W(\mathbf{q},\omega)=2\pi|V(q)|^2\ (1/\pi)\ \mathrm{Im}\chi(\mathbf{q},\omega), \tag{2.1}$$

in which $V(q)$ is the Fourier transform of the time-independent interaction potential $V(r)$ between the projectile and particles of the system. The function $\chi(\mathbf{q},\omega)$ is the density-density response function; it is a basic quantity in the description of the dynamics in many-body systems.

Concerning the approximations for $\chi(\mathbf{q},\omega)$ the simplest one refers to an ideal electron gas $\chi^0(\mathbf{q},\omega)$.[6,7] In going beyond the ideal electron gas result, mean-field approaches for $\chi(\mathbf{q},\omega)$ have proved very powerful. The basic theory, the random-phase approximation (RPA), is given by

$$\chi^{\mathrm{RPA}}(\mathbf{q},\omega)=\chi^0(\mathbf{q},\omega)/[1+v(q)\chi^0(\mathbf{q},\omega)], \tag{2.2}$$

in which $v(q)$ is the Fourier transform of the bare electron-electron interaction potential $v(r)$. In the RPA the collective aspects of the system-particle motion are included at the mean-field level. By taking $\mathrm{Im}\chi^{\mathrm{RPA}}(\mathbf{q},\omega)$ into Eq. (2.1) an

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important manifestation of the electron-electron interaction appears in the expression for $W(\mathbf{q},\omega)$. This manifestation is the screening which is one of the most fundamental concepts in conventional many-body theory.

Within the framework of generalized mean-field approaches;[8] one may modify Eq. (2.2) by the inclusion of a static local-field correction $G(\mathbf{q})$. Formally, $G(\mathbf{q})$ is incorporated in Eq. (2.2), beyond RPA, as

$$\chi^{\mathrm{LF}}(\mathbf{q},\omega)=\frac{\chi^0(\mathbf{q},\omega)}{1+v(q)[1-G(\mathbf{q})]\chi^0(\mathbf{q},\omega)}. \qquad (2.3)$$

In this paper we use a recent parametrized form for $G(\mathbf{q})$.[9] The energy-loss rate $(-dE/dt)$ is defined in the usual way[10]

$$-\frac{dE}{dt}=\frac{1}{(2\pi)^2}\int \mathbf{d}^2\mathbf{q}\ \omega W(\mathbf{q},\omega), \qquad (2.4)$$

where $\omega=\mathbf{q}\cdot\mathbf{v}-q^2/2M$. We will consider Coulomb interaction potentials, i.e., $V(q)=-2\pi Z_1/q$ in Eq. (2.1) and $v(q)=2\pi/q$ for the direct electron-electron term in Eqs. (2.2) and (2.3). Here Z_1 denotes the charge of the in-moving projectile. The integration limits in the (ω,$\mathbf{q}$) plane are constrained by energy and momentum conservation and the detailed behaviors of $\mathrm{Im}\chi^0(\mathbf{q},\omega)$.

III. RESULTS

The basic idea of the present work is entirely formulated in Eq. (2.4), as discussed in the preceding section. This section is devoted to the presentation and discussion of our results.

A. Heavy particles

By fixing the velocity $\mathbf{v}=\mathbf{K}/M$ of the projectile as input quantity, the mass-dependence of the energy transfer $\omega=\mathbf{q}\cdot\mathbf{v}-q^2/2M$ appears in the recoil term $q^2/2M$. In our comparative study we first neglect this recoil term. In fact, we suppose $K\gg p_F$, where p_F is the Fermi momentum defined by the host density n_0 as $p_F=(2\pi n_0)^{1/2}$. In this *restricted* case the evaluation of Eq. (2.4) with Eq. (2.1) and the ideal-gas response function, $\chi^0(\mathbf{q},\omega)$, is simple. The result for the reduced energy-loss rate $[(-1/v)dE/dt]$ is the following:

$$-\frac{1}{v}\frac{dE}{dt}=\begin{cases}\pi Z_1^2 v & \text{for } v<p_F\\ 2\pi^2 n_0 Z_1^2/v & \text{for } v>p_F.\end{cases} \qquad (3.1)$$

It is very important to note that the *same* result arises from the kinetic theory of the stopping for a bare Coulomb potential treated in the first Born approximation.[3]

The next natural step in the *restricted* case (i.e., neglecting recoil) is to use the representations given by Eqs. (2.2) and (2.3) in Eqs. (2.1) and (2.4). Both require numerical evaluation. The inclusion of the static local-field correction[9] enhances the low-velocity stopping in comparison with the RPA. This is due to the weaker screening (see Fig. 1). On the other hand the local-field modified description yields to results which are, at lower densities, higher than those obtained by the exact treatment for bare Coulomb potential (see Ref. 3). This is due to the first-order Born approximation inherent in a dielectric description (see. Fig. 1). In their nonlinear treatment Krakowski and Percus[4] used the low-velocity ($v \ll p_F$) expression[3] for the stopping

