Thermoelectric and thermophase effects in Josephson junctions

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We study the thermoelectric properties of a superconductor-insulator-superconductor Josephson junction. The total electrical current across the junction is composed of three parts: a normal current, a Josephson current, and an interference current. We show that only the normal part contributes to a thermocurrent (i.e., an electrical current that flows in response to a temperature drop). The fact that the interference current has no thermoelectric properties provides insight into the physical nature of this term. We distinguish between two mechanisms for the thermocurrent: one is the normal thermoelectric tunneling current; the other is a transport phenomenon, which ensues from a nonequilibrium (charge-imbalance) state in the bulk superconductors comprising the junction. The latter effect gives rise to a *phase-dependent* thermocurrent. Finally, we consider an open-circuit Josephson junction biased by a temperature drop. The possible steady states of the system are studied using the resistively shunted junction model. In particular, we consider the zero-voltage state which corresponds to canceling of the quasiparticle and condensate currents. We call this a *thermophase* effect. Experimental setups are suggested in order to detect this effect. [S0163-1829(97)11017-7]

I. INTRODUCTION

In normal metallic systems, be they bulk or tunnel junctions, the thermoelectric coefficient is a measure of the entropy transported by the carriers in the system.[1] The carriers are either electrons or holes. In superconducting systems, and in Josephson junctions in particular, the current is carried by normal quasiparticles and by pairs.[2] The quasiparticles (i.e., electronlike and holelike excitations) have normal-metal transport properties. Namely, the transport is a nonequilibrium process which involves dissipation. The pairs transport, on the other hand, is an equilibrium property and does not generate entropy.[3] Therefore, only a quasiparticle charge current will flow in the presence of a temperature drop across the junction. However, there exists a coupling between the quasiparticles and the condensate in superconductors. We will show that this coupling affects the transport properties of the conductor and introduces an interesting transport phenomenon.

One manifestation of this coupling is that in bulk superconductors and in Josephson junctions dc thermoelectric currents are shorted out by reverse supercurrents. For example, in homogeneous bulk superconductors the conventional thermoelectric effects, e.g., the Seebeck, Thompson, and Peltier effects, are absent.[4] This experimental fact is usually explained within the ''two-fluid'' picture[5,6] and is related to the Meissner effect. In an attempt to offer a more explicit explanation for this behavior, we propose a different interpretation for those experimental findings: in response to a temperature gradient, the superconductor develops a phase gradient which satisfies the constraint dictated by the Meissner effect. In other words, there exists a coupling between a temperature gradient and the phase of the superconducting order parameter. In order to study the consequences of the quasiparticle condensate coupling on a microscopic level, we study the thermoelectric properties of a Josephson junction. We also focus on the explicit relation between a temperature drop across the junction and the phase difference.

The coupling between quasiparticles and pairs is exhibited in superconducting systems in various ways. For example, in bulk superconductors it gives rise to the so-called charge-imbalance effect.[4] This is a nonequilibrium state in which the population of the two branches of the quasiparticle energy spectrum is different. A local net charge-density develops, which is sustained by the large reservoir of pairs. As we shall see, the generation of a charge-imbalance state in superconductor-insulator-superconductor (SIS) systems results in an interesting transport phenomenon. Another example of the coupling between quasiparticles and the condensate is exhibited in Josephson junctions. When performing the microscopic derivation of the total current through the junction, one can distinguish between three contributions: a quasiparticle current, a supercurrent, and an interference current.[7]

The fact that the normal and super ''fluids'' in the system are coupled presents a challenge when attempting to develop a consistent description of transport properties of a superconducting system. On one hand, the normal quasiparticle degrees of freedom are well described by thermodynamics. The transport of this ''fluid'' is accompanied by dissipation. On the other hand, one must also account for the macroscopic quantum-mechanical degree of freedom of the condensate, which yields supercurrents. A complete description must include all these degrees of freedoms. There are several approaches in the literature for doing this. One of these uses either the two-fluid theory or the Boltzmann equation coupled with the BCS gap equation.[6] Other possibilities include treatments that start at a more microscopic level, either applying perturbation theory to the BCS Hamiltonian,[2] or starting from the Bogliubov–de Gennes equations.[8] The system we shall study is an SIS, consisting of conventional superconductors modeled by the BCS Hamiltonian. An SIS Josephson junction is a convenient system to study since it is well approximated by a perturbative Hamiltonian model. The

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superconductor on each side is assumed to be a reservoir of particles in equilibrium. The system is driven out of equilibrium by biasing the two sides relative to each other. This description allows us to explicitly consider the different transport mechanisms, such as tunneling of pairs and tunneling of quasiparticles.

A more comprehensive approach for the study of Josephson junctions, including weak links, SIS, and superconductor-normal-superconductor (SNS) systems, is described in Ref. 8. This is a continuum scattering approach in which the particles are extended waves which scatter at the interfaces. The transport coefficients are derived using the Bogliubov–de Gennes equations. The disadvantage of this approach is that all the physical mechanisms are obscure, since they are implicit in the equations. Other approaches were utilized in order to study thermoelectric transport in SNS junctions[9] and superconducting weak links.[10] In these systems the behavior was explained by the presence of Andreev reflection (which is absent in SIS systems). None of the above approaches accounts for the inhomogeneity and relaxation of the superconductor order parameter. This turns out to be important with regard to the interference current in the SIS system. A consistent approach calls for the use of the time-dependent Ginzburg-Landau theory.[11] For simplicity, we shall assume a jump in the order parameter across the junction, with a magnitude which is constant in time.

In Sec. II we derive the total current through an SIS Josephson junction and discuss the thermoelectric properties of the system. We show that only the quasiparticle current flows in response to a temperature drop. We argue that this suggests that the interference current does not correspond to a dissipative process. In Sec. III we find a thermocurrent that is phase dependent and is generated only when charge imbalance occurs. Section IV includes a discussion of the possible steady states of an open-junction SIS system. In one case, an explicit relation between a temperature drop and a phase difference across the junction is defined as the thermophase response. Two experimental setups, designed to detect this effect, are suggested. Corresponding predictions are derived.

II. THERMOELECTRIC CURRENT IN A JOSEPHSON JUNCTION

A. The total current through the junction

In order to determine the thermoelectric properties of a Josephson junction we calculate the total current flowing through the junction within the following model. The junction is comprised of two BCS bulk superconductors separated by an insulating barrier. Experimentally, these electrodes are small compared to the leads. We assume each superconductor is in equilibrium and is characterized by the many-body BCS Hamiltonian $H_{\rm tot}$, a chemical potential μ, and a temperature T. The left-hand side (lhs) quantities are denoted by the subindex l and the right-hand side (rhs) quantities by r. The particle current is a tunneling current, therefore the total Hamiltonian includes a tunneling element H_T. The total Hamiltonian can be written as $H_{\rm tot}=H_l+H_r+H_T$. The particle current is calculated using a microscopic perturbation theory, expanding in the small tunneling matrix element. The total particle current is equal to the rate of change of the averaged electron-number operator in the lhs reservoir with respect to time. The electrical current is, according to the quantum-mechanical equation of motion,

$$I_{\rm tot}=-e\langle \dot{N}_l\rangle=\frac{-ei}{\hbar}\langle[H_{\rm tot},N_l]\rangle, \tag{1}$$

where $e>0$ denotes the electron charge. The outer brackets in Eq. (1) represent a thermodynamic average over a grand-canonical ensemble. The electron-number operator is

$$N_l=\sum_{k,\sigma} C^\dagger_{k,\sigma}C_{k,\sigma}, \tag{2}$$

where $C^\dagger_{k,\sigma}$ and $C_{k,\sigma}$ are single-electron creation and annihilation operators in the momentum (k) and spin (σ) representation. The momentum quantum-number of the lhs (rhs) superconductor is denoted by $k(q)$. The tunneling Hamiltonian is

$$H_T=\sum_{k,q,\sigma} T_{kq}C^\dagger_{k,\sigma}C_{q,\sigma}+\text{H.c.}, \tag{3}$$

where the tunneling matrix element is denoted by T_{kq}. Substituting the expressions for the operators into Eq. (1) we obtain

$$I_{\rm tot}=-\frac{2e}{\hbar}\text{Im}\left[\sum_{k,q,\sigma} T_{kq}\langle C^\dagger_{k,\sigma}C_{q,\sigma}\rangle\right]. \tag{4}$$

In the first order of perturbation theory[12] the total electrical current becomes

$$\begin{aligned} I_{\rm tot}=&-\frac{2e}{\hbar}\text{Im}\sum_{k,q,\sigma}\int_{-\infty}^{\infty}\int_{\infty}^{\infty}\frac{dw\,dw'}{(2\pi)^2}[f_l(w)-f_r(w')]\\ &\times\left[|T_{kq}|^2\frac{A_k(w)A_q(w')}{w-w'-\Delta\mu+i\eta}+\exp[-i(\Delta\theta+2\Delta\mu t)]\right.\\ &\left.\times T_{kq}T_{-k-q}\frac{B_k(w)B_q(w')}{w-w'-\Delta\mu+i\eta}\right], \end{aligned} \tag{5}$$

where $\Delta\theta\equiv\theta_l-\theta_r$ is the phase difference across the junction. The quasiparticle distribution function at temperature T is denoted by $f(w)=1/[\exp(w/k_BT)+1]$, where k_B is the Boltzmann constant. The spectral densities are

$$\begin{aligned} A_k(w)&=2\pi[|u_k|^2\delta(w-E_k)+|v_k|^2\delta(w+E_k)],\\ B_k(w)&=2\pi u_kv_k[\delta(w-E_k)-\delta(w+E_k)], \end{aligned} \tag{6}$$

where u_k and v_k are the coherence factors that satisfy the relations $|u_k|^2=1/2(1+\xi_k/E_k)$ and $|v_k|^2=1/2(1-\xi_k/E_k)$. $E_k=\sqrt{\xi_k^2+\Delta_k^2}$ is the BCS quasiparticle energy spectrum and ξ_k is the electron energy spectrum relative to the chemical potential. The difference between quasiparticle chemical potentials of the two superconductors is $\Delta\mu\equiv\mu_r-\mu_l$. The quasiparticle current pertains to the spectral-density operators $A_k(w)$, and the pair current to $B_k(w)$. Substituting the explicit expressions of the spectral densities into Eq. (5) and summing with respect to k, q and σ we find that

$$I_{\rm tot}=I_{\rm qp}+I_{\text{qp-pair}}+I_{\rm pair}=\frac{4\pi e}{\hbar}\int_{-\infty}^{\infty}dw\ \Theta(w^2-\Delta_{\rm max}^2)N_lN_r|T_{lr}|^2[f_l(w)-f_r(w-\Delta\mu)]\frac{|w||w-\Delta\mu|}{\sqrt{w^2-\Delta_l^2}\sqrt{(w-\Delta\mu)^2-\Delta_r^2}}$$
$$+\frac{4\pi e}{\hbar}\int_{-\infty}^{\infty}dw\ \Theta(w^2-\Delta_{\rm max}^2)N_lN_r|T_{lr}|^2[f_l(w)-f_r(w-\Delta\mu)]\cos[(\Delta\theta+2\Delta\mu t)]|\Delta_l||\Delta_r|$$
$$\times\frac{{\rm sgn}(w){\rm sgn}(w-\Delta\mu)}{\sqrt{w^2-\Delta_l^2}\sqrt{(w-\Delta\mu)^2-\Delta_r^2}}+\frac{4e}{\hbar}\int_{-\infty}^{\infty}\int_{\infty}^{\infty}dw\ dw'\ \Theta(w^2-\Delta_l^2)\Theta(w'^2-\Delta_r^2)N_lN_r|T_{lr}|^2[f_l(w)$$
$$-f_r(w')]P\frac{\sin[(\Delta\theta+2\Delta\mu t)]}{w'-w-\Delta\mu}|\Delta_l||\Delta_r|\frac{{\rm sgn}(w){\rm sgn}(w')}{\sqrt{w^2-\Delta_l^2}\sqrt{w'^2-\Delta_r^2}}. \tag{7}$$

Note that in all the integrals in Eq. (7) a Θ function, which is unity for a positive argument and zero for a negative argument, restricts the quasiparticle energy w to be above the BCS gap. In the first two integrals this is $\Delta_{\rm max}\equiv\max[\Delta_l,\Delta_r]$, where Δ_l amd Δ_r are assumed to be constant. The normal-metal densities of states (DOS) functions are denoted by $N_{l,r}$ and the tunneling matrix element is T_{lr}. We emphasize that Eq. (7) is obtained from Eq. (5) *only* if $N_{l,r}$ and T_{lr} are assumed to be constant with respect to the electronic momentum. For this reason the momentum subscripts k,q are omitted. In general, these functions are energy dependent and it is this dependence that gives rise to a thermoelectric current, as we show below. The third integral on the rhs is a principal value of the pole which is depicted by P. As evident in Eq. (7), the total current breaks up into three parts. The first integral is the normal current of quasiparticles tunneling. It vanishes when no thermodynamic generalized forces are applied, e.g., $\Delta\mu=\Delta T=0$. The second integral is known as the quasiparticle-pair interference current. Mathematically, this term originates from the $B(w)$ functions in Eq. (5) that produce pair tunneling.[2] Consequently, this current depends upon the phase difference across the junction. Yet, it satisfies features of a normal current, such as Ohmic behavior for small voltages. In the literature this term is interpreted as a product of the coupling between quasiparticles and the superconducting condensate and is related to loss in the system. A detailed discussion about the nature of this ''$\cos\Delta\theta$'' current can be found in Refs. 7, 13, 14, and references therein. The final part is the Josephson pair current which can flow even in the absence of the ordinary thermoelectric generalized forces.

B. Thermoelectric currents

Next, we study the thermoelectric properties of the expression for the total tunneling current. We confine the discussion to the case $\Delta\mu=0$. The main contribution to the thermoelectric transport in the junction ensues from the asymmetry in the electron-hole transport, induced by the insulating barrier. A temperature drop between the coupled superconductors will give rise to a normal electrical current. This current $I_{\rm qp}(\Delta T)=L_{12}^s\Delta T$ is analogous to the thermoelectric effect in the semiconductor model.[15] The transport coefficient L_{12}^s is the thermoelectric coefficient of the junction in the superconducting state. It is proportional to the energy derivative of the energy-dependent quantities, e.g., the product of the tunneling matrix element and the DOS of the bulks.