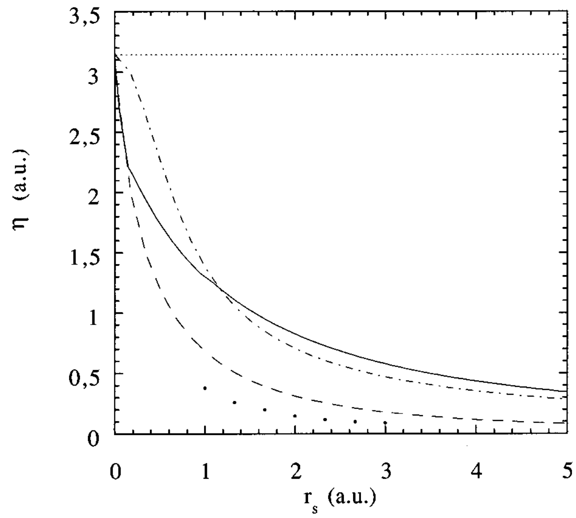

FIG. 1. Friction coefficient η, as a function of the density parameter r_s, for a recoilness, slows heavy-particle moving in a 2DEG. Dotted curve: based on firs-order Born treatment for a bare Coulomb potential; see Eq. (3.1). Dashed-dotted curve: based on exact treatment for a bare Coulomb potential (see Ref. 3). Dashed curve: based on the RPA for the linear-response function. Solid curve: based on the generalized mean-field approach of Ref. 9 for the linear-response function. Dots: based on nonlinear, scattering calculations for $Z_1=-1$ and are taken from Ref. 4.

$$S=n_0 v p_F \sigma_{tr}(2\mathrm{D},p_F). \qquad (3.2)$$

They evaluated the induced density and potential[11] for $Z_1=-1$ at various densities of the 2DEG and calculated the phase-shift values to $\sigma_{tr}(2\mathrm{D},p_F)$ by numerical solutions of scattering Schrödinger equation with their nonlinearly screened potential.

Figure 1 contains the *recoiless* (restricted) low-velocity results from the mentioned different approximations. In this figure we plot the so-called friction coefficient $[S=v\eta]$ as a function of the density parameter $r_s=\sqrt{2}/p_F$ for $Z_1=-1$. A simple comparison of curves in Fig. 1 shows that the effects of screening and nonperturbative treatment of scattering are important ingredients of a consistent description of friction.

In the remaining part of this section we present our numerical results obtained from Eq. (2.4) with Eqs. (2.1), (2.2), and (2.3) for $|Z_1|=1$, including the effect of recoil by means of the variable $\omega=\mathbf{q}\cdot\mathbf{v}-q^2/2M$. The reduced energy-loss rate $[(-1/v)dE/dt]$ is calculated for different values of the density parameter r_s, as a function of the given velocity. In Figs. 2 and 3 (for $r_s=1$ and $r_s=3$, respectively) we have separated the contributions arising from electron-hole and collective excitations. The sharp plasmon contribution essentially influences (in contrast to the statements of Refs. 1 and 2) the value of reduced total rates around the maximum, especially for the lower density $(r_s=3)$. In order to get a clear physical picture about dynamical screening and collective excitation, in Fig. 4 we plot results (at $r_s=1$) based on different approximations. It is important to note that beyond the maximum position, the unscreened result of Eq. (3.1) is

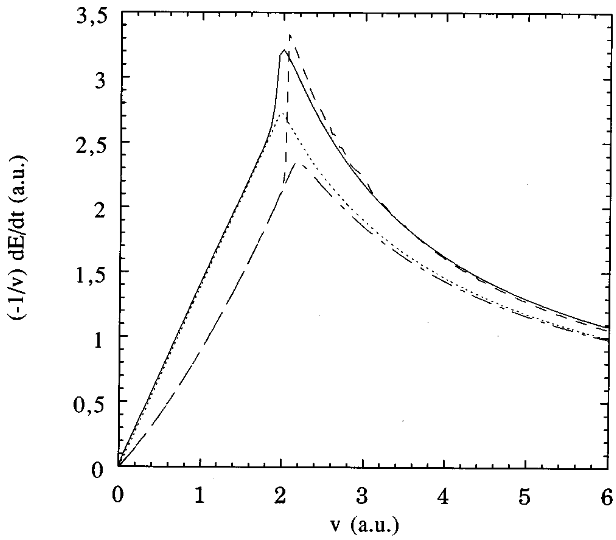

FIG. 2. Reduced energy-loss rate $[(-1/v)dE/dt]$ for heavy particles as a function of the velocity v at $r_s=1$ and $|Z_1=1|$. Dashed curve: total loss rate at the level of the RPA. Dash-dotted curve: contribution from electron-hole excitation within the RPA. Solid curve: total loss rate at the level of the generalized mean-field approach. Dotted curve: contribution from electron-hole excitation within this approximation.

very close to the numerical one (solid curve). With the help of Fig. 2 we can conclude from Fig. 4 that there is an almost a complete cancellation of screening effect in the electron-hole channel with collective contribution, at higher velocities.[1] On the other hand the curve based on the kinetic theory illustrates the importance of a proper treatment of scattering which goes beyond the perturbative treatment of Eq. (2.4).