This effect has been studied thoroughly in Ref. 16. In that paper the quasiparticle current was derived using the same Hamiltonian described above. The Hamiltonian, expressed in electron field operators, was transformed into quasiparticle operators using the Bogliubov transformation. The thermoelectric quasiparticle current was obtained by invoking the ''golden rule.'' As a result the normal current was identical to $I_{\rm qp}$ in Eq. (7) with one difference: the tunneling matrix element was assumed to be energy dependent and was related to the tunneling of *quasiparticles*. Therefore, the thermocurrent was nonzero when $|T_{lr}(w)|^2$ was expanded in powers of the *quasiparticle* energy w (assuming the DOS is constant). This approach yields the following expression for the thermocurrent in a symmetric junction

$$I_{\rm qp}|_{\Delta\mu=0}(\Delta T)=\frac{8\pi eaN_l(0)N_r(0)\Delta T}{\hbar T}\times\int_{\Delta_{\rm max}}^{\infty}dw\frac{w^4}{\sqrt{w^2-\Delta_l^2}\sqrt{w^2-\Delta_r^2}}\left(-\frac{\partial f}{\partial w}\right), \tag{8}$$

to first order in ΔT. In Eq. (8) we used the expansion $|T_{lr}(w)|^2\approx|T_{lr}(0)|^2+aw$ with $a=1/2\ (d|T_{lr}|_{w=0}^2/dw)$. The normal-metal DOS functions of the two bulks are approximated by their value at $\mu_l=\mu_r$.

The approach taken here is different. We derive the currents through the junction from the microscopic equation of motion, which is expressed by the electron field operators. The basic process is tunneling of *electrons* (rather than quasiparticles) and consequently, in order to obtain the thermocurrent we use Eq. (5). We expand $|T_{lr}(\xi)|^2$ in powers of the *electron* energy $\xi=\epsilon-\mu$, and only then do we transform ξ into the quasiparticle energy w. The expression obtained in this way is

$$I_{\rm qp}|_{\Delta\mu=0}(\Delta T)=\frac{8\pi eaN_l(0)N_r(0)\Delta T}{\hbar T}\int_{\Delta}^{\infty}dw\ w^2\left(-\frac{\partial f}{\partial w}\right), \tag{9}$$

where $a=1/2\ (d|T_{lr}|_{\xi=0}^2/d\xi)$. Note that the expression for $I_{\rm qp}$ as calculated by Eq. (9) is well behaved: it is finite and monotonic in the average temperature $T_{\rm av}\equiv(T_l+T_r)/2$, as

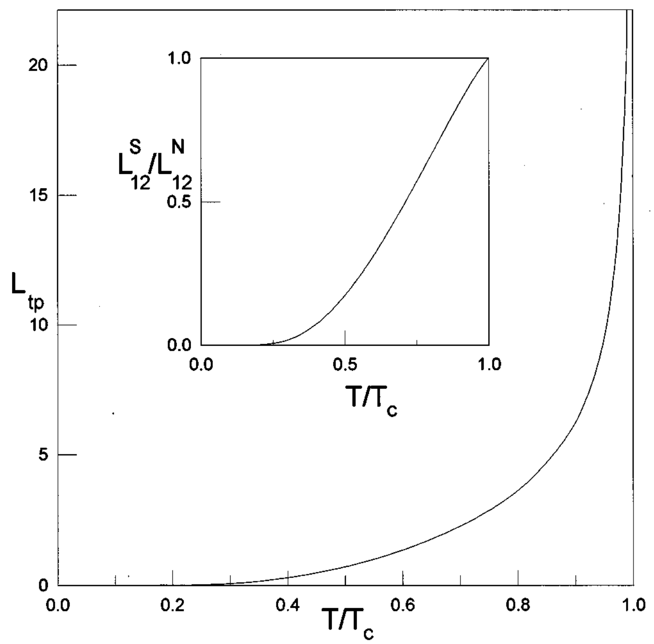

FIG. 1. A numerical calculation of the coefficient $L_{\rm tp}$ as function of temperature, Eq. (23). The units of $L_{\rm tp}$ are arbitrary. The inset illustrates the behavior of the superconducting junction thermopower $I_{\rm qp}/\Delta T$, normalized by the normal coefficient, as function of temperature.

illustrated in the inset of Fig. 1. The discrepancy between the two approaches arises form the fact that the golden rule is an *averaged* result. When the energy dependence of the expressions in Eq. (7) is important, one must go back to Eq. (5) and rederive the golden rule explicitly. As we have shown, it is precisely this energy dependence that is important for the thermoelectric effect. However, expansion of Eq. (7) in powers of the quasiparticle energy is tantamount to the neglect of this energy dependence—that is why such a procedure may lead to a wrong result.

This point is also relevant when studying $I_{\rm qp-pair}$ and $I_{\rm pair}$. Note that both terms stem from the integral that includes the functions $B(w)$ in Eq. (5), hence, the analysis is similar in both cases. Expanding $|T_{lr}|^2$ in the quasiparticle spectrum w in the expressions of Eq. (7) results in a nonzero thermocurrent $I_{\rm qp-pair}(\Delta T)$. Consequently, we obtain a quasiparticle thermoelectric coefficient $L_{12}^{\rm qp-pair}$ that depends on the phase difference $\Delta\theta$. Apparently, these calculations are wrong. A more careful analysis reveals that when expanding $|T_{lr}|^2$ and integrating over the *electron* energy in Eq. (5) we obtain $L_{12}^{\rm qp-pair}=0$. The reason is that in Eq. (5) the integrands are odd in ξ. We understand the absence of the thermoelectric effect in $I_{\rm qp-pair}$ in the following way: $I_{\rm qp-pair}$ is an effective transport of quasiparticles—it involves a superposition of tunneling pairs and events which break or create pairs in the bulk superconductors. Yet the physical process that underlies this current is the tunneling of pairs. Since a supercurrent is indifferent to the asymmetry in hole-electron transport, no thermoelectric effect can develop. Note also that for these boundary conditions the Josephson current can be shown to be approximately proportional to $I_{\rm pair}(T_l)+I_{\rm pair}(T_r)$ and is independent of ΔT.

In view of the above discussion, we suggest a different interpretation of the interference current. The vanishing of the thermoelectric coefficient may imply that the current of $I_{\rm qp-pair}$ is *nondissipative*. Consider the case of a nonzero voltage. We will show below that this is a ''running'' steady state, i.e., $V=V(t)$. For certain values of the voltage, $I_{\rm qp-pair}\sim V(t)$. In this case one can define a ''Joule heating'' generated by this term: $I_{\rm qp-pair}V=V(t)^2\cos(\Delta\theta(t))$. Note that this ''heating'' oscillates in time. This means that unlike an Ohmic resistor, which only dissipates energy, the system draws heat from the surroundings part of the time. We interpret this behavior as a *reversible* transformation of electrical energy, gained by the pairs traversing the voltage $V(t)$, into magnetic energy stored in the magnetic field. The change in the magnetic field is manifested as a change of the phase difference across the junction, giving rise to an ac pair current. From this point of view one may perceive $I_{\rm qp-pair}$ as an inductive response of a pair current and *not* as a normal resistive current. In this sense it is comparable to the Josephson current. The interpretation of $I_{\rm qp-pair}$ as a pair current is consistent with the mathematical origin of this term. Additional support for this view is found in a numerical calculation of the time-averaged Joule heating generated by $I_{\rm qp-pair}$. We calculated the time-dependent phase difference and voltage on the junction in the ''running'' state [including the ''$\cos\Delta\theta(t)$'' term] and inserted the solution into the expression for the Joule heating. The result is that the average over time of the heating due to $I_{\rm qp-pair}$ vanishes. Note that a small part of the electrical energy carried by the pairs is dissipated as radiation. We neglected this effect in our model.

Most of the experimental work done on almost every type of Josephson junction provides evidence for the existence of the interference current. However, the agreement with the theory of tunnel junctions, presented here, was not complete. The magnitude of the measured current was in line with the theory, whereas the sign was reversed.[14] Other theoretical approaches, which resolved this discrepancy, involved relaxation processes, suggesting that the interference current is dissipative.[11] However, in Ref. 14 the authors reported that they found evidence for both positive and negative interference currents in different temperature regions. They concluded that this must be a result of two mechanisms at play: one corresponds to the prediction of the perturbation theory given here; the other involves dissipative processes. Our understanding is that the first mechanism corresponds to nondissipative pair tunneling.

III. THE EFFECT OF CHARGE IMBALANCE

So far we have assumed that the bulk superconductors are in equilibrium. Next, we consider a modified SIS system in which the superconducting electrodes are driven out of equilibrium due to charge imbalance. This state could be induced in elongated electrodes to which a temperature gradient is applied. Another possibility is to inject quasiparticles into the superconductor via a normal-superconductor (NS) interface or a normal-insulator-superconductor junction (NIS).[4] We show that in the case of a temperature gradient in the bulk, a different type of thermoelectric transport takes place. According to Tinkham,[17] the symmetry in the quasiparticle energy spectrum at the Fermi surface is broken in the presence of a supercurrent. When, in addition, the populations of the electronlike branch and the holelike branch are not equal a net quasiparticle charge per unit volume Q^* develops in the superconductors. Charge neutrality is maintained by an adjustment of the charge of the condensate. The excess charge generation competes with a relaxation due to elastic and inelastic scattering. The effect is also understood within the phenomenological two-fluid theory. The theory predicts that the quasiparticle charge density that develops is proportional to the difference between the quasiparticle and pair chemical potentials, i.e., $Q^* \propto \mu_{\rm qp} - \mu_{\rm pair}$. Experimental evidence is in agreement with theory, see Ref. 4 and references therein. In particular, a temperature gradient within either bulk superconductor accompanied by a flow of supercurrent will induce the phenomenon.

A. Incorporating charge imbalance on the microscopic level

In order to study the effect of charge imbalance on $I_{\rm tot}$ we can insert the branch-population asymmetry into Eq. (5). Note that the sum in this equation is on the electronlike and holelike branches. Hitherto, we implicitly assumed that the distribution function $f(w)$ for the different branches was identical. This is not the case for the nonequilibrium state, hence we must sum over the two branches separately. To do so, we define a distribution function for each branch: $f^{>(<)}(w)$ is the distribution function of the electron(hole)like branch; in other words, $f^{>(<)}(w)$ is the the population of quasiparticles in the degenerate quasiparticle states $w = \sqrt{\Delta^2 + \xi_k^2}$ which corresponds to $\xi_k > (<) 0$. In nonequilibrium $f^{>}(w) \neq f^{<}(w)$. We also distinguish between the coherence factors $u_k^{>(<)}$ and $v_k^{>(<)}$. These factors satisfy

$$|u_k^{>}|^2 = |v_k^{<}|^2, \quad |u_k^{<}|^2 = |v_k^{>}|^2$$

$$|u_k^{>}|^2 - |v_k^{>}|^2 = -(|u_k^{<}|^2 - |v_k^{<}|^2) = \frac{|\xi_k|}{E_k}. \tag{10}$$

In order to calculate the charge imbalance we rewrite Eq. (5), distinguishing between the two branches in the sum: first we separate the rhs of Eq. (5) into a term including $f_k(w)$ and a term including $f_q(w)$; then the sum in the first term, associated with the lhs electrode of the junction, is separated into $\Sigma_k \rightarrow \Sigma_{k^>} + \Sigma_{k^<}$. The sum over the rhs momentum q does not contribute to charge imbalance in the lhs electrode, and, hence, it can be transformed into an integral over the electron spectrum ξ_q. The second term, corresponding to $f_q(w)$, is treated similarly. Inserting Eq. (6) and using the relations in Eq. (10) we find that charge imbalance is relevant only to the quasiparticle current. Mathematically, charge imbalance corresponds to the way the coherence factors enter in the normal spectral densities $A(w)$ and $B(w)$ in Eq. (6). In the expressions for $I_{\rm qp-pair}$ and $I_{\rm pair}$ the coherence factors enter only as products like $u_k v_k$. Such terms are even in the electron energy ξ, and therefore no charge imbalance is produced. After some manipulations we obtain the following expression for the normal current:

$$I_{\rm qp} = \frac{4\pi e N_l N_r}{\hbar} \int_{-\infty}^{\infty} dw \Theta(w^2 - \Delta_{\rm max}^2) \times \frac{|w||\tilde{w}||T_{lr}(w)|^2}{\sqrt{w^2-\Delta_l^2}\sqrt{\tilde{w}^2-\Delta_r^2}} \frac{1}{2} \Bigg\{ [f_l^{>}(w) + f_l^{<}(w)] + \frac{\sqrt{w^2-\Delta_l^2}}{w} [f_l^{>}(w) - f_l^{<}(w)] - [f_r^{>}(\tilde{w}) + f_r^{<}(\tilde{w})] - \frac{\sqrt{\tilde{w}^2-\Delta_r^2}}{\tilde{w}} [f_r^{>}(\tilde{w}) - f_r^{<}(\tilde{w})] \Bigg\}, \tag{11}$$

where $\tilde{w} = w - \Delta\mu$. Note that we assumed that the tunneling matrix element T_{lr} is insensitive to the branch. In any case, cross-branch tunneling is forbidden. The charge-imbalance state is manifested in the terms which include $f^{>}(w) - f^{<}(w) \neq 0$. In analogy to a normal-superconducting junction,[17] we define the nonequilibrium excess quasiparticle charge per unit volume as

$$Q_l^* \equiv -2eN_l(0) \int_{\Delta_r}^{\infty} dw D_r(\tilde{w}) [f_l^{>}(w) - f_l^{<}(w)],$$

$$Q_r^* \equiv -2eN_r(0) \int_{\Delta_l}^{\infty} dw D_l(w) [f_r^{>}(\tilde{w}) - f_r^{<}(\tilde{w})]. \tag{12}$$