A nonperturbative calculation, with the inclusion of the collective channel is, therefore, highly desirable. Similar conclusions can be drawn for other r_s values.

B. Light particles

As we mentioned in Sec. III A, the effect of recoil is expected to be more important for light projectiles (positron) than for heavy ones (proton, antiproton). In the following we use the *unrestricted* case for evaluation of Eq. (2.4), with Eqs. (2.1), (2.2), and (2.3), i.e., we apply $\omega=\mathbf{q}\cdot\mathbf{v}-q^2/2M$. We consider the case of positrons ($M=1$ and $Z_1=1$) which is of most physical interest.

The reduced energy-loss rate $[(-1/v)dE/dt]$ is calculated for different values of the density parameter r_s, as a function of the given velocity. Illustrative results are plotted in Figs. 5 and 6, for $r_s=1$ and 3, respectively. The contributions arising from electron-hole and collective excitations are again separated in these figures. It is important to note that the *onset* and the *shape* of the plasmon contribution is the same as the corresponding results obtained[12] for an external electron. The relative contribution of the collective channel to the total (reduced) loss rate is enhanced in comparison to the heavy-particle case (see Figs. 2 and 3). At low velocities the combined effects of Pauli restriction in the excitation and recoil lead to a substantial reduction of the energy-loss rates. Well beyond the position of the maxima, i.e., at high velocities one can observe a similar cancellation of screening effect in the electron-hole channel with collective contribution, as

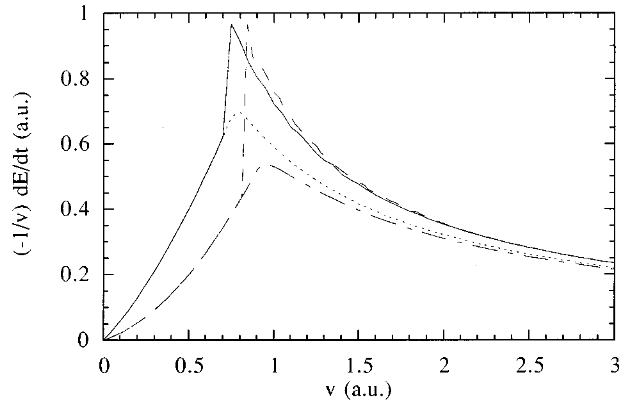

FIG. 3. The same as in Fig. 2, at $r_s=3$.

in the heavy-particle case. Furthermore, the reduced energy-loss rate tends, asymptotically, to a value which is one-half of the corresponding value obtained for heavy projectile. This kinematical, i.e., reduced-mass dependence of rates is absent in the three-dimensional version of our problem. In the following, we give a simple explanation of this observation.

For high velocities of the projectile one may consider the electron gas as effectively at rest. The effect of the Pauli restriction is then removed and the behavior is independent of the particle statistics. In fact, the results are also applicable to a two-dimensional charged Bose gas.[13] By using unscreened Coulomb potential in Eq. (2.1) we perform the integrations in Eq. (2.4) with $\omega=\mathbf{q}\cdot\mathbf{v}-q^2/2M$. The result is as follows ($\omega>0$; at zero temperature):

$$-(1/v)\ dE/dt=(1/v)\ 2\pi^2 Z_1^2 n_0\ M/(1+M)\,. \quad (3.3)$$

This simple expression shows, that the energy-loss rate is influenced by the reduced mass for high-velocity projectiles moving *in plane* of a 2D gas.