We assume the normal DOS, $N(0)$, and the tunneling matrix elements are taken at the electronic chemical potential. Note that the excess charge that develops on the lhs (rhs) superconductor depends on the normalized DOS of the rhs(lhs) superconductor

$$D_r(\tilde{w}) \equiv \frac{|\tilde{w}|}{\sqrt{\tilde{w}^2-\Delta_r^2}},$$

$$D_l(w) \equiv \frac{|w|}{\sqrt{w^2-\Delta_l^2}}. \tag{13}$$

The terms in Eq. (11) that include the sum $f^{>}(w) + f^{<}(w)$ can be approximated, in the limit of linear response, by $2f(w)$. $f(w)$ depicts the average branch distribution function. Having defined the above quantities, we can rewrite Eq. (11) in the following way:

$$I_{\rm qp} = \frac{4\pi e}{\hbar} N_l(0) N_r(0) \int_{-\infty}^{\infty} dw \ \Theta(w^2 - \Delta_{\rm max}^2) \times D_l(w) D_r(\tilde{w}) |T_{lr}(w)|^2 \, [\overline{f_l}(w) - \overline{f_r}(\tilde{w})] - \frac{4\pi |T_{lr}(0)|^2}{\hbar} [N_r(0) Q_l^* - N_l(0) Q_r^*]. \tag{14}$$

The first part of Eq. (14) is just the quasiparticle tunneling current in Sec. II. The second part is the current that flows through the junction due to the charge imbalance in each of the bulk superconductors. Note that if the system is symmetric then the current due to charge imbalance cancels out.

B. Phenomenological theory of charge imbalance

At this stage we can incorporate the phenomenological theory of charge imbalance into the expression we obtained for I_{qp}. From the two-fluid theory we have the relation

$$Q^* = -2eN(0)(\mu_{qp} - \mu_{pair}) \tag{15}$$

for either side of the junction. The bulk nonequilibrium state in which $\mu_{qp} \neq \mu_{pair}$ has been described phenomenologically within the thermodynamic theory of irreversible processes. Schmid[18] has shown that in addition to the coupled quasiparticle current and heat current, a third scalar ''current,'' $\mu_{qp} - \mu_{pair}$, can be added consistently. The corresponding generalized force is the scalar quantity $\nabla \cdot \mathbf{j}_s$ where $\mathbf{j}_s$ is the superconducting current density. This quantity is analogous to the quasiparticle chemical-potential gradient and temperature gradient. The resulting transport matrix is therefore (3×3), and satisfies the relevant Onsager reciprocity relations. In particular, we can write the following equation for the charge imbalance:

$$\mu_{qp} - \mu_{pair} = L_{31}\mathbf{v}_s \cdot \nabla \mu_{qp} + L_{32}\mathbf{v}_s \cdot \nabla T + L_{33}\nabla \cdot \mathbf{j}_s, \tag{16}$$

where $\mathbf{v}_s$ is the condensate velocity and L_{ij} are the transport coefficients that correspond to the thermodynamic generalized forces. One can calculate the coefficients by microscopic theory and then compare the prediction to experiment. This has been done with L_{32}, which represents charge imbalance induced by the simultaneous presence of a supercurrent and a temperature gradient in the bulk. Experimental results are in reasonable agreement with theory.[19] Since we are interested in thermoelectric effects, we focus on the case of a temperature gradient in the two bulk superconductors comprising the Josephson junction, and disregard the contribution from last L_{33} term. Note that in this system the generalized forces and currents are one-dimensional. For simplicity, we consider the case $\nabla \mu_{qp} = 0$ in both superconductors. We also impose a temperature drop between the two superconductors. With these boundary conditions we can rewrite Eq. (14): the first part of the equation is expanded to linear order in the temperature drop across the junction; regarding the second part, we invoke the phenomenological relations Eqs. (15) and (16) on both superconductors. For a nonsymmetrical junction, the quasiparticle current then becomes

$$I_{qp} = I_{qp}(\Delta T) + \frac{\pi\Delta}{2sn_s e^3 R^2}[L_{32}^l \nabla_l T - L_{32}^r \nabla_r T]\sin(\Delta\theta), \tag{17}$$

where $R = [4\pi e^2 N_l(0) N_r(0) |T_{lr}(0)|^2/\hbar]^{-1}$ is the normal-junction resistance, s is the cross section of the junction, and n_s is the bulk pair density. The notation $\nabla_{l(r)}$ denotes the gradient on the lhs (rhs) of the junction.

The first part of the rhs of Eq. (17) is just the normal thermoelectric current through a tunnel junction. It is proportional to the temperature drop across the junction and to $a = 1/2(dT_{lr}/d\xi)|_{\xi=0}$. The latter quantity is a measure of the asymmetry in the transport of electrons and holes. The second part describes a different effect which also produces a thermocurrent, i.e., an electric current that flows in response to a temperature drop. This effect does not correspond to the conventional quasiparticle thermoelectric current in the bulk. Indeed, the second part of Eq. (17) does not include a term proportional to L_{12}^s. The physical origin of this term, like the normal effect, is based on an asymmetry. However, in this case it is an asymmetry of the excitation spectrum about the Fermi surface. An interesting property of this current is that it depends linearly on the temperature gradient and also on $\Delta\theta$. The latter dependence results from current conservation: the supercurrent flowing through the lhs bulk tunnels to the rhs and is, therefore, proportional to $\sin(\Delta\theta)$. In the derivation of Eq. (17) we used the relation $\mathbf{v}_s = \pi\Delta\sin(\Delta\theta)/(2e)^2 Rn_s s$. The phenomenological transport coefficient L_{32} appearing in Eq. (17) has been calculated in different ways, using the Boltzmann equation (see Ref. 4 and references therein). The resulting expression depends on the assumed quasiparticle relaxation mechanisms. We use the result for L_{32} in the case of a dirty superconductor near T_c from the review by Schön.[4] Also, in order to get an order of magnitude estimate for the charge-imbalance induced thermocurrent [the second term of the rhs of Eq. (17) denoted by I^{CI}], we assume for simplicity that a temperature gradient exists only on the left side of the junction. In this way we get

$$I^{CI} = \frac{\beta T_c \Delta(T)\nabla_l T}{eR^2 t(1-t)s}\sin(\Delta\theta). \tag{18}$$

Here we denote the reduced temperature by $t = T/T_c$, where T_c is the transition temperature, while β is a numerical factor of order 10^{-15}. Since $\Delta(T) \sim \sqrt{1-t}$ near T_c, we find that the current diverges near the transition like $1/\sqrt{1-t}$. For a numerical estimate of the thermocurrent in a dirty Sn-O-Sn tunnel junction we approximate $\nabla_l T \approx \Delta_l T/l$, where $\Delta_l T$ is the temperature difference between the edges of the lhs electrode of length l. For a junction of cross section $s \sim 0.1\text{mm}^2$ the resistance is $R \sim 0.1\ \Omega$. We choose l to be several millimeters in order to allow for a realistic temperature gradient in the electrode. Taking $t = 0.9$, we obtain a maximal current of the order of 10^{-10} (A/K) $\Delta_l T$. This means that even for a small temperature difference across the electrode, of the order several K, there will be a measurable effect. The effect is enhanced as T_c is approached.

We have distinguished between two processes that lead to a normal thermocurrent through a Josephson junction: the first is a consequence of the ''discrete'' nonequilibrium between the two sides of the junction—it was discussed in Sec. II B. The second, which is discussed here, is a result of a nonequilibrium state in the bulk superconductors comprising the junction, and of the Josephson coupling between the bulks. The second process is independent of the first process and can be measured if one applies a temperature gradient of the type shown in Fig. 2, where there is no temperature difference across the oxide barrier. Such a profile is achievable if one can control the temperature of each electrode separately. If a temperature drop is applied to the opposite sides of the system, a temperature drop will occur on the barrier as well. In this case the thermocurrent will have two independent contributions. However, as we have shown, only I^{CI} will be sensitive to the phase drop across the junction. The phase dependence of I^{CI} can be measured by controlling either the supercurrent (in an open circuit) or a magnetic field (in a ring configuration). In Ref. 10 it is shown that a similar

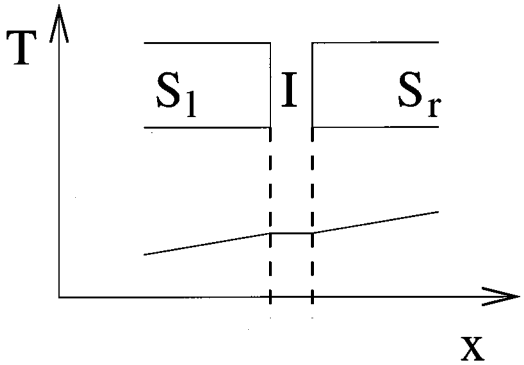

FIG. 2. Suggested temperature gradient across the system along the direction of the one-dimensional coordinate X. This particular configuration of an SIS junction gives rise to a thermoelectric current which is exclusively due to charge imbalance generated in the superconducting bulks S_l and S_r.

effect occurs in a weak link Josephson junction. The current was calculated for a superconducting weak-link with a temperature drop applied between the two edges of the system. In this case the entire superconducting system was in a nonequilibrium state. The resulting current was interpreted as a thermoelectric response. The thermoelectric coefficient was found to depend on $\Delta\theta$, but in a different way than was found in our system. Andreev reflections were suggested as the explanation of the phase dependence. We believe that our derivation may imply that the origin of the phase-dependent thermoelectric effect in the weak-link system is actually a result of the charge-imbalance phenomenon. This mechanism is obscured when the system is treated as a continuum, as was done in Ref. 10. Our model is discrete in the sense that each of the bulk superconductors is treated separately. In this case the physical mechanisms are explicit and distinguishable.

Another point to be noted is that in nonequilibrium bulk superconductors the normal current, given by the two-fluid model, is proportional to ∇Q^*. This is the continuum limit of the result obtained here. We considered the tunneling quasiparticle current between two superconductors and found it to be proportional to $N_r(0)Q_l^* - N_l(0)Q_r^*$. The physical difference between the systems is in the charge-transport mechanism. In the bulk transport is via diffusion, whereas in the junction it is by tunneling. The latter mechanism induces the $\sin(\Delta\theta)$ dependence in Eq. (17). Finally, we already mentioned that charge imbalance can be induced by injecting quasiparticles into a superconductor. This has been measured for NS and NIS systems.[4] In our system we *assumed* that the charge imbalance was induced externally (e.g., by a temperature gradient) and Eq. (5) was modified accordingly. We suspect that a self-consistent treatment of an SIS system may show that the inevitable tunneling of quasiparticles between the superconductors (in response to a current source at $T \neq 0$) results in charge imbalance. Consequently, invoking Eq. (16), this may lead to a temperature gradient in the bulks.

IV. THE THERMOPHASE RESPONSE

Consider a superconductor biased by a temperature gradient. As explained in the introduction, the steady state of an open circuit is a normal thermoelectric current which is canceled by a reverse supercurrent. We interpret this situation, in analogy to the thermoelectric effect, as a response of the system to the applied temperature gradient. From this point of view, the temperature gradient stimulates a superconducting phase gradient, which in turn drives the reverse supercurrent. In other words, there exists a coupling between ∇T and $\nabla\theta$. We are careful not to characterize $\nabla\theta$ as a thermodynamic generalized force, since in all theories the phase is a *mechanical* parameter. The flow of a supercurrent is an *equilibrium* state, and is not associated with the production of entropy. Therefore, within a conventional theory, the coupling between ∇T and $\nabla\theta$ cannot be done systematically by the thermodynamic theory of irreversible processes. We will refer to the mechanism which couples these gradients as a *thermophase* effect.

A. The RSJ model for a Josephson junction

In order to study this effect we propose to consider a discrete system such as the Josephson junction modeled in the previous section. In this system we want to understand the relation between a temperature drop and the phase difference between the two equilibrium bulk superconductors. Note that due to gauge invariance the voltage across the junction, which is also a thermodynamic generalized force, is proportional to the time derivative of the phase difference across the junction. Consider the following experiment. An open-circuit Josephson junction (i.e., the junction is driven by a current source which sets $I_{\mathrm{tot}}=0$) is biased by a temperature drop. The currents in the junction have been obtained in Sec. II. Applying these boundary conditions, we can write the total current in the following form:

$$I_{\mathrm{qp}}(V,T)|_{\Delta T=0} + I_{\mathrm{qp}}(\Delta T,T)|_{V=0} + I_{\mathrm{qp\text{-}pair}}(V,T)\cos\Delta\theta + I_c \sin\Delta\theta = 0, \tag{19}$$

where $V=\Delta\mu_{\mathrm{qp}}/e$. The amplitudes of the currents on the lhs of Eq. (19) are given in Eq. (7). I_c is the critical Josephson current. Under the condition of an open junction with a fixed temperature drop, we may regard the quasiparticle thermocurrent $I_{\mathrm{qp}}(\Delta T,T)|_{V=0}$ as an external current bias. In this case we recognize Eq. (19) to be the overdamped limit of the resistively shunted-junction (RSJ) model equation of motion[20] with the addition of $I_{\mathrm{qp-pair}}$. Therefore, in order to study the relation between the temperature drop on the junction and the phase difference, we must find the steady states of the system in this model. For simplicity, we shall consider the limit of linear response: $I_{\mathrm{qp}}(V,T)|_{\Delta T=0} \approx V/R$ and $I_{\mathrm{qp}}(\Delta T,T)|_{V=0} \approx L_{12}^s \Delta T$. Although experimental I-V curves of Josephson junctions are nonlinear in the vicinity of $V=2\Delta/e$, the linear response is a good approximation above and below this region. In addition, we neglect $I_{\mathrm{qp-pair}}$, since a numerical solution of this model that we carried out has shown that this term does not qualitatively affect the steady-state solutions of the system. We also neglect charge-imbalance effects.