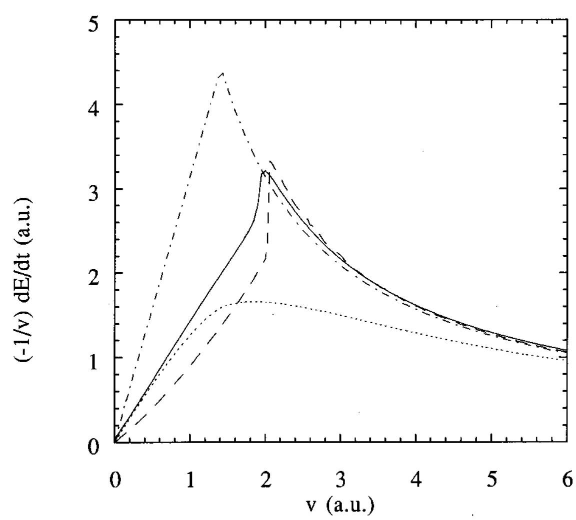

FIG. 4. Reduced energy-loss rates obtained from different approximations at $r_s=1$ and $|Z_1|=1$, for heavy particles. Dash-dotted curve: based on first-order Born treatment for a bare Coulomb potential; see Eq. (3.1). Dashed curve: total loss-rate obtained using the RPA response function. Solid curve: total loss rate obtained within the generalized mean-field approach of Ref. 9. Dotted curve: based on the kinetic theory (see Ref. 3) with the exact treatment for Coulomb scattering.

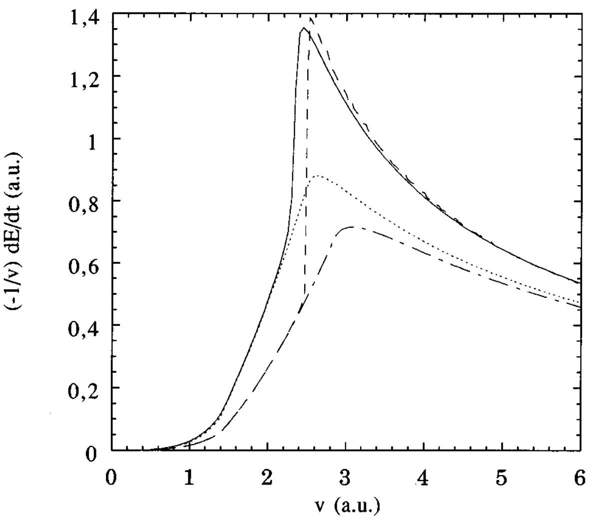

FIG. 5. The same as in Fig. 2, for a light particle, at $r_s=1$.

IV. SUMMARY AND COMMENTS

In this paper we have investigated the energy-loss rates for different-mass external projectiles moving in plane of a two-dimensional quantum electron gas. Our description is based on the golden-rule expression of the transition rate for excitations in the host system. In this way, the Pauli principle and the recoil effect are, naturally, incorporated in the calculation. For the case of heavy external particles a comparison is made with earlier results obtained with the recoiless assumption. Contributions arising from the electron hole and collective excitations are separated at the levels of random-phase approximation and a generalized mean-field approach. Phase-space reduction in the excitation of the fermion system and a mass-dependent kinematical effect are pointed out. The electron-electron interaction beyond the RPA was incorporated in our first-order Born treatment by a static local-field correction. This static correction is related to the equilibrium pair-correlation function of the homogeneous system. Furthermore, one may characterize the external particle-electron interaction in the medium by another pair-correlation function.[18] This latter is related to the density enhancement or depletion around the projectile. In fact, it is related to the question of nonlinear screening and will be investigated in subsequent works. The detailed and separated roles of these above-mentioned pair-correlation functions (exchange-correlation and nonlinearity) need further investigation in 2D.

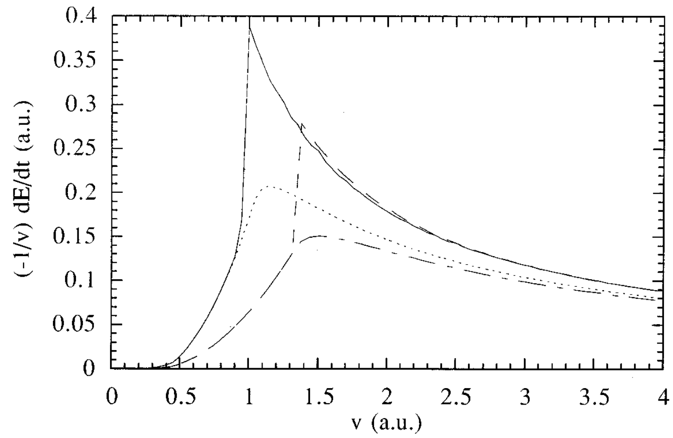

FIG. 6. The same as in Fig. 2, at $r_s=3$.

The comparative study, presented here for heavy particles, signals the importance of the nonperturbative treatments of screening[3] and scattering.[4] On the other hand, a nonlinear treatment of screening at higher velocities is a nontrivial task.[14] It would be desirable to investigate the capability of a higher-order response function treatment[15] for 2DEG.

For light particles the problem of nonlinearity seems to be even more complicated. The effect of recoil excludes[16] a straightforward application of standard methods for screening, and the description of scattering beyond the first-order Born approximation needs a more careful and detailed study.[10,17]

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