In the RSJ model one considers an equivalent circuit consisting of a nonlinear inductance, a capacitance, and a resistor in parallel. The nonlinear inductance represents the nonlinear Josephson coupling, and the capacitor represents the geometric capacitance of the junction C. Loss in the system is represented by a resistance R. Note that in our description of the Josephson junction we neglected the charging energy due to the capacitance. The total current in the circuit is therefore

$$I_{\rm ext}=C\dot{V}+\frac{V}{R}+I_c\sin\Delta\theta, \tag{20}$$

where $I_{\rm ext}$ is an external current bias. Recalling the relation $2eV/\hbar=d(\Delta\theta)/dt$, Eq. (20) can be viewed as an equation of motion for a classical ''particle'' with coordinate $\Delta\theta$ moving in a ''potential'' $U(\theta)=-I_c\cos\Delta\theta$. The steady-state solutions of this system have been thoroughly studied and the results have been verified experimentally.[21]

Solutions of Eq. (20) fall into two regimes: an overdamped regime ($1/R\gg\sqrt{2CI_c/eR_Q}$ where R_Q is the quantum resistance) and an underdamped regime in the opposite case. In the overdamped regime the inertia term $C\dot{V}$ can be neglected. When the external current is smaller than the critical Josephson current I_c, the steady state of the system is static: the quasiparticle current is shorted by a reverse Josephson current and no voltage develops on the junction. This state is analogous to the behavior of a bulk superconductor. When $I_{\rm ext}>I_c$ a ''running solution'' develops in which an oscillating voltage $V(t)$ is generated across the junction. In the underdamped regime, when $I_{\rm ext}>I_c$ the ''running state'' occurs as before. Below a threshold current $I_{\rm min}$, only the static solution exists. When the external current satisfies $I_{\rm min}<I_{\rm ext}<I_c$, both solutions exists.

B. The definition of a thermophase coefficient

Consider an open-circuit Josephson junction. We impose the constraint $I_{\rm tot}=0$ and apply an external temperature drop. As explained above, this system can be described by the RSJ equation of motion with the normal thermoelectric current acting as an effective external current. For clarity, we limit ourselves to the static solution $V=0$. Neglecting charge-imbalance generation, we can define the thermophase ''coefficient,'' in analogy with the definition of the thermoelectric transport coefficients. In the limit of small ΔT and small $\Delta\theta$ we can write

$$I_{\rm qp}(\Delta T)=L_{12}^s\Delta T=-I_c\sin\Delta\theta\approx-I_c\Delta\theta. \tag{21}$$

We have set $I_{\rm qp-pairs}=0$ as is the case for $\Delta\mu=0$. We define

$$L_{\rm tp}\equiv-\left.\frac{\delta(\Delta\theta)}{\Delta T}\right|_{I_{\rm tot}=0,\Delta\mu=0,\Delta T\to0}=\frac{L_{12}^s}{I_c}. \tag{22}$$

The notation $\delta(\Delta\theta)$ is used in order to emphasize the phase difference that develops due to the temperature drop. The explicit form of $L_{\rm tp}$ includes the microscopic parameters as given by Eq. (7). The final expression for a symmetric Josephson junction is given by

$$L_{\rm tp}(T)=\frac{2a}{T|T_{lr}(0)|^2\Delta(T)}\int_\Delta^\infty dw\ w^2\left(-\frac{\partial f}{\partial w}\right), \tag{23}$$

where we have expanded $|T_{lr}|^2$ as in Eq. (9). Equation (23) was evaluated numerically as function of temperature. The results are given in Fig. 1. An estimate of the magnitude of $L_{\rm tp}$ can be given using Eq. (22). Near T_c the thermopower approaches the normal values. Thus, $L_{12}^s\sim10^{-6}/R$, where R is the resistance in Ω and L_{12} is in V/Ω K. Taking $R\sim0.1\ \Omega$ for a typical Sn-O-Sn junction of cross section 0.1 mm^2, we obtain $L_{\rm tp}\sim10^{-5}/I_c$ where I_c is in A and $L_{\rm tp}$ is in K^{-1}. Note that for this junction I_c is of the order of mA for $T\ll T_c$, but it vanishes as we approach the transition temperature. Therefore, as we approach T_c we find $L_{\rm tp}\to\infty$. The divergence indicates that the definition of $L_{\rm tp}$ breaks down at the superconducting phase transition. We expect this behavior, based on the understanding that the effect is a result of a coupling between the condensate and the quasiparticle populations. The phase difference is generated by a transient Seebeck voltage induced by the temperature drop. The steady-state value of the phase difference is determined by the constraint that $I_{\rm pair}$ exactly cancels the quasiparticle current and $V=0$. When an oscillating state is possible, the relation between the phase difference and the temperature drop is more complicated but the mechanism is the same. The temperature dependence of $L_{\rm tp}$ ($V=0$) is now evident. Since at low temperatures few excitations exist, the coupling between the condensate and the quasiparticle current is small. As the temperature is increased, a larger $\Delta\theta$ is needed to cancel the thermoelectric current.

We emphasize that the coefficient $L_{\rm tp}$ is *not* a well-defined transport coefficient since it cannot be obtained within a systematic use of thermodynamics. The initial state $\Delta T\neq0$ corresponds to a nonequilibrium state, however, the state of $\Delta\theta\neq0$ is an equilibrium state. As a consequence, it is not clear that we can invert $L_{\rm tp}$, in the sense of the Onsager relations. Under the assumptions made, Eq. (21) only predicts that fixing a temperature drop across a junction will generate a phase difference. The physical mechanism of this process is understood. However, it is not clear if an applied phase difference (in a closed circuit) will induce a temperature drop on the junction. Note that we have already seen that the phase-dependent currents $I_{\rm qp-pair}$ and $I_{\rm pair}$ do not depend on ΔT, because these currents do not involve the transport of electrons and holes but rather of pairs. Hence, these currents cannot contribute to the generation of a temperature drop. If the effect exists only in the direction $\Delta T\to\Delta\theta$, then $L_{\rm tp}$ is no more than a technical definition that is proportional to the thermoelectric coefficient. However, if the reverse process indeed exists, then the thermophase effect constitutes a real physical effect.

C. Experimental detection of the thermophase effect

The thermophase response, defined in an open circuit, can be realized and measured experimentally in a closed-circuit system. Indeed, this has been done indirectly in Ref. 16. A closed-circuit superconductor tunnel junction was heated on one side by a laser. A thermoelectric current was measured above a certain temperature and disappeared below it. The interpretation was that a quasiparticle thermoelectric current was canceled by a reverse Josephson current only beneath a threshold temperature, at which the supercurrent was large enough. This result corresponds to a thermophase effect. When the junction was biased by an additional ac current (which was interpreted as an external voltage by the authors), a thermoelectric current was measured even below the threshold temperature. The explanation was that the Joseph-

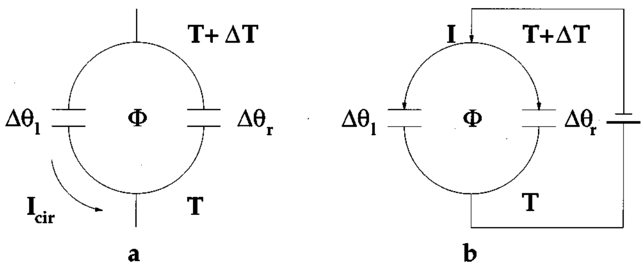

FIG. 3. Two experimental setups of dc SQUID circuits. An initial phase difference is set on each junction of the SQUID by a magnetic flux Φ. The top part of the ring is heated homogeneously to a temperature $T+\Delta T$. Thus, we obtain a temperature drop on the Josephson junctions, with no temperature gradients in the superconducting leads. (a) An open circuit with a circulating current $I_{\rm cir}(\Phi)$. (b) A closed circuit in which an interference current flows $I(\Phi)$.

son current oscillated as a result of the external bias. Thus, it did not contribute to the average dc signal, and could not cancel the thermoelectric quasiparticle current. This experimental setup enables an indirect measurement of the coupling between a temperature drop and the phase difference across a junction. There was no direct control over the phase.

In order to measure this coupling directly we propose the following systems. Consider a dc superconducting quantum interferenced device (SQUID) configuration,[2] illustrated in Fig. 3. Such a system enables one to control the phase difference on the Josephson junctions via the magnetic flux through the ring. This is reflected in the following relation:

$$\Delta\theta_l-\Delta\theta_r=2\pi\frac{\Phi}{\Phi_0}+\int_{\rm lhs}dr\nabla\theta+\int_{\rm rhs}dr\nabla\theta, \qquad (24)$$

where Φ is the total flux threading the ring and Φ_0 is the quantum unit flux. $\nabla\theta$ is the continuous superconducting-phase gradient along the wires of the ring, and the lhs and rhs integrals are along counterclockwise paths. The effect of temperature on $\Delta\theta$ can be measured in two ways. The first experimental setup is an open circuit illustrated in Fig. 3(a). If the two junctions comprising the ring are equivalent, the current circulating $I_{\rm cir}$ in the ring is given by[2]

$$I_{\rm cir}(\Phi)=I_c\sin\left(\frac{\Delta\theta_l-\Delta\theta_r}{2}\right)\cos\left(\frac{\Delta\theta_l+\Delta\theta_r}{2}\right), \qquad (25)$$

where I_c is the critical current of either junction. $I_{\rm cir}(\Phi)$ can be controlled by an external flux $\phi_{\rm ext}$. In order to attain a nonzero circulating supercurrent, we apply an external flux. According to Eq. (25) the resulting current will be smaller than the critical current of the ring. Suppose we heat the upper half of the ring with respect to the lower half (avoiding temperature gradients in the wires). As long as the temperature drop is not too large, the static steady state (i.e., $V=0$ on both junctions) will be sustained. In this case no additional circulating current is generated by the thermoelectric effect, since the thermoelectric current is canceled by a reverse supercurrent on both junctions. However, an additional phase drop will develop on both junctions due to the thermophase effect, which is proportional to the temperature drop. This phase shift will alter the circulating supercurrent and can thus be detected. Note that the argument of the cosine term in Eq. (25) is a sum. Hence, the response of the two junctions is added. We can, therefore, use a homogeneous ring to measure the effect. A quantitative prediction for such an experiment is obtained by substituting $\Delta\theta(\Delta T)\approx\Delta\theta(0)+\delta$ in Eq. (25). Here, δ is the (linear-response) thermophase response and it satisfies Eq. (22). In the static steady state, both junctions will produce the same phase correction δ. Expanding the cosine in Eq. (25) in δ, and invoking the definition Eq. (22), we find

$$\Delta I|_{\Delta T\to 0}=I_c\sin\left(\frac{\Delta\theta_l+\Delta\theta_r}{2}\right)\sin\left(\frac{\pi\Phi}{\Phi_0}\right)L_{\rm tp}\Delta T. \qquad (26)$$

This can be detected as an additional flux $\Phi_{\rm tp}(\Delta T)=L\Delta I$, where L is the geometric self-inductance of the ring. Note that ΔI is periodic in the flux threading the ring with a period of $2\Phi_0$. When the temperature drop exceeds a critical value a ''running state'' will develop in the ring. This critical temperature depends on the flux Φ through the ring as $\sin(\pi\Phi/\Phi_0)$. This is essentially the same as the function $I_{\rm cir}(\Phi)$ which appears in Eq. (25). This running state was predicted to occur in and measured in SNS SQUID systems.[22] A similar effect was also predicted and demonstrated in single SNS junctions.[23,24]

Another experimental setup is illustrated in Fig. 3(b). It comprises of a closed-circuit system in which a supercurrent can flow. The current will flow in parallel, through the two Josephson junctions, and interfere. The interference pattern is realized when measuring the current as function of the magnetic flux threading the ring. Suppose we fix the external magnetic flux, in the absence of a temperature drop, so that the current is smaller than the critical current of the ring. Now we heat the upper half of the ring with respect to the lower half, as before. In the static steady state this will affect the interference pattern. A quantitative prediction for such an experiment can be calculated by following the derivation of the open-circuit setup. The change of the current in response to the temperature drop is given by

$$\left.\frac{\Delta I}{\Delta T}\right|_{\Delta T\to 0}=-2I_c\cos\left(\frac{\Delta\theta_l+\Delta\theta_r}{2}\right)\cos\left(\frac{\pi\Phi}{\Phi_0}\right)L_{\rm tp}. \qquad (27)$$

Equation (27) relates the experimental measurement $(\Delta I/\Delta T)$ to the theoretical prediction. As before, the excess current generated by the temperature drop is periodic in the magnetic field threading the ring. If the experimental setup corresponds to an underdamped junction, an ac voltage can develop on each junction. The effect can be calculated as before and compared to experiment.

As discussed above, the definition of $L_{\rm tp}$ reflects the coupling between the currents of the condensate and the quasiparticles. Such a coupling is manifested in other phenomena, e.g., the charge-imbalance state and $I_{\rm qp-pair}$. Our understanding of the thermophase coupling is the following: the macroscopic normal thermoelectric voltage couples to the phase difference on the junction via the quantum-mechanical relation $V\sim d(\Delta\theta)/dt$. This mechanism bridges the macroscopic thermodynamic properties of the system with the macroscopic quantum-mechanical phase. We note that the thermophase effect in Josephson junctions differs from the situation in homogeneous bulk superconductors. In an open-circuit system subjected to a temperature drop, the static $V=0$ state is observed in both Josephson junctions and bulk superconductors. However, the physics is different: in bulks the steady state must satisfy the Meissner effect, whereas in Josephson junctions this does not necessarily apply. The two superconducting electrodes are isolated from each other, and the behavior of the system is understood within the RSJ model, as discussed above. Furthermore, in contrast to bulk superconductors, a Josephson junction has solutions other than the static solution. In these cases the thermophase coupling is manifested by the running state. Finally, we do not know if the reverse effect, in which a phase difference across the junction induces a temperature drop, exists. This question can be resolved experimentally in setups similar to those introduced here. However, it has been brought to our attention that type-II bulk superconductors in a magnetic field exhibit a thermoelectrically induced phase slippage effect—see Ref. 25 and references therein. This effect might be related to the thermophase effect discussed here. Moreover, the reverse effect was also measured in such systems.[26] These experiments were conducted in high-T_c superconductors.

ACKNOWLEDGMENTS

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